Introduction to Time Series Analysis

Kritchavat Ploddi

Section 1

Introduction to Time Series Analysis

Learning objectives

After completing this tutorials, students should be able to:

- Understand basic concepts of time series analysis
- Apply statistical models to forecast health outcomes with R
- Evaluate predictive performance of time series models
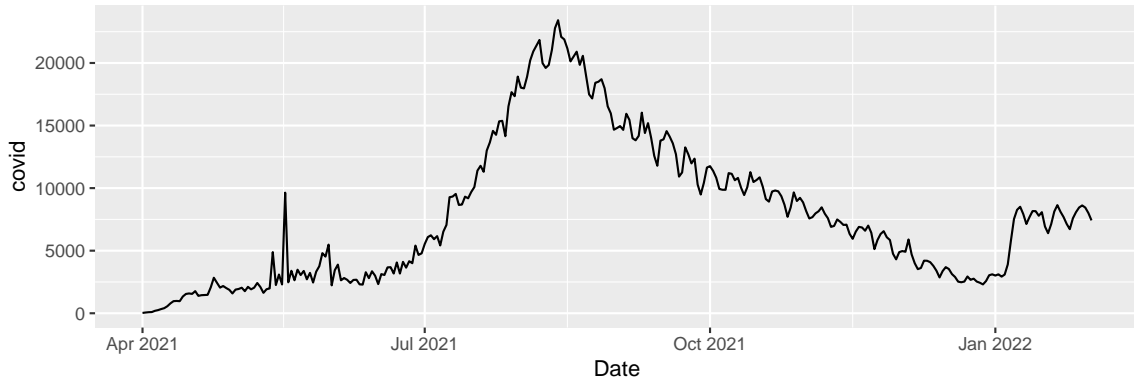
What is a time series?

What is a time series?

- A time series is a series of data points indexed (or listed or graphed) in time order.
- Most commonly, a time series is a sequence taken at successive equally spaced points in time.

What is a time series? (Examples)

```
##          date covid
## 1  2022-01-01  3011
## 2  2022-01-02  3112
## 3  2022-01-03  2927
## 4  2022-01-04  3091
## 5  2022-01-05  3899
## 6  2022-01-06  5775
## 7  2022-01-07  7526
## 8  2022-01-08  8263
## 9  2022-01-09  8511
## 10 2022-01-10  7926
## 11 2022-01-11  7133
## 12 2022-01-12  7681
## 13 2022-01-13  8167
## 14 2022-01-14  8158
## 15 2022-01-15  7793
```

# What is a time series? (Examples)

Daily confirmed covid cases (1 April 2021 – 31 January 2022)

# What is a time series? (Examples)

```
Year Month   DHF
1 2558   1 2639
2 2558   2 2183
3 2558   3 2716
4 2558   4 3431
5 2558   5 8065
6 2558   6 12913
7 2558   7 17735
8 2558   8 21852
9 2558   9 21181
10 2558   10 18058
11 2558   11 23472
12 2558   12 10707
13 2559   1 7068
14 2559   2 4060
15 2559   3 3712
```
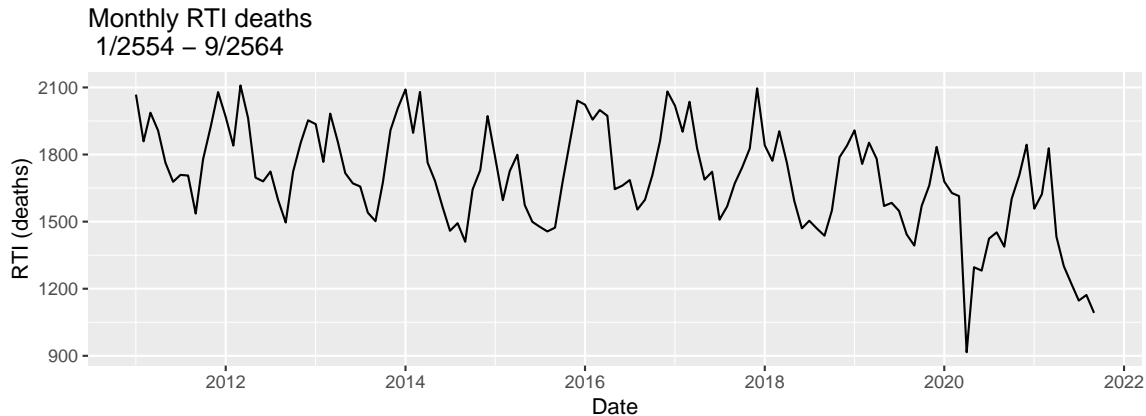
# What is a time series? (Examples)

Monthly Dengue cases
1/2558 – 9/2564
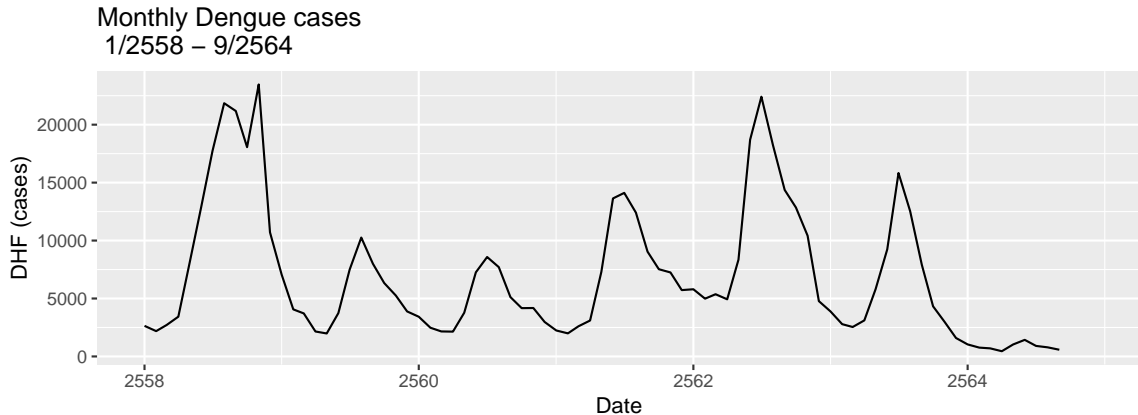
## What is a time series? (Examples)

```
date rti
1   2011-01-01 2068
2   2011-02-01 1859
3   2011-03-01 1987
4   2011-04-01 1907
5   2011-05-01 1763
6   2011-06-01 1678
7   2011-07-01 1709
8   2011-08-01 1706
9   2011-09-01 1536
10 2011-10-01 1779
11 2011-11-01 1925
12 2011-12-01 2079
13 2012-01-01 1968
14 2012-02-01 1840
15 2012-03-01 2109
```

# What is a time series? (Examples)



Monthly RTI deaths
1/2554 – 9/2564

## What is a time series? (Examples)

```
##                 DATETIMEDATA     PM25
## 1    2021-11-10 00:00:00 14.07865
## 2    2021-11-10 01:00:00 13.37079
## 3    2021-11-10 02:00:00 13.28090
## 4    2021-11-10 03:00:00 13.11236
## 5    2021-11-10 04:00:00 13.16854
## 6    2021-11-10 05:00:00 13.73034
## 7    2021-11-10 06:00:00 14.19101
## 8    2021-11-10 07:00:00 15.35227
## 9    2021-11-10 08:00:00 16.25000
## 10   2021-11-10 09:00:00 16.26136
## 11   2021-11-10 10:00:00 15.98864
## 12   2021-11-10 11:00:00 15.09091
## 13   2021-11-10 12:00:00 14.41573
## 14   2021-11-10 13:00:00 14.39326
## 15   2021-11-10 14:00:00 13.48864
```

What is a time series? (Examples)

Hourly PM 2.5 concentration
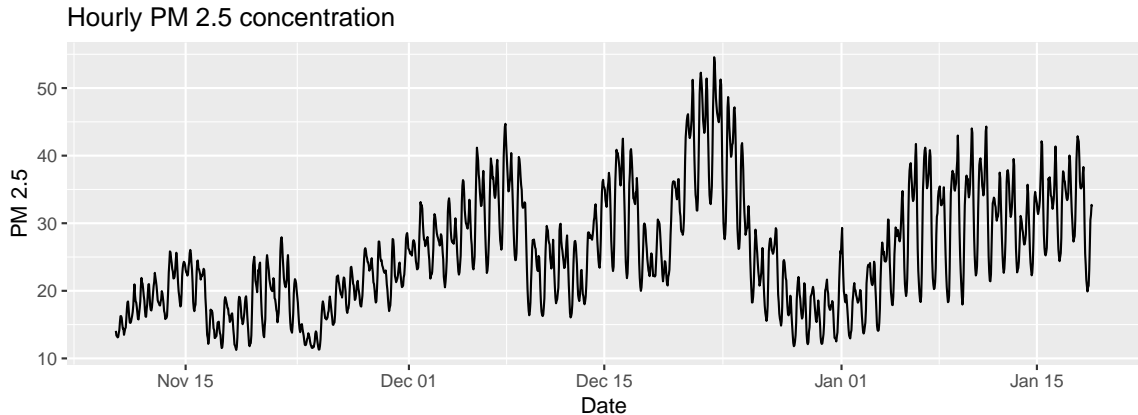
Other time series data?

Other time series data?

- **Medicine**: EKG, Vital signs, Plasma glucose

Other time series data?

- **Medicine**: EKG, Vital signs, Plasma glucose
- **Business and finance**: Sales, Stock prices

Other time series data?

- **Medicine**: EKG, Vital signs, Plasma glucose
- **Business and finance**: Sales, Stock prices
- **Meteorology**: Temperature, Rainfall levels

Common characteristics of time series data

Common characteristics of time series data

- **Trend** pattern exists when there is a long-term increase or decrease in the data

Common characteristics of time series data

- **Trend** pattern exists when there is a long-term increase or decrease in the data
- **Seasonal** pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Common characteristics of time series data

- **Trend** pattern exists when there is a long-term increase or decrease in the data
- **Seasonal** pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).
- **Cyclic** pattern exists when data exhibit rises and falls that are not of fixed period (duration usually of at least 2 years).
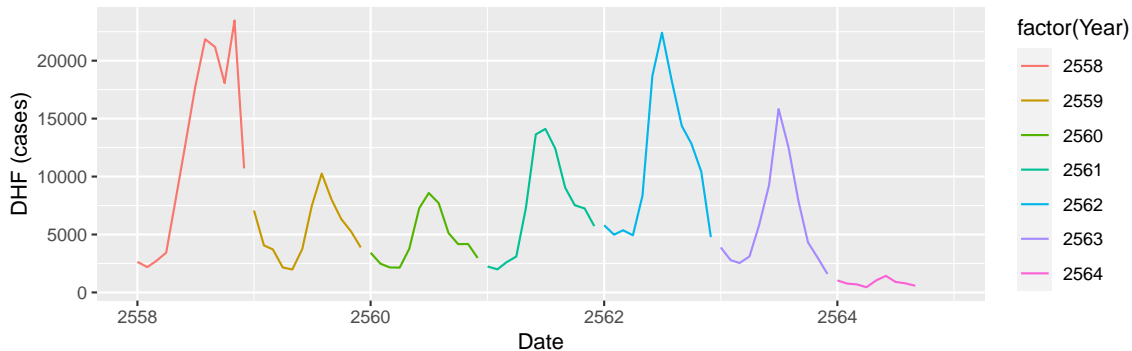
Comparing seasonality and cycle

