Energy-Efficient Design in Wireless OFDMA

Guowang Miao [†], Nageen Himayat [*], Ye (Geoffrey) Li [†], and David Bormann [*]
[†] School of ECE, Georgia Institute of Technology
Atlanta, Georgia 30332–0250, email: gmiao3@gatech.edu and liye@ece.gatech.edu
[*] Wireless Commun. Lab./Commun. Tech. Lab., Corp. Tech. Group, Intel Corp.
Santa Clara, CA, email: nageen.himayat@intel.com and david.bormann@intel.com

***Abstract*—Energy-efficient transmission is an important aspect of wireless system design due to limited battery power in mobile devices. We consider uplink energy-efficient transmission in OFDMA systems since mobile stations are battery powered. We account for both circuit and transmit power when designing energy-efficient communication mechanisms and emphasize energy efficiency over peak rates or throughput. Both link adaptation and resource allocation schemes are developed to optimize the overall bits transmitted per Joule of energy, which allows for maximum energy savings in a network. Our simulation results show that the proposed schemes significantly improve energy efficiency.**

Index Terms– energy efficiency, OFDMA, bits per Joule, link adaptation, resource allocation

I. Introduction

Power efficiency is becoming increasingly important for wireless communication systems due to limited battery resources in mobile devices. Unfortunately, battery technology has not progressed as fast as silicon technology [1]. Hence, recent energy-efficient management schemes [2]–[4] have focused on minimizing energy consumption rather than throughput maximization [5]. Additionally, *orthogonal frequency division multiple access* (OFDMA) has emerged as one of the prime multiple access schemes for next generation multi-user broadband wireless networks. However, limited research has been done for energy-efficient communication in OFDMA systems. In this paper, we consider uplink energy-efficient transmission in OFDMA systems to improve battery consumption at the mobiles. We account for both circuit and transmit power when designing link adaptation and resource allocation schemes, and emphasize energy efficiency over peak rates or throughput. We initially focus on the case of flat-fading OFDMA channels, and defer the frequency selective case to future work.

The rest of the paper is organized as following. In Section II, we briefly describe the system model. Then we develop optimal energy-efficient link adaptation and network resource allocation schemes in Sections III and IV respectively. Finally, we conclude the paper in Section V.

II. System Description

Multiple access is achieved in OFDMA by assigning subchannels to individual users based on *quality of service* (QoS) requirement and channel condition. This allows simultaneous data transmission for several users. For simplicity, we investigate energy-efficient OFDMA communication with flat fading channels in this paper.

Consider uplink transmission in an OFDMA network with one *base station* (BS) and multiple users, i.e. mobile stations. Denote N and K as the numbers of users and subchannels, respectively. Denote c_i as the number of subchannels assigned to User i. Each subchannel will be assigned to one user exclusively at each frame slot. Hence,

$$\sum_{i=1}^{N} c_i \leq K. \tag{1}$$

Denote r_i as the achievable data rate on each subchannel by User i, then the data rate of User i is

$$R_i = r_i c_i. \tag{2}$$

The BS allocates subchannels to improve the overall network energy efficiency, which is measured by the number of bits transmitted per Joule. Additionally, each user also adjusts transmit power and modulation order for further optimization.

III. Optimal Energy-Efficient Link Adaptation

This section considers per link adaptation schemes that will result in minimum energy consumption, or equivalently, maximum energy-efficiency. Throughout this section, assume c subchannels are assigned. Since we focus on per link energy-efficient optimization, subscripts indicating different users will be dropped subsequently.

A. Energy-Efficient Transmission Rate

Power consumption of a mobile station in transmission mode consists of two parts. The first is *circuit power*, denoted as P_C, which is independent of data rate and exists whenever the user is in transmission mode. The second is *transmit power*, $P_T(R)$, which depends on data transmission rate, R. For example, we consider an *additive white Gaussian noise* (AWGN) channel with signal bandwidth W, the achievable data rate is given by the Shannon capacity as

$$R = W \log(1 + \frac{P_T g}{N_o W}), \tag{3}$$

where g is the channel power gain, N_o is the power spectral density. Hence,

$$P_T(R) = (e^{\frac{R}{W}} - 1)N_oW/g, \tag{4}$$

which is monotonically increasing and strictly convex in R. In general, we assume $P_T(R)$ to be monotonically increasing and strictly convex and $P_T(0) = 0$.

