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Geometric interpretation of the general POE model for a serial-link robot via conversion into D-H parameterization

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Key Findings

1. The general D-H model and the general POE (Product of Exponentials) formula (general means covering the type of helical joints) are interconvertible automatically.
2. The maximum number of identifiable parameters in a general model is 5h+4r+2t+n+6 where h, r, t, and n stand for the number of helical, revolute, prismatic, and general joints.
3. The identifiability of the base frame and the tool frame in the D-H model is restricted rather than the arbitrary 6 parameters as assumed previously.
4. Matlab implementation of the interconversion between the two models can be found at https://github.com/drliaowu/GeneralPOE2DH

Motivation

1. The POE model has been proved as more effective for the calibration of serial-link robots than the D-H model.
2. However, the D-H model is more widely used with a long history and many algorithms and libraries developed based on it.
3. If the two models are interconvertible automatically, it will be very convenient to take advantage of both models.

General D-H Model

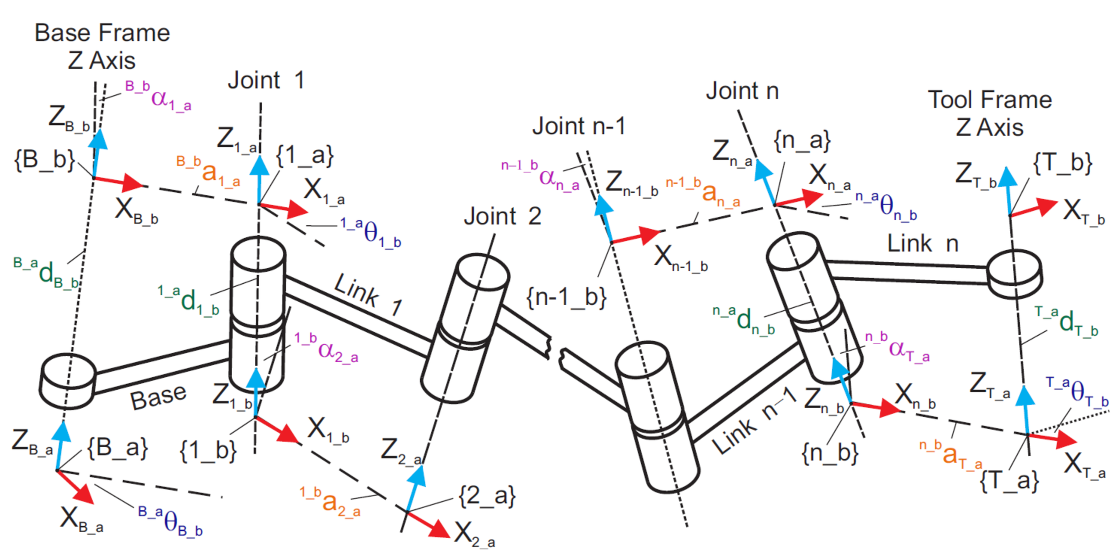


Fig. 1. D-H parameters for an n-joint serial-link robot with an arbitrarily located base frame and tool frame.

The forward kinematics with D-H parameters:

$$
\begin{aligned}
{}^{B}\boldsymbol{H}_T \quad = \quad & \underbrace{R_Z({}^{B\_a}\theta_{B\_b})T_Z({}^{B\_a}d_{B\_b})R_X({}^{B\_b}\alpha_{1\_a})T_X({}^{B\_b}a_{1\_a})}_{{}^{B}\boldsymbol{H}_0}Q_{j_1,k_1}(q_1)\\
& \underbrace{R_Z({}^{1\_a}\theta_{1\_b})T_Z({}^{1\_a}d_{1\_b})R_X({}^{1\_b}\alpha_{2\_a})T_X({}^{1\_b}a_{2\_a})}_{{}^{0}\boldsymbol{H}_1}Q_{j_2,k_2}(q_2)\\
& \cdots Q_{j_n,k_n}(q_n)\underbrace{R_Z({}^{n\_a}\theta_{n\_b})T_Z({}^{n\_a}d_{n\_b})R_X({}^{n\_b}\alpha_{T\_a})T_X({}^{n\_b}a_{T\_a})}_{{}^{n-1}\boldsymbol{H}_n}\\
& \underbrace{R_Z({}^{T\_a}\theta_{T\_b})T_Z({}^{T\_a}d_{T\_b})}_{{}^{n}\boldsymbol{H}_T}.
\end{aligned}
$$