Patterns	Variability	Length	Magnitude
Seasonality	Constant	Shorter	Lower
Cycle	Variable	Longer	Higher
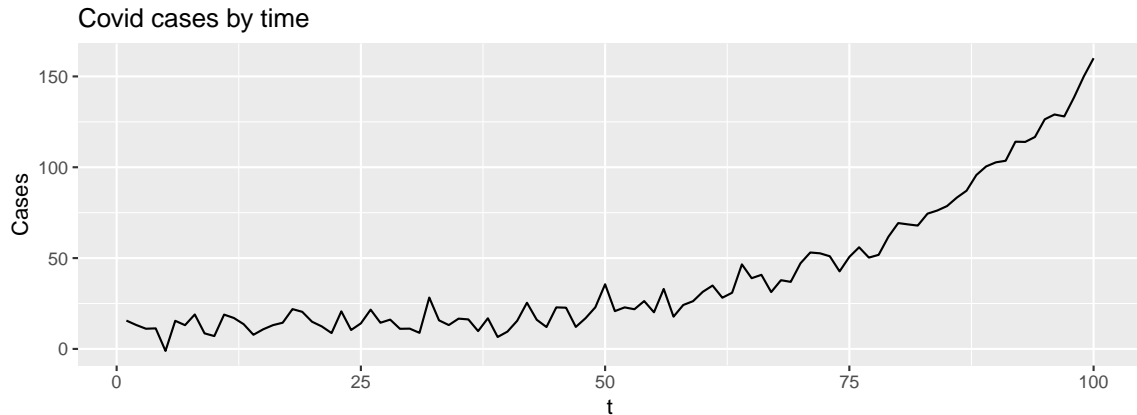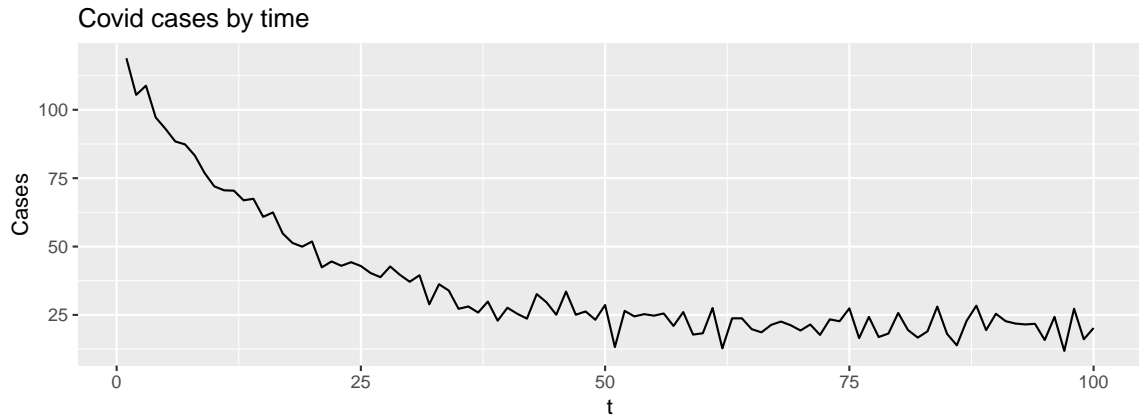
Trend

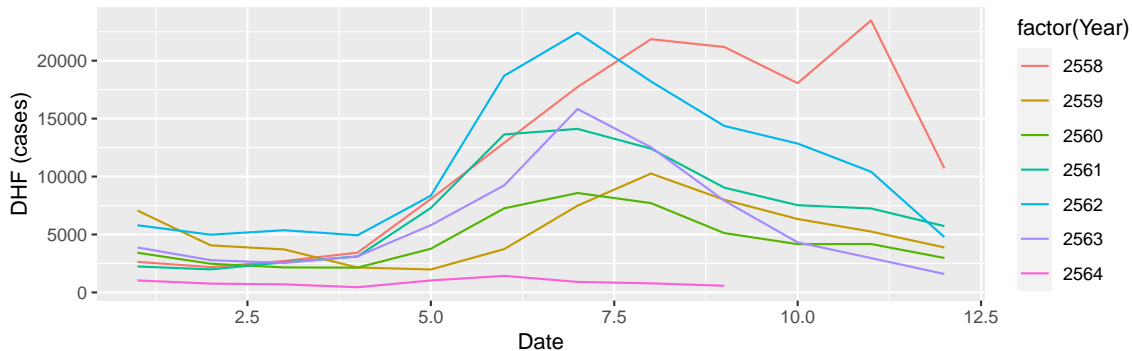Covid cases by time

Trend

Covid cases by time

Seasonality

Monthly Dengue cases
1/2558 – 9/2564
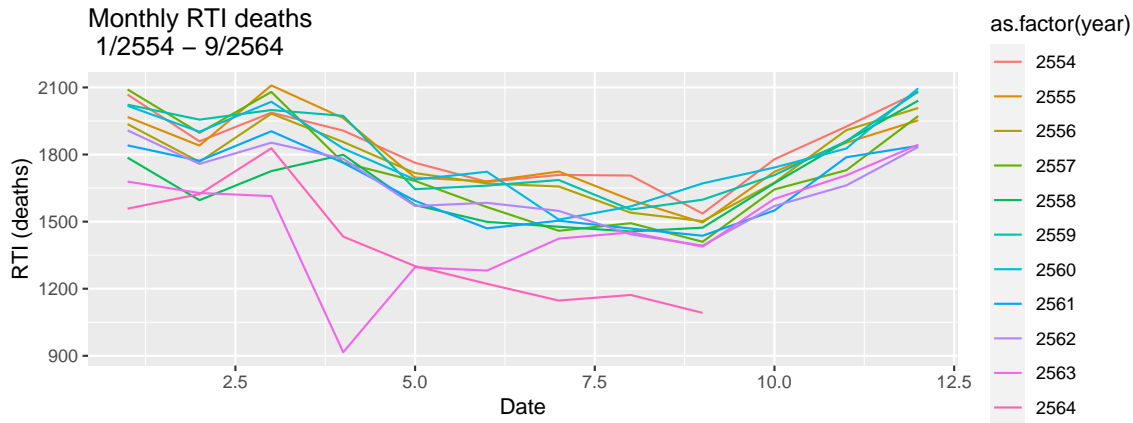
Seasonality

Monthly Dengue cases
1/2558 – 9/2564
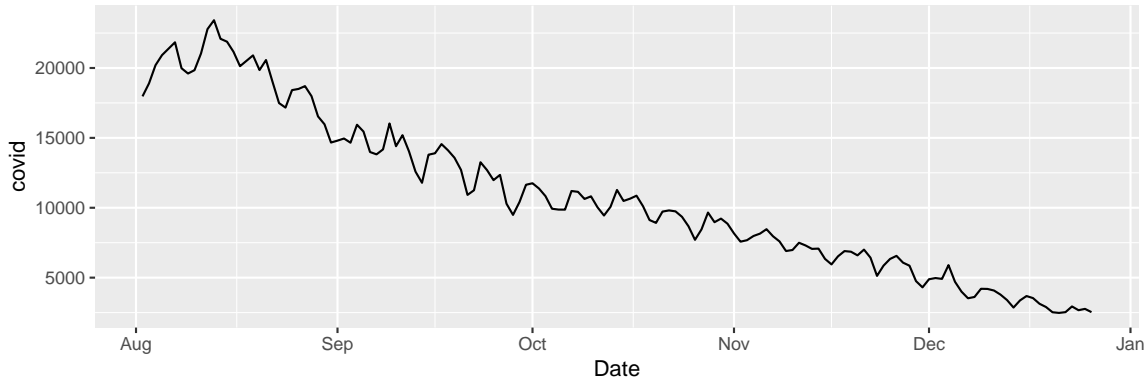
Seasonality

Monthly RTI deaths
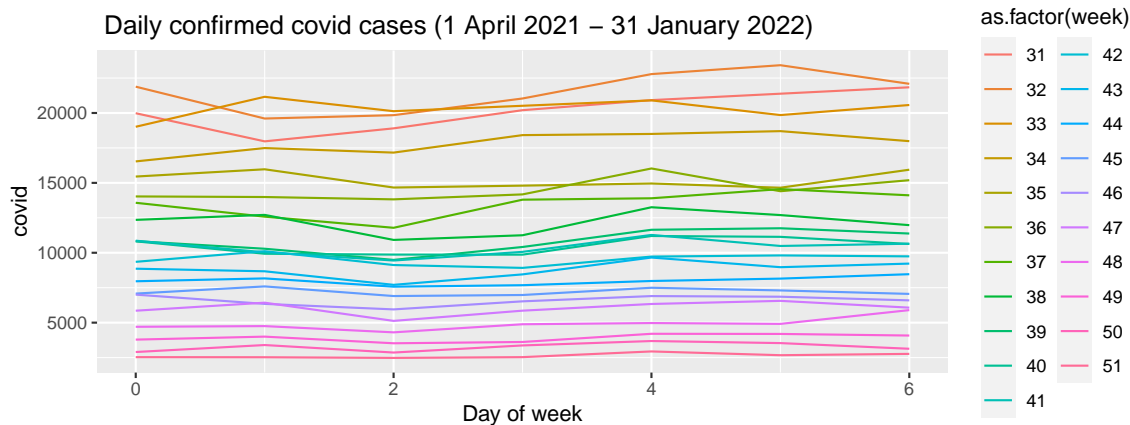1/2554 – 9/2564

Seasonality

Daily confirmed covid cases (Aug – Dec)

Seasonality

Daily confirmed covid cases (1 April 2021 – 31 January 2022)

Section 2

Time Series Visualization

Plotting time series data with R ggplot2

Before working with RStudio, please set working directory to the workshop folder.

Rstudio > Session > Set Working Directory > Choose Directory

Make sure that the workshop directory is displayed in the Files pane (Right-Lower pane of RStudio)

Please install ggplot2 and tidyquant with the following commands:

install.packages("ggplot2")

install.packages("tidyquant")

Once finished import both packages into R workspace:

```
library(ggplot2)
library(tidyquant)
```

Plotting time series data with R ggplot2

You can load csv data into R workspace with the command read.csv().

```
covid <- read.csv('data/covidts.csv')
head(covid)
```

```
##           date day month year dow week covid
## 1 2021-04-01   1     4 2021   4   13    26
## 2 2021-04-02   2     4 2021   5   13    58
## 3 2021-04-03   3     4 2021   6   13    84
## 4 2021-04-04   4     4 2021   0   13    96
## 5 2021-04-05   5     4 2021   1   14   194
## 6 2021-04-06   6     4 2021   2   14   250
```

Plotting time series data with R ggplot2

Once loaded, we create a time series plot having date as X and covid as Y.

