1 Math Problem B

Define X_n as a random variable such that,

$$X_n = \begin{cases} A & \text{bus arrival at time n} \\ NA & \text{no bus arrival at time n} \end{cases}$$

We can then define a DTMC on X_n with transition probabilities,

$$P(X_{t+1} = A|X_t = A)$$
$$P(X_{t+1} = NA|X_t = A)$$
$$P(X_{t+1} = A|X_t = NA)$$
$$P(X_{t+1} = A|X_t = NA)$$

We are interested in finding the pmf,

$$P(D_t = w)$$

Representing the amount of time, w, a traveler has to wait for an arriving bus given the traveler starts waiting at time t.

For the case $P(D_t = 0)$, a bus arriving when the traveler starts waiting, a bus appeared at time t after there had been no bus at $t-1$ or the buses arrived consecutively; i.e.,

$$P(D_t = 0) = P(X_t = A|X_{t-1} = NA) + P(X_t = A|X_{t-1} = A)$$

Using the independence of bus arrivals, we can then generalize the waiting time from the initial time the traveler starts waiting to the time when a bus finally arrives (for $w \geq 1$),

$$P(D_t = w) = P(A|NA)P(NA|NA)^{w-1}P(NA|NA) + P(A|NA)P(NA|NA)^{w-1}P(NA|A)$$
$$= P(A|NA)P(NA|NA)^{w-1}[P(NA|NA) + P(NA|A)]$$

2 Math Problem C

2.1 Continuous Markov Chain

The continuous Markov chain is defined for (i,j,k) where i represents the number of customers being served, j represents the queue for the manager, and k represents the number of people in the general queue. This model follows the model described on the blog. If the queue is empty and i is not equal to the number of servers available then any new customer goes immediately to a server with rate alpha*(1-p) and to the manager with alpha*p rate. The values of i,j,k are bounded by i + j + k <= queue buffer. When the queue is full, no new customers can show up.

2.2 Proportion of calls denied

The proportion of calls denied is equal to the number of calls denied divided by the total number of calls. This is equal to the $P(i + j + k = b$ AND we get a call$) = P(i+j+k=b)*P($we get a call$)$. The probability we are in a full state is equal to:

$$\sum_{i+j+k=b} \pi_{ijk} * P(call) = \sum_{i+j+k=b} \pi_{ijk} * \frac{\alpha}{\alpha+\sigma+\omega+\mu}$$

2.3 Proportion of customers leaving due to impatience

The proportion of customers leaving due to impatience is equal to the number of people would leave divided by the total number of people. The argument flows similar to above.

$$\sum_{k>0 or j>0} \pi_{ijk} * \frac{\omega}{\alpha+\sigma+\omega+\mu} * (k+j)$$

3 Simulation

Using DES package, we created a discrete event simulation of the client-manager-server queue ("S2Q3.R") where the monitored discrete events are arrivals (simple and complex jobs), job completions, and line drop outs. The complex jobs are immediate recognized and forwarded to the manager queue without any time delays. The analytical results ("ProblemC.R") were calculated by forming P_{ij} matrix and the holding times and uses the stationary distribution with the equations above to solve for the proportion of calls denied and the proportion of customers leaving due to impatience.There are some major differences between simulated and analytical results which is potentially a result of subtle program bugs.

	Servers	Buffer Size	p	α	σ	ω	μ	Proportion Denied	Proportion Left
Analytical Solution	3	6	0.3	0.7	0.5	0.3	0.2	0.0256	0.1818
Analytical Solution	4	8	0.2	3	2	0.2	0.05	0.381	0.067
DES Simulation Solution	3	6	0.3	0.7	0.5	0.3	0.2	0.2206	0
DES Simulation Solution	4	8	0.2	3	2	0.2	0.05	0.6650	1.67e-5