where

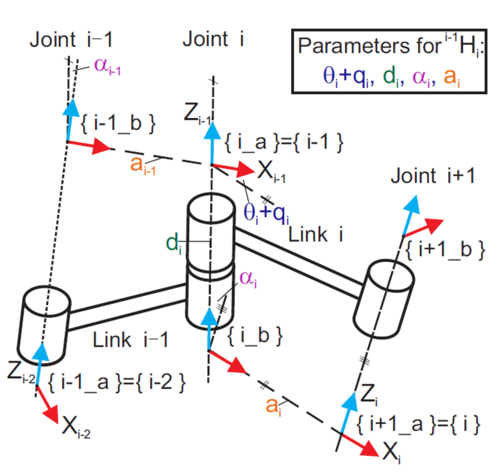
$$Q_{j_i,k_i}(q_i) = R_Z(j_i q_i)T_Z(k_i q_i)$$

and

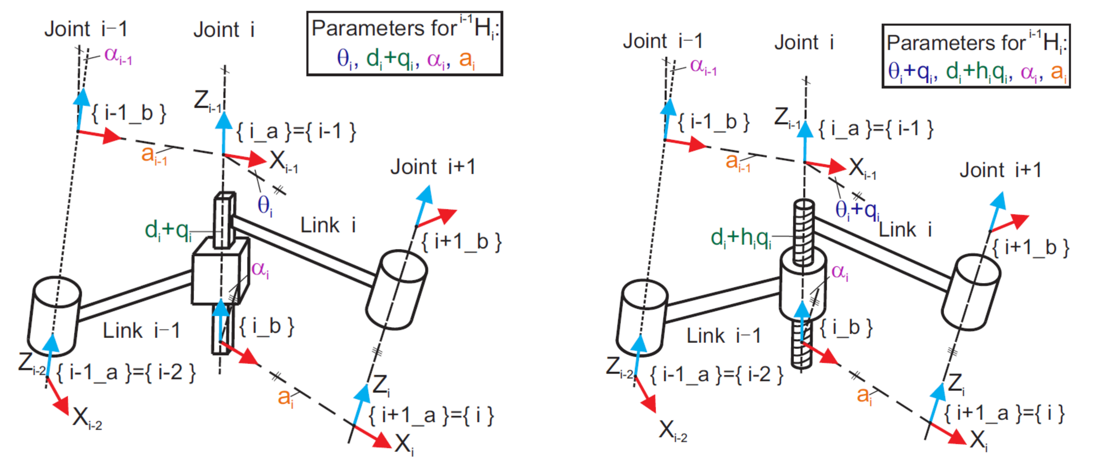
for revolute joint $j_i = 1$ and $k_i = 0$

for prismatic joint $j_i = 0$ and $k_i = 1$

for helical joint $j_i = 1$ and $k_i = h_i$



(a) Revolute joint.



(b) Prismatic joint.   (c) Helical joint.

Fig. 2. Standard D-H parameters for three basic 1-DOF lower pair joints.

