Music in Theory and Practice

CHAPTER 1: INTRODUCTION

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Welcome to your first piano lesson! Throughout this chapter, we will start to explore the world of music from both theoretical and practical aspects. The piano is the best instrument for one to quickly grasp the theoretical concepts, because modern music theory is based heavily on the piano keyboard.

LEARNING OBJECTIVES

- Pitch
- Identify keys on the piano
- Finger numbers
- Reading notes from the staff
- Ledger lines
- The treble and bass clefs
- Bars and Bar lines, the time signatures $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$
- Semibreves, minims and crotchets, dotted notes

1.1 Introduction to Pitch

The most fundamental principle in music is that of the **octave equivalence**. We will introduce it below, but first we give some historic background about the piano.

The term *piano* is an abbreviation for *pianoforte*, a portmanteau of the terms *piano* and *forte* coming from the Italian for *soft* and *loud* respectively. It is named as such for the simple fact that the harder you press a key, the louder the sound—this is not to be taken for granted—for it was the first keyboard instrument which had this quality (predecessors include the pipe organ, harpsichord and the clavichord). In fact, a harpsichord often has two *manuals* (i.e. two identical keyboards, one above of the other) where one of them plays notes louder than the other, as this is the only way the instrument can produce differences in volume.

The inventor of the piano or as he called it, the *gravicembalo col piano e forte* (soft and loud keyboard instrument), Bartolomeo Cristofori, achieved this breakthrough by the invention of what is known as the **piano action** in 1709. This is fairly recent, considering that music for the keyboard predates the seventh century. Great composers such as Bach never laid eyes on a piano, and the later composer Mozart played on the *fortepiano* (an earlier version of the pianoforte). Music which brings out the potential of the modern piano we know today was first written by Ludwig van Beethoven, a composer who lived shortly after Mozart.

The piano action for the modern upright piano (which has long since been improved since Cristofori's day) can be seen in figure 1.

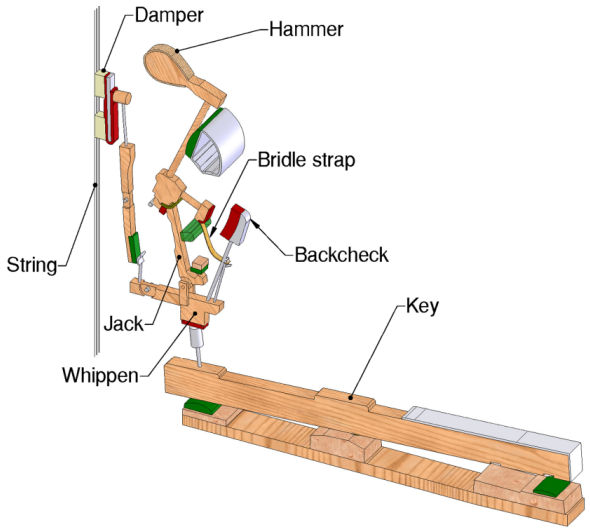


FIGURE 1: Modern upright piano action

When the pianist strikes a key, the action inside the piano pushes the corresponding hammer forwards (or upwards in the case of a grand piano) to strike the strings. A piano hammer is a round wooden stick with soft felt around the head; its resemblance to a hammer gives it its name. When a string vibrates, it resonates a musical sound, and the soundboard the string is attached to amplifies this sound much more than the string could solely. When the key is released, a felt pad called the damper drops on the string to stop it from resonating. The rightmost pedal on the piano, called the **sustain pedal**, keeps the dampers raised so that notes continue to resonate after keys are released.

There are three characteristics which vary between strings (i.e. from one note to another): the *length* of the string (the shorter the wire, the higher the pitch) *mass* per unit length (the thinner the wire, the higher the pitch) and *tension* (the tighter the wire, the higher the pitch).

It is by exploiting these three factors that the string can be modified to the correct pitch. We can delve more into the physics[1] behind this (Pythagorean tuning and so on), but it does not concern us.