With ggplot, we use geom_line to create a line plot.

```
# convert date as date variable
covid$date = as.Date(covid$date)
ggplot(covid) +
  geom_line(aes(x=date, y=covid))
```
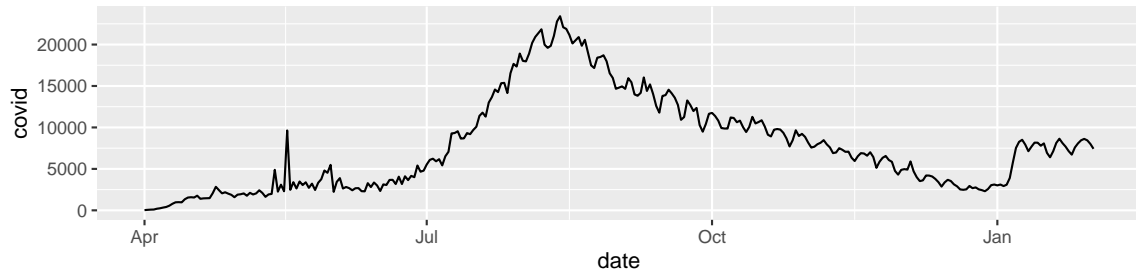
Plotting time series data with R ggplot2

Now let's create a time series plot of RTI death.

Plotting time series data with R ggplot2

Also, load the data with read.csv() and convert `date` to date variable.
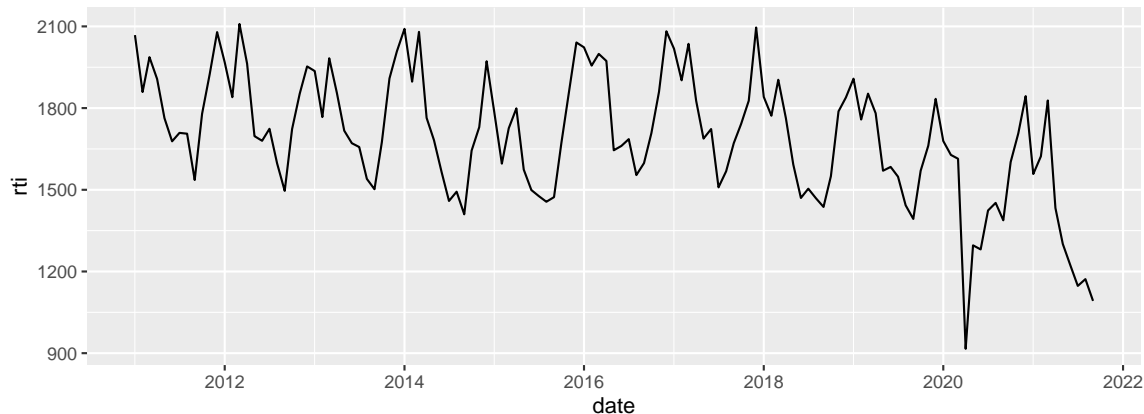
```
rti <- read.csv("data/rti.csv")
rti$date <- as.Date(rti$date)
head(rti)
```

```
##          date  rti year month
## 1 2011-01-01 2068 2554     1
## 2 2011-02-01 1859 2554     2
## 3 2011-03-01 1987 2554     3
## 4 2011-04-01 1907 2554     4
## 5 2011-05-01 1763 2554     5
## 6 2011-06-01 1678 2554     6
```

Plotting time series data with R ggplot2

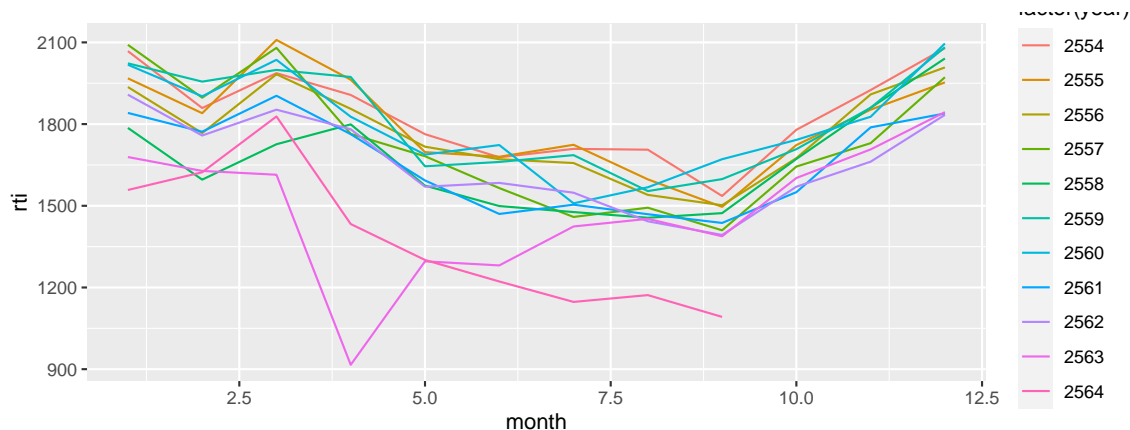Plotting the time series plot is also straightforward.

```
ggplot(rti) +
  geom_line(aes(x=date, y=rti))
```

Plotting time series data with R ggplot2

Next, plotting monthly RTI each year is recommended to examine seasonality.

```
ggplot(rti) +
  geom_line(aes(x=month, y=rti, color=factor(year)))
```
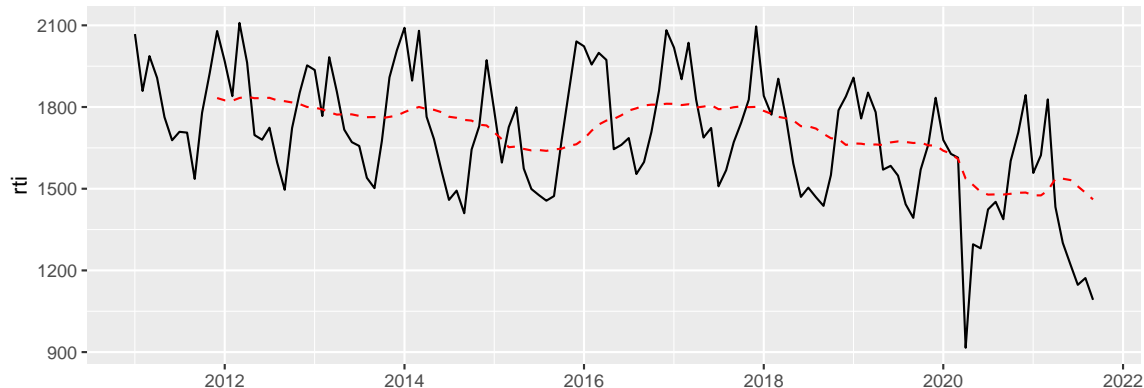
Plotting time series data with R ggplot2

To examine trend pattern, a moving average is recommended.

In this exercise, we use geom_ma of tidyquant package to create the moving average (of 12 months) plot.

```
ggplot(rti) +
  geom_line(aes(x=date, y=rti)) +
  tidyquant::geom_ma(aes(x=date, y=rti), ma_fun=SMA, n=12, color='red', type='l')
```

STL Decomposition

Another way of visualizing time series components is by decomposing them.

Additive: $Y_t = Trend_t + Seasonal_t + Cyclic_t + Others_t$

Multiplicative: $Y_t = Trend_t \times Seasonal_t \times Cyclic_t \times Others_t$

There are methods we used to decompose the time series, e.g. X11, STL.

STL Decomposition

Now we'll use the method called STL decomposition to decompose the RTI death.

Firstly, we have to create a time series object from existing data.

```
ts_rti <- ts(rti$rti, start=c(2011,1), freq=12) # create a time series object
ts_rti
```

```
##         Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
## 2011  2068 1859 1987 1907 1763 1678 1709 1706 1536 1779 1925 2079
## 2012  1968 1840 2109 1963 1697 1680 1724 1597 1496 1723 1853 1953
## 2013  1936 1767 1983 1856 1717 1671 1657 1540 1502 1675 1909 2008
## 2014  2091 1897 2080 1764 1682 1565 1459 1493 1410 1644 1730 1972
## 2015  1786 1596 1726 1799 1574 1499 1477 1456 1473 1672 1860 2041
## 2016  2023 1956 1999 1973 1645 1661 1686 1554 1598 1708 1860 2082
## 2017  2018 1902 2036 1827 1688 1723 1509 1568 1671 1742 1827 2096
## 2018  1841 1772 1904 1763 1593 1470 1504 1469 1437 1550 1788 1839
## 2019  1908 1758 1853 1781 1570 1584 1548 1443 1393 1570 1662 1834
```

STL Decomposition

We use the command `stl` to decompose the time series RTI data.

```
decomposition <- stl(ts_rti, t.window=12, s.window="periodic", robust=TRUE) #
plot(decomposition)
```
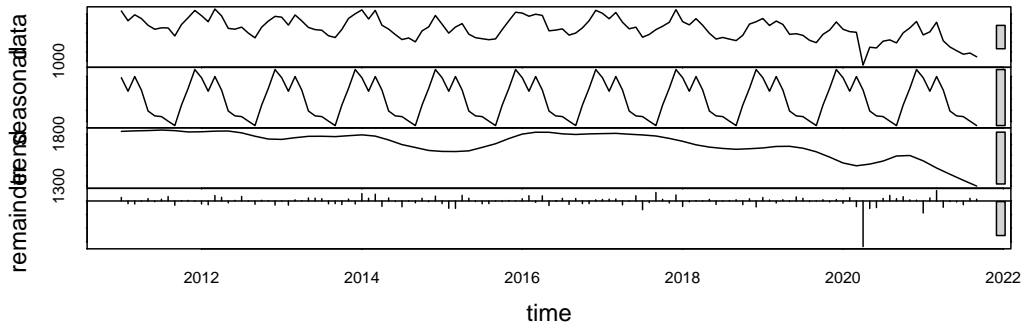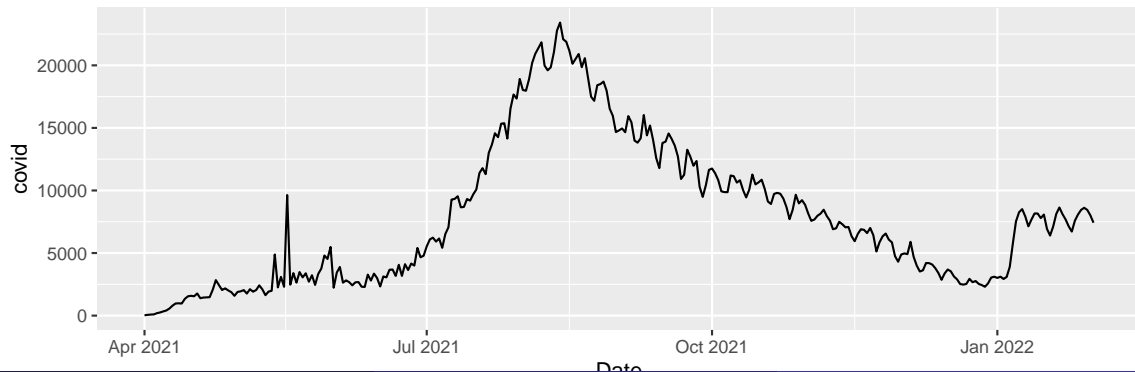
Time series correlation

Most time series data are correlated with themselves.

