Interference-Aware Energy-Efficient Power Optimization*

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Abstract

Since battery technology has not kept up with the demand for battery capacity on mobile devices, power optimization techniques are becoming increasingly important in wireless system design. Power optimization schemes are critical to interference management in wireless systems as interference usually results from aggressive spectral reuse and high power transmission and severely limits system performance. In this paper, we develop energy-efficient power optimization schemes for interference-limited communications. We consider both circuit and transmit powers and focus on energy efficiency over throughput. Because power optimization problem in the presence of interference is intractable, we first study a simple two-user network with ideal user cooperation and then develop a noncooperative game for energy-efficient power optimization. We further study the existence and uniqueness of equilibrium for the noncooperative power optimization. Then we study the tradeoff between energy efficiency and spectral efficiency and show by simulation results that the proposed scheme improves not only energy efficiency but also spectral efficiency in an interference-limited cellular network.

Index Terms

interference, energy efficiency, power optimization, OFDM, noncooperative power control

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I. INTRODUCTION

As more users need to share the same spectrum for wideband multimedia communications and cellular networks move towards aggressive full frequency reuse scenarios [1], the performance of wireless cellular networks is heavily impaired by interference. This motivates the use of multi-cell power control optimization for interference management [1]–[3]. On the other hand, as higher capacity wireless links are designed to meet increasing demand from multimedia applications, the device power consumption also increases. In contrast, the improvement in battery technology is much slower, leading to an exponentially increasing gap between the required and available battery capacity [4]. Hence, power optimization is also important for maximizing the battery life for mobile devices. Although, power optimization plays a pivotal role in both interference management and energy utilization, little research has addressed their joint interaction. Only an implicit discussion can be found in [5], which summarizes existing approaches that addresses either throughput or energy efficiency separately in the context of power control for CDMA networks. In this paper, we will address this topic and develop energy-efficient power optimization, specifically for interference-limited environments.

Our previous work in [6]–[8] has studied energy efficiency issue in uplink communications in single-cell *orthogonal frequency division multiple access* (OFDMA) systems to improve mobile battery consumption, which has considered both circuit and transmit powers. Using throughput per Joule as a performance metric, we have studied both link adaptation and resource allocation techniques. We have observed that in an interference free environment, a tradeoff between *energy efficiency* (EE) and *spectral efficiency* (SE) exists, as increasing transmit power always improves throughput but not necessarily EE. In this paper, we consider multi-cell interference-limited scenarios and develop power optimization and resource allocation schemes to improve EE. We note that the general power optimization problem in the presence of interference is intractable even when ideal user cooperation is assumed. We first study this problem for a simple two-user network with ideal user cooperation. Then we develop a noncooperative game for energy-efficient power optimization. We show that the equilibrium always exists. Furthermore, when there is only one subchannel or the channel experiences flat fading, there will be a unique equilibrium. However, in frequency-selective channels, this is not true in general as demonstrated by a counter example. We reveal a sufficient condition that assures the uniqueness. Then we investigate the tradeoff between EE and SE. We show that in interference-limited scenarios, since increased transmit power also brings higher interference to the network, SE is not necessarily increased. Energy-efficient power optimization not only improves system EE but also improves the tradeoff between EE and SE due to the conservative nature of power allocation, which effectively controls interference from other cells to improve network throughput. Later, we also design an implementation of the noncooperative power optimization game. Simulation results show that the proposed scheme improves not only energy efficiency but also spectral efficiency in an interference-limited cellular network.

The rest of the paper is organized as follows. We first formulate the interference aware power control problem in Section II. In Section III, a two-user network with ideal user cooperation is discussed to gain insights into energy-efficient power control. Noncooperative energy-efficient power optimization is discussed in Section IV and the performance improvement is demonstrated by simulations in Section V. Finally, we conclude the paper in Section VI.

II. Problem Description

We introduce interference-aware energy-efficient power optimization in this section.

Consider a system with K subchannels. Each of them experiences independent and flat fading and *additive white Gaussian noise* (AWGN). There are N users, each consisting of a pair of transmitter and receiver and operating on these subchannels. All users interfere with each other. Accurate channel state information is available to any pair of transmitter and receiver. Denote the signal power attenuation of User i at Subchannel k to be $g_{ii}^{(k)}$ and the interference power gain from the transmitter of User i to the receiver of User j at Subchannel k to be $g_{ij}^{(k)}$. The noise power on each subchannel is σ^2. The power allocation of User n on all subchannels is denoted by vector $\mathbf{p}_n = [p_n^{(1)} p_n^{(2)} \cdots p_n^{(K)}]^T$, where T is the transpose. The interference on all subchannels of User n is denoted by vector $\mathbf{I}_n = [I_n^{(1)} I_n^{(2)} \cdots I_n^{(K)}]^T$, where

$$I_n^{(k)} = \sum_{i=1, i\neq n}^{N} p_i^{(k)} g_{in}^{(k)}. \tag{1}$$

Consequently, the *signal-to-interference-plus-noise ratio* (SINR), $\eta_n^{(k)}$, of User n at Subchannel k can be expressed as

$$\eta_n^{(k)} = \frac{p_n^{(k)} g_{nn}^{(k)}}{\sum_{i=1, i\neq n}^{N} p_i^{(k)} g_{in}^{(k)} + \sigma^2}. \tag{2}$$

The data rate at Subchannel k of User n, $r_n^{(k)}$, is assumed to be a function of η_n and can be expressed as

$$r_n^{(k)} = R(\eta_n^{(k)}), \tag{3}$$

where $R()$ depends on the modulation and coding and is assumed to be strictly concave and increasing with $R(0) = 0$. For capacity approaching coding [9], $r_n^{(k)} = w \log(1 + \eta_n^{(k)})$, where w

is the bandwidth of each subchannel, and the above assumptions are obviously satisfied.

Let the data rate vector of User n across the K subchannels be $\mathbf{r}_n = [r_n^{(1)}, r_n^{(2)}, \cdots, r_n^{(K)}]^T$, then the overall data rate is

$$r_n = \sum_{k=1}^{K} r_n^{(k)}. \tag{4}$$

The total transmit power is

$$p_n = \sum_{k=1}^{K} p_n^{(k)}. \tag{5}$$

Note that as in [6], [7], both transmit power and circuit power, p_c, are important for energy-efficient communications. While transmit power is used for reliable data transmission, circuit power represents average energy consumption of device electronics. As in [7], we optimize the energy efficiency, which can be expressed as

$$u_n = \frac{r_n}{\triangle e / \triangle t} = \frac{r_n}{p_n + p_c}, \tag{6}$$

where r_n is given by (4) and p_n by (5). u_n is called EE of User n.

Note that if we fix the overall transmit power, the objective of Equation (6) is equivalent to maximizing the overall throughput of all subchannels and existing water-filling power allocation approach [9] gives the solution. However, besides power distributions on all subchannels, the overall transmit power needs to be adapted according to the states of all subchannels to maximize energy efficiency. Hence, the solution to Equation (6) is in general different from existing power allocation schemes that maximize throughput with power constraints. The power control in a multi-cell setting to optimize the overall network energy efficiency is also different from traditional power control schemes that emphasize throughput improvement.

We define EE of the overall network to be

$$u = \sum_{n=1}^{N} u_n, \tag{7}$$

which is a function of $p_n^{(k)}$ for all n and k. This definition is based on summation of EE of all users rather than the ratio of sum network throughput to sum network power consumption because powers of different users can not be shared and so are their throughput and EE.

We now need to determine power allocation of all users to optimize overall network EE subject to the interference scenario.