Every note has a **pitch**, which is defined by the *frequency* of the sound wave produced when the string is hit. If a note has frequency f, then twice the frequency, $2f$, is said to be of the same **pitch class** as the other note. This is denoted

$$f \simeq 2f$$

Now in Western music, we subdivide each interval between f and $2f$ into 8 tones (this number is arbitrary), each denoted by A, B, C, D, E, F and G (sometimes we refer to them using *solfége*, i.e. do, re, mi, fa, so, la, and ti instead). These are the white keys on the piano. In fact, you may have noticed that the keys on the piano have a pattern which repeats after every 8 white keys, and this is the reason for it. The note after G is another A, but it sounds higher because it is exactly twice the frequency of the previous A. We say that it is of the same pitch class, but it differs in **timbre** (octave and tone colour, think of the difference between a flute and an organ).

So if we denote the first A on the piano (this is in fact the first key on the piano) by A_0 (with frequency of 27.5 Hz), we can move up 8 white keys and we would have found A_1 (55 Hz), with double the frequency.

[1] If you are interested, refer to *Physics of the Piano* by Nicholas J. Gioardano (2010)

Similarly we can find A_2, then A_3, and so on, all the way up to A_7, which has a frequency of 2^7 times that of A_0 (3520 Hz).

Exercise. *Try and play two white keys on the piano which are 8 apart (or 12, if you count the black keys). We call this distance between notes an* ***octave****.*

Exercise (For the mathematically inclined)**.**

(i) Prove that $\simeq$ defines an equivalence relation on the set of notes

$$\mathbf{N} = \{A_0, B_0, C_1, D_1, E_1, \ldots, A_7, B_7, C_8\}.$$

(ii) Prove that the equivalence classes of the relation $\simeq$ are the notes of the same pitch class.

(iii) For $\ell \in \{A, B, C, D, E, F, G\}$, let $f(\ell_n)$ denote the frequency of the note ℓ_n. show that $k \geqslant 0$,

$$f(\ell_{n+k}) = 2^k f(\ell_n).$$

Hence, given that C_1 has a frequency of 132.7 Hz, deduce the frequency of C_5.

(iv) Show that the number of octaves between two notes of frequencies f_1 and f_2 (where $f_1 \leqslant f_2$) is given by

$$\left\lfloor \log_2 \left(\frac{f_2}{f_1} \right) \right\rfloor.$$

How many octaves are there between A_0 with frequency 27.5 Hz and C_2 with frequency 65.41?

1.2 The Piano Keyboard

The piano keyboard is perhaps the most important pedagogical tool in aiding us to understand music theory. Even though our aim is to finally play the piano, if we were to learn to play a different instrument, we would still be introduced to music theory with reference to the piano keyboard.

Almost all modern pianos have 88 keys, 52 white and 36 black, labelled A_0 to C_8. Only the first two white keys (A_0 and B_0) are labelled with the

subscript 0. So the notes by subscript are A_0 to B_0, then C_1 to B_1, then C_2 to B_2 and so on, up to C_7 to B_7 followed by a solitary C_8.

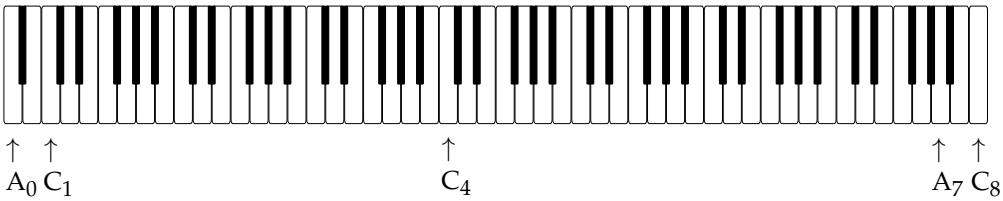


FIGURE 2: The keys of a modern piano keyboard

Notice how the black keys are grouped in twos and threes. These allow us to easily identify octaves, as well as individual notes. A C note is always to the left of the *two* black keys. The C note roughly in the centre of the keyboard is called the **middle C** (C_4).

Exercise. *Play all the 'C's you can find, using your right thumb.*

When reading music, you will come across numbers beneath certain notes. These guide us on which **fingering** to use, as illustrated in figure 3.