In statistics, this is called autocorrelation.

For example, cases tend to be correlated (similar) with others, especially at the nearest time point.
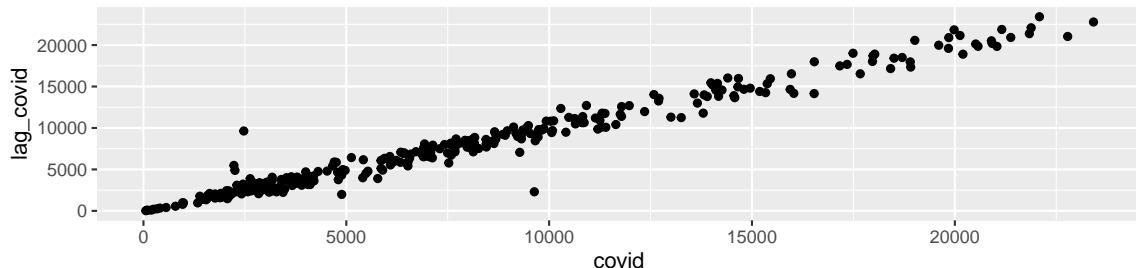
Daily confirmed covid cases (1 April 2021 – 31 January 2022)

Time series correlation

We could use a scatter plot to visually examine the correlation of Covid-19 cases with its lag (yesterday cases).

```
covid$lag_covid <- lag(covid$covid, 1)
ggplot(covid) +
  geom_point(aes(x=covid, y=lag_covid))
```

Time series correlation

We could also use the Pearson's correlation coefficient.

```
with(covid, cor(covid, lag_covid, use='complete.obs'))
```

```
## [1] 0.985227
```

The value 0.98 suggested a strong correlation between today Covid cases and yesterday cases.

Autocorrelation function (ACF)

The autocorrelation function (ACF) is essentially a function to determine the Pearson's correlation of a time series data and its previous lag at any time points.
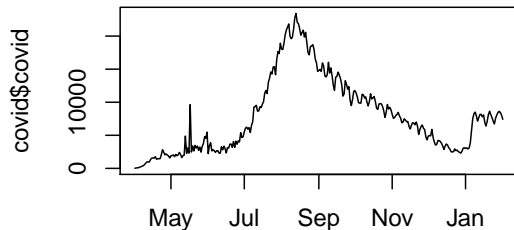
In R, the ACF is called by the acf command.

Autocorrelation function (ACF)

```
# format subplots having 1 row and 2 columns
par(mfrow=c(1,2))

plot(covid$date, covid$covid, type='l')
acf(covid$covid)
```
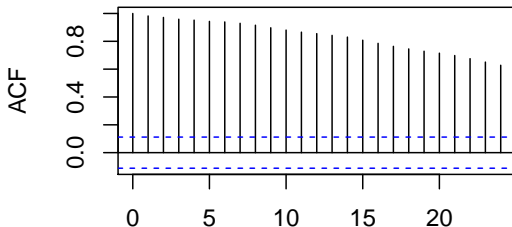
Series covid$covid

Autocorrelation function (ACF)

```
# format subplots having 1 row and 2 columns
par(mfrow=c(1,2))

plot(rti$date, rti$rti, type='l')
acf(rti$rti)
```


Series rti$rti
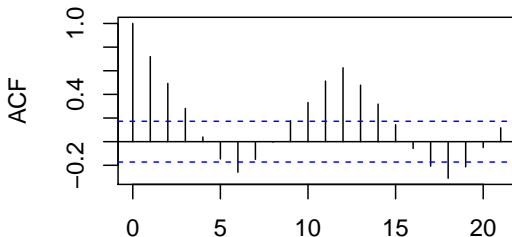
Autocorrelation function (ACF)

After examining the ACF, it is obvious that most time series data are correlated with its lag.

However, the ACF cannot exactly determined the correlation with it, say 7th lag, given all other lags.

This is because the correlation with the 7th lag may be confounded with other lags.

Partial Autocorrleation Function (PACF)

The Partial Autocorrelation Function (PACF) also shows the Pearson Correlation Coefficient of the time series and its lags.

It is different from the ACF in that the function is given by all other lag (meaning that there would be no confounding effects from other lags).

In R, we use the command, pacf.

Partial Autocorrelation function (PACF)

```r
# format subplots having 1 row and 2 columns
par(mfrow=c(1,2))

plot(covid$date, covid$covid, type='l')
pacf(covid$covid)
```

Series covid$covid
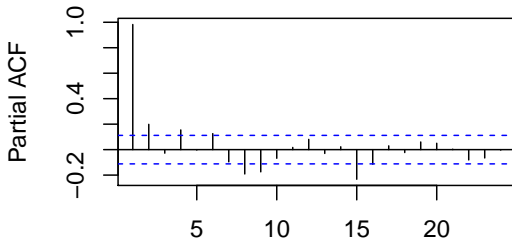
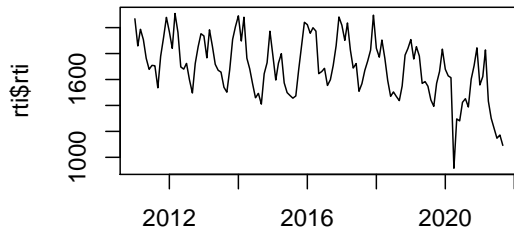# Partial Autocorrelation function (PACF)

```
# format subplots having 1 row and 2 columns
par(mfrow=c(1,2))

plot(rti$date, rti$rti, type='l')
pacf(rti$rti)
```
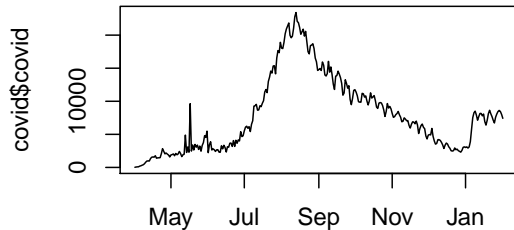
Series rti$rti
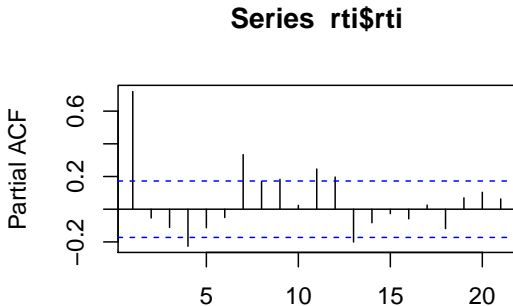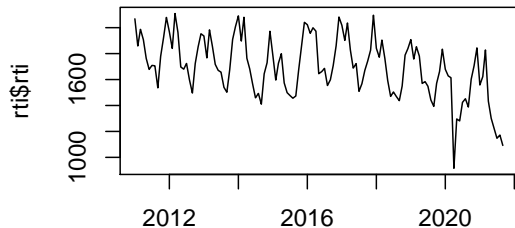
Stationary

Another important concept in time series analysis is stationary.

This is because most time series forecasting models require stationary assumptions.

Stationary

A time series Y is said to be stationary if all of its values y_t do not depend on time t.

In other words, the distribution of y_t has constant mean and variance.

Stationary ?

The numbers of Covid–19 cases

Stationary ?

Daily changes in numbers of Covid–19 cases
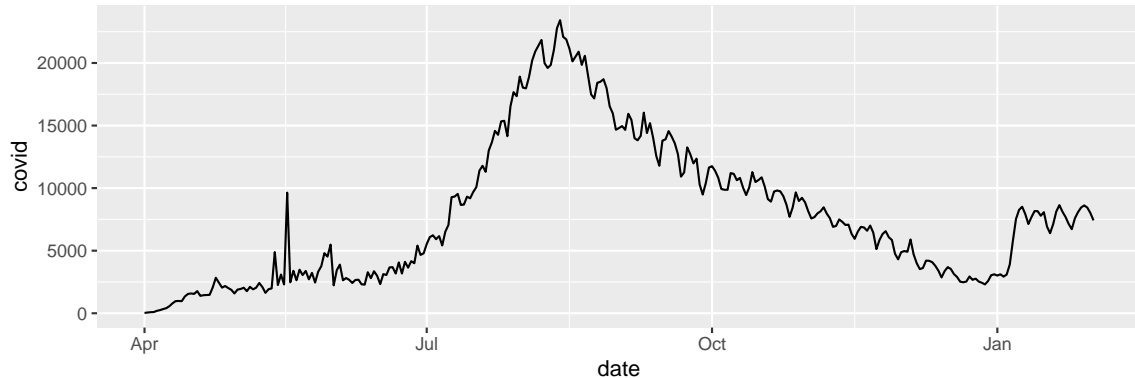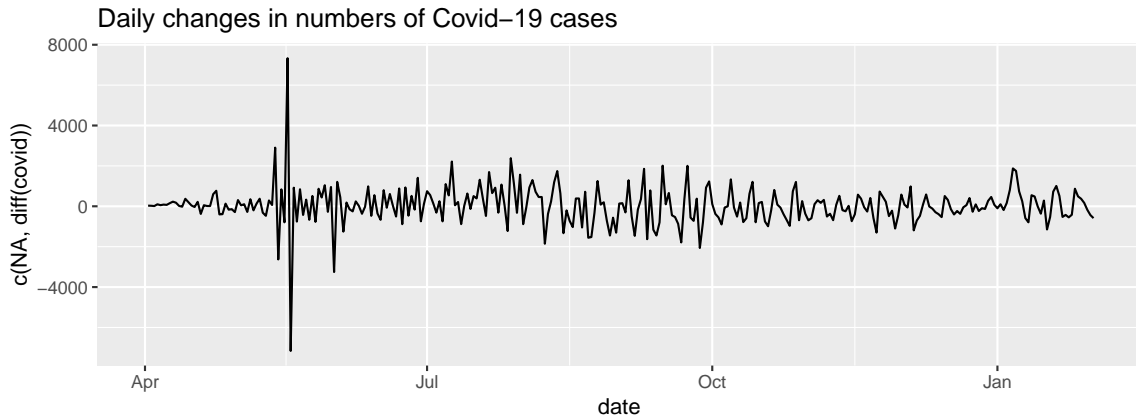
How to transform non-stationary time series to be stationary

Stationary is characterized by constant mean and variance.

Transformations help to stabilize the variance.