The overall power used for data transmission is

$$P(R) = P_C + P_T(R). \tag{5}$$

The number of bits transmitted per Joule of energy, called *energy efficiency*, is used as a performance measure, and it is defined as

$$U(R) = \frac{R}{P(R)} = \frac{R}{P_C + P_T(R)}. \tag{6}$$

The network is optimized for the highest energy efficiency. Thus, the intended data rate is

$$R^* = \arg\max_R U(R) = \arg\max_R \frac{R}{P_C + P_T(R)}. \tag{7}$$

The optimal transmission data rate is given by the following theorem, which is proved in Appendix A.

Theorem 1. *If $P_T(R)$ is monotonically increasing and strictly convex in R, there exists a unique globally optimal transmission data rate for (7) given by*

$$R^* = \frac{P_C + P_T(R^*)}{P_T'(R^*)}, \tag{8}$$

where $P_T'(\cdot)$ is the first derivative of function $P_T(\cdot)$.

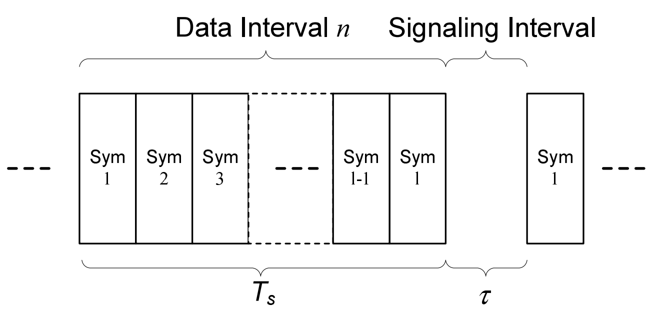


Fig. 1: Frame structure

To illustrate the application of Theorem 1, we derive the optimal link settings for uncoded *multiple quadrature amplitude modulation* (M-QAM) in AWGN channel. The frame structure of the system is shown in Figure 1. Each transmission slot consists of a data interval, T_s, and a signalling interval, τ. Assume block fading, that is, the channel state remains constant during each data interval and is independent from one to another. For M-QAM, the number of bits transmitted per symbol is $b = \log_2 M$, where M is the modulation order. In each data interval, l symbols are transmitted on each subchannel. The data rate on each subchannel is $r = \frac{bl}{T_s+\tau}$, and the overall data rate is

$$R = cr = \frac{cbl}{T_s + \tau}. \tag{9}$$

Consequently, for a given data transmission rate, the number of bits transmitted per symbol will be $b = \frac{R(T_s+\tau)}{cl}$.

The *bit-error rate* (BER) for coherently detected M-QAM with Gray mapping over an AWGN channel is approximated by [8]

$$P_e(\gamma) \approx 0.2 \exp\left(-\frac{1.5\gamma}{M-1}\right), \tag{10}$$

where γ is the *signal-to-noise ratio* (SNR). Denote g to be the power gain of the channel. The SNR on each subchannel will be

$$\gamma = \frac{P_T(R)g}{cN_oW}, \tag{11}$$

where N_o is the power spectral density and W is the signal bandwidth in each subchannel. For a given BER, P_e, the required SNR can be determined by (10). Consequently, the required transmit power will be

$$P_T(R) = \frac{\gamma c N_o W}{g} = A(1 - 2^{BR}), \tag{12}$$

where $A = \frac{2c\ln(5P_e)N_oW}{3g}$ and $B = \frac{T_s+\tau}{cl}$. Usually, $5P_e < 1$ for effective transmission, therefore, $A < 0$. It can be seen that $P_T(R)$ is monotonically increasing and strictly convex in R. P_C characterizes circuit power consumption in both the data and signalling intervals. According to Theorem 1, the desired data rate is

$$R^* = \frac{A(2^{BR^*} - 1) - P_c}{AB2^{BR^*}\ln 2}. \tag{13}$$

Correspondingly, $b^* = \frac{R^*(T_s+\tau)}{cl}$ and $M^* = 2^{\frac{R^*(T_s+\tau)}{cl}}$.

