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Source: *Publications of the Astronomical Society of the Pacific*, Vol. 124, No. 913 (March 2012), pp. 274-279

Published by: [The University of Chicago Press](#) on behalf of the [Astronomical Society of the Pacific](#)

Stable URL: <http://www.jstor.org/stable/10.1086/664796>

Accessed: 13/05/2014 09:31

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Can Self-Organizing Maps Accurately Predict Photometric Redshifts?

M. J. WAY^{1,2,3} AND C. D. KLOSE⁴

Received 2011 October 27; accepted 2012 January 17; published 2012 February 15

ABSTRACT. We present an unsupervised machine-learning approach that can be employed for estimating photometric redshifts. The proposed method is based on a vector quantization called the self-organizing-map (SOM) approach. A variety of photometrically derived input values were utilized from the Sloan Digital Sky Survey’s main galaxy sample, luminous red galaxy, and quasar samples, along with the PHAT0 data set from the Photo- z Accuracy Testing project. Regression results obtained with this new approach were evaluated in terms of root-mean-square error (RMSE) to estimate the accuracy of the photometric redshift estimates. The results demonstrate competitive RMSE and outlier percentages when compared with several other popular approaches, such as artificial neural networks and Gaussian process regression. SOM RMSE results (using $\Delta z = z_{\text{phot}} - z_{\text{spec}}$) are 0.023 for the main galaxy sample, 0.027 for the luminous red galaxy sample, 0.418 for quasars, and 0.022 for PHAT0 synthetic data. The results demonstrate that there are nonunique solutions for estimating SOM RMSEs. Further research is needed in order to find more robust estimation techniques using SOMs, but the results herein are a positive indication of their capabilities when compared with other well-known methods.

1. INTRODUCTION

There is a pressing need for accurate estimates of galaxy photometric redshifts (photo- z ’s), as demonstrated by the increasing number of articles on this topic, and especially by recent attempts to objectively compare methods (e.g., Hildebrandt et al. 2010; Abdalla et al. 2011). The need for photo- z ’s will only increase as larger and deeper surveys such as the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS; Kaiser 2004), Large Synoptic Survey Telescope (LSST; Ivezić et al. 2008), and Euclid (Sorba & Sawicki 2011) come online in the coming decade. The photometric-only surveys (Pan-STARRS and LSST) will have relatively small numbers of follow-up spectroscopic redshifts and will rely upon either template-fitting methods such as Bayesian photo- z ’s (Benítez 2000), Le Phare (Ilbert et al. 2006), or training-set methods such as those discussed herein. The Euclid mission may include a slitless spectrograph offering far more training-set galaxies.

A diverse set of regression techniques using training-set methods have been applied to the problem of estimating photometric redshifts in the past 10 years. These include artificial neural networks (Firth et al. 2003; Tagliaferri et al. 2003; Ball et al. 2004; Collister & Lahav 2004; Vanzella et al. 2004), decision trees (Suchkov et al. 2005), Gaussian process regression

(Way & Srivastava 2006; Foster et al. 2009; Way et al. 2009; Bonfield et al. 2010; Way 2011), support vector machines (Wadadekar 2005), ensemble modeling (Way et al. 2009), random forests (Carliles et al. 2008), and k D trees (Csabai et al. 2003), to name but a few.

On the other hand, even though self-organizing maps (SOMs) have been used extensively in a number of other scientific fields (the article that opened the field, Kohonen [1982], currently has over 2000 citations), they have been used sparingly thus far in astronomy (e.g., Mahdi 2011; Naim et al. 1997; Way et al. 2011) and only last year in estimating photometric redshifts (Geach 2011).

In this work we attempt to use SOMs to estimate photometric redshifts for several catalogs of different galaxy types, derived from the Sloan Digital Sky Survey (SDSS; York et al. 2000), including quasars along with the PHAT0 data set of Hildebrandt et al. (2010). In § 2 we describe the input data sets used, in § 3 we give an overview of SOMs, and we give some conclusions in § 4.

2. DATA

Three different data sets derived from the SDSS Data Release 7 (DR7; Abazajian et al. 2009) were used. They include the main galaxy sample (MGS; Strauss et al. 2002) the luminous red galaxy sample (LRG; Eisenstein et al. 2001), and the QSO sample (QSO; Schneider et al. 2007). Data from the Galaxy Zoo⁵ (Lintott et al. 2008) Data Release 1 (Lintott et al. 2011) survey results were used to segregate galaxies as spiral or elliptical in

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⁵ See <http://www.galaxyzoo.org>.

