Energy-Efficient Transmission in Frequency-Selective Channels*

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***Abstract*—Energy efficiency is becoming increasingly important for small form factor mobile devices, as battery technology has not kept up with the growing requirements stemming from ubiquitous multimedia applications. This paper addresses link adaptive transmission for maximization of energy efficiency rather than throughput. We extend our previous results for flat fading OFDMA to frequency-selective channels. Different from existing water-filling power allocation schemes that maximize throughput subject to overall transmit power constraints, our scheme adapts both overall transmit power and its allocation according to the states of all subchannels and circuit power consumption to maximize energy efficiency. We demonstrate the existence of a unique globally optimal link adaptation solution and provide iterative algorithms to obtain this optimum. Simulation results show at least a 15% improvement in energy utilization when frequency selectivity is exploited.**

***Index Terms*–** energy efficiency, OFDM, bits per Joule, link adaptation, frequency-selective

I. Introduction

Due to fading of wireless channels, channel qualities differ as a function of time and frequency. Therefore, link adaptation can be used to improve transmission performance. With link adaptation, modulation order, coding rate, and transmit power can be selected according to *channel state information* (CSI). Earlier research on link adaptation focused on power allocation to improve channel capacity. Optimal power allocation for frequency-selective channels was implied when information theorists derived channel capacity [1]. Besides power allocation, throughput can be also improved if modulation order code date rate are changed according to channel quality and transmit power [2]–[5].

In addition to throughput improvement, energy efficiency is becoming increasingly important for mobile communications due to slow progress of battery technology [6] and fast growing requirements of anytime and anywhere multimedia applications. With limited battery capacity, link adaptation could be adapted toward energy conservation to minimize battery drain. Hence, recent research has focused on energy-efficient link adaptation techniques [7]–[11]. The minimum signal energy per bit for reliable communication is -1.59 dB [7] when the bandwidth tends to infinity for *additive white Gaussian noise* (AWGN) channels. For bandlimited transmission, lowest order modulation should be used [8]. This research assumes negligible circuit power. Energy dissipation of both transmitter circuits and radio frequency output is investigated in [9] and modulation level is adapted to minimize energy consumption according to simulation results. In [10], these ideas are extended to a detailed analysis of circuit and transmit powers for both adaptive *multiple quadrature amplitude modulation* (M-QAM) and *multiple frequency shift keying* (MFSK).

Orthogonal frequency division multiplexing (OFDM) has emerged as the primary modulation scheme for next generation broadband wireless communications [12]. While extensive research has been conducted to improve throughput performance [5], [13], limited work has been done to address energy-efficient communication for OFDM systems. We have studied uplink energy-efficient transmission in *orthogonal frequency division multiple access* (OFDMA) systems for flat fading channels [11] and obtained the following conclusions on energy-efficient transmission:

1) Both the transmission data rate, determined by the modulation order, and the energy efficiency increase with channel power gain;
2) The modulation order on each subchannel decreases with the increase of the number of subchannels assigned to a user, while the energy efficiency increases with it.

In general, different subchannels of OFDM experience different channel fadings, resulting in different modulation order, code rate, and power allocation for optimal energy-efficient transmissions. In this paper, we address energy-efficient link adaptation for frequency-selective fading channels. The rest of the paper is organized as follows. In Section II, we investigate optimal conditions and develop algorithms to obtain the globally optimal solution. As an example, we apply the energy-efficient scheme in OFDM systems with subchannelization in Section III and provide simulation results. Finally, we conclude the paper in Section IV.

II. Energy-Efficient Transmission

In this section, we will investigate optimal energy-efficient transmission in frequency-selective channels.

A. Conditions for Global Optimum

Assume there are K subchannels. The transmit power and modulation order on each subchannel can be adjusted to

* This work was supported by Intel Corp., the U.S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-20-0011, and SABA Program of Motorola.