B. Characteristics of Energy-Efficient Transmission

Theorems 2 and 3 describes the characteristics of energy-efficient link transmission and are proved in Appendices B and C respectively.

Theorem 2. *For energy-efficient transmission, both the transmission data rate, determined by modulation order, and the energy efficiency increase with channel power gain.*

Theorem 3. *For energy-efficient transmission, the modulation order on each subchannel decreases with the increase of the number of subchannels assigned to a user, while the energy efficiency increases with it.*

To demonstrate Theorem 2 and 3, we present energy-efficient link transmission for an uncoded M-QAM system with the frame structure as in Figure 1. System parameters are listed in Table I. Each user is assigned 10 subchannels, unless otherwise specified.

Figure 2(a) shows the energy efficiency of users located at different distances from the BS. The lower axis shows the data rate achieved given the modulation order indicated by the top

TABLE I: System Parameters

Carrier frequency	1.5 GHz
Subchannel bandwidth	10 kHz
BER	10^{-6}
Symbol number of data interval, l	100
Time duration of data interval, T_s	0.01s
Time duration of signalling interval, τ	0.001s
Thermal noise power, N_o	-141 dBW/MHz
User antenna height	1.6 m
BS antenna height	40 m
Environment	Macro cell in urban area
Circuit power, P_C	100 mW
Maximum transmit power	33 dBm
Propagation Model	Okumura-Hata model
Fading	Flat fading
Modulation	Uncoded M-QAM

axis. The figure shows that by selecting an optimal modulation scheme, energy efficiency increases as the user moves closer to the BS. Furthermore, the optimal modulation for energy-efficient transmission varies with the distance between the user and BS. In general, for transmission with maximum energy efficiency, the closer the user is to the BS, the higher the modulation order should be. Figure 2(b) shows energy efficiency of a user located 1 km away from the BS with different numbers of assigned subchannels. From Figure 2(b), the more the number of subchannels assigned to a user, the higher the maximum energy efficiency is, and the more sensitive to modulation order the energy efficiency is.

Figure 3 shows the energy consumed for sending one megabit. Figure 3(a) compares energy consumption for a system with fixed modulation and with optimal modulation order determined by the proposed energy-efficient transmission. For fixed modulation, the transmit power is allocated such that the BER is 10^{-6}. Figure 3(b) compares the optimal energy-efficient scheme with traditional adaptive modulation. In traditional adaptive modulation, the transmit power is fixed to be 15 dBm, 20 dBm, 25 dBm, or 30 dBm. The energy values are normalized with those of the proposed optimal energy-efficient scheme. From the figures, the proposed scheme always achieves the lowest energy consumption.

IV. Energy-Efficient Resource Allocation

The BS allocates subchannels to improve overall network energy efficiency. Subscripts are added to distinguish users.

A. Resource Allocation without Fairness

Denote set $\mathfrak{c} = \{c_1, c_2, \cdots, c_N\}$ to be the set of numbers of subchannels assigned to each user. With subchannel assignment, the total energy efficiency across the whole network should be maximized, i.e.,

$$\mathfrak{c}^* = \arg\max_{\mathfrak{c}} \sum_i \frac{R_i}{P_i(R_i)} = \arg\max_{\mathfrak{c}} \sum_i U_i(R_i) \tag{14a}$$

subject to

$$R_i = r_i c_i, \tag{14b}$$

and

$$\sum_{i=1}^{N} c_i \leq K. \tag{14c}$$

Denote $P_{T_i}(r_i)$ as the transmit power on each subchannel when the supported data rate is r_i for User i. $P_i(R_i) = P_C + c_i P_{T_i}(r_i)$. We have,

$$\begin{aligned} U_i(R_i) = \frac{R_i}{P_i(R_i)} &= \frac{c_i r_i}{P_C + c_i P_{T_i}(r_i)} \\ &= \frac{r_i}{\frac{P_C}{c_i} + P_{T_i}(r_i)}. \end{aligned} \tag{15}$$

Let $V_i(c_i) = \frac{r_i}{\frac{P_C}{c_i} + P_{T_i}(r_i)}$, which is strictly concave in c_i. Problem (14) is equivalent to

$$\mathfrak{c}^* = \arg\max_{\mathfrak{c}} \sum_i V_i(c_i) \tag{16a}$$

subject to

$$\sum_{i=1}^{N} c_i \leq K. \tag{16b}$$

Since $V_i(c_i)$ is strictly concave and therefore, unique globally optimal subchannel assignment exists.