TABLE 1
RESULTS

Data ^a	Method ^b	50%	σ_{RMSE}^c 10%	90%	Outlier ^d
(1)	(2)	(3)	(4)	(5)	(6)
MGS (455803)	GPR	0.02087	0.02072	0.02096	0.11629
MGS (455803)	ANNz	0.02044	0.14482
MGS (455803)	SOM	0.02339	0.1689
MGS (455803)	Linear	0.02742	0.02729	0.02758	0.35986
MGS (455803)	Quadratic	0.02494	0.02412	0.02762	0.29184
LRG (143221)	GPR	0.02278	0.02256	0.02309	0.41898
LRG (143221)	ANNz	0.02138	0.41176
LRG (143221)	SOM	0.02689	0.64292
LRG (143221)	Linear	0.02896	0.02896	0.02897	0.71516
LRG (143221)	Quadratic	0.02382	0.02376	0.02402	0.45510
MGS-ELL (45521)	GPR	0.01455	0.01434	0.01473	0.06591
MGS-ELL (45521)	ANNz	0.01442	0.06591
MGS-ELL (45521)	SOM	0.02044	0.10984
MGS-ELL (45521)	Linear	0.01745	0.01731	0.01756	0.19772
MGS-ELL (45521)	Quadratic	0.01612	0.01609	0.01629	0.10984
MGS-SP (120266)	GPR	0.02078	0.02061	0.02093	0.13305
MGS-SP (120266)	ANNz	0.01991	0.05821
MGS-SP (120266)	SOM	0.02426	0.04158
MGS-SP (120266)	Linear	0.02539	0.02529	0.02555	0.28272
MGS-SP (120266)	Quadratic	0.02326	0.02296	0.02607	0.20788
LRG-SP (13708)	GPR	0.01416	0.01397	0.01436	0.00000
LRG-SP (13708)	ANNz	0.01516	0.00000
LRG-SP (13708)	SOM	0.01848	0.07299
LRG-SP (13708)	Linear	0.01635	0.01627	0.01649	0.07299
LRG-SP (13708)	Quadratic	0.01469	0.01462	0.01477	0.00000
LRG-ELL (27378)	GPR	0.01186	0.01162	0.01224	0.00000
LRG-ELL (27378)	ANNz	0.01298	0.10961
LRG-ELL (27378)	SOM	0.01568	0.00000
LRG-ELL (27378)	Linear	0.01362	0.01361	0.01364	0.10961
LRG-ELL (27378)	Quadratic	0.01263	0.01254	0.01274	0.07307
QSO (56923)	GPR	0.37342	0.03967	0.37626	50.96627
QSO (56923)	ANNz	0.65802	88.54533
QSO (56923)	SOM	0.41821	54.23401
QSO (56923)	Linear	0.57061	0.57010	0.57102	84.64512
QSO (56923)	Quadratic	0.53972	0.53679	0.54539	81.27196
phat0 (169520)	GPR	0.01487	0.01436	0.01532	0.03539
phat0 (169520)	ANN	0.01805	0.05309
phat0 (169520)	SOM	0.02236	0.37754
phat0 (169520)	Linear	0.08703	0.08702	0.08704	19.34875
phat0 (169520)	Quadratic	0.02436	0.02433	0.02438	0.19467

^a MGS is the main galaxy sample (Strauss et al. 2002), LRG are luminous red galaxies (Eisenstein et al. 2001), SP is classified as spiral by Galaxy Zoo, ELL is classified as elliptical by Galaxy Zoo, and QSO is the quasar sample (Schneider et al. 2007)

^b GPR is Gaussian process regression (Foster et al. 2009), ANNz is artificial neural network code (Collister & Lahav 2004), SOM is a self-organizing map (SOM-4100 and SOM-5100; see Fig. 2 for details), and phat0 is a PHAT (photo- z accuracy testing) synthetic sample.

^c We quote the bootstrapped 50%, 10%, and 90% confidence levels as in Way et al. (2009) for the RMSE when available.