III. Cooperative Two-User Case

Note that the solution maximizing sum network EE is difficult to obtain as the objective function, in general, is non-concave in $p_n^{(k)}$. To gain some insight, we investigate the case where two users transmit simultaneously on a single channel in this section. We assume both users have complete network knowledge and cooperate to maximize the sum energy efficiency,

$$u(p_1, p_2) = \frac{r_1}{p_1 + p_c} + \frac{r_2}{p_2 + p_c}, \tag{8}$$

where

$$r_1 = w \log(1 + \frac{p_1 g_1}{p_2 g_{21} + \sigma^2}) \qquad \text{and} \qquad r_2 = w \log(1 + \frac{p_2 g_2}{p_1 g_{12} + \sigma^2}). \tag{9}$$

As u is non-concave in p_1 and p_2, finding the global maximum is intractable. However, we can get some effective approaches by restricting our attention to some special regimes.

A. Circuit Power Dominated Regime

In this regime, circuit power dominates power consumption, i.e. $p_c \gg p_n$ for $n = 1, 2$. This is usually true for short-range communications as small transmit power is needed to compensate for path loss. In this case, we have

$$u(p_1, p_2) \approx \frac{w}{p_c} \left(\log(1 + \frac{p_1 g_1}{p_2 g_{21} + \sigma^2}) + \log(1 + \frac{p_2 g_2}{p_1 g_{12} + \sigma^2}) \right). \tag{10}$$

Hence, maximizing EE is equivalent to maximizing sum network capacity, which has been discussed in literature [2], [3]. The optimal solution takes on the form of binary power control where each user either shuts down or transmits with full power [3]. Whether two users transmit simultaneously or exclusively depends on interference strength.

B. Transmit Power Dominated Regime

When the circuit power is negligible, e.g. in extremely long distance communications where transmit power should be strong enough to compensate for large path loss,

$$u(p_1, p_2) \approx \frac{w \log(1 + \frac{p_1 g_1}{p_2 g_{21} + \sigma^2})}{p_1} + \frac{w \log(1 + \frac{p_2 g_2}{p_1 g_{12} + \sigma^2})}{p_2}. \tag{11}$$

By examining derivatives of $u(p_1, p_2)$ in Appendix I, we can see that $u(p_1, p_2)$ is strictly decreasing with both p_1 and p_2. Hence, the optimal solution is to allocate as low power as possible. However, the above conclusion holds only when the circuit power is negligible. When the transmit power is comparable to the circuit power, other approaches are needed to determine the optimal power.

C. Noise Dominated Regime

Now we look at the problem from a different perspective. When noise is much stronger than interference, we have

$$u(p_1, p_2) \approx \frac{w \log(1 + \frac{p_1 g_1}{\sigma^2})}{p_1 + p_c} + \frac{w \log(1 + \frac{p_2 g_2}{\sigma^2})}{p_2 + p_c}. \tag{12}$$

Hence, the problem is decoupled and the sum network EE is maximized when each user selects power to maximize their own EE, which has already been given in [6], [7].

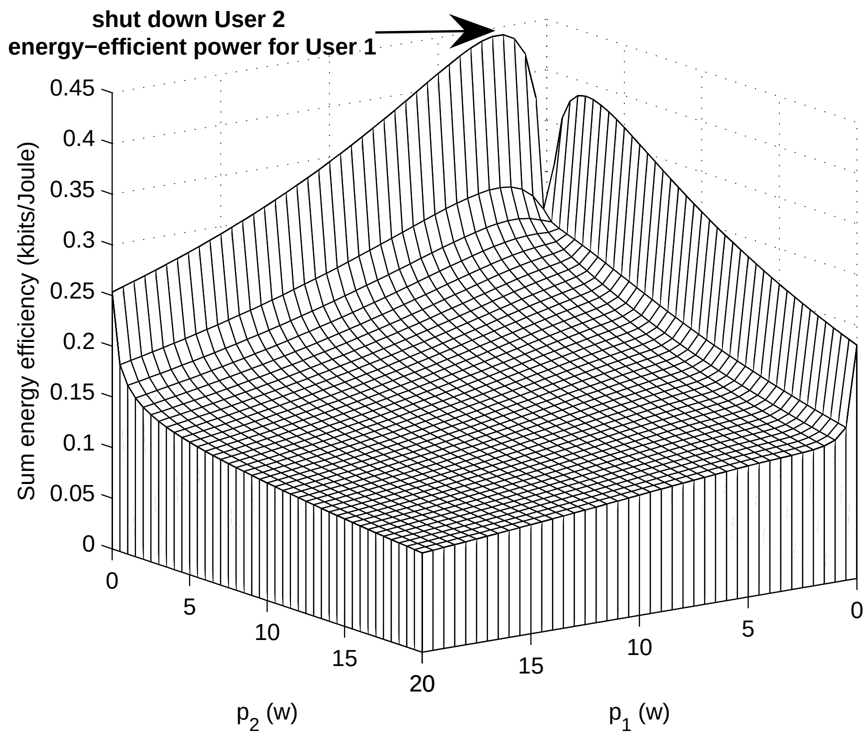
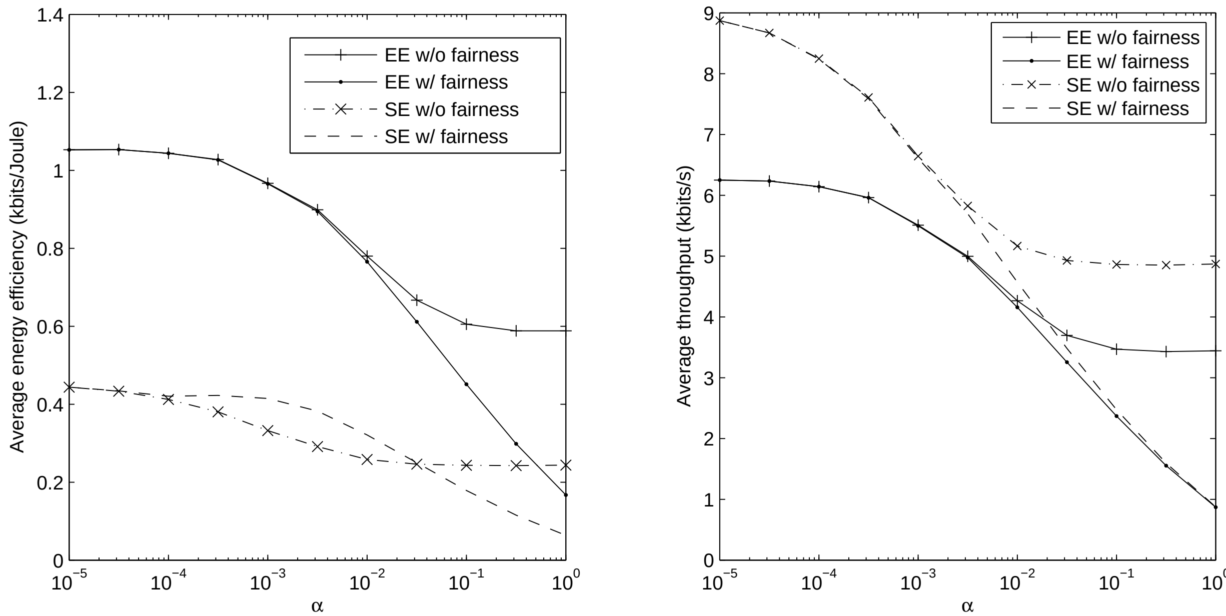


Fig. 1: Sum energy efficiency and transmit powers in interference dominated regime.

D. Interference Dominated Regime

In the interference dominated regime, interference is much stronger than noise, i.e. $p_1 g_{12} \gg \sigma^2$ and $p_2 g_{21} \gg \sigma^2$ for any feasible p_1 and p_2 that support reliable transmission. To be specific, we require that $p_1 g_{12} \gg \sigma^2$ and $p_2 g_{21} \gg \sigma^2$ are significant enough that the *interference-to-noise ratio* (INR) and SINR of each user satisfies

$$INR > 1 + SINR. \tag{13}$$



(a) Energy efficiency.

(b) Spectral efficiency.

Fig. 2: Comparison of cooperative EE and SE.