Lagrange multiplier can be used to find the solution of the above optimization problem. The Lagrange function associated with problem (16) is

$$L(\mathfrak{c}, \lambda) = \sum_{i=1}^{N} V_i(c_i) - \lambda\left(\sum_{i=1}^{N} c_i - K\right). \tag{17}$$

Let $\frac{\partial L}{\partial c_i} = V_i^{'}(c_i) - \lambda = 0$, the optimal assignment for User i is

$$c_i^* = V_i^{'-1}(\lambda^*), \tag{18}$$

where $V_i^{'-1}(\cdot)$ is the inverse function of $V_i^{'}(\cdot)$, and λ^* is given by

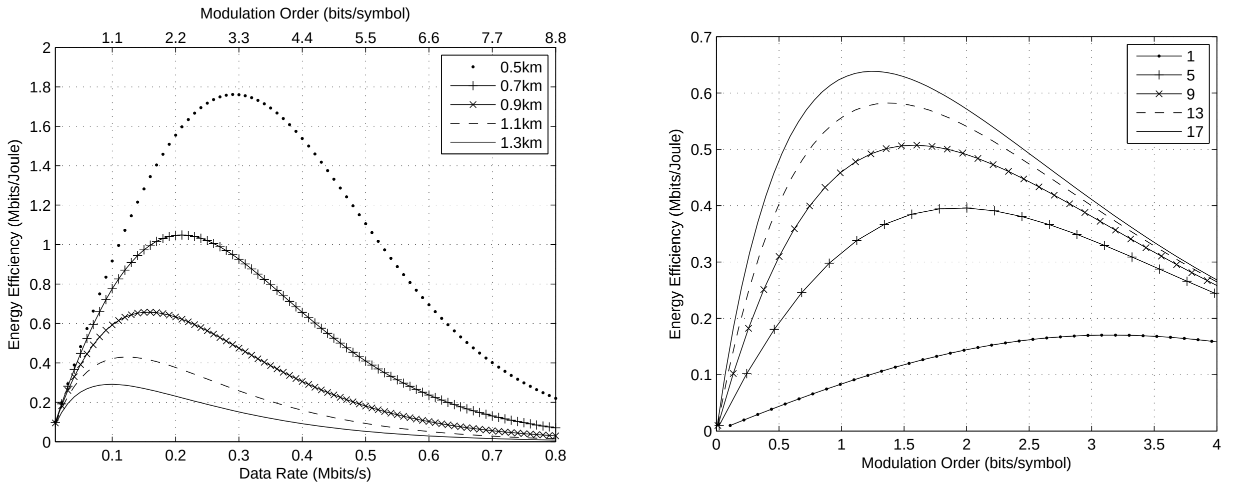
$$\sum_i c_i^* = \sum_i V_i^{'-1}(\lambda^*) = K. \tag{19}$$

The optimal solution in (18) may produce fractional subchannel assignment, which is not desired. If we search integers, $c_i^{'}$s, for problem (14), then it can be treated as a utility based resource allocation, and has been thoroughly investigated in [9]. The practical "sorting-search" algorithm in [9] can be used to assign subchannels for the purpose of energy efficiency.