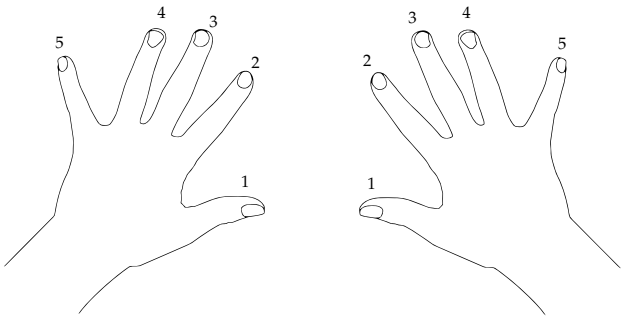


FIGURE 3: Finger numbering

The thumbs are both numbered 1; so if you see a note with a '1' beneath it, it means you should play it with your thumb.

Exercise. *Start from the middle C, and play C, D, E, F, G with fingers 1, 2, 3, 4 and 5 on the right hand.*

1.3 The Staff

Music is written on the **lines** and **spaces** of the **staff**.

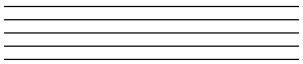


FIGURE 4: The staff

The staff is two dimensional. Up/down tells us the *pitch* of the note, left/right tells us when to play the note. We can think of it as a graph with pitch on the y-axis and time on the x-axis. Notes on the staff tell us which keys to play. There are different kinds of notes which represent different durations (i.e. how long to hold down the key), but first we will discuss the **semibreve**.

A semibreve looks like this: 𝅝 and is held for 4 counts (count 1 to 4, then release the key).

Every line and space on the staff represents a note, so we have 5 lines and 6 spaces, which allows us to represent 11 notes. But we have 88 notes on the piano! How can we represent all of them? Well, firstly, we 'cheat' by adding extra lines called **ledger lines**, as illustrated in figure 5,

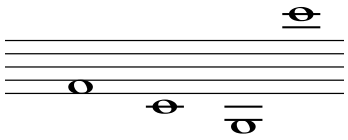


FIGURE 5: Ledger lines

so we can keep going up and down as we please. This partially solves the issue, however it is still difficult to represent all 88 keys this way without an excessive number of ledger lines. So we introduce the notion of **clefs**. A clef tells us where we are on the keyboard. Consider, starting from the middle C (C_4), the notes C, D, E.

Using the **treble clef**, 𝄞, these notes played in succession are denoted as in figure 6.

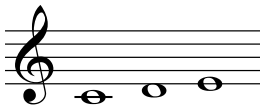


FIGURE 6: The notes C, D, E in the treble clef, starting from the middle C

On the other hand, using the **bass clef**, 𝄢, these very same three notes are denoted as in figure 7.

FIGURE 7: The notes C, D, E in the bass clef, starting from the middle C

Note that we are only concerned with representing white keys for now. The clefs allow us to shift our point of view up/down the keyboard as necessary. Note that if we wish to represent the C higher than middle C, it is convenient to use the treble, whereas for the C lower than middle C, it is better to use the bass.

FIGURE 8: The middle C (C_4), and the C above (C_5) and below (C_3) in both the treble and bass clefs.

1.4 Time Signatures, Bars and Bar Lines

As you've probably guessed, the piano is played with two hands. Therefore we use two staffs—one for the left hand, and the other for the right hand. Most often, the left hand plays lower notes than the right hand, so we use the bass clef for the left hand staff, and the treble clef for the right hand staff.

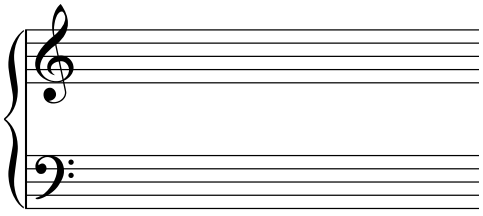


FIGURE 9: Right-hand and left-hand staffs

This is not always the case however. If we are playing high notes with both hands we might have two treble-staffs. Similarly, for low notes we might have two bass-staffs.

The **brace** (curly bracket) joining the two staffs shows that they are played at the same time. Otherwise, we'd assume it's simply a continuation from one line to the next. When reading sheet music for orchestral arrangements, we use one large brace to group all the instruments, thus showing they are playing at the same time.

Music is divided into **bars** using **bar lines**. We can think of bars as units of time, as every bar has the same duration.