Difference help to stabilize the mean

Transformations

Variance could be stabilized by taking:

- (Natural) Logarithm (log)
- Squared root (sqrt)

Transformations

```
par(mfrow=c(1,3))
plot(covid$date, covid$covid, type='l')
plot(covid$date, log(covid$covid), type='l')
plot(covid$date, sqrt(covid$covid), type='l')
```
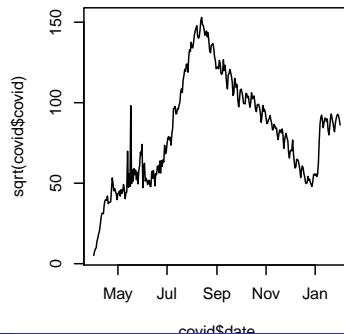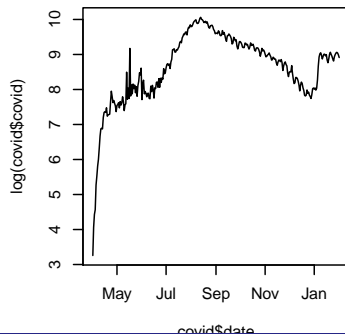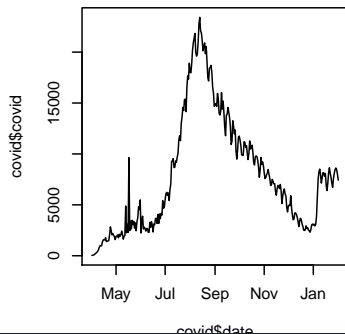
Transformations

We found that taking natural logarithm on Covid cases mostly stabilize the variance.

```
covid$log_covid <- log(covid$covid)
head(covid[, c('date', 'covid', 'log_covid')])
```

```
##          date covid log_covid
## 1 2021-04-01    26  3.258097
## 2 2021-04-02    58  4.060443
## 3 2021-04-03    84  4.430817
## 4 2021-04-04    96  4.564348
## 5 2021-04-05   194  5.267858
## 6 2021-04-06   250  5.521461
```

Differencing

Difference helps stabilize the mean.

In R, we could use the command diff() (plus NA offset at the first index).

```r
covid$diff_log_covid <- c(NA, diff(covid$log_covid))

par(mfrow=c(1,2))
plot(covid$date, covid$covid, type='l')
plot(covid$date, covid$diff_log_covid, type='l')
```
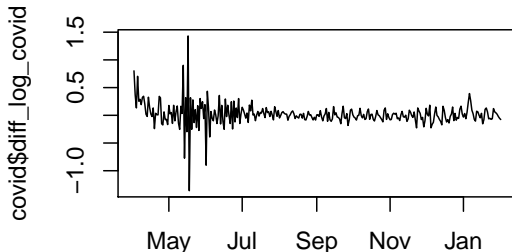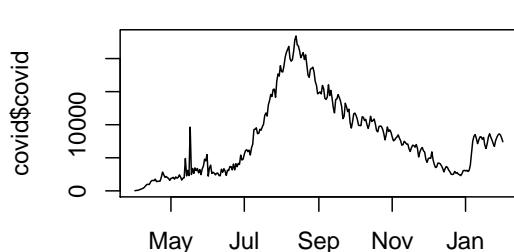
Section 3

Time Series Regression

Linear regression applied to time series data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + ... + \beta_n x_{n,t} + \epsilon_t$$

- y_t is the value of a time series Y we want to predict.

- $x_{n,t}$ is the n predictor, which could be any other values from other series, or the time t itself.

- ϵ_t is the error term called the white noise, where

$$\epsilon_t \sim \mathcal{N}(\mu, \sigma^2)$$

.

Linear regresssion in R

In R, linear regression is fitted with the command `lm()`.

The formula of the command is $lm(y \sim X, data)$, where y is the predicted variable and X are the predictors.

```
fit <- lm(rti ~ year + factor(month), data = rti)
summary(fit)
```

```
##
## Call:
## lm(formula = rti ~ year + factor(month), data = rti)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -680.54  -58.91   16.22   64.04  249.09
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    84484.761   8888.239   9.505 3.43e-16 ***
## year             -32.273      3.473  -9.292 1.08e-15 ***
## factor(month)2  -116.182     52.099  -2.230  0.02767 *
## factor(month)3    22.091     52.099   0.424  0.67234
## factor(month)4  -172.182     52.099  -3.305  0.00126 **
## factor(month)5  -304.545     52.099  -5.845 4.74e-08 ***
## factor(month)6  -349.273     52.099  -6.704 7.73e-10 ***
## factor(month)7  -366.545     52.099  -7.035 1.48e-10 ***
## factor(month)8  -402.364     52.099  -7.723 4.43e-12 ***
## factor(month)9  -443.636     52.099  -8.515 6.92e-14 ***
## factor(month)10 -247.455     53.414  -4.633 9.50e-06 ***
## factor(month)11 -101.855     53.414  -1.907  0.05901 .
## factor(month)12   60.845     53.414   1.139  0.25700
```

Time series regression

The most common application of time series regression is to predict time series based on another series.

For example, if we wish to predict the number of COPD visits based on asthma visits.

```
ncd <- read.csv('data/aci_month.csv')
ncd$date <- as.Date(ncd$date)
ncd
```

```
##          date asthma  copd  ihd month year
## 1  2014-01-01   4510 73038  700     1 2014
## 2  2014-02-01   3933 64946  612     2 2014
## 3  2014-03-01   3961 69802  570     3 2014
## 4  2014-04-01   3470 65011  712     4 2014
## 5  2014-05-01   3861 65504  703     5 2014
## 6  2014-06-01   3915 63536  709     6 2014
## 7  2014-07-01   3948 64355  774     7 2014
## 8  2014-08-01   4136 62949  620     8 2014
## 9  2014-09-01   4762 66514  725     9 2014
```

Time series regression

We could create the predicted variable: the number of COPD visits in the next month.

We do this by taking leading of time series.

```
##            date asthma  copd  ihd month year copd_nextmonth
## 1  2014-01-01   4510 73038  700     1 2014          64946
## 2  2014-02-01   3933 64946  612     2 2014          69802
## 3  2014-03-01   3961 69802  570     3 2014          65011
## 4  2014-04-01   3470 65011  712     4 2014          65504
## 5  2014-05-01   3861 65504  703     5 2014          63536
## 6  2014-06-01   3915 63536  709     6 2014          64355
## 7  2014-07-01   3948 64355  774     7 2014          62949
## 8  2014-08-01   4136 62949  620     8 2014          66514
## 9  2014-09-01   4762 66514  725     9 2014          87466
## 10 2014-10-01   6251 87466 1011    10 2014          80243
## 11 2014-11-01   5733 80243  874    11 2014          85998
## 12 2014-12-01   6759 85998  925    12 2014          89837
```

Time series regression

Before we begin, we should plot both time series (asthma and COPD) to see whether there are patterns associated with both series.

```
ggplot(ncd) +
  geom_line(aes(x=date, y=asthma, color='Asthma')) +
  geom_line(aes(x=date, y=copd, color='COPD')) +
  scale_colour_manual(breaks=c("Asthma", "COPD"), values=c("red", "blue")) +
  scale_y_log10()
```
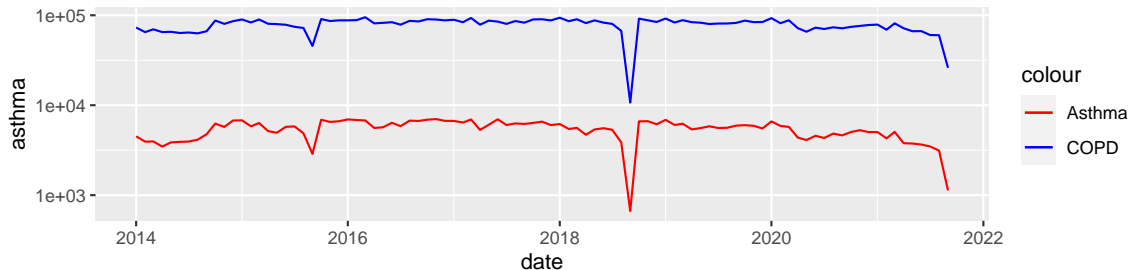
Time series regression

To identify any linear relationship between asthma and COPD, we could use the scatter plot.

```
ggplot(ncd) +
  geom_point(aes(x=asthma, y=copd_nextmonth))
```
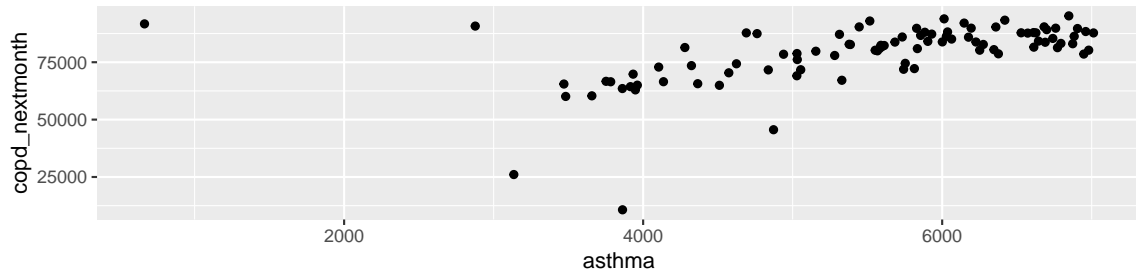

It is clearly seen that there is linear relationship between both series.

Time series regression

Now we could fit the linear regression between asthma and COPD.

```
lm_copd_asthma <- lm(copd_nextmonth ~ asthma, data=ncd)
summary(lm_copd_asthma)
```

```
##
## Call:
## lm(formula = copd_nextmonth ~ asthma, data = ncd)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -58397  -4282    683   3990  42137
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.551e+04  5.546e+03   8.206 1.53e-12 ***
## asthma      6.109e+00  9.924e-01   6.155 2.04e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11110 on 90 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.2963, Adjusted R-squared:  0.2884
## F-statistic: 37.89 on 1 and 90 DF,  p-value: 2.044e-08
```

Time series regression

For the prediction task, we are interested in the coefficients (not the standard error or p-value).

```
coef(lm_copd_asthma)
```

```
## (Intercept)      asthma
## 45510.918092    6.108965
```

$E[COPD] = 45510.918092 + 6.108965 \times Asthma$

Note the we use $E[...]$, which is denoted as the expected number of ..., in other words, we are predicting the mean of COPD visits next month.

Time series regression

We could use predict function to predict COPD cases using the fitted model.

```
ncd$predicted_copd <- c(predict(lm_copd_asthma), NA)
ggplot(ncd) +
    geom_line(aes(x=date, y=copd, color='actual')) +
    geom_line(aes(x=date, y=predicted_copd, color='predicted')) +
    scale_colour_manual(breaks=c("actual", "predicted"), values=c("blue", "red"))
```
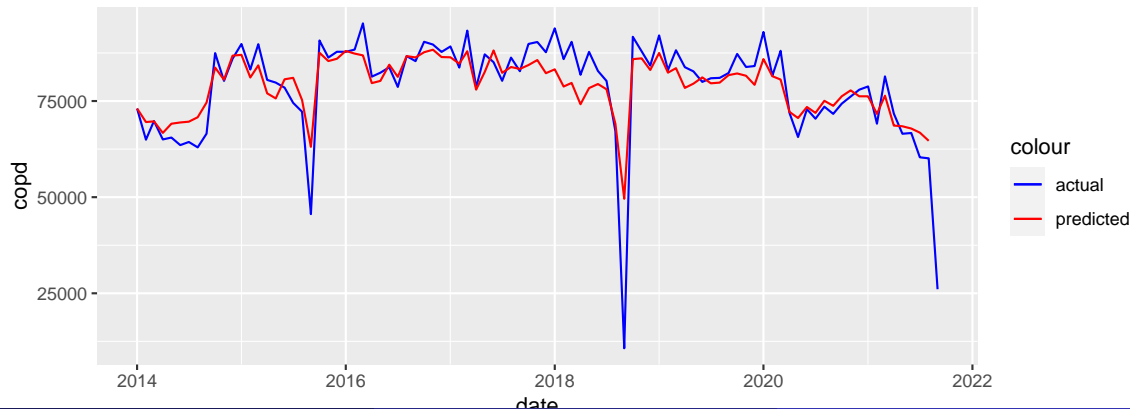
Time series regression

However, it is assumed that we have already know the actual asthma visits, in order to forecast the COPD visits in the future.