Note that Equation (13) does hold when the interference is strong engough since INR increases with interference power while SINR decreases with it. The interference dominated regime exists when different transmissions are close to each other, e.g. closely coupled. Hence,

$$u(p_1, p_2) \approx \frac{w \log(1 + \frac{p_1 g_1}{p_2 g_{21}})}{p_1 + p_c} + \frac{w \log(1 + \frac{p_2 g_2}{p_1 g_{12}})}{p_2 + p_c}. \tag{14}$$

In Appendix II, we show that $u(p_1, p_2)$ is maximized by an ON-OFF approach, i.e. letting the user with higher channel gain to transmit with energy-efficient power selection and shutting down the other. Figure 1 illustrates an example when the average interference-to-noise ratio is 20 dB. In Figure 1, the sum energy-efficiency is maximized by shutting down User 2 and choosing power for User 1 to maximize its EE.

E. Spectral Efficiency and Energy Efficiency Tradeoff with Cooperation

Our previous research [6] has shown that maximizing EE and maximizing SE usually disagree. Therefore, tradeoff between them exists. To examine the impact of interference on this tradeoff when ideal cooperation exists, we consider a symmetric two-user network and compare energy-efficient schemes with spectral-efficient ones. Both users experience Rayleigh fading. Different power optimizations result in different interference scenarios. To characterize interference level, we need to use a metric that is independent of transmit powers. Hence, define network coupling factor α,

$$\alpha = \frac{\text{average interference channel gain}}{\text{average signal channel gain}}. \tag{15}$$

α characterizes what level different transmissions interfere with each other and higher α represents heavier interfering scenario.

Consider two network performance metrics, arithmetic and geometric means. The power is optimized to maximize the arithmetic- or geometric-mean metric. It has been indicated in [10] that optimization based on the arithmetic average of SE leads to power allocation for sum throughput maximization that considers no fairness since some users may have zero throughput, and optimization based on the geometric average also assures proportional fairness among all users. Optimization based on the arithmetic and geometric averages of EE has similar characteristics and we call arithmetic- or geometric-mean metrics for EE optimization to be energy-efficient power optimization schemes without or with fairness.

In Figure 2, the EEs and throughputs of four schemes are compared when α has different values. The schemes are the energy-efficient and spectral-efficient power allocation either with or without proportional fairness. A peak power constraint is applied in the power allocation. The schemes without fairness allocate power to maximize the sum of either EE (energy-efficient) or throughput (spectral-efficient) while those with proportional fairness maximize the product. From Figure 2, we can see that the tradeoff between SE and EE depends on the network coupling.

IV. Noncooperative Energy-Efficient Communications

The above section discusses energy-efficient power optimization with ideal cooperation in a two-user network. Extension to special regimes for a multi-user network is straightforward and omitted. However, in general, it is difficult to determine the globally optimal power allocation due to the nonconcavity of sum energy-efficiency functions. More users and subchannels will result in more local maximums and searching the globally optimal power allocation would be a daunting task. Even if the globally optimal solution can be found, it is still impractical since it requires complete network knowledge, including interference channel gains. In this section, we consider a more practical case and assume no cooperation among users. To assure fairness, all users apply the same policy using local information. In the following, we first model the noncooperative energy-efficient control from a game-theory perspective and then discuss the existence and uniqueness of its equilibrium. Then we investigate SE and EE tradeoff assuming symmetric channel condition to obtain insights. After that, we further develop a noncooperative energy-efficient power control scheme to facilitate implementation.

A. Noncooperative Energy-Efficient Power Optimization Game

Since the network energy efficiency depends on the behaviors of two or more users, we model the power control to be a noncooperative game in game theory [11]. Rooted in economics, game theory has been broadly applied in wireless communications for random access and power control optimizations [2], [12].

In a noncooperative game, each user optimizes power allocation to maximize its energy efficiency. Consider the power allocation of User n and denote the power vectors of other users to be vector

$$\mathbf{p}_{-n} = (\mathbf{p}_1^T, \mathbf{p}_2^T, \cdots, \mathbf{p}_{n-1}^T, \mathbf{p}_{n+1}^T, \cdots, \mathbf{p}_N^T)^T. \tag{16}$$

Given $\mathbf{p}_{-n}$, the best response of the power allocation of User n is

$$\mathbf{p}_n^o = f_n(\mathbf{p}_{-n}) = \arg\max_{\mathbf{p}_n} u_n(\mathbf{p}_n, \mathbf{p}_{-n}), \tag{17}$$

where u_n is given by (6) and is a function of both $\mathbf{p}_n$ and $\mathbf{p}_{-n}$. $f_n(\mathbf{p}_{-n})$ is called the best response function of User n. The existence and uniqueness of $\mathbf{p}_n^o$, i.e. the best response, is assured by Theorem I in our previous work [7].

Note that noncooperative power control is not efficient in terms of SE optimization since users tend to act selfishly by increasing their transmit power beyond what is reasonable [12]. Hence, pricing mechanisms are introduced to regulate the aggressive power transmission by individuals to produce more socially beneficial outcome towards improving sum throughput of all users [2]. Different from SE optimal power control, energy-efficient power optimization desires a power setting that is greedy in EE but chary of power. Furthermore, Problem (17) is equivalent to

$$\begin{aligned} \mathbf{p}_n^o &= \arg\max_{\mathbf{p}_n} \log(u_n(\mathbf{p}_n, \mathbf{p}_{-n})) \\ &= \arg\max_{\mathbf{p}_n} \left(\log(r_n) - \log(p_n + p_c)\right), \end{aligned} \tag{18}$$

which implies that energy-efficient power control can be regarded as a variation of traditional spectral-efficient one with power pricing [2]. Since this power-conservative expression is socially favorable in interference-limited scenarios, energy-efficient power control is desirable to reduce interference and improve throughput in a noncooperative setting.

Each user optimizes their power independently. The variation of power allocation of one user impacts those of all others. Equilibrium is the condition of a network in which competing influences are balanced assuming invariant channel conditions. Its properties are important to network performance. Hence, we characterize the equilibrium of noncooperative energy-efficient power optimization in the following three sections.

B. Existence of Equilibrium

In a noncooperative game, a set of strategies is said to be at Nash equilibrium, referred as equilibrium in the following, if no user can gain individually by unilaterally altering its own strategy. Denote the equilibrium as

$$\mathbf{p}^* = (\mathbf{p}_1^*, \mathbf{p}_2^*, \cdots, \mathbf{p}_N^*). \tag{19}$$

Nash equilibrium can be described by the following definition.

Definition 1. *In an energy-efficient noncooperative game, an equilibrium is a set of power allocation that no user can unilaterally improve its energy efficiency by choosing a different set of power allocation, i.e.*

$$\mathbf{p}^* = f(\mathbf{p}^*) = (f_1(\mathbf{p}_{-1}^*), f_2(\mathbf{p}_{-2}^*), \cdots, f_N(\mathbf{p}_{-N}^*)), \tag{20}$$

where $f(\mathbf{p})$ is the network response function.

The network response relies on energy efficiency of all users. In the following, we first give the properties of energy efficiency function and then study the existence of equilibrium. To facilitate our discussion, we first introduce the concept of quasiconcavity.

Definition 2. *As defined in [13], a function z, which maps from a convex set of real n-dimensional vectors, $\mathcal{D}$, to a real number, is called strictly quasiconcave if for any $\mathbf{x}_1, \mathbf{x}_2 \in \mathcal{D}$ and $\mathbf{x}_1 \neq \mathbf{x}_2$,*

$$z(\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) > \min\{z(\mathbf{x}_1), z(\mathbf{x}_2)\}, \tag{21}$$

for any $0 < \lambda < 1$.

Lemma 1 characterizes the energy efficiency function and is proved in Appendix III.