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maximize energy efficiency. Denote the data rate on Subchannel i as r_i and the data rate vector on all subchannels as $\mathbf{R} = [r_1, r_2, \cdots, r_K]^T$, which depends on channel state, modulation, and power allocation. For a given channel state, transmit power on each subchannel is determined by the data rate. Denote the overall transmit power as $P_T(\mathbf{R})$. Similar to the argument in [11], $P_T(\mathbf{R})$ is assumed to be strictly convex in $\mathbf{R}$ and $P_T(\mathbf{0}) = 0$. The overall power consumption for a given data rate vector is

$$P(\mathbf{R}) = P_C + P_T(\mathbf{R}), \tag{1}$$

where P_C is denoted as the circuit power consumption. The energy efficiency is defined as

$$U(\mathbf{R}) = \frac{R}{P_C + P_T(\mathbf{R})}, \tag{2}$$

where $R = \sum_{i=1}^{K} r_i$. The optimal energy-efficient link adaptation achieves maximum energy efficiency, i.e.

$$\mathbf{R}^* = \arg\max_{\mathbf{R}} U(\mathbf{R}) = \arg\max_{\mathbf{R}} \frac{R}{P_C + P_T(\mathbf{R})}. \tag{3}$$

Note that if we fix the overall transmit power, the objective of Equation 3 is equivalent to maximizing the overall throughput and existing water-filling power allocation approach [1] gives the solution. However, besides adapting the power distributions on all subchannels, the overall transmit power can also be adapted according to the states of all subchannels to maximize energy efficiency. Hence, the solution to Equation (3) is in general different from existing power allocation schemes that maximize throughput with power constraints.

In the following, we will demonstrate that a unique globally optimal data rate vector exists and develop algorithms to reach it. However, before showing this property, the concept of quasiconcavity needs to be introduced first.

Definition 1. *As defined in [14], a function f, which maps from a convex set of real n-dimensional vectors, $\mathcal{D}$, to a real number, is called strictly quasiconcave if for any $\mathbf{x}_1, \mathbf{x}_2 \in \mathcal{D}$ and $\mathbf{x}_1 \neq \mathbf{x}_2$,*

$$f(\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) > \min\{f(\mathbf{x}_1), f(\mathbf{x}_2)\}, \tag{4}$$

for any $0 < \lambda < 1$.

In the following, Lemmas 1 and 2 characterize $U(\mathbf{R})$ and are proved in Appendices I and II respectively.

Lemma 1. *If $P_T(\mathbf{R})$ is strictly convex in $\mathbf{R}$, $U(\mathbf{R})$ is strictly quasiconcave.*

According to Lemma 1, if a local maximum exists for $U(\mathbf{R})$, it is also globally optimal [14]. Lemma 2 further tells the existence of the local maximum.

Lemma 2. *If $P_T(\mathbf{R})$ is strictly convex in $\mathbf{R}$, $U(\mathbf{R})$ is either strictly decreasing or first strictly increasing and then strictly decreasing in any r_i of $\mathbf{R}$, i.e. the local maximum of $U(\mathbf{R})$ for each r_i exists at either 0 or a positive finite value.*

From Lemmas 1 and 2, a unique globally optimal transmission rate vector exists and its characteristics are summarized in Theorem 1 according to the proofs in Appendix II.

Theorem 1. *If $P_T(\mathbf{R})$ is strictly convex, there exists a unique globally optimal transmission data rate vector $\mathbf{R}^* = [r_1^*, r_2^*, \cdots, r_K^*]^T$ for (3), where r_i^* is given by*

(i) when $\frac{P_C + P_T(\mathbf{R}_i^{(0)})}{R_i^{(0)}} \geq \left.\frac{\partial P_T(\mathbf{R})}{\partial r_i}\right|_{\mathbf{R}=\mathbf{R}_i^{(0)}}$, $\left.\frac{\partial U(\mathbf{R})}{\partial r_i}\right|_{\mathbf{R}=\mathbf{R}^*} = 0$, i.e. $\frac{1}{\frac{\partial P_T(\mathbf{R}^*)}{\partial r_i^*}} = \frac{R^*}{P_C + P_T(\mathbf{R}^*)} = U(\mathbf{R}^*)$;