^d Percentage of points defined as outliers. Following a prescription similar to that of Hildebrandt et al. (2010), we quote the percentage of points outside of $\Delta z = z_{\text{phot}} - z_{\text{spec}} \pm 0.1$.

the case of the MGS and LRG. Way (2011) outlines how this was accomplished, while the visualization software Viewpoints

(Gazis et al. 2011) was also relied upon. Dereddened magnitudes (u , g , r , i , and z) were used as inputs in all scenarios. The same

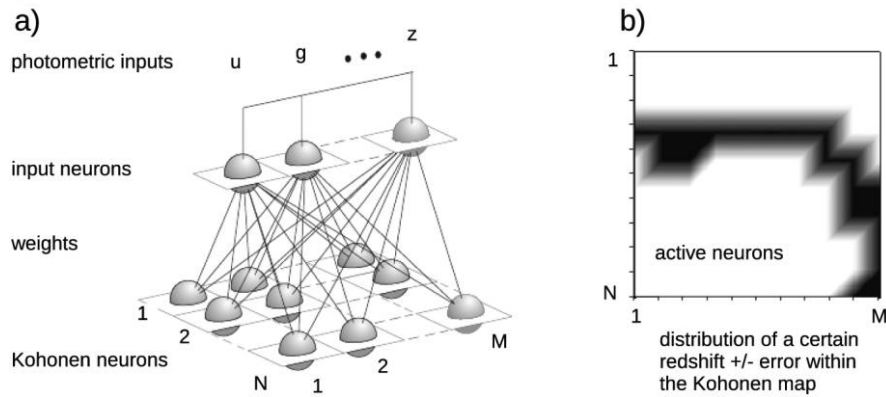


FIG. 1.—Schematic illustration of the (a) structure and (b) functionality of a SOM with I input neurons and $M \times N$ Kohonen neurons. The SOM visualizes the structure of the I -dimensional input space. In this case, the SOM illuminates a certain redshift \pm error within the Kohonen map and as a function of the input space.

SDSS photometric and redshift quality flags on the input variables were used as in Way (2011). In addition, we used the simulation-based PHAT0 data set (see Hildebrandt et al. 2010), which was constructed to test a variety of different photo- z estimation methods. The PHAT0 data set consists of five SDSS-like filters (u , g , r , i , and z) used on MegaCam at Canada-France-Hawaii Telescope (Boulade et al. 2003), with an additional

six input filters (Y , J , H , K , *Spitzer* IRAC [3.6], and *Spitzer* IRAC [4.5]), giving a total of 11 filters spanning a range of 4000 Å to 50,000 Å. This large range should help to avoid color-redshift degeneracies that can occur if ultraviolet or infrared bandpasses are not used (Benítez 2000). The PHAT0 synthetic photometry was created from the Le Phare photo- z code (Arnouts et al. 2002; Ilbert et al. 2006). Initially, Le Phare creates noise-free data, but given the desire to test more real-world conditions, we utilized the PHAT0 data with added noise. A parametric form was used for the signal-to-noise ratio as a function of magnitude, where it acts as an exponential at fainter magnitudes and as a power law at brighter ones. The magnitude cut between these two regimes is filter-dependent and is given in Table 2 of Hildebrandt et al. (2010). The larger of the two catalogs was used herein (as suggested for training-set methods), which contains $\sim 170,000$ objects.

Since we use a training-set method, our original data sets are split into 89% training, 10% testing, and 1% validation. Validation was only used in the artificial neural network algorithm, discussed in the next section. The full size of each input data set is listed in parentheses in column (1) of Table 1.

3. METHODS

Several methods in use for calculating photometric redshifts were compared with the SOM results: the artificial neural network (ANNz) code of Collister & Lahav (2004), the Gaussian process regression (GPR) code of Foster et al. (2009), and simple linear and quadratic regression. The latter is comparable with that of the polynomial fits used by Li & Yee (2008). Both the ANNz and GPR codes are freely downloadable.⁶ Details on the ANNz and GPR algorithms can be found in their respective preceding citations.