Lemma 1. *$u_n(\mathbf{p}_n, \mathbf{p}_{-n})$ is strictly quasiconcave in $\mathbf{p}_n$.*

Based on Lemma 1, the existence of equilibrium $\mathbf{p}^*$ is given by Theorem 1. A necessary and sufficient condition for a set of power allocation to be an equilibrium is also summarized in Theorem 1. The proof can be found in Appendix IV.

Theorem 1 (Existence)**.** *There exists at least one equilibrium $\mathbf{p}^*$ in the noncooperative energy-efficient power optimization game defined by (17). A set of power allocation of all users, $\mathbf{p}^* = (\mathbf{p}_1^*, \mathbf{p}_2^*, \cdots, \mathbf{p}_N^*)$, is an equilibrium if and only if it satisfies that, for any Subchannel i of any User n,* *(i) if* $\frac{\sum_{j\neq i} r_n^{(j)*}}{p_c+\sum_{j\neq i} p_n^{(j)*}} \leq R^{'}(0)\gamma_n^{(i)*}$, $\left.\frac{\partial u_n(\mathbf{p}_n,\mathbf{p}_{-n}^*)}{\partial p_n^{(i)}}\right|_{\mathbf{p}_n=\mathbf{p}_n^*} = 0$, *i.e.* $R^{'}(\gamma_n^{(i)*}p_n^{(i)*})\gamma_n^{(i)*} = u(\mathbf{p}_n^*, \mathbf{p}_{-n}^*)$*;*

(ii) otherwise, $p_n^{(i)*} = 0$*,*

where $\gamma_n^{(i)*} = \frac{g_{nn}^{(i)}}{\sum_{j=1,j\neq n}^{N} p_j^{(i)*} g_{jn}^{(i)}+\sigma^2}$.

C. Uniqueness of Equilibrium in Flat Fading Channels

In this section, we discuss the uniqueness of the equilibrium. First, we consider a special case when there is a single subchannel in a network and

$$p_n^o = f_n(\mathbf{p}_{-n}) = \arg\max_{p_n} u_n(p_n, \mathbf{p}_{-n}). \tag{22}$$

Proposition 1 shows the properties of the response functions and is proved in Appendix V.

Proposition 1. *When there is only one subchannel, the power allocation, i.e. the response functions, of all users satisfy*

- *Concavity:* $f_n(\mathbf{p}_{-n})$ *is strictly concave in* $\mathbf{p}_{-n}$*;*
- *Positivity:* $f_n(\mathbf{p}_{-n}) > 0$*;*
- *Monotonicity: If* $\mathbf{p}_{-n} \succ \mathbf{q}_{-n}$, $f_n(\mathbf{p}_{-n}) > f_n(\mathbf{q}_{-n})$*;*
- *Scalability: For all* $\alpha > 1$, $\alpha f_n(\mathbf{p}_{-n}) > f_n(\alpha\mathbf{p}_{-n})$*,*

where $\succ$ *denotes vector inequality and each element of the vector satisfies the inequality.*

Note that the monotonicity indicates that increasing interference results in increasing transmit power while the scalability indicates that variation of transmit power is always smaller than that of the interference power. These assure the convergence to a unique equilibrium.

The properties in Proposition 1 can be extended to networks with multiple subchannels where all subchannels experience the same channel gain, i.e. flat-fading channels. This can be done by defining $f_n(\mathbf{p}_{-n})$ to be the optimal total transmit power on all subchannels and the four properties can be easily verified by the approaches in Appendix V.

Theorem 2 (Uniqueness)**.** *When the channel experiences flat fading, there exists one and only one equilibrium* $\mathbf{p}^*$ *in the noncooperative energy-efficient power optimization game defined by (17).*

Proof: It has been shown in [14] that a noncooperative power control with positivity, monotonicity, and scalability has a unique fixed point $\mathbf{p} = f(\mathbf{p})$. Hence, we have the above theorem. ■

D. Uniqueness of Equilibrium in Frequency-Selective Channels

When there are multiple subchannels which experience frequency-selective fading, whether there is a unique equilibrium depends on channel conditions.

Consider a network with two users as an example. Let $p_c = 1, w = 1, \sigma^2 = 1, g_{11}^{(1)} = g_{11}^{(2)} = g_{22}^{(1)} = g_{22}^{(2)} = 1, g_{12}^{(1)} = g_{21}^{(1)} = 1e^{-10}, g_{12}^{(2)} = g_{21}^{(2)} = 1e^{10}$. We show in Appendix VI that one of the equilibrium has the form $\mathbf{p}_1^* = [p_a\ p_b]$ and $\mathbf{p}_2^* = [p_c\ 0]$, where p_a, p_b, and p_c are positive. Due to the symmetry of network conditions, $\mathbf{p}_1 = [p_c\ 0]$ and $\mathbf{p}_2 = [p_a\ p_b]$ also form an equilibrium. Hence, the network has at least two equilibria. When there are more users and subchannels, more equilibria will exist in general. However, in the following, we will show that when the interfering channels satisfy a certain condition, there will be a unique equilibrium.

We consider a general noncooperative power control over multiple subchannels where each user selfishly chooses power allocation to maximize its own utility in an interference-limited environment. The utility, denoted by $U_n(\mathbf{p}_n, \mathbf{I}_n(\mathbf{p}_{-n}))$, is assumed to be quasiconcave in $\mathbf{p}_n$ given $\mathbf{I}_n$, interferences on all subchannels. $\mathbf{I}_n$ is a function of $\mathbf{p}_{-n}$ and is determined by (1). The best response of power allocation of User n is denoted to be

$$\mathbf{p}_n^o = F_n(\mathbf{p}_{-n}) = \tilde{F}_n(\mathbf{I}_n(\mathbf{p}_{-n})) = \arg\max_{\mathbf{p}_n} U_n(\mathbf{p}_n, \mathbf{I}_n(\mathbf{p}_{-n})). \tag{23}$$

The noncooperative energy-efficient power optimization in (17) is an example of (23).

Denote the Jacobian matrix of $\tilde{F}_n$ at $\mathbf{I}_n$ to be $\frac{\partial \tilde{F}_n}{\partial \mathbf{I}_n}$ and the Jacobian matrix of $\mathbf{I}_n$ at $\mathbf{p}_{-n}$ to be $\frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}}$. Denote $||A||$ to be the Frobenius norm of matrix $A = (a_{ij})$, i.e. $||A|| = \sqrt{\sum_{i,j} a_{ij}^2}$. We know that when a contraction mapping has a fixed point, the fixed point is unique [15]. Readily, we have the following sufficient condition, which comes from [16], that assures a unique equilibrium.

Theorem 3 (Uniqueness)**.** *In frequency selective channels, if for any User* n*,* $||F_n(\mathbf{p}_{-n}) - F_n(\check{\mathbf{p}}_{-n})|| < ||\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}||$ *for any different* $\mathbf{p}_{-n}$ *and* $\check{\mathbf{p}}_{-n}$*, there exists one and only one equilibrium* $\mathbf{p}^*$ *in the noncooperative power control game defined by (23).*

Intuitively, Theorem 3 says that if other users change their transmit powers by some amount, the best power allocation of the user is altered by a lesser amount, then the equilibrium is unique. Note that the transmit powers of other users and the best response $F_n(\mathbf{p}_{-n})$ in (23) are related through interference channel gains, which therefore determines the variation of the best response and whether the sufficient condition can be guaranteed. Stronger interference channel gains result in higher correlation and vice versa. The above two-user network illustrates an example where one subchannel has extremely strong interference channel gains. In this case, the sufficient condition is violated and there are multiple equilibria.

Based on Theorem 3, Theorem 4 explicitly shows the impact of interference channel gains on the number of equilibria and is proved in Appendix VII.

Theorem 4 (Uniqueness)**.** *In frequency selective channels, if for any User n,*

$$\left|\left|\frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}}\right|\right| < \frac{1}{\sup_{\mathbf{I}_n}\left|\left|\frac{\partial \tilde{F}_n}{\partial \mathbf{I}_n}\right|\right|}, \tag{24}$$

where $\sup_{\mathbf{I}_n}$ is the supremum on all feasible $\mathbf{I}_n$, there exists one and only one equilibrium $\mathbf{p}^*$ in the noncooperative power control game defined by (23).