(ii) when $\frac{P_C + P_T(\mathbf{R}_i^{(0)})}{R_i^{(0)}} < \left.\frac{\partial P_T(\mathbf{R})}{\partial r_i}\right|_{\mathbf{R}=\mathbf{R}_i^{(0)}}$, $r_i^* = 0$,

where $\mathbf{R}_i^{(0)} = [r_1^*, r_2^*, \cdots, r_{i-1}^*, 0, r_{i+1}^*, \cdots, r_K^*]$ and $R_i^{(0)} = \sum_{j \neq i} r_j^*$, i.e. the overall data rate on all other subchannels except i.

Theorem 1 has clear physical insights. $P_C + P_T(\mathbf{R}_i^{(0)})$ is the power consumption of both circuit and all other subchannels when subchannel i is not used. $\frac{P_C + P_T(\mathbf{R}_i^{(0)})}{|\mathbf{R}_i^{(0)}|}$ is the per-bit energy consumption when subchannel i is not used and the overall per-bit energy consumption needs to be minimized for energy-efficient communications. $\left.\frac{\partial P_T(\mathbf{R})}{\partial r_i}\right|_{\mathbf{R}=\mathbf{R}_i^{(0)}}$ is the per-bit energy consumption transmitting infinitely small data rate on subchannel i conditioning on the optimal status of all other subchannels. Hence, subchannel i should not transmit anything when $\frac{P_C + P_T(\mathbf{R}_i^{(0)})}{|\mathbf{R}_i^{(0)}|} < \left.\frac{\partial P_T(\mathbf{R})}{\partial r_i}\right|_{\mathbf{R}=\mathbf{R}_i^{(0)}}$. Otherwise, there should be a tradeoff between the desired data rate on subchannel i and the incurred power consumption. The tradeoff closely depends on the power consumption of both circuits and transmission on all other subchannels and can be found through the unique zero derivative of $U(\mathbf{R})$ with respect to r_i. Note that when only one subchannel exists, $\mathbf{R}_1^{(0)} = 0$ and Condition 1 gives the optimal transmission for flat-fading channels in [11]. According to condition i of Theorem 1, it is easy to show that the power allocation for energy-efficient link adaptation is a dynamic water-filling and the water level is determined by the optimal energy efficiency $U(\mathbf{R}^*)$.

B. Algorithms

Theorem 1 provides necessary and sufficient conditions for the unique and globally optimum data rate vector. However, to obtain the optimal data rate vector $\mathbf{R}^*$, it is usually difficult to directly solve joint nonlinear equations according to Theorem 1, since the characteristics of those nonlinear equations are complicated. Therefore, we develop iterative methods to search the optimal $\mathbf{R}$ for maximizing $U(\mathbf{R})$. The global optimality is guaranteed by Lemma 1. In the following, we describe the low-complexity iterative algorithms.

1) Gradient Assisted Binary Search: When there is only one subchannel, Lemma 2 shows that function $U(\mathrm{r})$ has a unique r^* such that for any $r < r^*$, $\frac{dU(\mathrm{r})}{dr} > 0$, and for any $r > r^*$, $\frac{dU(\mathrm{r})}{rt} < 0$. Hence, we have the following lemma to seek two points r_1 and r_2 such that $r_1 \leq r^* \leq r_2$.