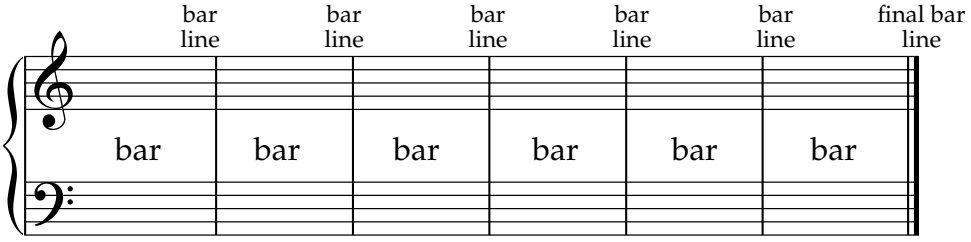


FIGURE 10: Bars and bar lines

The duration of each bar is given once at the start of a piece of music by the **time signature**. The time signature is a concept similar to a fraction in mathematics. In this chapter, we only concern ourselves with the top number. This tells us how many counts there are to each bar.

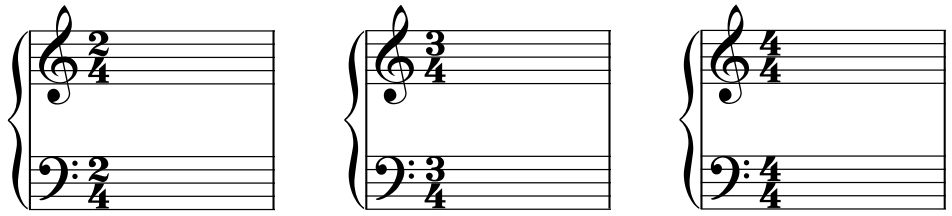
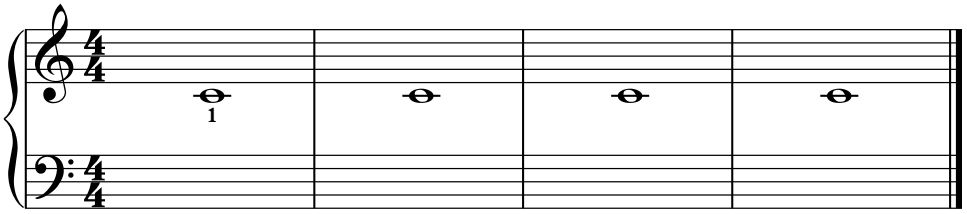


FIGURE 11: Different time signatures

1.5 A First Go on the Piano

Time for your debut. Play the following pieces on the piano, and make sure to observe all the points given.

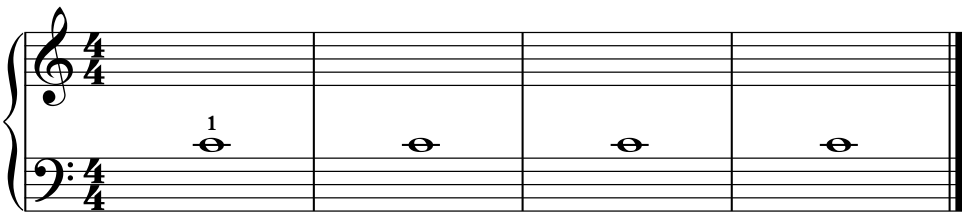
Piece 1: Church Bells in the Right Tower



- The note is the *middle C* in the treble clef.
- The note is in the *right hand staff*, so we play it with the right hand.
- The number '1' beneath the note is a fingering suggestion, telling us to play the note with our *right thumb*.
- Each note is a semibreve, so we count from *1 to 4* before releasing it.

Note that semibreves can only fit in bars whose time signature is $\frac{4}{4}$ (since it takes up the whole 4 counts in the bar).

Piece 2: Church Bells in the Left Tower

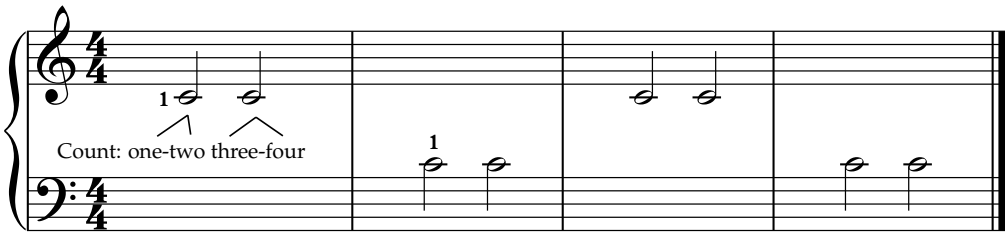


- The note is the *middle C* in the bass clef.
- The note is in the *left hand staff*, so we play it with the left hand.
- The number '1' above the note is a fingering suggestion, telling us to play the note with our *left thumb*.