After examining the Jacobian matrices, we see that the left hand side of (24) depends on interference channel gains only, while the right hand side is independent of interference channel gains. Hence, interference channel gains directly impacts the number of equilibria. Consider an example where different users are sufficiently far away and all interference channel gains are close to zero. It is easy to see that transmit powers of other users have almost no effect on the best response of the user and there is a unique equilibrium.

Note that while a sufficient condition of a unique equilibrium for distributed power control over a single channel is given in [14], we provide sufficient conditions of a unique equilibrium for distributed multichannel power controls in Theorems 3 and 4, which can be applied to different kinds of distributed *multiple input multiple output* (MIMO) and *orthogonal frequency-division multiplexing* (OFDM) systems.

Given Theorems 3 and 4, a sufficient condition to assure a unique equilibrium of the noncooperative energy-efficient power optimization follows immediately.

Theorem 5 (Uniqueness)**.** *In frequency selective channels, the noncooperative energy-efficient power optimization game defined by (17) has a unique equilibrium if for any User n, $||f_n(\mathbf{p}_{-n}) - f_n(\check{\mathbf{p}}_{-n})|| < ||\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}||$ for any different $\mathbf{p}_{-n}$ and $\check{\mathbf{p}}_{-n}$ or $\left|\left|\frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}}\right|\right| < \frac{1}{\sup_{\mathbf{I}_n}\left|\left|\frac{\partial f_n}{\partial \mathbf{I}_n}\right|\right|}$.*

Note that the above theorem only gives sufficient conditions of uniqueness that may not be necessary ones. For example for a single-channel network, due to the strict concavity of $f_n(\mathbf{p}_{-n})$, $\sup_{I_n}\left|\left|\frac{\partial f_n}{\partial I_n}\right|\right| = \left.\frac{\partial f_n}{\partial I_n}\right|_{I_n=0}$. However, for all interference channel gains, there is always a unique equilibrium, as shown in Theorem 2.

E. SE and EE Tradeoff without Cooperation

In this section, we investigate the tradeoff between noncooperative energy-efficient power optimization and noncooperative spectral-efficient power control schemes. Here, no peak power constraint is assumed to investigate performance limit.

Consider a symmetric single-channel network to simplify analysis and to get insights. There are N users, all experiencing the same channel power gain g. All interference channels have the same power gain $\widetilde{g}$. The network coupling factor is

$$\alpha = \frac{\widetilde{g}}{g}. \tag{25}$$

Consider the equilibrium, which is unique according to Theorem 2. Due to the assumption of network symmetry, all users transmit with the same power in the equilibrium. Denote the transmit power of all users to be p.

The overall network EE is

$$u(p) = \sum_{n=1}^{N} \frac{w \log\left(1 + \frac{pg}{\sum_{i,i\neq n} p\widetilde{g} + \sigma^2}\right)}{p + p_c} = \frac{Nw \log\left(1 + \frac{p}{(N-1)\alpha p + \frac{\sigma^2}{g}}\right)}{p + p_c}, \tag{26}$$

and the network SE is

$$r(p) = N \log\left(1 + \frac{p}{(N-1)\alpha p + \frac{\sigma^2}{g}}\right). \tag{27}$$

With noncooperative spectral-efficient power control, every user allocates power to selfishly maximize its SE. Without power limit, the transmit power tends to infinity in the equilibrium. Besides, we can see that $r(p)$ is strictly increasing in p. Hence, the maximum network SE is obtained in the equilibrium and the upperbound is

$$r_{SE} = \lim_{p\to\infty} r(p) = N \log(1 + \frac{1}{(N-1)\alpha}) \tag{28}$$

with the corresponding EE $u_{SE} = \lim_{p\to\infty} u(p) = 0$, which is completely energy inefficient and noncooperative SE optimal power control is not desired for energy efficiency.

With noncooperative energy-efficient power optimization, the network energy efficiency at the equilibrium is $u_{EE} = u(p^*)$ with the corresponding SE $r_{EE} = r(p^*)$. Hence, the SE penalty of energy-efficient power optimization is

$$r_{tr} = r_{SE} - r_{EE} = N \log(1 + \frac{1}{(N-1)\alpha}) - r(p^*). \tag{29}$$

In an interference-free scenario, i.e. $N = 1$ or $\alpha = 0$, the penalty is infinite. Otherwise, whenever interference exists, it is bounded.

To further understand the tradeoff, Figure 3 illustrates a case when two users transmit with the same power and interfere with each other. Curves with markers draw the relationship between transmit power and SE when the network has different couplings while those without markers draw the corresponding energy efficiency. When $\alpha = 0$, arbitrary SE can be obtained by choosing enough

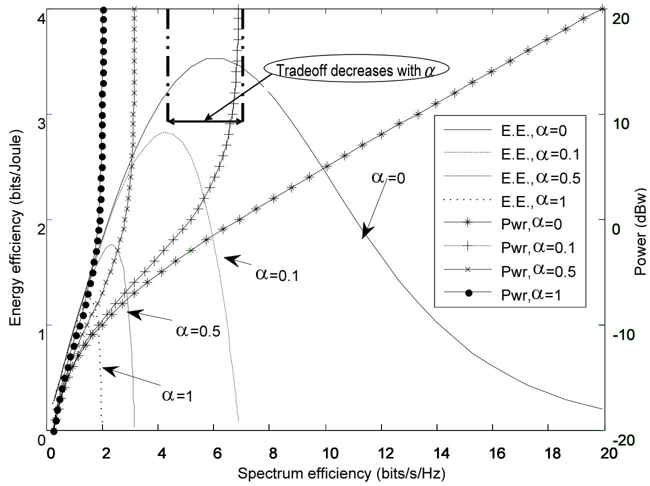


Fig. 3: Tradeoff of EE and SE with different interfering scenarios ($p_c = 1, g = 1, \sigma^2 = 0.01, N = 2$).

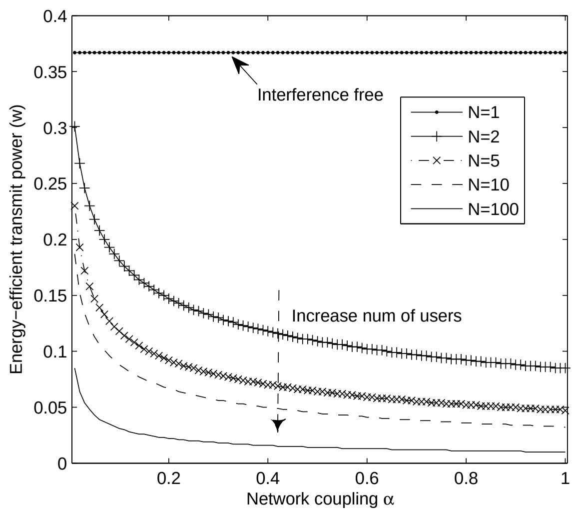


Fig. 4: Noncooperative energy-efficient power optimization in the equilibrium($P_c = 1, g = 1, \sigma^2 = 0.01$).

transmit power. When $\alpha > 0$, regions beyond the SE upperbound is not achievable. Furthermore, EE is much more sensitive to power selection than SE. In interference-limited scenarios, increasing transmit power beyond the optimal power for EE has little SE improvement but significantly hurts EE. Furthermore, power optimization to achieve the highest energy efficiency will also have reduced SE tradeoff with the increase of α. Figure 4 shows the transmit power in the equilibrium when the network has different couplings and numbers of users. The equilibrium power decreases with either user number or α and automatically alleviates network interference.

F. Implementation of Noncooperative Energy-Efficient Power Optimization

In the previous section we know that energy-efficient power optimization is advantageous in interference-limited scenarios due to its conservative power allocation nature. In this section, we will develop practical approaches for noncooperative energy-efficient power optimization.