Lemma 3. *Let initial setting $r^{[0]} > 0$ and set $\alpha > 1$. For any $i \geq 0$, let*

$$r^{[i+1]} = \begin{cases} \frac{r^{[i]}}{\alpha} & \left.\frac{dU(r)}{dr}\right|_{r^{[0]}} < 0 \\ \alpha r^{[i]} & otherwise \end{cases}. \tag{5}$$

Repeat (5) until $r^{[I]}$ such that $\left.\frac{dU(r)}{dr}\right|_{r^{[I]}}$ has a different sign from $\left.\frac{dU(r)}{dr}\right|_{r^{[0]}}$. Then r^* must be between $r^{[I]}$ and $r^{[I-1]}$.

To locate r^* between r_1 and r_2, let $\widehat{r} = \frac{r_1 + r_2}{2}$. If $\left.\frac{dU(r)}{dr}\right|_{\widehat{r}} = 0$, r^* is found. If $\left.\frac{dU(r)}{dr}\right|_{\widehat{r}} < 0$, $r_1 < r^* < \widehat{r}$ and replace r_2

with $\widehat{r}$; otherwise, replace r_1 with $\widehat{r}$. This leads to the *gradient assisted binary search* (GABS) algorithm for maximizing $U(r)$, which is summarized in Table I. Furthermore, Theorem 2 clearly characterizes the global convergence of GABS and is proved in Appendix III.

Theorem 2. *GABS converges to the globally optimal transmission data rate r^*. A rate r, which satisfies $|r - r^*| \leq \epsilon$, can be found within at most M iterations, where M is the minimum integer such that $M \geq \log_2(\frac{(\alpha-1)r^*}{\epsilon} - 1)$.*

Algorithm *GABS*(r_o)
($*$ algorithm for single-subchannel transmission. $*$)
Input: initial guess: $r_o > 0$
Output: optimal transmission rate: r^*
$r_1 = r_o$, $h_1 \leftarrow \frac{dU(r)}{dr}\Big|_{r_1}$, initialize $\alpha > 1$ (e.g.10)
if $h_1 < 0$
($*$ seek r_1 and r_2 such that $r_1 < r^* < r_2$ $*$)
then $r_2 \leftarrow r_1$, $r_1 \leftarrow \frac{r_1}{\alpha}$, and $h_1 \leftarrow \frac{dU(r)}{dr}\Big|_{r_1}$
while $h_1 < 0$
do $r_2 \leftarrow r_1$, $r_1 \leftarrow \frac{r_1}{\alpha}$, and $h_1 \leftarrow \frac{dU(r)}{dr}\Big|_{r_1}$
else $r_2 \leftarrow r_1 * \alpha$ and $h_2 \leftarrow \frac{dU(r)}{dr}\Big|_{r_2}$
while $h_2 > 0$
do $r_1 \leftarrow r_2$, $r_2 \leftarrow r_2 * \alpha$, and $h_2 \leftarrow \frac{dU(r)}{dr}\Big|_{r_2}$
while no convergence
($*$ seek r^* between r_1 and r_2 $*$)
do $\widehat{r} \leftarrow \frac{r_2+r_1}{2}$; $\widehat{h} \leftarrow \frac{dU(r)}{dr}\Big|_{\widehat{r}}$
if $\widehat{h} > 0$
then $r_1 = \widehat{r}$;
else $r_2 = \widehat{r}$
return $\widehat{r}$

TABLE I: Gradient assisted binary search

2) Binary Search Assisted Ascent: To find optimal transmission data rate vector for multiple subchannel case, we use gradient ascent method to produce a maximizing sequence $\mathbf{R}^{[i]}$, $n = 0, 1, \cdots$, and

$$\mathbf{R}^{[i+1]} = \mathbf{R}^{[i]} + \mu \nabla U(\mathbf{R}^{[i]}), \tag{6}$$

where μ is the step size and $\nabla U(\mathbf{R}^{[i]})$ is the gradient at iteration i. With sufficiently small step size, $U(\mathbf{R}^{[i+1]})$ will always be bigger than $U(\mathbf{R}^{[i]})$ except when $\nabla U(\mathbf{R}^{[i]}) = 0$ [15]. However, small step size leads to slow convergence. Besides, each element of the gradient depends on the corresponding subchannel power gain, which could potentially differ from each other by orders of magnitude. Hence, a line search of the optimal step size needs to cover a large range to assure global convergence on all subchannels, which is computationally expensive. Therefore, at each $\mathbf{R}^{[i]}$, an efficient algorithm is needed to find the optimal step size.