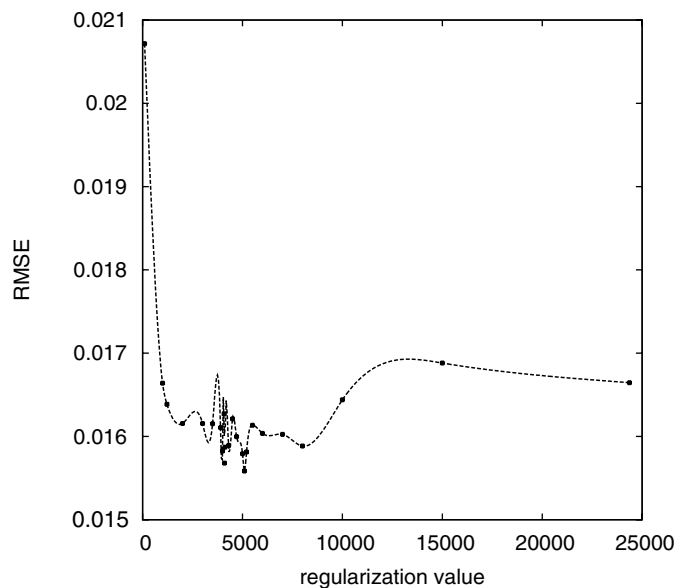


FIG. 2.—Accuracy (RMSE) vs. regularization parameter value ξ for the LRG-ELL data set (see Table 1). Different classifications will result from different choices of the ξ value. The regularization value is defined by the number of Kohonen neurons, which is maximum with respect to the training data set. The convex curve has a two local minima at $\xi = 4100$ and $\xi = 5100$. The roughness of this RMSE cost function shows that traditional gradient-based optimization strategies, e.g., deterministic annealing, might result in suboptimal solutions. Other methods, such as genetic programming, might find the global minimum much faster.

⁶ GPR: <http://dashlink.arc.nasa.gov/algorithm/stableGP> and ANNz: <http://www.star.ucl.ac.uk/~lahav/annz.html>