B. Resource Allocation with Fairness

In this section, we consider energy-efficient scheduling with proportional fairness constraint [10]. The BS assigns subchannels to maximize the product of energy efficiency of all users, i.e.,

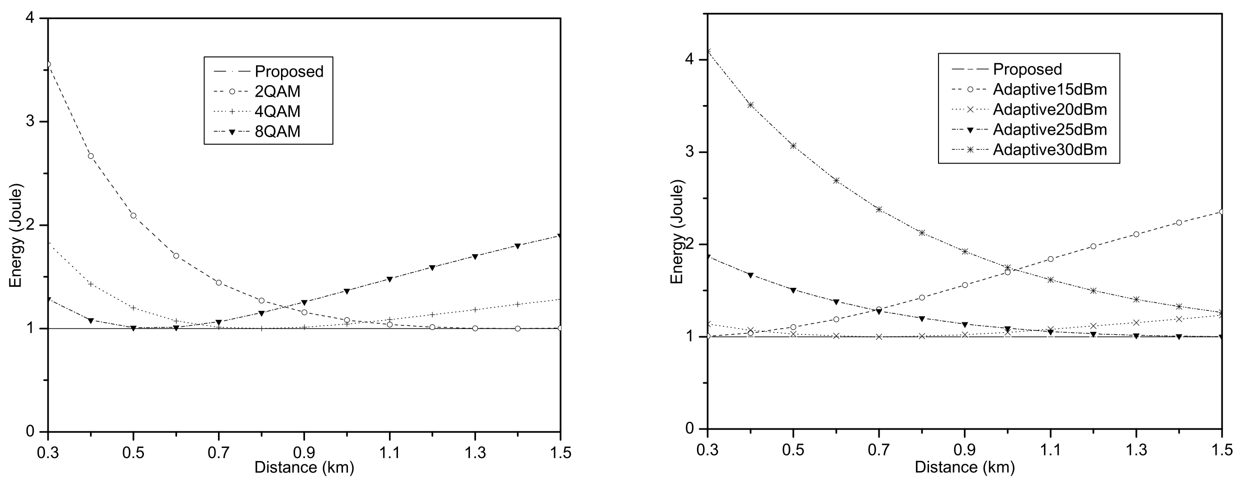
$$\mathfrak{c}^* = \arg\max_{\mathfrak{c}} \prod_i \frac{R_i}{P_i(R_i)} = \arg\max_{\mathfrak{c}} \sum_i \log\left(\frac{R_i}{P_i(R_i)}\right) \tag{20a}$$



(a) Relationship of energy efficiency, distance, modulation and transmission data rate

(b) Relationship of energy efficiency, modulation and subchannel assignment

Fig. 2: Energy-efficiency relationship of per link transmission



(a) Comparison with fixed modulation (normalized energy)

(b) Comparison with adaptive modulation (normalized energy)

Fig. 3: Energy consumed for transmitting one megabit

subject to

$$R_i = c_i r_i \tag{20b}$$

and

$$\sum_{i=1}^{N} c_i \leq K. \tag{20c}$$

Denote $W_i(c_i) = \log(V_i(c_i))$. Problem (20) is equivalent to

$$\mathfrak{c}^* = \arg\max_{\mathfrak{c}} \sum_i \log(V_i(c_i)) = \arg\max_{\mathfrak{c}} \sum_i W_i(c_i), \tag{21a}$$