If it is not the case, we have to forecast the COPD visits based on its trend and seasonality.

Assuming that the trend is continuing downward further from 2019.

Time series regression

Firstly, we split the training data only on and after 2019.

```
ncd2019 <- ncd[ncd$year >= 2019, ]
ncd2019
```

```
##              date asthma  copd  ihd month year copd_nextmonth predicted_copd
## 61    2019-01-01   6873 92060 1389     1 2019          83059       87497.84
## 62    2019-02-01   6036 83059 1209     2 2019          88208       82384.63
## 63    2019-03-01   6226 88208 1287     3 2019          83807       83545.34
## 64    2019-04-01   5386 83807 1283     4 2019          82714       78413.81
## 65    2019-05-01   5567 82714 1188     5 2019          79974       79519.53
## 66    2019-06-01   5834 79974 1230     6 2019          80929       81150.62
## 67    2019-07-01   5582 80929 1157     7 2019          81027       79611.16
## 68    2019-08-01   5612 81027 1084     8 2019          82285       79794.43
## 69    2019-09-01   5930 82285 1072     9 2019          87250       81737.08
## 70    2019-10-01   6001 87250 1320    10 2019          83857       82170.82
## 71    2019-11-01   5904 83857 1116    11 2019          84100       81578.25
```

Time series regression

Next, we create a new forecasting horizon (2021 to 2022).

```
newdata2021 <- data.frame(year=2021, month=1:12) # 2021
newdata2022 <- data.frame(year=2022, month=1:12) # 2022
newdata <- rbind(newdata2021, newdata2022) # Appending 2021 and 2022
newdata$date <- as.Date(ISOdate(newdata$year, newdata$month, 1)) # create a new date variable
newdata
```

```
##    year month       date
## 1  2021     1 2021-01-01
## 2  2021     2 2021-02-01
## 3  2021     3 2021-03-01
## 4  2021     4 2021-04-01
## 5  2021     5 2021-05-01
## 6  2021     6 2021-06-01
## 7  2021     7 2021-07-01
## 8  2021     8 2021-08-01
## 9  2021     9 2021-09-01
## 10 2021    10 2021-10-01
## 11 2021    11 2021-11-01
## 12 2021    12 2021-12-01
## 13 2022     1 2022-01-01
## 14 2022     2 2022-02-01
## 15 2022     3 2022-03-01
## 16 2022     4 2022-04-01
## 17 2022     5 2022-05-01
## 18 2022     6 2022-06-01
## 19 2022     7 2022-07-01
## 20 2022     8 2022-08-01
## 21 2022     9 2022-09-01
```

Time series regression

Next, the linear model is fitted to the training data (ncd2019) with the following formula:

$$E[COPD] = Date + Jan + Feb + Mar + ... + Dec$$

```
lm2 <- lm(copd ~ date + factor(month), data=ncd2019)
summary(lm2)
```

```
##
## Call:
## lm(formula = copd ~ date + factor(month), data = ncd2019)
##
## Residuals:
##      Min      1Q   Median      3Q      Max
## -24369.6  -1716.1    433.8  2176.7  12705.2
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)      566311.672  88024.475   6.434 2.83e-06 ***
## date                -26.195      4.814  -5.442 2.51e-05 ***
## factor(month)2    -9213.302   6346.108  -1.452  0.16206
## factor(month)3     -489.455   6350.779  -0.077  0.93933
## factor(month)4    -9751.757   6359.237  -1.533  0.14083
## factor(month)5   -13169.587   6370.742  -2.067  0.05190 .
## factor(month)6   -10790.221   6386.039  -1.690  0.10662
## factor(month)7   -12624.384   6404.120  -1.971  0.06269 .
## factor(month)8   -10825.353   6426.160  -1.685  0.10761
## factor(month)9   -21562.321   6451.578  -3.342  0.00325 **
## factor(month)10   -4773.192   7106.617  -0.672  0.50949
## factor(month)11   -4737.660   7117.340  -0.666  0.51324
```

Time series regression

We could use the fitted model to predict COPD cases in 2021 and 2022.

```
newdata$forecast <- predict(lm2, newdata=newdata)
newdata
```

```
##    year month       date forecast
## 1  2021     1 2021-01-01 78359.18
## 2  2021     2 2021-02-01 68333.85
## 3  2021     3 2021-03-01 76324.25
## 4  2021     4 2021-04-01 66249.92
## 5  2021     5 2021-05-01 62046.25
## 6  2021     6 2021-06-01 63613.58
## 7  2021     7 2021-07-01 60993.58
## 8  2021     8 2021-08-01 61980.58
## 9  2021     9 2021-09-01 50431.58
## 10 2021    10 2021-10-01 66434.87
## 11 2021    11 2021-11-01 65658.37
## 12 2021    12 2021-12-01 66667.87
## 13 2022     1 2022-01-01 68798.17
## 14 2022     2 2022-02-01 58772.83
## 15 2022     3 2022-03-01 66763.23
## 16 2022     4 2022-04-01 56688.90
## 17 2022     5 2022-05-01 52485.23
## 18 2022     6 2022-06-01 54052.56
## 19 2022     7 2022-07-01 51432.56
## 20 2022     8 2022-08-01 52419.56
## 21 2022     9 2022-09-01 40870.56
## 22 2022    10 2022-10-01 56873.86
## 23 2022    11 2022-11-01 56097.36
## 24 2022    12 2022-12-01 57106.86
```

Time series regression

We could also visualize the forecast.

```
ggplot() +
  geom_line(data=ncd, aes(x=date, y=copd)) +
  geom_line(data=newdata, aes(x=date, y=forecast), color='red')
```
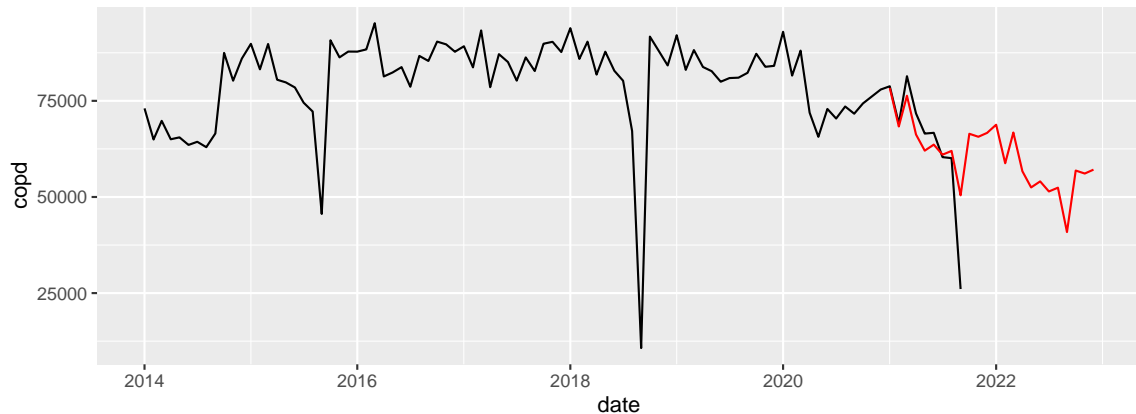
We finally summarise predicted cases in 2022.

```
newdata[newdata$year==2022, ]
```

```
##    year month       date forecast
## 13 2022     1 2022-01-01 68798.17
## 14 2022     2 2022-02-01 58772.83
## 15 2022     3 2022-03-01 66763.23
## 16 2022     4 2022-04-01 56688.90
## 17 2022     5 2022-05-01 52485.23
## 18 2022     6 2022-06-01 54052.56
## 19 2022     7 2022-07-01 51432.56
## 20 2022     8 2022-08-01 52419.56
## 21 2022     9 2022-09-01 40870.56
## 22 2022    10 2022-10-01 56873.86
## 23 2022    11 2022-11-01 56097.36
## 24 2022    12 2022-12-01 57106.86
```

Time series regression: diagnostic

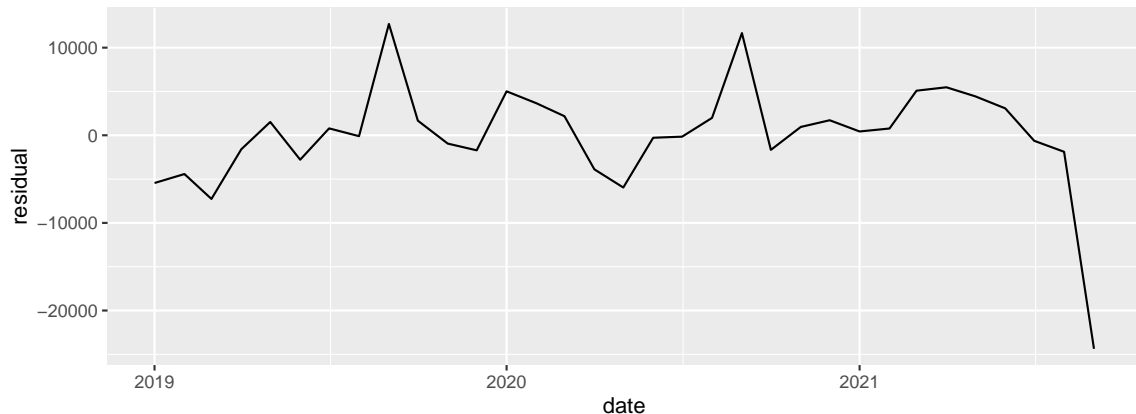Residual plots

```
ncd2019$fit <- lm2$fit
ncd2019$residual <- lm2$residual
ncd2019[, c("date", "copd", "fit", "residual")]
```

```
##            date  copd      fit    residual
## 61 2019-01-01 92060 97507.42  -5447.4170
## 62 2019-02-01 83059 87482.08  -4423.0836
## 63 2019-03-01 88208 95472.48  -7264.4818
## 64 2019-04-01 83807 85398.15  -1591.1485
## 65 2019-05-01 82714 81194.48   1519.5182
## 66 2019-06-01 79974 82761.82  -2787.8152
## 67 2019-07-01 80929 80141.82    787.1848
## 68 2019-08-01 81027 81128.82   -101.8152
## 69 2019-09-01 82285 69579.82  12705.1848
## 70 2019-10-01 87250 85583.11   1666.8933
```
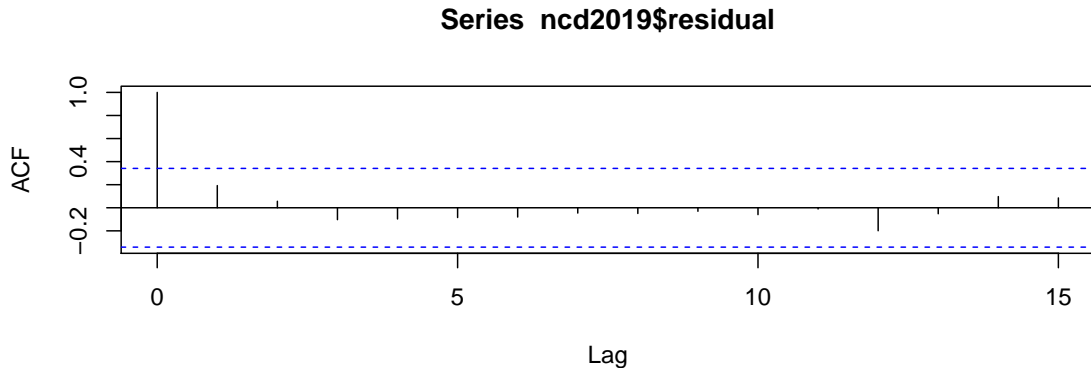
Time series regression: Diagnostics