Denote

$$g_i(\mu) = U(\mathbf{R}^{[i]} + \mu \nabla U(\mathbf{R}^{[\mathbf{i}]})). \tag{7}$$

Similar to the proof of Lemma 1, it is easy to show that $g_i(\mu)$ is also strictly quasiconcave in μ and has a unique globally maximum μ^* such that for any $\mu < \mu^*$, $\frac{dg_i(\mu)}{d\mu} > 0$, and for any $\mu > \mu^*$, $\frac{dg_i(\mu)}{d\mu} < 0$. Thus, by replacing $\frac{dU(r)}{dr}$ to be

$$\frac{dg_i(\mu)}{d\mu} = [\nabla U(\mathbf{R}^{[i]} + \mu \nabla U(\mathbf{R}^{[i]}))]^T \nabla U(\mathbf{R}^{[i]}), \tag{8}$$

where $[]^T$ is the transpose, GABS can be used for quick location of the optimal step size. This leads to the *binary search assisted ascent* (BSAA) algorithm in Table II.

Algorithm *BSAA*($\mathbf{R}_o$)
($*$ algorithm for multi-subchannel transmission. $*$)
Input: initial guess: $\mathbf{R}_o$(default transmission rate can be used)
Output: optimal transmission rate vector: $\mathbf{R}^*$
$\mathbf{R} = \mathbf{R}_o$,
while no convergence
do use GABS to find the optimal step size μ^*;
$\mathbf{R} = \mathbf{R} + \mu^* \nabla f(\mathbf{R})$
return $\mathbf{R}$

TABLE II: Binary search assisted ascent

III. Energy-Efficient Transmission for OFDM with Subchannelization

The link adaptation in Section II is general and can be applied to different kinds of OFDM, MIMO, and MIMO+OFDM systems. The only problem to apply it is to find the transmit power relationship $P_T(\mathbf{R})$ of those systems. In this section, we apply the optimal energy-efficient link adaptation to OFDM with subchannelization as an example.

A. Modeling of OFDM with Subchannelization

In OFDM systems with subchannelization, subcarriers are grouped into subchannels and the subcarriers forming one subchannel may, but not necessarily be adjacent, such as the contiguous and distributed subchannelization schemes in 802.16e [12]. Each subchannel is treated to be flat fading and the *effective channel power gain*, $\bar{h}$, rather than physical channel power gain of each subcarrier, is used as a metric. For simplicity, $\bar{h}$ is the average of channel power gains of all subcarriers within the subchannel. Note that classical OFDM is a special case when each subchannel has one subcarrier.

We use the same frame structure on each subchannel as the one in [11]. Each transmission slot consists of a data interval, T_s, and a signalling interval, τ. In each data interval, l symbols are transmitted. Assume block fading, that is, the channel state remains constant during each data interval and is independent from one to another. By using uncoded M-QAM, the transmit power on subchannel i to achieve a given *bit-error rate* (BER) requirement, P_e, is given by [11]

$$P_{T_i}(r_i) = A_i(1 - 2^{B_i r_i}), \tag{9}$$

where

$$A_i = \frac{2c_i \ln(5P_e) N_o W}{3\bar{h}_i} \tag{10}$$

and

$$B = \frac{T_s + \tau}{c_i l}. \tag{11}$$

c_i is the number of subcarriers within Subchannel i, N_o is power spectral density, and W is the signal bandwidth of each subcarrier. Assuming no coupling between transmit powers