In (17), the best response of User n depends on the transmit power vectors of all other users, $\mathbf{p}_{-n}$, which can not be obtained in a noncooperative setting. Instead, we observe that $\mathbf{p}_{-n}$ affects the best response in the form of interference, which thus contains sufficient information of $\mathbf{p}_{-n}$ to determine the best response and can be acquired locally. Hence, we let each user measure interferences on all subchannels to determine the power optimization.

At time $t-1$, the measured interference powers on all subchannels of User n are denoted by $\mathbf{I}_n[t-1] = [I_n^{(1)}[t-1], I_n^{(2)}[t-1], \cdots, I_n^{(K)}[t-1]]^T$. Denote the predicted SINR to be

$$\widehat{\eta}_n^{(k)}[t] = \frac{p_n^{(k)}[t] g_{nn}^{(k)}}{\widehat{I}_n^{(k)}[t-1] + \sigma^2} = \frac{p_n^{(k)}[t] g_{nn}^{(k)}}{I_n^{(k)}[t-1] + \sigma^2}. \tag{30}$$

Hence, the predicted EE is

$$\widehat{u}_{n[t]}(\mathbf{p}_n[t]) = \frac{\widehat{r}_n[t]}{p_n[t] + p_c} = \frac{\sum_k R(\widehat{\eta}_n^{(k)}[t])}{\sum_k p_n^{(k)}[t] + p_c}. \tag{31}$$

The best response at time t of User n is

$$\mathbf{p}_n^o[t] = \arg\max_{\mathbf{p}_n[t]} \widehat{u}_{n[t]}(\mathbf{p}_n[t]). \tag{32}$$

Due to the strictly quasi-concavity of $\widehat{u}_{n[t]}$, numerical methods like gradient ascent algorithms can be used to find the optimal power allocation at each time slot. A *Binary Search Assisted Ascent* algorithm has been developed in [7]. However, if we obtain the optimal power allocation at each time slot, it requires intensive computations. Instead, we introduce a *temporal iterative binary search* (TIBS) algorithm to track channel temporal variation and search for the optimal power allocation with reduced complexity.

The basic idea of TIBS is to search a better power allocation along the gradient at each time slot and enable iterative search along time. The power at t is updated by

$$\mathbf{p}_n[t] = \mathbf{p}_n[t-1] + \mu(\nabla \widehat{u}_{n[t]})_{\mathbf{p}_n[t-1]}, \tag{33}$$

where $(\nabla \widehat{u}_{n[t]})_{\mathbf{p}_n[t-1]}$ is the gradient of $\widehat{u}_{n[t]}$ at $\mathbf{p}_n[t-1]$ and μ is a small step size. Fixing channel states and transmit powers of all other users, the EE at t will always be bigger than that at $t-1$ with sufficiently small step size except when the gradient is zero, i.e. $\mathbf{p}_n[t-1]$ is already optimal

[17]. However, small step size leads to slow convergence and channel tracking capability. Denote

$$g(\mu) = \widehat{u}_{n[t]}(\mathbf{p}_n[t-1] + \mu(\nabla\widehat{u}_{n[t]})_{\mathbf{p}_n[t-1]}). \tag{34}$$

It is easy to show that $g(\mu)$ is also strictly quasi-concave in μ and binary search can be used for rapid location of the optimal step size μ^* [7]. TIBS is summarized in the following algorithm.

Algorithm *Temporal Iterative Binary Search (TIBS)*

($*$ noncooperative energy-efficient power optimization $*$)

Input: $\mathbf{p}[t-1]$, $\mathbf{I}[t-1]$

Output: $\mathbf{p}[t]$

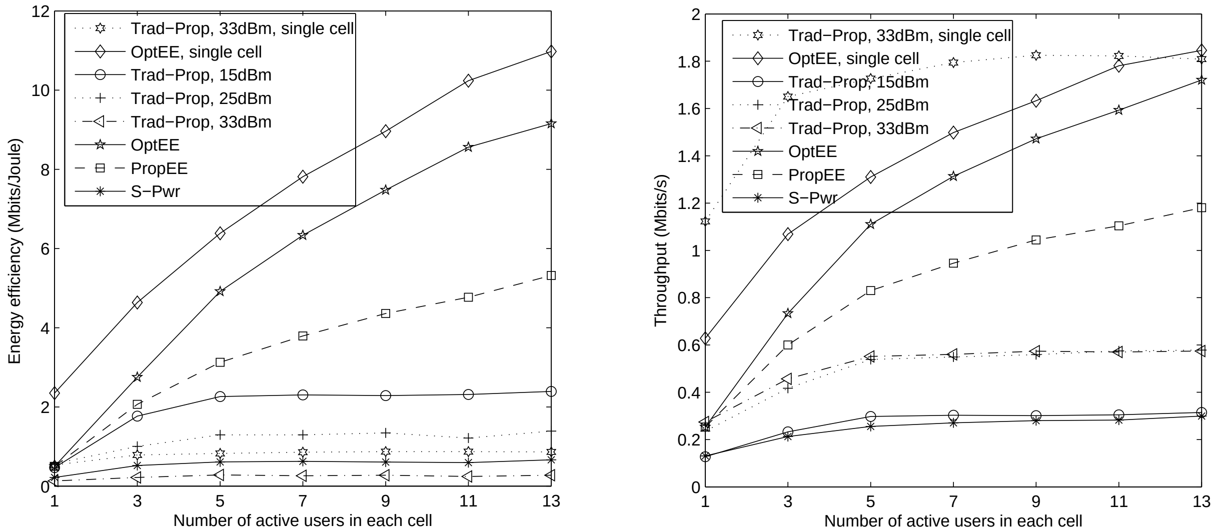
1. use *Gradient Assisted Binary Search*([7]) to find the optimal step size μ^*;
2. $\mathbf{p}[t] = \mathbf{p}[t-1] + \mu^*(\nabla\widehat{u}[t])_{\mathbf{p}[t-1]}$,
3. **return** $\mathbf{p}[t]$

TABLE I: System Parameters

Carrier frequency	1.5 GHz
Number of subchannels	96
Subchannel bandwidth	10 kHz
Target BER	10^{-3}
Thermal noise power, N_o	-141 dBW/MHz
Circuit power, P_C	100 mW
Maximum transmit power	33 dBm
Propagation model	Okumura-Hata model
Shadowing	Log-normal
Fading	Rayleigh flat fading
Modulation	Uncoded M-QAM

V. SIMULATION RESULTS

In this section we present simulation results for an interference-limited uplink OFDMA cellular network with reuse one. The network consists of seven hexagonal cells and the center cell is surrounded by the other six. Users are uniformly dropped into each cell at each simulation trial. The system parameters are listed in Table I. The base station schedules subchannels to maximize different network performance metrics. All schedulers and corresponding power control schemes are listed in Table II. Since the energy-efficient scheduling for frequency-selective channels is still an open problem, we use Rayleigh flat-fading channel and apply the energy-efficient schedulers for flat-fading OFDMA in [6]. The traditional proportional fair scheduler assigns subchannels to the user with the highest $\frac{r}{T}$, where r is the instantaneous data rate on that subchannel and T the average total throughput [18]. While energy-efficient schedulers assign subchannels to different users to maximize EE either with or without fairness, the traditional proportional fair scheduler



(a) Energy efficiency.

(b) Throughput.

Fig. 5: Performance comparison of different schemes.

assigns all subchannels to one user at each time slot due to flat fading. We also implement a traditional soft power control scheme [19]. In this scheme, parameters are selected to maximize the throughput of cell-edge users while not hurting the throughput of other users too much.