subject to

$$\sum_{i=1}^{N} c_i \leq K. \tag{21b}$$

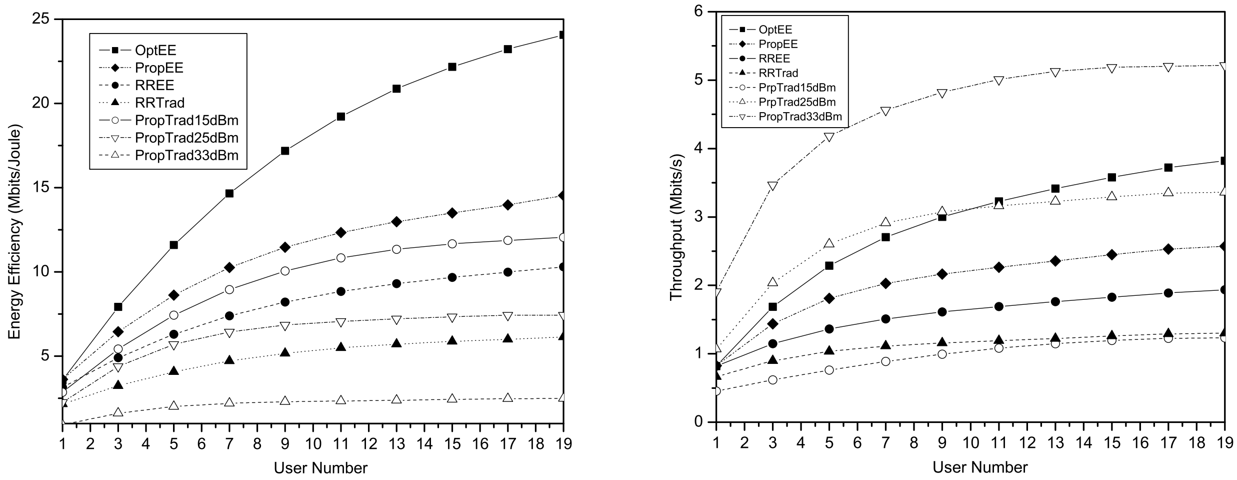
Since $V_i(c_i) > 0$ is strictly concave,

$$\begin{aligned} \frac{\partial^2 W_i(c_i)}{\partial c_i^2} &= \frac{\partial^2 \log(V_i(c_i))}{\partial c_i^2} \\ &= \frac{V_i^{''}(c_i))V_i(c_i)) - \left[V_i^{'}(c_i))\right]^2}{V_i^2(c_i))} < 0. \end{aligned} \tag{22}$$

Hence, $W_i(c_i)$ is strictly concave in c_i. Problem (21) is strictly concave and unique globally optimal assignment exists. Similar to (18), the optimal assignment is given by

$$c_i^* = W_i^{'-1}(\lambda^*), \tag{23}$$

where λ^* satisfies $\sum_i W_i^{'-1}(\lambda^*) = K$.



(a) Network energy efficiency comparison

(b) Network throughput comparison

Fig. 4: Comparisons of different algorithms

C. Performance Comparisons

In this section we present performance results for energy-efficient resource allocation schemes. System parameters are the same as those in Table I. All the schedulers and corresponding transmission schemes are listed in Table II. In PropTrad, we set the transmit power to be 15 dBm, 25 dBm, and 33 dBm respectively. Users are randomly dropped within the cell and the channels experience both log-normal shadowing with standard deviation of 10 dB, and Rayleigh fading with unit average power gain. There are 96 subchannels, each with 10 kHz.

Figure 4 compares energy efficiency and the corresponding throughput, respectively. We note that the energy-efficient scheduler without fairness performs with highest energy efficiency and with similar throughput to the proportional scheduler with 25 dBm transmit power for adaptive modulation. Comparing the proportional scheduler with 25 dBm transmit power and the energy-efficient scheduler with proportional fairness, both of which guarantee fairness amongst users, we note that the energy-efficient scheduler has around 20% less instantaneous throughput than the proportional scheduler, however, it transmits about 100% more data than the proportional scheduler given a fixed amount of energy. Or equivalently, given a fixed amount of data, the energy-efficient scheduler saves 50% energy. Comparing the two round-robin schedulers, we note that the one with energy-efficient transmission always performs approximately 50% better than the one with fixed modulation for both energy efficiency and throughput and this comes from the adaptivity of both modulation and power by energy-efficient transmission to the channel status. We note that while energy-efficient scheduling can optimize the energy utilization, overall throughput is not optimized. Observing the performance of proportional scheduler with different values of transmit power in Figures 4(a) and 4(b), we note that the throughput increases as the transmit power, while the energy efficiency decreases. Energy efficiency and throughput efficiency do not necessarily agree.

TABLE II: Scheduling and Transmission Schemes

Legend	Scheduler	Modulation
OptEE	energy-efficient scheduler without fairness	energy-efficient transmission
RREE	round-robin scheduler	energy-efficient transmission
RRTrad	round-robin scheduler	2, 4, 8-QAM selected with equal probability
PropTrad	proportional scheduler	adaptive modulation with fixed transmit power
PropEE	energy-efficient scheduler with proportional fairness	energy-efficient transmission

Figure 5 compares the cumulative distribution functions of energy efficiency when 13 users are active. While the energy-efficient scheduler without fairness has the most percentage of users at high energy efficiency, the one with proportional fairness achieves the best fairness across different users with lowest percentage at low energy efficiency range. The energy-efficient scheduler with proportional fairness performs better than both round-robin schedulers, which means that while assuring better fairness among all users, it also achieves higher energy efficiency.

