1 Math Problem B

Define X_n as a bernoulli random variable such that,

$$X_n = \begin{cases} A & \text{bus arrival at time n} \\ NA & \text{no bus arrival at time n} \end{cases}$$

We can then define a DTMC on X_n with transition probabilities,

$$P(X_{t+1} = A | X_t = A)$$
$$P(X_{t+1} = NA | X_t = A)$$
$$P(X_{t+1} = A | X_t = NA)$$
$$P(X_{t+1} = A | X_t = NA)$$

We are interested in finding the pmf,

$$P(D_t = w)$$

Representing the amount of time, w, a traveler has to wait for an arriving bus given the traveler starts waiting at time t.

For the case $P(D_t = 0)$, a bus arriving when the traveler starts waiting, either a bus appeared at time t after there had been no bus at $t-1$ or the buses arrived consecutively; i.e.,

$$P(D_t = 0) = P(X_t = A | X_{t-1} = NA) + P(X_t = A | X_{t-1} = A)$$

Using the independence of bus arrivals, we can then generalize the waiting time from the initial time the traveler starts waiting to the time when a bus finally arrives (for $w \geq 1$),

$$\begin{aligned} P(D_t = w) &= P(A|NA)P(NA|NA)^{w-1}P(NA|NA) + P(A|NA)P(NA|NA)^{w-1}P(NA|A) \\ &= P(A|NA)P(NA|NA)^{w-1}[P(NA|NA) + P(NA|A)] \end{aligned}$$

2 Math Problem C

2.1 Continuous Markov Chain

The continuous Markov chain is defined for (i,j,k) where i represents the number of customers being served, j represents the queue for the manager, and k represents the number of people in the general queue. This model follows the model described on the blog. If the queue is empty and i is not equal to the number of servers available then any new customer goes immediately to a server with rate alpha*(1-p) and to the manager with alpha*p rate. The values of i,j,k are bounded by i + j + k <= queue buffer. When the queue is full, no new customers can show up.

2.2 Proportion of calls denied

The proportion of calls denied is equal to the number of calls denied divided by the total number of calls. This is equal to the P(i + j + k = b AND we get a call) = P(i+j+k=b)*P(we get a call). The probability we are in a full state is equal to:
$\sum\limits_{i+j+k=b} \pi_{ijk} * P(call) = \sum\limits_{i+j+k=b} \pi_{ijk} * \frac{\alpha}{\alpha+\sigma+\omega+\mu}$

2.3 Proportion of customers leaving due to impatience

The proportion of customers leaving due to impatience is equal to the number of people would leave divided by the total number of people. The argument flows similar to above.
$\sum\limits_{k>0 or j>0} \pi_{ijk} * \frac{\omega}{\alpha+\sigma+\omega+\mu} * (k+j)$

3 Simulation