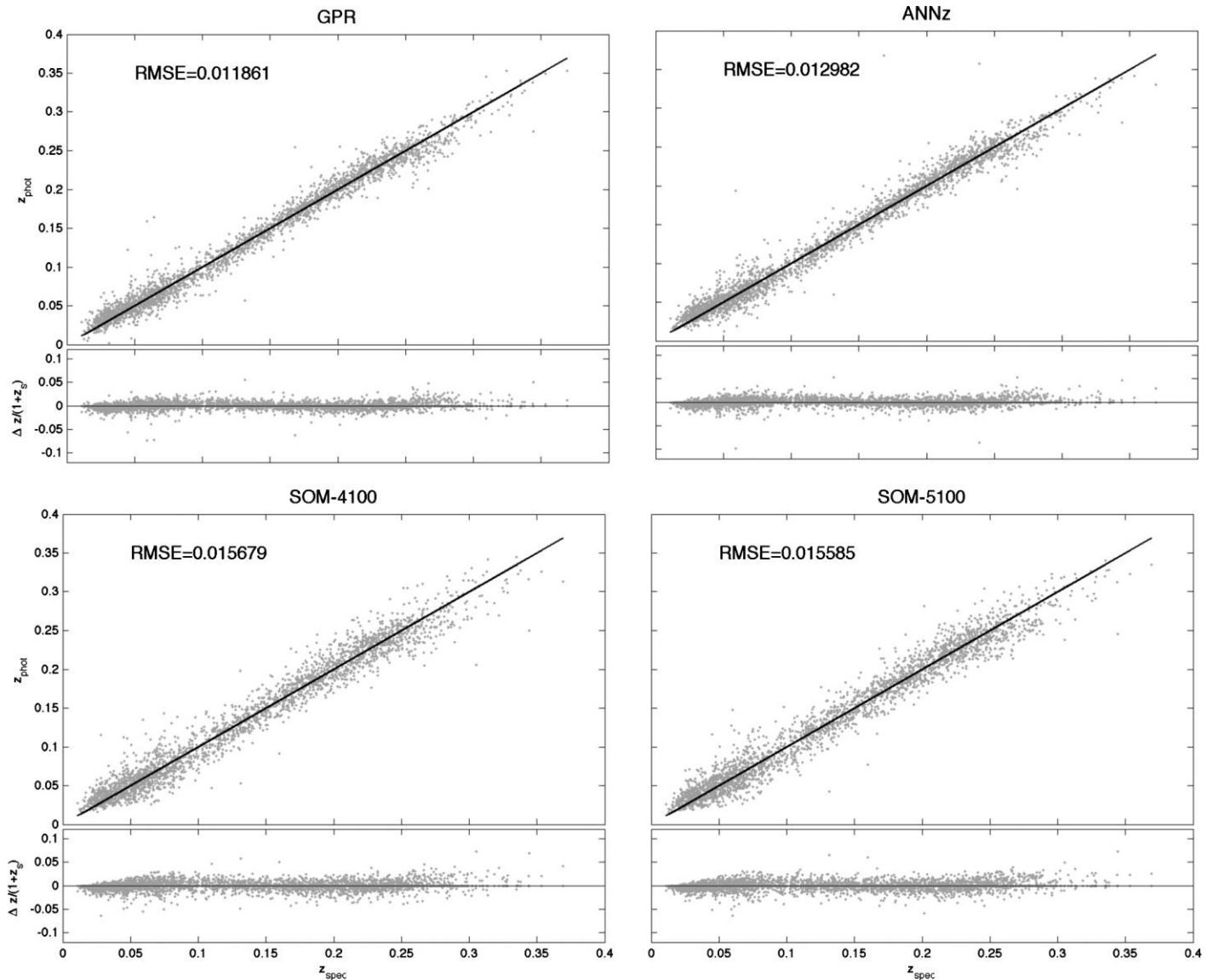


FIG. 3.—Results from the three methods using SDSS *ugriz* dereddened magnitudes as inputs for the SDSS DR7 luminous red galaxies classified as ellipticals by the Galaxy Zoo team. The bottom two plots show the SOM results for the two local minima described in § 3 and shown in Fig. 2.

The main purpose of self-organized mapping is the ability of SOMs to transform a feature vector of arbitrary dimension drawn from the given feature space of photometric inputs (e.g., the SDSS *u*, *g*, *r*, *i*, and *z* magnitudes) into simplified one- or two-dimensional discrete maps. The method was originally developed by Kohonen (1982, 2001) to organize information in a logical manner. This type of machine learning utilizes an unsupervised learning scheme of vector quantization, known as competitive learning, in the field of neural information processing. It is useful for analyzing complex data with a priori unknown relationships that are visualized by the self-organization process (Kohonen 2001).

A SOM is structured in two layers: an input layer and a Kohonen layer (Fig. 1). For example, the Kohonen layer could

represent a structure with a single two-dimensional map (lattice) consisting of neurons arranged in rows and columns. Each neuron of this discrete lattice is fixed and is fully connected with all source neurons in the input layer. For the given task of estimating photometric redshifts, a five-dimensional feature vector of the *u*, *g*, *r*, *i*, and *z* magnitudes is defined. One feature vector (*u*, *g*, *r*, *i*, and *z*) is presented to five input-layer neurons. This typically activates (stimulates) one neuron in the Kohonen layer. Learning occurs during the self-organizing procedure as feature vectors drawn from a training data set are presented to the input layer of the SOM network (Fig. 1a). These feature vectors are also referred to as input vectors. Neurons of the Kohonen layer compete to see which neuron will be activated by the weight vectors that connect the input neurons and Kohonen neurons.

In other words, the weight vectors identify which input vector can be represented by a single Kohonen neuron. Hence, they are used to determine only one activated neuron in the Kohonen layer, after the winner-takes-all principle (Fig. 1b).

The SOM is considered as being trained after learning, at which time the weights of the neurons have stored the interrelations of all five-dimensional u , g , r , i , and z feature vectors. Then, known spectroscopic redshift values for all input vectors of a test data set that are represented by a single Kohonen neuron are averaged (Fig. 1b). The redshift mean value represents all 5D u , g , r , i , and z vectors that are similar to the weight vector of the activated Kohonen neuron. The more Kohonen neurons there are, the more precisely each input vector can be represented by a weight vector. However, the total number of Kohonen neurons are optimized for each data set (see Fig. 2). A practical overview about the learning/training process is described by Klose (2006) and Klose et al. (2008, 2010) and in much greater detail by Kohonen (2001).

After training, the u , g , r , i , and z input vectors of a test data set are presented to a trained SOM. At the end of a classification step, every Kohonen neuron approximates an input vector whereby similar/dissimilar input data were represented by neighboring/distant neurons. One neuron could even classify several input vectors if these input vectors were very similar compared with the other given input vectors. Results from the photometric redshift approximations are then compared with known spectroscopic redshift data. Regression performance is estimated based on the root-mean-square error (RMSE) of the predicted photometric redshifts and the known spectroscopic redshifts (using $\Delta z = z_{\text{phot}} - z_{\text{spec}}$). To reiterate, during the training phase, each Kohonen neuron identifies a certain number of galaxies that are characterized by similar u , g , r , i , and z intensities. Photometric redshift data were then averaged for these intensity values.