1.6 The Minim and the Crotchet

The semibreve 𝅝 is the longest note which we tend to come across. We will now introduce two shorter notes: The **minim**, 𝅗𝅥 or 𝅗𝅥 is held for 2 counts; and the **crotchet**, ♩ or ♩ is held for 1 count.

Piece 3: Grandfather's Clock

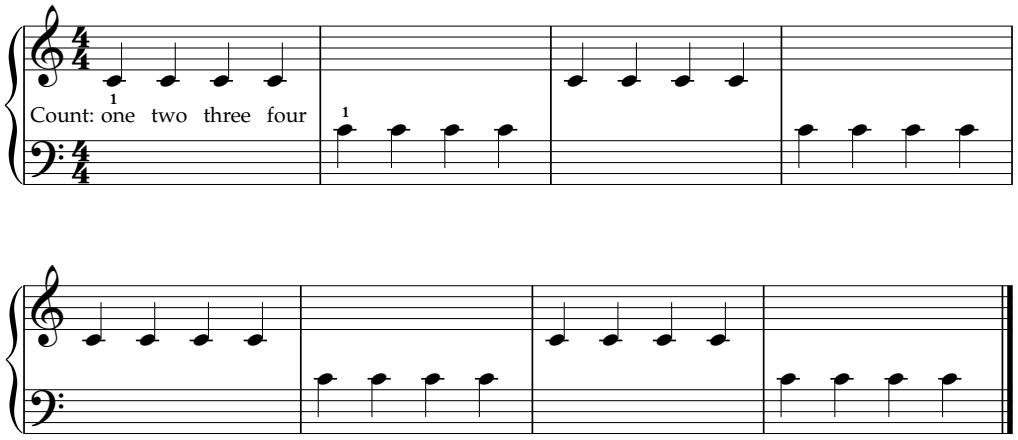


- The note is the *middle C* in both the treble and bass clefs.
- The notes alternate between staffs, so we play using both the right

and left hands.

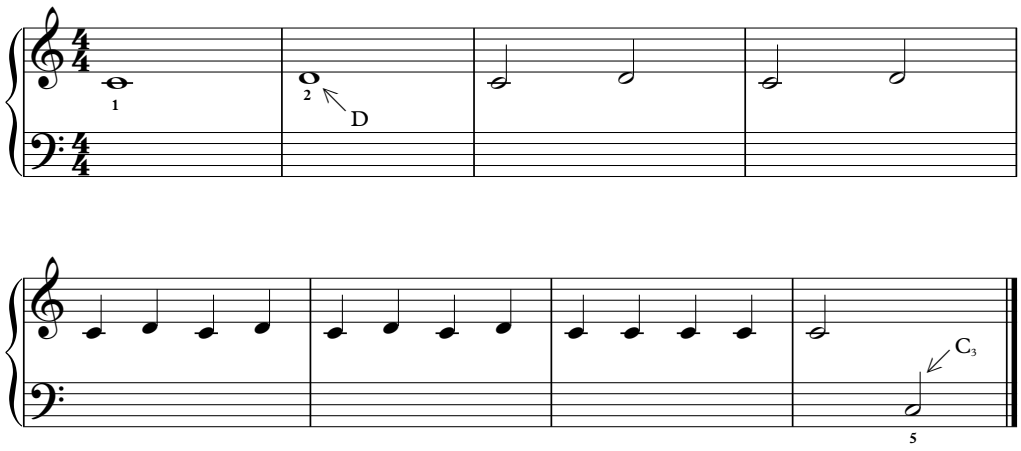
- The number '1' next to the note is a fingering suggestion, telling us to play the note with our *right/left thumb*.
- Each note is a minim, so we give it two counts before releasing it.