Carrier frequency	1.5 GHz
Subcarrier number	256
Subcarrier bandwidth	10 kHz
BER requirement	10^{-3}
Symbol number of data interval, l	100
Time duration of data interval, T_s	0.01s
Time duration of signalling interval, τ	0.001s
Thermal noise power, N_o	-141 dBW/MHz
User antenna height	1.6 m
BS antenna height	40 m
Environment	Macro cell in urban area
Circuit power, P_C	100 mW
Modulation	Uncoded M-QAM
Subchannelization	Fixed-interval and contiguous
Propagation Model	Okumura-Hata model
Shadowing	Log-normal with standard deviation of 10 dB
Frequency-selective fading	ITU pedestrian channel B
User speed	3 km/h

TABLE III: System parameters

among subchannels, the overall transmit power will be the cumulative of the transmit powers of all subchannels, that is,

$$P_T(\mathbf{R}) = \sum_{i=1}^{K} P_{Ti}(r_i), \tag{12}$$

which is monotonically increasing and strictly convex in each element of $\mathbf{R}$. The energy-efficient link adaptation immediately follows from Section II.

B. Performance comparison

In this section, we demonstrate performance of energy-efficient OFDM transmission through comparison with traditional transmission schemes. The system parameters are listed in Table III. The *International Telecommunication Union* (ITU) pedestrian channel model B is used to implement the multipath frequency-selective fading. We implemented two subchannelization schemes, fixed-interval and contiguous, both of which group 10 subcarriers into a subchannel. In the fixed-interval subchannelization, one draws subcarriers out of all subcarriers with a fixed interval to form a subchannel, while in the contiguous one, each subchannel consists of a block of contiguous subcarriers.

Figures 1 and 2 compare energy efficiency and throughput of different transmission schemes with contiguous subchannelization while Figure 3 compares energy efficiency of schemes with fixed-interval subchannelization. Two energy-efficient OFDM transmission schemes are implemented. The first one, FS EE, is the optimal energy-efficient transmission developed in this paper, and the second one, flat EE, treats the channel as flat fading [11]. Transmissions with both fixed and adaptive QAM modulations are also implemented for comparison. For fixed modulation, the transmit power is adapted to meet BER requirement while not exceeding 15 dBm maximum power constraint. In adaptive modulation, transmit power is equally distributed onto all subchannels and the modulation is adapted to meet BER requirement. Observing Figures 1 and 2, fixed and adaptive modulations perform closely to each other, especially when far away from BS, for both energy efficiency and throughput, when the maximum transmit power is 15 dBm. By increasing the transmit power from 15 dBm to 25 dBm, the throughput of adaptive modulation increases, however, the energy efficiency first increased and then decreases. Due to the global optimality, the proposed energy-efficient transmission for frequency-selective channels always achieves the highest energy efficiency, and outperforms the others by at least 15%. However, the throughput is not necessarily maximum and the other schemes, especially the adaptive QAM modulation with 25 dBm transmit power, sacrifice power to obtain higher throughput. In Figure 3, we note that when fixed-interval subchannelization is used, the energy-efficient transmission treating the channel to be flat fading performs the same as the one considering the difference of different subchannels.

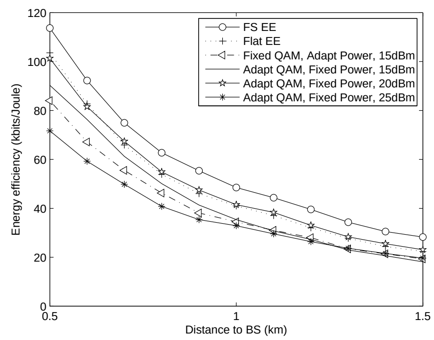

Fig. 1: Energy efficiency comparison (contiguous subchannelization)