The SOM approximates the input feature space and maps it into an output space. The output space shows the SOM approximation as a 2D map (Haykin 2009). Best results can be obtained with an optimization scheme such that the RMSE of the test data set is minimal, as illustrated in Figure 2. Accuracy (e.g., RMSE) depends on the size of the Kohonen map. The number of neurons in the Kohonen map can be considered a regularization parameter (ξ), as shown in Figure 2.

Figure 2 shows that RMSE is high when the number of Kohonen neurons is too small ($\xi < 2000$) or too large ($\xi > 10,000$) and, hence, that the five-dimensional u , g , r , i , and z input space is underfit or overfit. Theoretically, a global minimum of the RMSE curve might exist. However, the input feature space for the given photometric redshift problem shows a very rough RMSE curve (Fig. 2), with at least two local minima. In this case, it is clear that SDSS redshift estimation tends to have several local minima, which makes it important to choose the right optimization method to determine the SOM network size. On the other hand, the smoother the RMSE curve is, the

better gradient methods can be utilized. Evolution strategies or genetic programming could be applied to rougher curves with many local minima. This in turn can make it cumbersome to find fast back-propagation artificial neural network (ANN) structures, especially when data sets are small.

Another advantage of SOMs in comparison with ANNs is that there is no need to optimize the structure of SOMs (e.g., number of hidden layers), since it is based on unsupervised learning.

Only the size of the Kohonen map needs to be optimized for each data set. SOMs also allow nonexperts to visualize the redshift estimates in relation to the multidimensional input space. This eliminates the often criticized “black box” problem of ANNs. As mentioned previously, SOMs approximate the input feature space, while ANNs typically separate them into subregions. Finally, SOMs are known to be powerful when very small data sets are available for training (see Haykin 2009).

4. CONCLUSION

SOMs offer a competitive choice in terms of low-RMSE, algorithm comprehension (also see Göppert & Rosenstiel 1993), and percentage of outliers. The final results are presented in Table 1, and plots for the LRG-ELL data set for the SOM, ANNz, and GPR methods are shown in Figure 3

As mentioned previously, obtaining the global minimum is important and, not surprisingly, can affect the results. Figure 2 shows the two local minima ($\xi = 4100$ and 5100) listed for the LRG-ELL (luminous red galaxies classified as ellipticals by Galaxy Zoo) data set in Table 1. Clearly, there are a number of other ξ -values and the RMSE will be greatly affected by the choice, as shown on the y -axis of Figure 2 for a given ξ -value. Given these facts, the roughness of the RMSE cost function in Figure 2 shows that traditional gradient-based optimization strategies, e.g., deterministic annealing, might yield suboptimal solutions. Other methods, such as genetic programming, might find the “global” minimum much faster if a global minimum exists with respect to the uncertainties of the RMSE.

During completion of this article, another article using SOMs for classification and photometric estimation was released (Geach 2011). Our work differs in that we mostly focus on a wider variety of low-redshift samples drawn from the SDSS, while Geach (2011) focuses more on the higher-redshift samples, akin to those used in Hildebrandt et al. (2010).

We have shown that SOMs are a powerful tool for estimating photometric redshifts and that, with additional work, they are sure to be useful in future surveys with limited numbers of follow-up spectroscopic redshifts.

M. J. W. would like to thank the Astrophysics Department at Uppsala University for their generous hospitality while part of this work was completed. C. D. K. thanks Think Geohazards for providing the computational resources needed for estimating photometric redshifts via self-organizing mapping. Thanks go to Joe Bredekamp and the NASA Applied Information Systems

Research Program for support and encouragement. Funding for the Sloan Digital Sky Survey (SDSS) has been provided by the Alfred P. Sloan Foundation, the participating institutions, the National Aeronautics and Space Administration, the National Science Foundation, the US Department of Energy, the Japanese Monbukagakusho, and the Max Planck Society. The SDSS is managed by the Astrophysical Research Consortium for the participating institutions. The participating institutions are The

University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, The Johns Hopkins University, Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy, the Max-Planck-Institute for Astrophysics, New Mexico State University, University of Pittsburgh, Princeton University, the US Naval Observatory, and the University of Washington. This research has made use of NASA's Astrophysics Data System Bibliographic Services.

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