Figure 5 compares the average sum network EE and the corresponding throughput performance respectively. For fixed-power transmission, the transmit powers are shown in the legend. To see performance loss due to interference, the energy-efficient scheduler without fairness and the traditional proportional scheduler with the maximum transmit power is also simulated in a single cell network. We can see that transmitting with the highest power brings the highest interference and causes significant throughput loss for the traditional scheduler. In contrast, energy-efficient power control effectively reduces network interference and has much less throughput loss. While our previous results in [6] show that EE and throughput efficiency do not necessarily agree for an interference-free single cell scenario, the situation is different for a multi-cell interference-limited network. Here energy-efficient schemes optimize both throughput and energy utilization and exhibit an improved SE tradeoff. Figure 6 further shows the *cumulative distribution functions* (CDFs) of energy efficiency and throughput when there are nine users in the network. Observe the throughput CDF of the soft power control scheme. Compared with other traditional schemes, it maximizes cell-edge throughput that is illustrated in low-throughput range. However, it performs much worse than other traditional schemes in high-throughput range. From the CDFs, we can see that the proposed EE schemes not only improve the sum energy efficiency and throughput, but also uniformly improve the performance of all users in the cell.

TABLE II: Scheduling and Power Control

Legend	Scheduler	Power control
OptEE	Energy-efficient scheduler w/o fairness	TIBS
PropEE	Energy-efficient scheduler w/ proportional fairness	TIBS
Trad-Prop	Traditional proportional fair	Fixed power
S-Pwr	Traditional proportional fair	Traditional power control

VI. Conclusion

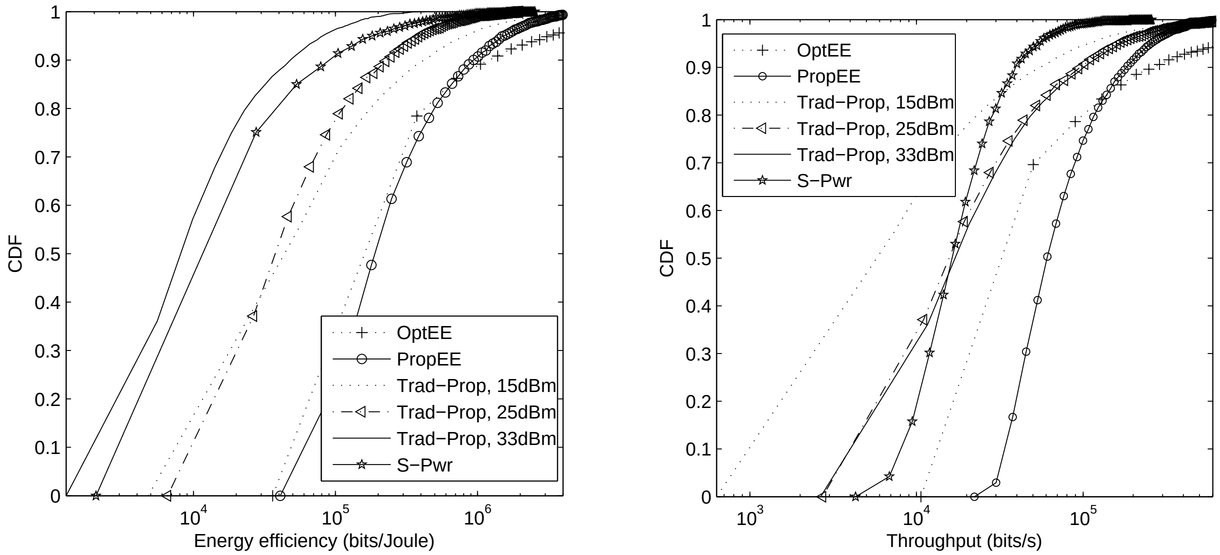
Since power optimization is critical for both interference management and energy utilization, we investigate energy-efficient power optimization schemes for interference-limited communications. The optimal power allocation solution in an interference-limited setting is intractable due to the non-convexity of the objective function. To gain insight into this problem, we first study a two-user network with ideal user cooperation and get effective approaches for specific regimes. Then we develop a noncooperative energy-efficient power optimization game. We show that the equilibrium always exists. Furthermore, when there is only one subchannel or the channel experiences flat fading, there will be a unique equilibrium. However, in frequency-selective channels, this is not true in general. We give a sufficient condition that assures the uniqueness. We further show that the spectral efficiency tradeoff of energy-efficient power control is reduced in interference-limited scenarios. Then we develop a practical approach of the noncooperative power optimization game. Simulation results show that the proposed scheme improves not only energy efficiency but also spectral efficiency uniformly for all users due to the conservative nature of power allocation achieved, which reduces other-cell interference to improve the overall network throughput.

Appendix I
Proof for Transmit Power Dominated Regime

Proof: It is obvious that the second term in $u(p_1, p_2)$ is strictly decreasing in p_1. To determine the first term, we need to verify that $F(p) = \frac{\log(1+ap)}{p}, \forall a > 0$, is strictly decreasing in p, i.e.

$$\frac{\partial F(p)}{\partial p} = \frac{ap - \log(1+ap) - ap\log(1+ap)}{p^2(1+ap)} < 0 \tag{I.35}$$

which is equivalent to $G(p) = ap - \log(1+ap) - ap\log(1+ap) < 0$. $G(0) = 0$. Besides $\frac{\partial G(p)}{\partial p} = -a\log(1+ap) < 0$. Hence $G(p) < 0, \forall p > 0$. Thus $F(p)$ is strictly decreasing in p. ∎



(a) Energy efficiency.

(b) Throughput.

Fig. 6: Comparison of different schemes.

APPENDIX II
PROOF FOR INTERFERENCE DOMINATED REGIME

Proof: Since we are considering interference dominated regime, whenever Users 1 and 2 are sending data, $p_1 g_{12} \gg \sigma^2$ and $p_2 g_{21} \gg \sigma^2$ and $INR > 1 + SINR$ due to close coupling between these transmissions. In wireless communications, radio links exhibit a threshold effect where link quality is acceptable where signal-to-noise ratio must exceed certain thresholds [20]. This indicates that the power allocation should not be too small. We assume that feasible p_1 and p_2 satisfies $p_1 \geq \widehat{p}_1$ and $p_2 \geq \widehat{p}_2$; otherwise, the user is shut down. Besides, in the interference dominated regime, $\widehat{p}_1 g_{12} \gg \sigma^2$ and $\widehat{p}_2 g_{21} \gg \sigma^2$. We compare two schemes. The first is to let both users send data simultaneously and the other is to shut down one user. First, we will show that when both users transmit, for User 1,

$$\frac{2w \log(1+\frac{p_1 g_1}{p_2 g_{21}+\sigma^2})}{p_1 + p_c} < \frac{w \log(1+\frac{p_1 g_1}{\sigma^2})}{p_1 + p_c}, \tag{II.36}$$

which is equivalent to show that $(1+\frac{p_1 g_1}{p_2 g_{21}+\sigma^2})^2 < 1+\frac{p_1 g_1}{\sigma^2}$. This inequality equals to

$$1+\frac{p_1 g_1}{p_2 g_{21} + \sigma^2} < \frac{p_2 g_{21}}{\sigma^2}. \tag{II.37}$$

Since $INR > SINR + 1$, (II.37) holds and so is (II.36). Similarly we have the same result for User 2. When both users are sending, the maximum energy efficiency is

$$\max_{\substack{p_1 \geq \hat{p}_1 \gg \frac{\sigma^2}{g_{12}} \\ p_2 \geq \hat{p}_2 \gg \frac{\sigma^2}{g_{21}}}} \left(\frac{w \log(1 + \frac{p_1 g_1}{p_2 g_{21} + \sigma^2})}{p_1 + p_c} + \frac{w \log(1 + \frac{p_2 g_2}{p_1 g_{12} + \sigma^2})}{p_2 + p_c} \right). \tag{II.38}$$