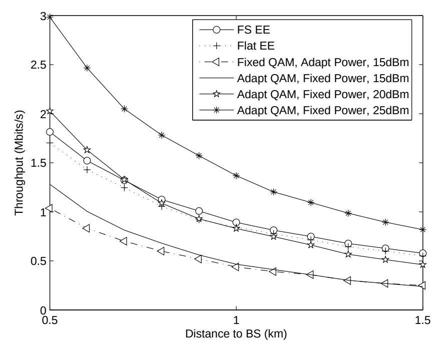

Fig. 2: Throughput comparison (contiguous subchannelization)

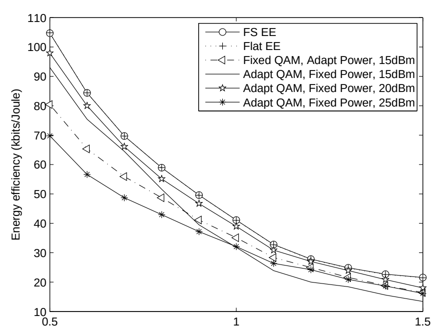

Fig. 3: Energy efficiency comparison (fixed-interval subchannelization)

IV. Conclusion

In this paper, we obtain unique globally optimal link adaptation for energy-efficient OFDM transmission in frequency-selective channels. Joint circuit and transmit power consumptions are taken into account for link adaptation to maximize energy efficiency. From the simulation results, we observed at least 15% improvement in energy utilization when frequency selectivity is exploited and the improvement depends on how much frequency diversity exists within the channels.

Appendix I
Proof of Lemma 1

Proof: Denote the upper contour sets of $U(\mathbf{R})$ as

$$S_\alpha = \{\mathbf{R} \succeq \mathbf{0} | U(\mathbf{R}) \geq \alpha\}, \tag{I.13}$$

where symbol $\succeq$ denotes vector inequality and $\mathbf{R} \succeq \mathbf{0}$ means each element of $\mathbf{R}$ is nonnegative. According to Proposition C.9 of [14], $U(\mathbf{R})$ is strictly quasiconcave if and only if S_α is strictly convex for any real number α. When $\alpha < 0$, no points exist on the contour $U(\mathbf{R}) = \alpha$. When $\alpha = 0$, only $\mathbf{0}$ is on the contour $U(\mathbf{0}) = \alpha$. Hence, S_α is strictly convex when $\alpha \leq 0$. Now we investigate the case when $\alpha > 0$. Since

$$U(\mathbf{R}) = \frac{R}{P_C + P_T(\mathbf{R})} \geq \alpha, \tag{I.14}$$

S_α is equivalent to

$$S_\alpha = \{\mathbf{R} \succeq \mathbf{0} | \alpha P_C + \alpha P_T(\mathbf{R}) - R \leq 0\}. \tag{I.15}$$

Since $P_R(\mathbf{R})$ is strictly convex in $\mathbf{R}$, S_α is also strictly convex. Hence, we have Lemma 1. ■

Appendix II
Proof of Lemma 2

Proof: The partial derivative of $U(\mathbf{R})$ with r_i is

$$\begin{aligned}\frac{\partial U(\mathbf{R})}{\partial r_i} &= \frac{P_C + P_T(\mathbf{R}) - RP_T^{'}(\mathbf{R})}{(P_C + P_T(\mathbf{R}))^2} \\ &\triangleq \frac{\beta(r_i)}{(P_C + P_T(\mathbf{R}))^2},\end{aligned} \tag{II.16}$$

where $P_T^{'}(\mathbf{R})$ is the first partial derivative of $P_T(\mathbf{R})$ with respect to r_i. According to Lemma 1, if r_i^* exists such that $\left.\frac{\partial U(\mathbf{R})}{\partial r_i}\right|_{r_i = r_i^*} = 0$, it is unique, i.e. if there is a r_i^* such that $\beta(r_i^*) = 0$, it is unique. In the following, we investigate the conditions when r_i^* exists.