Residual plots

```
ggplot(ncd2019) +
  geom_line(aes(x=date, y=residual))
```

ACF plot of the residuals

```
acf(ncd2019$residual)
```

Series ncd2019$residual

Forecasting PM 2.5

Forecasting PM 2.5 concentration capturing trend and seasonality (hour and day-of-week)

```
pm <- read.csv("data/pm_ts.csv")
pm$DATETIMEDATA <- as.POSIXct(pm$DATETIMEDATA)
pm$hour <- format(pm$DATETIMEDATA, "%H")
pm$dow <- weekdays(pm$DATETIMEDATA)
pm
```

```
##           DATETIMEDATA     PM25 hour       dow
## 1  2021-11-10 00:00:00 14.07865   00 Wednesday
## 2  2021-11-10 01:00:00 13.37079   01 Wednesday
## 3  2021-11-10 02:00:00 13.28090   02 Wednesday
## 4  2021-11-10 03:00:00 13.11236   03 Wednesday
## 5  2021-11-10 04:00:00 13.16854   04 Wednesday
## 6  2021-11-10 05:00:00 13.73034   05 Wednesday
## 7  2021-11-10 06:00:00 14.19101   06 Wednesday
## 8  2021-11-10 07:00:00 15.35227   07 Wednesday
## 9  2021-11-10 08:00:00 16.25000   08 Wednesday
## 10 2021-11-10 09:00:00 16.26136   09 Wednesday
## 11 2021-11-10 10:00:00 15.98864   10 Wednesday
## 12 2021-11-10 11:00:00 15.09091   11 Wednesday
## 13 2021-11-10 12:00:00 14.41573   12 Wednesday
## 14 2021-11-10 13:00:00 14.39326   13 Wednesday
## 15 2021-11-10 14:00:00 13.48864   14 Wednesday
## 16 2021-11-10 15:00:00 13.94318   15 Wednesday
## 17 2021-11-10 16:00:00 14.39326   16 Wednesday
## 18 2021-11-10 17:00:00 14.56180   17 Wednesday
## 19 2021-11-10 18:00:00 15.87640   18 Wednesday
## 20 2021-11-10 19:00:00 17.30337   19 Wednesday
## 21 2021-11-10 20:00:00 18.31461   20 Wednesday
```
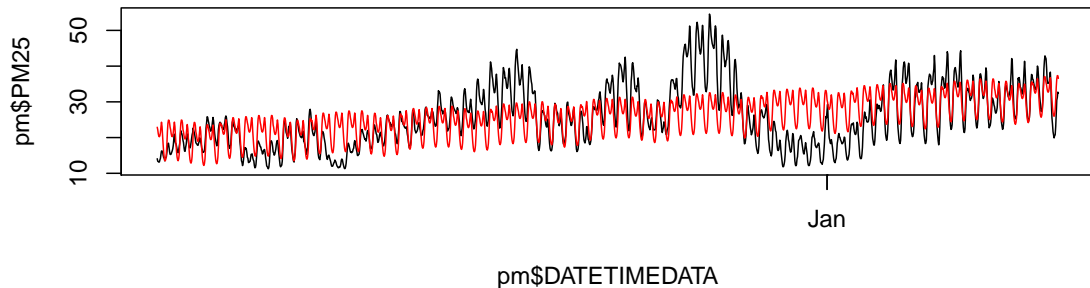
Forecasting PM 2.5

```r
lm_pm <- lm(PM25 ~ DATETIMEDATA+hour+dow, pm)

pm$fit <- lm_pm$fit
plot(pm$DATETIMEDATA, pm$PM25, type='l')
lines(pm$DATETIMEDATA, pm$fit, col='red')
```

Case study: Forecasting Dengue with rainfall data

In this tutorial, we will predict Dengue cases, based on rainfall data.

Case study: Forecasting Dengue with rainfall data

Firstly, loaded the data containing dengue cases and rainfall data.

```
# read the data
dhfrain <- read.csv('data/dhfrain.csv')
dhfrain$date <- as.Date(dhfrain$date)
head(dhfrain)
```

```
##           date year month cases mean_rain max_rain year_from_last_epidemic
## 1 2009-01-01 2009     1  2614 0.6542994    176.2                       2
## 2 2009-02-01 2009     2  2057 0.2529173     46.4                       2
## 3 2009-03-01 2009     3  2324 2.8785509    132.0                       2
## 4 2009-04-01 2009     4  2947 4.5172608    216.8                       2
## 5 2009-05-01 2009     5  6234 7.4758991    178.7                       2
## 6 2009-06-01 2009     6  8569 5.2782202    157.1                       2
```

Case study: Forecasting Dengue with rainfall data
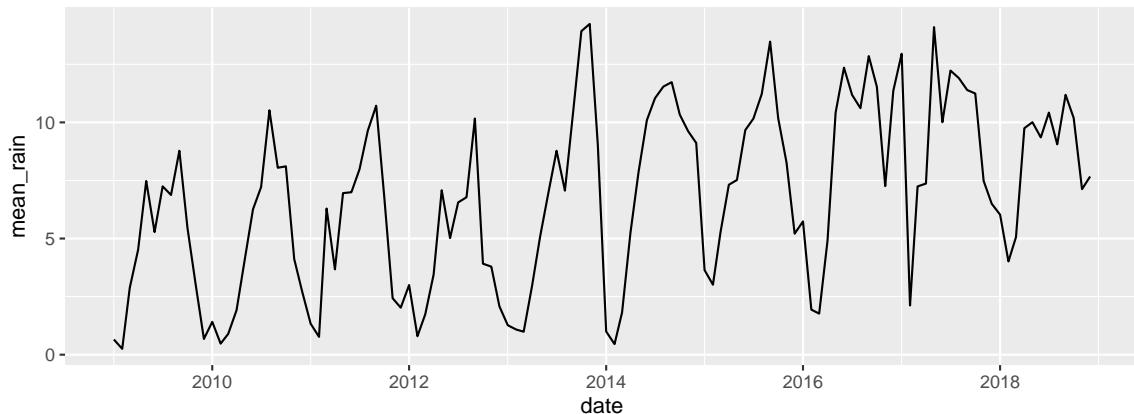
We visualize time series of monthly dengue cases.

```
ggplot(dhfrain) + geom_line(aes(x=date, y=cases))
```
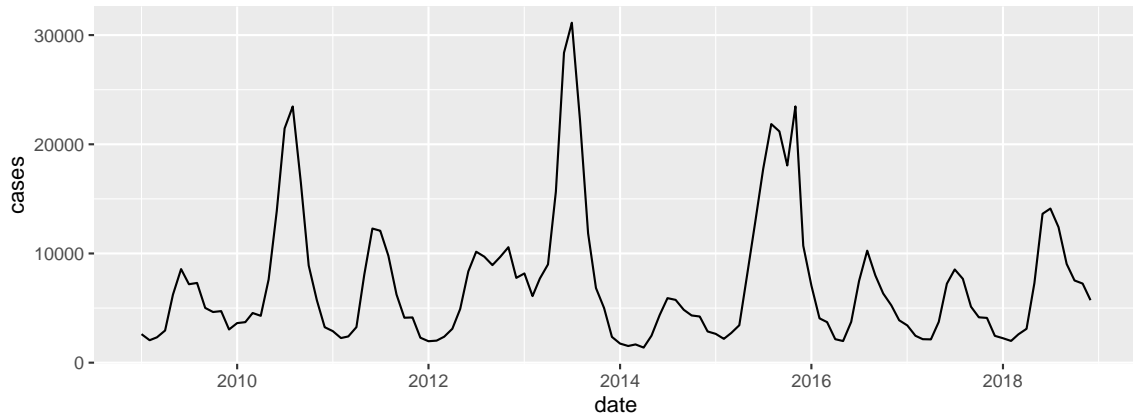
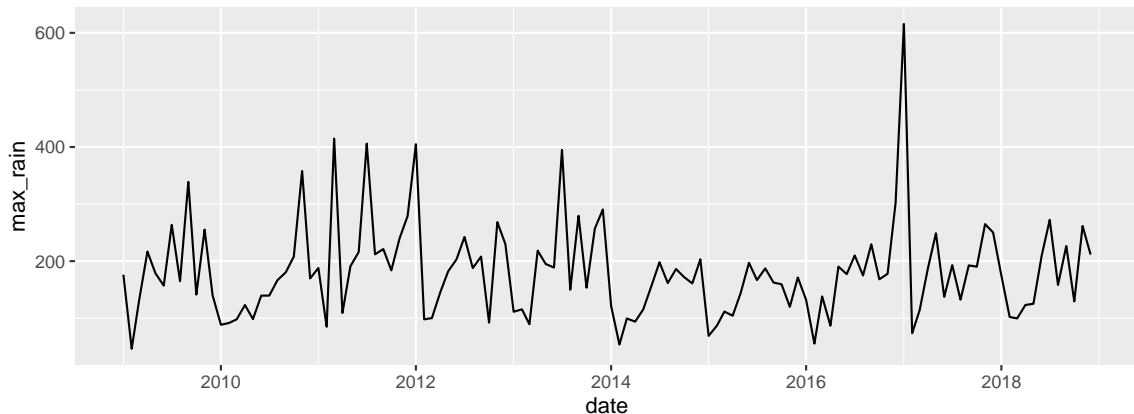Next, mean monthly rainfall is shown.

```
ggplot(dhfrain) +
  geom_line(aes(x=date, y=mean_rain))
```

Case study: Forecasting Dengue with rainfall data

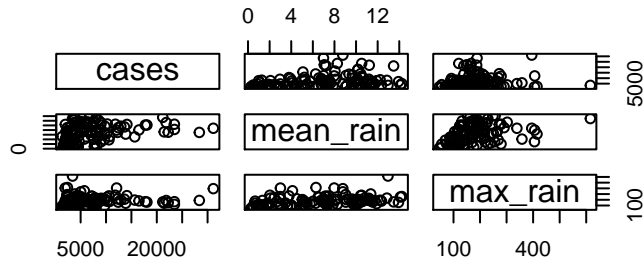Next, visualizing max rainfall each month

```
ggplot(dhfrain) +
  geom_line(aes(x=date, y=max_rain))
```

Case study: Forecasting Dengue with rainfall data

We could also examine the relationship between DHF cases and rainfalls.

```
pairs(dhfrain[, c('cases', 'mean_rain', 'max_rain')])
```

Case study: Forecasting Dengue with rainfall data

Now, use lm() to fit linear regression.