General POE Model

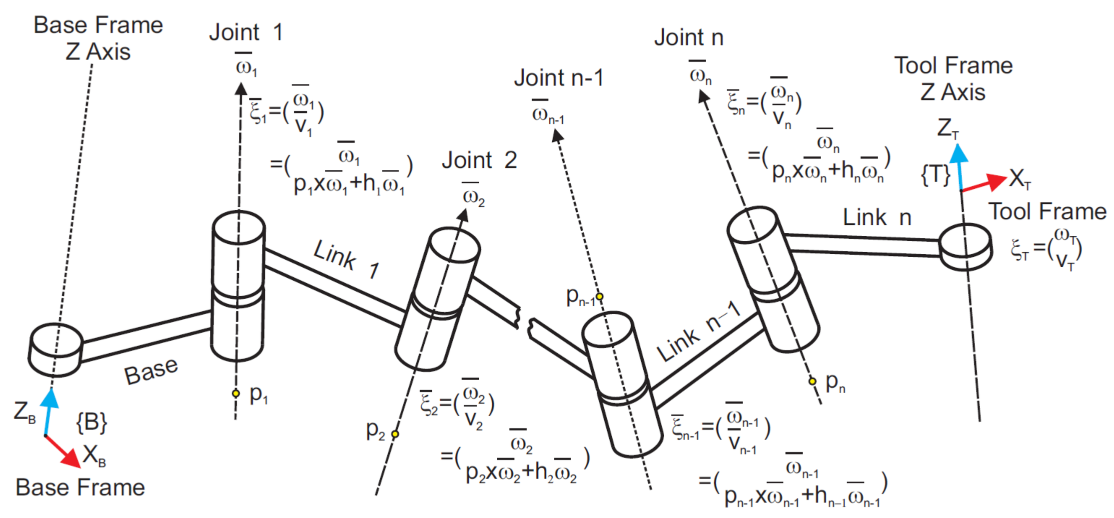


Fig. 3. A general POE model for an n-DOF serial-link robot.

Three equivalent expressions:

1. The *base* POE formula:

$$\boldsymbol{H} = e^{\hat{\boldsymbol{\xi}}_1(q_1+\Delta q_1)}e^{\hat{\boldsymbol{\xi}}_2(q_2+\Delta q_2)}\ldots e^{\hat{\boldsymbol{\xi}}_n(q_n+\Delta q_n)}e^{\hat{\boldsymbol{\xi}}_T}$$

2. The *tool* POE formula:

$$\boldsymbol{H} = e^{\hat{\boldsymbol{\xi}}_T}e^{\hat{\boldsymbol{\xi}}_1^T(q_1+\Delta q_1)}e^{\hat{\boldsymbol{\xi}}_2^T(q_2+\Delta q_2)}\ldots e^{\hat{\boldsymbol{\xi}}_n^T(q_n+\Delta q_n)}$$

3. The *local* POE formula:

$$\boldsymbol{H} = \boldsymbol{H}_1 e^{\hat{\boldsymbol{\xi}}_1^{H_1}(q_1+\Delta q_1)}\boldsymbol{H}_2 e^{\hat{\boldsymbol{\xi}}_2^{H_2}(q_2+\Delta q_2)}\ldots\boldsymbol{H}_n e^{\hat{\boldsymbol{\xi}}_n^{H_n}(q_n+\Delta q_n)}\boldsymbol{H}_{n+1}$$

Conversion

Lemma 1. *For a helical joint, the exponential $e^{\hat{\boldsymbol{\xi}}q}$ can be converted into the form of $\boldsymbol{H}R_Z(q)T_Z(hq)\boldsymbol{H}^{-1}$, where $h = \bar{\boldsymbol{\omega}}^T\boldsymbol{v}$ and $\boldsymbol{H} = R_Z(\theta)T_Z(d)R_X(\alpha)T_X(a)$.*

Lemma 2. *For a revolute joint, the exponential $e^{\hat{\boldsymbol{\xi}}q}$ can be converted into the form of $\boldsymbol{H}R_Z(q)\boldsymbol{H}^{-1}$, where $\boldsymbol{H} = R_Z(\theta)T_Z(d)R_X(\alpha)T_X(a)$.*

Lemma 3. *For a prismatic joint, the exponential $e^{\hat{\boldsymbol{\xi}}q}$ can be converted into the form of $\boldsymbol{H}T_Z(q)\boldsymbol{H}^{-1}$, where $\boldsymbol{H} = R_Z(\theta)T_Z(d)R_X(\alpha)T_X(a)$.*