The derivative of $\beta(r_i)$ is

$$\beta^{'}(r_i) = -RP_T^{''}(\mathbf{R}) < 0, \tag{II.17}$$

where $P_T^{''}(\mathbf{R})$ is the second partial derivative of $P_T(\mathbf{R})$ with respect to r_i. Hence, $\beta(r_i)$ is strictly decreasing.

According to the L'Hopital's rule, it is easy to show that

$$\begin{aligned}\lim_{r_i -> \infty} \beta(r_i) &= \lim_{r_i -> \infty} (P_C + P_T(\mathbf{R}) - RP_T^{'}(\mathbf{R})) \\ &= \lim_{r_i -> \infty} \left(\frac{P_C + P_T(\mathbf{R}) - RP_T^{'}(\mathbf{R})}{r_i} r_i \right) \\ &= \lim_{r_i -> \infty} \left(\frac{P_T^{'}(\mathbf{R}) - P_T^{'}(\mathbf{R}) - RP_T^{''}(\mathbf{R})}{1} r_i \right) \\ &= \lim_{r_i -> \infty} -P_T^{''}(\mathbf{R}) R r_i < 0.\end{aligned} \tag{II.18}$$

Besides,

$$\begin{aligned}\lim_{r_i -> 0} \beta(r_i) &= \lim_{r_i -> 0} (P_C + P_T(\mathbf{R}) - RP_T^{'}(\mathbf{R})) \\ &= P_C + P_T(\mathbf{R}_i^{(0)}) - R_i^{(0)} P_T^{'}(\mathbf{R}_i^{(0)}),\end{aligned} \tag{II.19}$$

where $\mathbf{R}_i^{(0)} = [r_1, r_2, \cdots, r_{i-1}, 0, r_{i+1}, \cdots, r_K]^T$ and $R_i^{(0)} = \sum_{j \neq i} r_j$.

(1^o)When $P_C + P_T(\mathbf{R}_i^{(0)}) - R_i^{(0)} P_T^{'}(\mathbf{R}_i^{(0)}) \geq 0$, $\lim_{r_i -> 0} \beta(r_i) \geq 0$. Together with (II.18), we see that t_i^* exists and $U(\mathbf{R})$ is first strictly increasing and then strictly decreasing in r_i.

(2^o)When $P_C + P_T(\mathbf{R}_i^{(0)}) - R_i^{(0)} P_T^{'}(\mathbf{R}_i^{(0)}) < 0$, $\lim_{r_i -> 0} \beta(r_i) < 0$. Together with (II.17) and (II.18), t_i^* does not exist. However, $U(\mathbf{R})$ is always strictly decreasing in r_i. Hence, $U(\mathbf{R})$ is maximized at $r_i = 0$.

Lemma 2 is readily obtained. ■

Appendix III
Proof of Theorem 2

Proof: The global convergence is straightforward from Lemmas 1 and 2. Since $r_2^{[0]} = \alpha r_1^{[0]}$ and $r_1^{[i]} \leq r^* \leq r_2^{[i]}$, with induction, we have $r_2^{[i]} - r_1^{[i]} = \frac{r_2^{[0]} - r_1^{[0]}}{2^i} \leq \frac{(\alpha-1)r^*}{2^i}$. Hence, $\widehat{r}^{[i]} = \frac{r_1^{[i]} + r_2^{[i]}}{2} \geq (2r_2^{[i]} - \frac{(\alpha-1)r^*}{2^i})/2 \geq r^* - \frac{(\alpha-1)r^*}{2^{i+1}}$ and $\widehat{r}^{[i]} \leq r^* + \frac{(\alpha-1)r^*}{2^{i+1}}$. Then $|\widehat{r}^{[i]} - r^*| \leq \frac{(\alpha-1)r^*}{2^{i+1}}$. Let $\frac{(\alpha-1)r^*}{2^{i+1}} \leq \epsilon$. We have $i \geq \log_2(\frac{(\alpha-1)r^*}{\epsilon} - 1)$. Theorem 2 follows immediately. ■

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