V. Conclusion

In this paper, we design link adaptation and resource allocation schemes that emphasize energy efficiency. Both circuit and transmit power are taken into account. Scheduling policies are also designed for BSs to improve the overall network energy efficiency. Simulation results show that the proposed schemes significantly improve mobile energy efficiency per link as well as across the network. Here, we focused on the case of flat fading OFDMA channels. Extension of energy-efficient design for frequency selective OFDMA channels, will be addressed in future work.

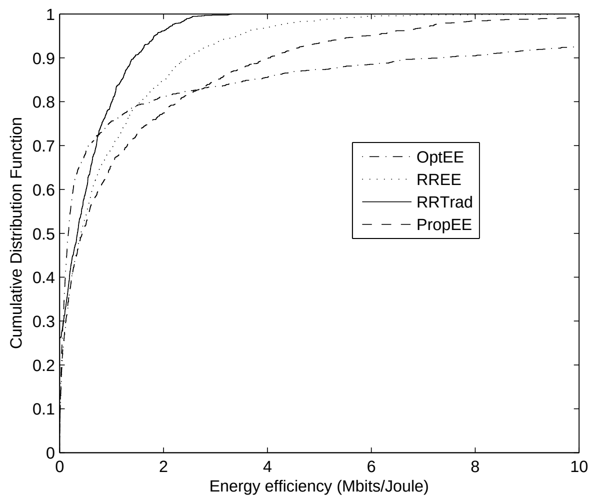


Fig. 5: Fairness comparisons

Appendix A
Proof of Theorem 1

Proof: $R^* = \arg\max_R \frac{R}{P_C+P_T(R)} = \arg\min_R \frac{P_C+P_T(R)}{R}$. Denote $f(R) = \frac{P_C+P_T(R)}{R}$. Let $R = \frac{1}{t} > 0$, and $g(t) = f(\frac{1}{t}) = P_C t + P_T(\frac{1}{t})t$. Then $R^* = \frac{1}{t^*}$ and $t^* = \arg\min_t g(t)$. Since $\frac{\partial^2 g(t)}{\partial t^2} = \frac{1}{t^3} P_T^{''}(\frac{1}{t}) > 0$, $g(t)$ is strictly convex in t. Since $P_T(R)$ is monotonically increasing and strictly convex in R, the derivative satisfies $\lim_{R->\infty} P_T^{'}(R) = \infty$. According to the L'Hopital's rule, $\lim_{t->0} g(t) = \lim_{t->0} P_T(\frac{1}{t})t = \lim_{R->\infty} \frac{P_T(R)}{R} = \lim_{R->\infty} \frac{P_T^{'}(R)}{1} = \infty$. Besides, $\lim_{t->\infty} g(t) = \infty$. Since $g(t) < \infty$ for $0 < t < \infty$, t^* uniquely exists and is globally optimal. By letting $\frac{\partial g(t)}{\partial t} = 0$ and $R = \frac{1}{t}$, we have the solution in Equation (8). ■

Appendix B
Proof of Theorem 2

Proof: Denote $P_R(r)$ to be the received power on a subchannel for reliable detection when the data rate on the subchannel is r. We have

$$P_T(R) = \frac{cP_R(r)}{g} = \frac{cP_R(\frac{R}{c})}{g}, \tag{B.24}$$

where g is the channel power gain. It is easy to see that $P_R(r)$ is monotonically increasing and strictly convex, and $P_T(0) = P_R(0) = 0$. According to Theorem 1, we have $R^* P_T^{'}(R^*) = P_C + P_T(R^*)$, which is equivalent to

$$R^* P_R^{'}(\frac{R^*}{c}) - cP_R(\frac{R^*}{c}) = P_C g. \tag{B.25}$$

By differentiating the left hand side of (B.25) with respect to R^*, $\frac{\partial\left(R^* P_R^{'}(\frac{R^*}{c}) - cP_R(\frac{R^*}{c})\right)}{\partial R^*} = \frac{R^*}{c} P_R^{''}(\frac{R^*}{c}) > 0$. Hence, the left hand side of (B.25) is strictly increasing in R_i^*. Therefore, higher modulation order should be used when the channel has higher power gain.