Lemma 4. *For an arbitrary twist $\hat{\boldsymbol{\xi}}$, the exponential $e^{\hat{\boldsymbol{\xi}}}$ can be decomposed into a product $e^{\hat{\boldsymbol{\xi}}} = \boldsymbol{H}_1\boldsymbol{H}_2$, where $\boldsymbol{H}_1 = R_Z(\theta_1)T_Z(d_1)R_X(\alpha_1)T_X(a_1)$ and $\boldsymbol{H}_2 = R_Z(\theta_2)T_Z(d_2)$.*

$$
\begin{aligned}
{}^{B}\boldsymbol{H}_T &= e^{\hat{\boldsymbol{\xi}}_1 q_1}e^{\hat{\boldsymbol{\xi}}_2 q_2}\ldots e^{\hat{\boldsymbol{\xi}}_n q_n}e^{\hat{\boldsymbol{\xi}}_T}\\
&\overset{\text{Lemma 1-3}}{=} {}^{B}\boldsymbol{H}_0 Q(q_1)\,{}^{B}\boldsymbol{H}_0^{-1}e^{\hat{\boldsymbol{\xi}}_2 q_2}\ldots e^{\hat{\boldsymbol{\xi}}_n q_n}e^{\hat{\boldsymbol{\xi}}_T}\\
&\overset{(3)}{=} {}^{B}\boldsymbol{H}_0 Q(q_1)e^{{}^{B}\boldsymbol{H}_0^{-1}\hat{\boldsymbol{\xi}}_2\,{}^{B}\boldsymbol{H}_0 q_2}\,{}^{B}\boldsymbol{H}_0^{-1}\ldots e^{\hat{\boldsymbol{\xi}}_n q_n}e^{\hat{\boldsymbol{\xi}}_T}\\
&\overset{(3)}{=} {}^{B}\boldsymbol{H}_0 Q(q_1)e^{\hat{\boldsymbol{\xi}}'_2 q_2}\,{}^{B}\boldsymbol{H}_0^{-1}\ldots e^{\hat{\boldsymbol{\xi}}_n q_n}e^{\hat{\boldsymbol{\xi}}_T}\\
&\overset{\text{Lemma 1-3}}{=} {}^{B}\boldsymbol{H}_0 Q(q_1)\,{}^{0}\boldsymbol{H}_1 Q(q_2)\,{}^{0}\boldsymbol{H}_1^{-1}\,{}^{B}\boldsymbol{H}_0^{-1}\ldots e^{\hat{\boldsymbol{\xi}}_n q_n}e^{\hat{\boldsymbol{\xi}}_T}\\
&\;\vdots\\
&= {}^{B}\boldsymbol{H}_0 Q(q_1)\,{}^{0}\boldsymbol{H}_1 Q(q_2)\cdots{}^{n-2}\boldsymbol{H}_{n-1}Q(q_n)\,{}^{n-2}\boldsymbol{H}_{n-1}^{-1}\\
&\qquad\ldots{}^{0}\boldsymbol{H}_1^{-1}\,{}^{B}\boldsymbol{H}_0^{-1}e^{\hat{\boldsymbol{\xi}}_T}\\
&= {}^{B}\boldsymbol{H}_0 Q(q_1)\,{}^{0}\boldsymbol{H}_1 Q(q_2)\cdots{}^{n-2}\boldsymbol{H}_{n-1}Q(q_n)e^{\hat{\boldsymbol{\xi}}'_T}\\
&\overset{\text{Lemma 4}}{=} {}^{B}\boldsymbol{H}_0 Q(q_1)\,{}^{0}\boldsymbol{H}_1 Q(q_2)\cdots{}^{n-2}\boldsymbol{H}_{n-1}Q(q_n)\,{}^{n-1}\boldsymbol{H}_n\,{}^{n}\boldsymbol{H}_T
\end{aligned}
$$

Matlab Implementation

1. Convert norminal POE into D-H

```
POE_nominal =

     0     0     0     0     0     0     0
     0    -1    -1     0    -1     0     0
     1     0     0    -1     0    -1     0
     0     0     0   -50   -20   -50   250
     0     0     0   250     0   250    50
     0     0  -100     0  -250     0   -20

>> [DH_nominal, h_nominal, q_bar_nominal]= POE2DH_H(POE_nominal)

DH_nominal =                                      h_nominal =   q_bar_nominal =

         0         0         0         0             0             1
         0         0    1.5708         0             0             1
         0         0         0  100.0000             0             1
         0  -50.0000    1.5708  150.0000             0             1
    3.1416   20.0000    1.5708         0             0             1
    3.1416    0.0000    1.5708         0             0             1
         0         0    3.1416         0
   -0.0000         0       NaN       NaN
```