Piece 4: Mocassin Dance



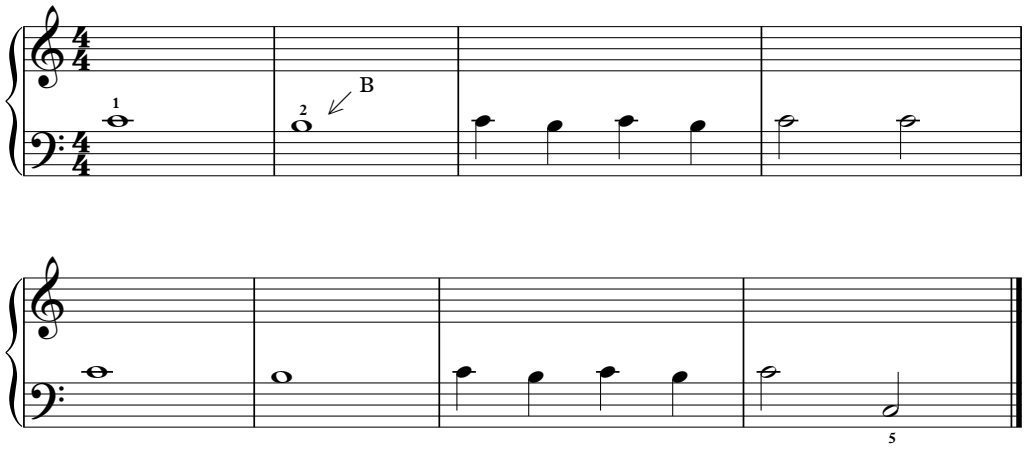
- The note is the *middle C* in both the treble and bass clefs.
- The notes alternate between staffs, so we play it with both the right and left hands.
- The number '1' next to the note is a fingering suggestion, telling us to play the note with our *right/left thumb*.
- Each note is a crotchet, so we give it one count before releasing it.

Piece 5: The Train



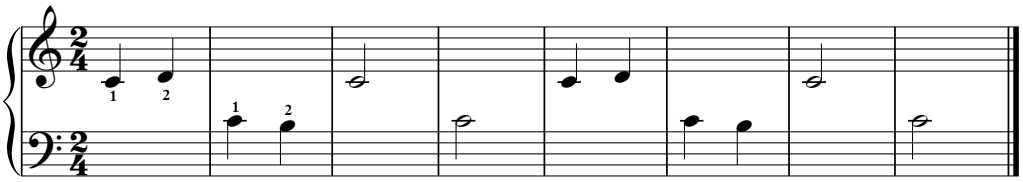
- The second note is *D above middle C* (the next white key) in the treble clef.
- The numbers '1' and '2' next to the notes are fingering suggestions, telling us to play the note with our *right thumb* and *index finger*.
- We have a combination of semibreves, minims and crotchets. Be sure to give each four, two and one counts respectively.
- The final note is the *C below middle C* (i.e. one octave below). We are told to play it with the *left little finger*.

Piece 6: The Seabees



- The second note is *B below middle C* (the previous white key) in the bass clef.
- The numbers '1' and '2' next to the notes are fingering suggestions, telling us to play the note with our *left thumb* and *index finger*.
- Just as in the previous piece, the final note is the *C below middle C* played with finger 5.

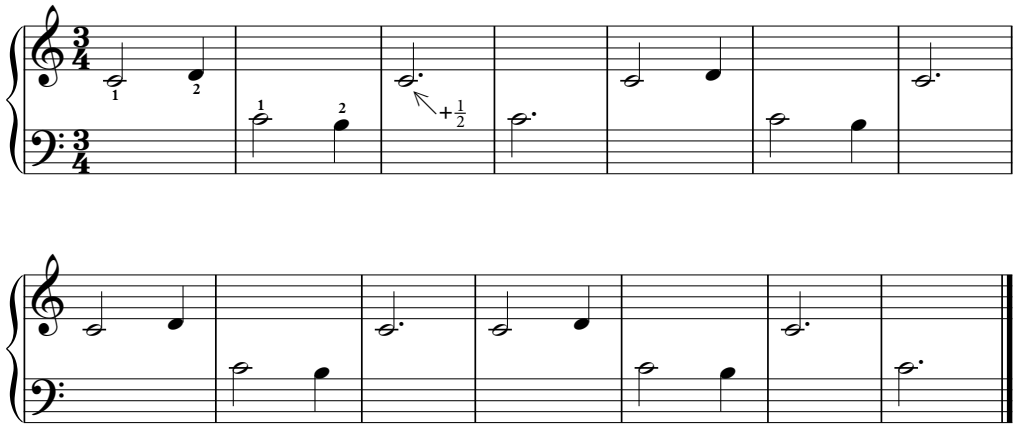
Piece 7: March of the Gnomes



- Note the time signature is now $\frac{2}{4}$, so each bar is given *two counts*.
  This time signature is typical for marching music—we can think of the beats in each bar as "left-foot, right-foot".
- The numbers '1' and '2' next to the notes are fingering suggestions, telling us to play the note with our *right/left thumb* and *index finger*.
- We have a combination of minims and crotchets. Be sure to give each two and one counts respectively.

