1 Problem A

1.1 Analytical Solution

$$cov(X_n, X_{n+1}) = E[X_n X_{n+1}] - EX_n EX_{n+1} \tag{1}$$

$$E[X_n X_{n+1}] = \sum_{i=0}^{\infty} E[X_n X_{n+1}|X_n = i]P(X_n = i) \tag{2}$$

$$= \sum_{i=0}^{\infty} iE[X_{n+1}|X_n = i]P(X_n = i)$$

$$= \sum_{i=0}^{\infty} i(E[X_n|X_n = i] + E[I_n|X_n = i])P(X_n = i)$$

$$= \sum_{i=0}^{\infty} i(i + E[I_n|X_n = i])P(X_n = i)$$

$$= \sum_{i=0}^{\infty} i(i + (\alpha_i - \beta_i))P(X_n = i)$$

$$EX_n = \sum_{i=0}^{\infty} iP(X_n = i) \tag{3}$$

$$EX_{n+1} = EX_n + EI_n \tag{4}$$

$$= EX_n + \sum_{i=0}^{\infty} E[I_n|X_n = i]P(X_n = i)$$

$$= EX_n + \sum_{i=0}^{\infty} (\alpha_i - \beta_i)P(X_n = i)$$

$$\text{From (1), (2)} - (3) * (4) \Rightarrow$$

$$\lim_{n\to\infty} cov(X_n, X_{n+1}) = \sum_{i=0}^{\infty} i(i + \alpha_i - \beta_i)\pi_i - \sum_{i=0}^{\infty} i\pi_i \sum_{i=0}^{\infty} (i + \alpha_i - \beta_i)\pi_i \tag{5}$$

1.2 Simulation Results

(todo)