2. Convert actual POE into D-H

```
POE_actual =

    0.0400         0    0.1780    0.0620    0.0010    0.0950    0.0200
   -0.0200   -1.0000   -0.9840    0.0130   -1.0000    0.0310   -0.0100
    0.9990         0   -0.0010   -0.9980         0   -0.9950    0.0100
    0.0200   -0.0200   -0.0700  -51.0000  -20.6000  -51.0000  249.0000
    0.0400         0    0.0090  249.0000   -0.0206  249.0000   51.0000
         0    0.0500 -101.0000    0.0752 -249.0000         0  -20.6000

>> [DH_actual, h_actual, q_bar_actual]= POE2DH_H(POE_actual)

DH_actual =                                        h_actual =    q_bar_actual =

    1.1071    1.0000    0.0447         0               0         1.0000
    2.0340   -1.0214   -1.5508    0.0092               0         1.0000
   -1.5364 -567.6554    0.1790    0.5114          0.0797         1.0000
   -1.5019  552.7518    1.5716  153.3035         -0.0000         1.0000
   -2.9640   30.1922    1.5837    1.8365          0.0000         1.0000
   -3.1084   -0.7927    1.6017    3.1761          2.8740         1.0000
    1.4452  -37.0573    3.0556   -0.7202
    1.4519  -35.4328       NaN       NaN
```

Inferences

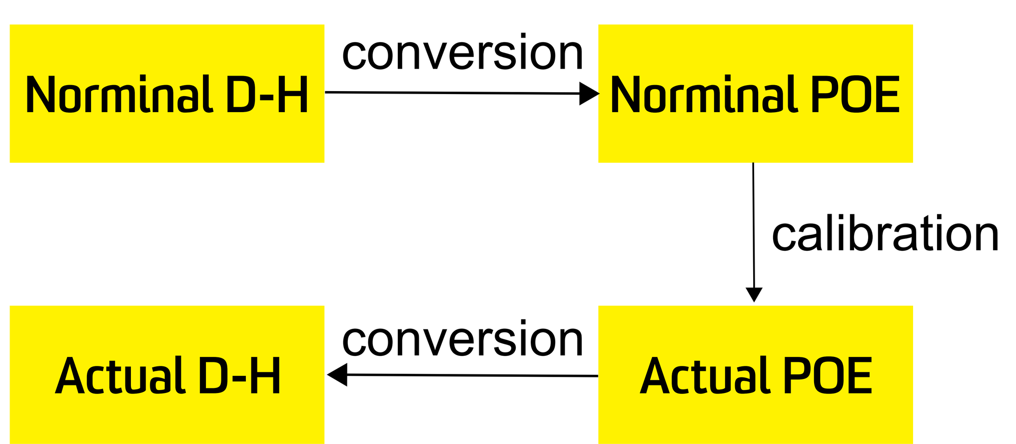
1. For an inaccurate n-link robot, the identifiability of the kinematic parameters can be given as

$$C_3 = \underbrace{5h+4r+2t}_{\boldsymbol{\xi}_i} + \underbrace{n}_{\bar{q}_i} + \underbrace{6}_{\boldsymbol{\xi}_T}$$

When excluding helical motion and error of joint position reading, it degrades to the widely agreed expression

$$C_2 = 4r + 2t + 6.$$

2. The identifiability of the base frame and the tool frame in the D-H model is restricted rather than the arbitrary 6 parameters as assumed previously.
Specifically in the shown case, the identifiable six parameters are the four with the base and the two with the tool.
3. Small variation of the robot structure may not cause large variation of POE parameters, but could result in drastic change in D-H model. Therefore, when calibration is needed and the D-H model has to be taken for some reason, a possible approach could be



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