Suppose $g_1 > g_2$, and the corresponding optimal modulations and codings result in data rates R_1^* and R_2^* respectively. Hence, $U_1(R_1^*) > U_1(R_2^*)$. Besides, $U_1(R_2^*) = \frac{R_2^*}{P_C + \frac{cP_R(R_2^*/c)}{g_1}} > \frac{R_2^*}{P_C + \frac{cP_R(R_2^*/c)}{g_2}} = U_2(R_2^*)$. Hence, the energy efficiency increase with channel gain. ■

Appendix C
Proof of Theorem 3

Proof: $R = cr$ and $P_T(R) = c\overline{P}_T(r) = c\overline{P}_T(\frac{R}{c})$, where $\overline{P}_T(r)$ is the transmit power on each subchannel, and is monotonically increasing and strictly convex in r. According to Theorem 1, we have $R^* \overline{P}_T^{'}(\frac{R^*}{c}) = P_C + c\overline{P}_T(\frac{R^*}{c})$, which is equivalent to $r^* \overline{P}_T^{'}(r^*) - P_T(r^*) = \frac{P_C}{c}$. The left hand side is increasing in r^* while the right hand side is decreasing in c. Hence, the modulation order on each subchannel should decrease with increasing number of subchannels assigned.

The proof that the energy efficiency increases with the number of subchannels assigned is similar to the proof in B and is omitted here. ■

References

[1] K. Lahiri, A. Raghunathan, S. Dey, and D. Panigrahi, "Battery-driven system design: A new frontier in low power design," in *Proc. Intl. Conf. on VLSI Design*, Bangalore, India, pp. 261-267, Jan. 2002.

[2] Y. Xiao, "Energy saving mechanism in the IEEE 802.16e wireless MAN," *IEEE Commun. Letters*, vol. 9, no. 7, pp. 595 - 597, July 2005.

[3] F. Meshkati, H. V. Poor, S. C. Schwartz, and N. B. Mandayam, "An energy-efficient approach to power control and receiver design in wireless networks," *IEEE Trans. Commun.*, vol. 5, no. 1, pp. 3306-3315, Nov. 2006.

[4] N. Feng, S. C. Mau, and N. B. Mandayam, "Pricing and Power Control for Joint Network-Centric and User-Centric Radio Resource Management," *IEEE Trans. Commun.*, vol. 52, no. 9, pp. 1547-1557, Sep. 2004.

[5] G. Miao, and Z. Niu, "Practical Feedback Design based OFDM Link Adaptive Communications over Frequency Selective Channels," in *Proc. IEEE Conf. Commun. 2006*, Istanbul, Turkey, June 2006, pp. 4624-4629.

[6] IEEE, "IEEE 802.16e-2004, part 16: air interface for fixed and mobile broadband wireless access systems - amendment for physical and medium access control layers for combined fixed and mobile operation in licensed bands," Nov., 2004.

[7] IEEE, "IEEE 802.16e-2005, part 16: air interface for fixed and mobile broadband wireless access systems - amendment2: physical and medium access control layers for combined fixed and mobile operation in licensed bands," Feb., 2006.

[8] A.J.Goldsmith and S. G. Chua "Variable-rate variable-power MQAM for fading channels," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1218-1230, Oct. 1997.

[9] G. Song, and Y. Li, "Cross-layer optimization for OFDM wireless networks-part II: algorithm development," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 625-634, March 2005.

[10] R. Mazumdar, L. G. Mason, and C. Douligieris, "Fairness in network optimal flow control: optimality of product forms," *IEEE Trans. Commun.*, vol 39, no. 5, pp. 775-782, May 1991.