

Differential equations: the pendulum

Solve ordinary differential equation for a simple harmonic oscillating system using 3 methods.

Equation of a simple pendulum: $d^2\theta/dt^2 = \ddot{\theta} = -g/L \sin(\theta)$.

Methods for solving equation:

-Euler

$$\theta(t + \delta) = \theta(t) + \dot{\theta}(t) \delta,$$

$$\dot{\theta}(t + \delta) = \dot{\theta}(t) + \ddot{\theta}(t) \delta$$

- Midpoint

$$\dot{\theta}(t + \delta) = \dot{\theta}(t) + \ddot{\theta}(t) \delta,$$

$$\theta(t + \delta) = \theta(t) + \dot{\theta}(t + \delta) \delta$$

-Verlet

$$\dot{\theta}(t + \delta/2) = \dot{\theta}(t) + 1/2 \ddot{\theta}(t) \delta,$$

$$\theta(t + \delta) = \theta(t) + \dot{\theta}(t + \delta/2) \delta,$$

$$\dot{\theta}(t + \delta) = \dot{\theta}(t + \delta/2) + 1/2 \ddot{\theta}(t + \delta) \delta.$$

Plot the pendulum trajectory $\theta(t)$ for 3 time steps $\delta = 0.1$, 0.01 , and 0.001 . Zoom in on the curve at one of the coarse points (say, $t = 1$) and visually compare the values from the three time steps.

Initial conditions: $\theta_0 = 2\pi/3$, $\dot{\theta} = 0$, $g = 9.8 \text{ m/s}^2$ and $L = 1 \text{ m}$.

All graphs should have a title, axis description and be saved to a file.

Example solution graphs are shown in Figure 1.

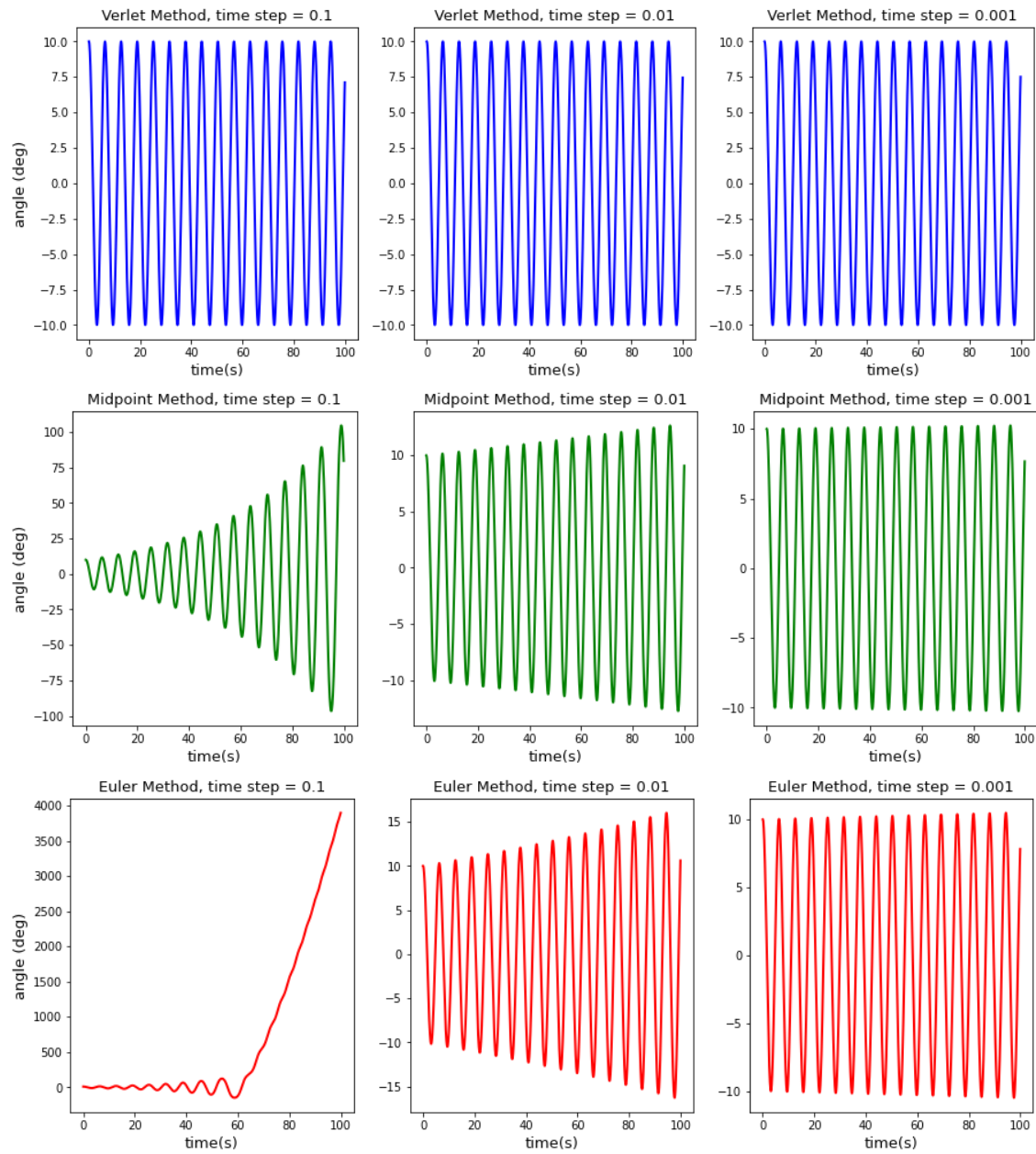


Figure 1. Example solution graphs.