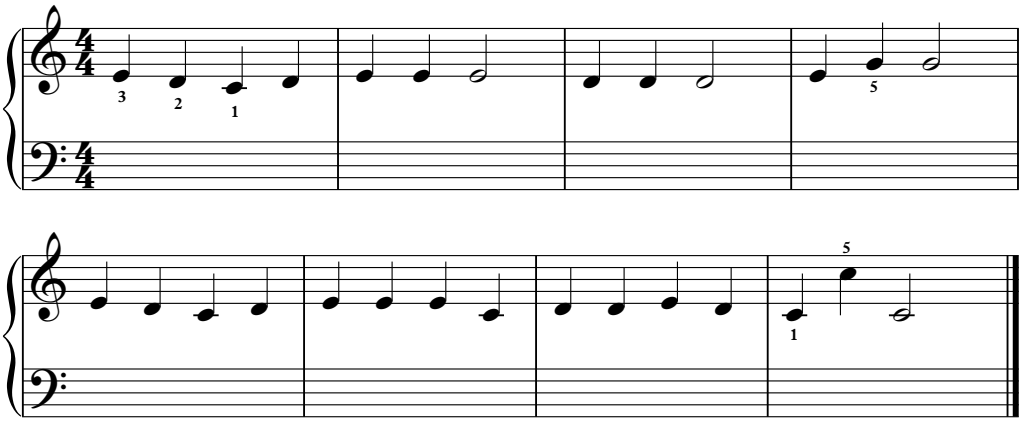
Piece 8: Dance of the Gnomes

Note: A **dotted note** is a note which is followed by a dot. This signifies adding half to the original value of the note. For example, a minim has two counts. Half of two is one, so a dotted minim has a total of three counts. Similarly, a dotted semibreve has six counts, and a dotted crotchet has one and a half counts.



- Note the time signature is now $\frac{3}{4}$, so each bar is given *three counts*. This time signature is typical for dancing music, particularly waltzes: we can think of the beats in each bar as "one, two, three" or "um, pa, pa".
- The numbers '1' and '2' next to the notes are fingering suggestions, telling us to play the note with our *right/left thumb* and *index finger*.
- We have a combination of dotted minims, minims and crotchets. Be sure to give each three, two and one counts respectively.

Piece 9: Mary Had a Little Lamb

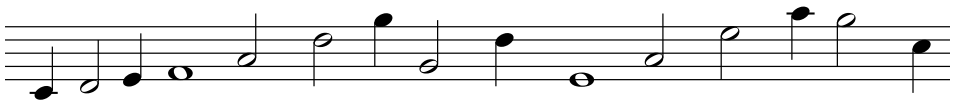


- We are playing notes E, D, middle C and G with fingers 3, 2, 1 and 5 in the right hand. The note before last is the C above middle C, played with finger 5.
- It is important to always keep counting to yourself when playing the notes. It is good to get into the habit of **accenting** (slightly stressing) the first note of each bar.

Additional Exercises

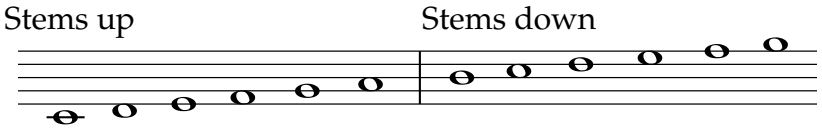
1. Define the following terms: pitch & pitch class, piano action, sustain pedal, timbre, octave, middle C, staff, semibreve, minim, crotchet, clef, time signature.

2. Write down the time value and names of the following notes.



   E.g. the first note: C, 1 count (crotchet)

3. Turn the following into minims by adding stems:



4. In the following extract, first add bar lines where they belong (note the time signature). Next write the name of the notes, and finally mark their time values (as in question 2).