Firstly, having only rainfall parameters as predictors

```
model_dhfrain_1 <- lm(cases ~ max_rain+mean_rain, dhfrain)
summary(model_dhfrain_1)
```

```
##
## Call:
## lm(formula = cases ~ max_rain + mean_rain, data = dhfrain)
##
## Residuals:
##    Min      1Q  Median      3Q     Max
##  -7730   -3633   -1654    1984   22483
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3549.278   1349.418   2.630  0.00968 **
## max_rain       2.035      6.608   0.308  0.75861
## mean_rain    489.410    146.535   3.340  0.00113 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5655 on 117 degrees of freedom
## Multiple R-squared:  0.1057, Adjusted R-squared:  0.09043
## F-statistic: 6.916 on 2 and 117 DF,  p-value: 0.00145
```

Case study: Forecasting Dengue with rainfall data

Use fitted model to produce fitted data.

```
dhfrain$fit_dhfrain_1 <- predict(model_dhfrain_1)
ggplot(dhfrain) + geom_line(aes(x=date, y=cases)) + geom_line(aes(x=date, y=fit_dhfrain_1), col='red')
```
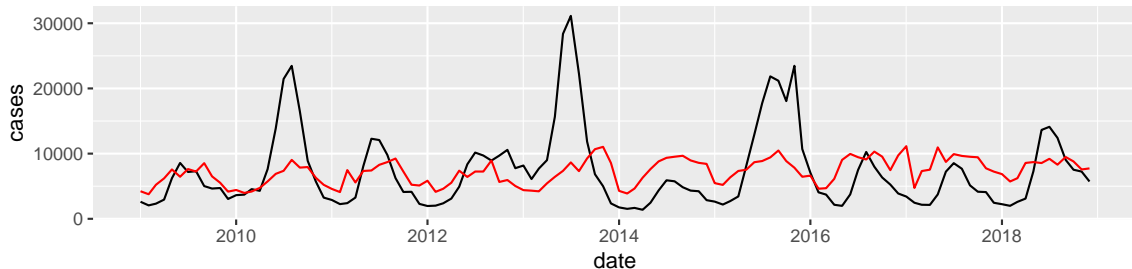

It is clearly seen that this forecast isn't adequate.

Case study: Forecasting Dengue with rainfall data

Next, we could add month-of-the-year to capture seasonal pattern of the disease.

```
model_dhfrain_2 <- lm(cases ~ max_rain+mean_rain+factor(month), dhfrain)
dhfrain$fit_dhfrain_2 <- predict(model_dhfrain_2)
ggplot(dhfrain) + geom_line(aes(x=date, y=cases)) + geom_line(aes(x=date, y=fit_dhfrain_2, col='red')
```
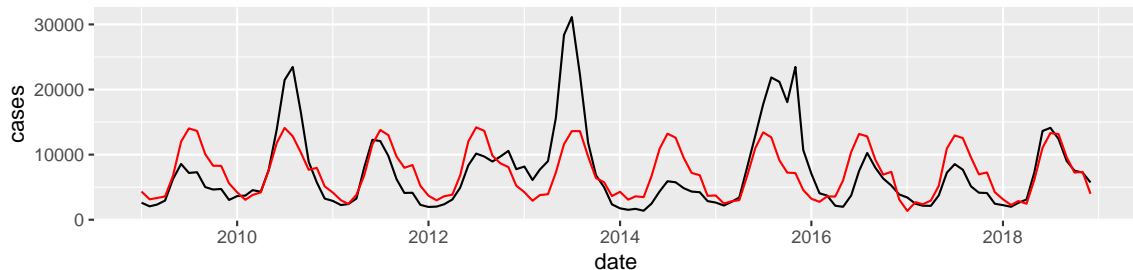

From this plot, it is clear that adding months improve the forecast but still not capture cyclical pattern.

Case study: Forecasting Dengue with rainfall data

Next, we use the number of years from last epidemic as proxy indicator for the cyclical pattern

```
model_dhfrain_3 <- lm(cases ~ max_rain+mean_rain+factor(month)+factor(year_from_last_epidemic), dhfrain)
dhfrain$fit_dhfrain_3 <- predict(model_dhfrain_3)
ggplot(dhfrain) + geom_line(aes(x=date, y=cases)) + geom_line(aes(x=date, y=fit_dhfrain_3), col='red')
```
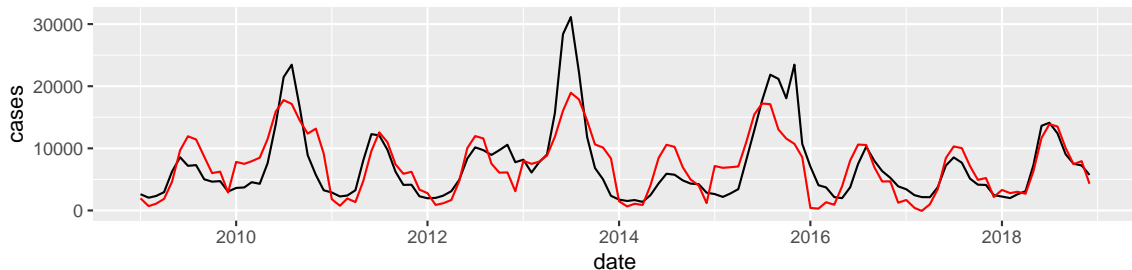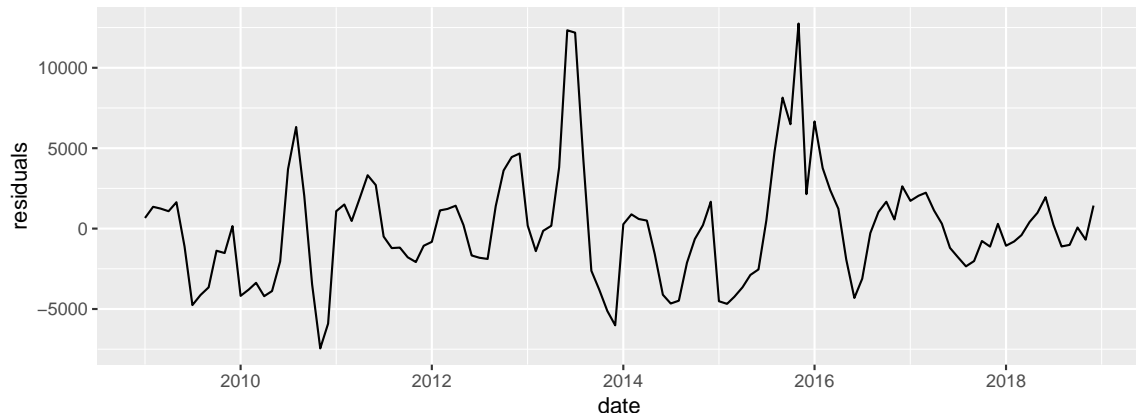

Including the number of years partially capture cyclical pattern.

Case study: Forecasting Dengue with rainfall data

Now, we should diagnose if the model violate linear regression assumptions.

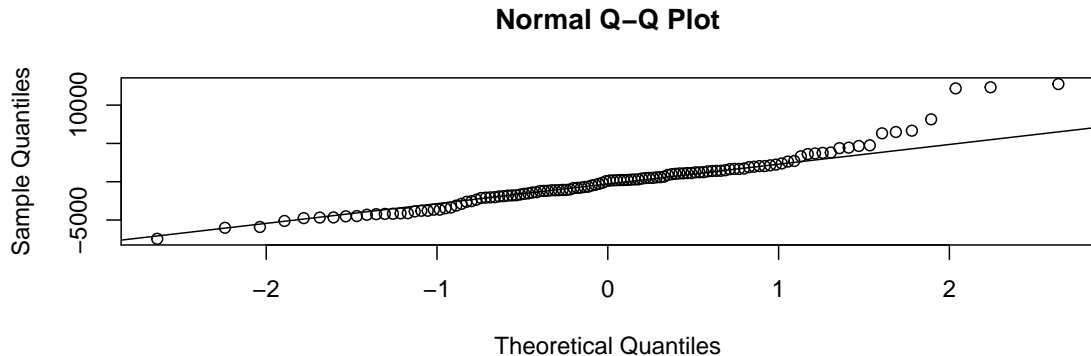We calculate residuals, which is unlikely normally distributed.

```
dhfrain$residuals <- model_dhfrain_3$residuals
ggplot(dhfrain) + geom_line(aes(x=date, y=residuals))
```

Case study: Forecasting Dengue with rainfall data

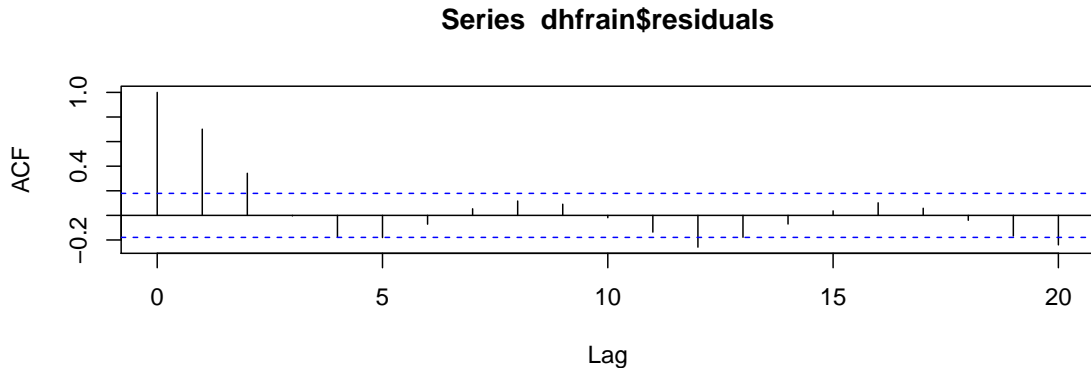Q-Q plot also suggest that the normality assumption is violated.

```
qqnorm(dhfrain$residuals)
qqline(dhfrain$residuals)
```


Normal Q–Q Plot

Case study: Forecasting Dengue with rainfall data

ACF plot reveals autocorrelation at lag 1 and 2. Thereby independence is also violated.

```
acf(dhfrain$residuals)
```

Series dhfrain$residuals

Case study: Forecasting Dengue with rainfall data

Final remarks:

- Linear regression is a great method to forecast, especially with incorporating external information.

- However, most of the time series data violate linear regression assumption so that there would be some information that didn't be captured by the model.

- The are several methods addressing this problems. The most common ways to handle this is to also predict the residuals themselves with other methods, such as ARIMA.