Assume the above maximum energy efficiency is obtained by p_1° and p_2°. According to (II.36),

$$\frac{w \log(1 + \frac{p_1^\circ g_1}{p_2^\circ g_{21} + \sigma^2})}{p_1^\circ + p_c} < \frac{w \log(1 + \frac{p_1^\circ g_1}{\sigma^2})}{2(p_1^\circ + p_c)} \leq \frac{1}{2} \max_{p_1} \left(\frac{w \log(1 + \frac{p_1 g_1}{\sigma^2})}{p_1 + p_c} \right) \tag{II.39}$$

Similarly, $\frac{w \log(1 + \frac{p_2^\circ g_2}{p_1^\circ g_{12} + \sigma^2})}{p_2^\circ + p_c} < \frac{1}{2} \max_{p_2} \left(\frac{w \log(1 + \frac{p_2 g_2}{\sigma^2})}{p_2 + p_c} \right)$. Suppose $g_1 \geq g_2$. It is easy to see that

$$\max_{p_1} \left(\frac{w \log(1 + \frac{p_1 g_1}{\sigma^2})}{p_1 + p_c} \right) \geq \max_{p_2} \left(\frac{w \log(1 + \frac{p_2 g_2}{\sigma^2})}{p_2 + p_c} \right). \tag{II.40}$$

Comparing the above inequalities, we can see that

$$\max_{p_1} \left(\frac{w \log(1 + \frac{p_1 g_1}{\sigma^2})}{p_1 + p_c} \right) > \frac{w \log(1 + \frac{p_2^\circ g_2}{p_1^\circ g_{12} + \sigma^2})}{p_2^\circ + p_c} + \frac{w \log(1 + \frac{p_2^\circ g_2}{p_1^\circ g_{12} + \sigma^2})}{p_2^\circ + p_c}. \tag{II.41}$$

The conclusion follows immediately. Extension to multi-user case is straightforward. ∎

Appendix III
Proof of Lemma 1

Proof: Denote the upper contour sets of $u_n(\mathbf{p}_n, \mathbf{p}_{-n})$ as $S_\alpha = \{\mathbf{p}_n \succeq \mathbf{0} | u_n(\mathbf{p}_n, \mathbf{p}_{-n}) \geq \alpha\}$, where symbol $\succeq$ denotes vector inequality and $\mathbf{R} \succeq \mathbf{0}$ means each element of $\mathbf{R}$ is nonnegative. According to Proposition C.9 of [13], $u_n(\mathbf{p}_n, \mathbf{p}_{-n})$ is strictly quasiconcave in $\mathbf{p}_n$ if and only if S_α is strictly convex for any real number α. It is obvious that when $\alpha \leq 0$, S_α is strictly convex when $\alpha \leq 0$. Now we investigate the case when $\alpha > 0$. Since $u_n(\mathbf{p}_n, \mathbf{p}_{-n}) = \frac{\sum_{k=1}^K R(\frac{p_n^{(k)} g_{nn}^{(k)}}{\sum_{i=1, i\neq n}^N p_i^{(k)} g_{in}^{(k)} + \sigma^2})}{p_c + \sum_{k=1}^K p_n^{(k)}} \geq \alpha$, S_α is equivalent to $S_\alpha = \{\mathbf{p}_n \succeq \mathbf{0} \middle| \sum_{k=1}^K R(\frac{p_n^{(k)} g_{nn}^{(k)}}{\sum_{i=1, i\neq n}^N p_i^{(k)} g_{in}^{(k)} + \sigma^2}) - (p_c + \sum_{k=1}^K p_n^{(k)})\alpha \geq 0\}$. Since $R()$ is strictly concave, S_α is also strictly convex. Hence, we have Lemma 1. ∎

Appendix IV
Proof of Theorem 1

Proof: In [16], it has been shown a Nash equilibrium exists in a noncooperative game if for any n, (1) $\mathbf{p}_n$ is a nonempty, convex, and compact subset of some Euclidean space $\Re^L$ and (2) $u_n(\mathbf{p}_n, \mathbf{p}_{-n})$ is continuous and quasi-concave in $\mathbf{p}_n$, both of which are satisfied in our noncooperative energy-efficient control game. Hence, the existence of the equilibrium immediately follows. According to our previous work in [7], in a point-to-point energy-efficient transmission, the necessary and sufficient condition for a data rate vector of User n, $\mathbf{r}_n^o = [r_n^{(1)o}, r_n^{(2)o}, \cdots, r_n^{(K)o}]^T$, to be globally optimal is given by, for any Subchannel i,

(i) if $\frac{p_c+\sum_{j\neq i} p_n^{(j)}}{\sum_{j\neq i} r_n^{(j)}} \geq \left.\frac{\partial(\sum_j p_n^{(j)})}{\partial r_n^{(i)}}\right|_{\mathbf{r}_n=\mathbf{r}_n^{(i0)}}$, $\left.\frac{\partial u_n(\mathbf{p}_n,\mathbf{p}_{-n})}{\partial r_n^{(i)}}\right|_{\mathbf{r}_n=\mathbf{r}_n^o} = 0$, i.e. $\left.\frac{\partial(\sum_j p_n^{(j)})}{\partial r_n^{(i)}}\right|_{\mathbf{r}_n=\mathbf{r}_n^o} = \frac{1}{u(\mathbf{p}_n^o,\mathbf{p}_{-n})}$;

(ii) otherwise, $r_n^{(i)o} = 0$,

where $\mathbf{r}_n^{(i0)} = [r_n^{(1)o}, r_n^{(2)o}, \cdots, r_n^{(i-1)o}, 0, r_n^{(i+1)o}, \cdots, r_n^{(K)o}]$.

By transformation of parameters, $\frac{\partial f}{\partial r_n^{(i)}} = \frac{\partial f}{\partial p_n^{(i)}} \Big/ \frac{\partial r_n^{(i)}}{\partial p_n^{(i)}} = \frac{\partial f}{\partial p_n^{(i)}} \frac{1}{R'(\eta_n^{(i)})\gamma_n^{(i)}}$, where $R'()$ is the first order derivative of $R()$ and $\gamma_n^{(i)} = \frac{\eta_n^{(i)}}{p_n^{(i)}} = \frac{g_{nn}^{(i)}}{\sum_{j=1,j\neq n}^N p_j^{(i)} g_{jn}^{(i)} + \sigma^2}$. Hence, we have the following equivalent condition for each user. For any Subchannel i,

(i) if $\frac{\sum_{j\neq i} r_n^{(j)}}{p_c+\sum_{j\neq i} p_n^{(j)}} \leq R'(0)\gamma_n^{(i)}$, $\left.\frac{\partial u_n(\mathbf{p}_n,\mathbf{p}_{-n})}{\partial p_n^{(i)}}\right|_{\mathbf{p}_n=\mathbf{p}_n^o} = 0$, i.e.

$$R'(\gamma_n^{(i)} p_n^{(i)o})\gamma_n^{(i)} = u(\mathbf{p}_n^o, \mathbf{p}_{-n}); \tag{IV.42}$$

(ii) otherwise, $p_n^{(i)o} = 0$.

It is easy to see that the network achieves an equilibrium if and only if the power settings of all users satisfy the above conditions. Theorem 1 is readily obtained.

■

APPENDIX V
PROOF OF PROPOSITION 1

Proof: $p_n^o = f_n(\mathbf{p}_{-n}) = \arg\max_{p_n} u_n(p_n, \mathbf{p}_{-n})$. Since $u_n(0, \mathbf{p}_{-n}) = 0$ and $u_n(p_n, \mathbf{p}_{-n}) > 0$ for any $p_n > 0$, $f_n(\mathbf{p}_{-n}) > 0$ and we have the positivity. Denote $I_n = \sum_{j=1,j\neq n}^N p_j g_{jn}$ **and** $\gamma_n = \frac{g_{nn}}{I+\sigma^2}$. According to (IV.42), p_n^o satisfies

$$R'(\gamma_n p_n^o)\gamma_n = u(p_n^o, \mathbf{p}_{-n}) = \frac{R(\gamma_n p_n^o)}{p_c + p_n^o}. \tag{V.43}$$

Substituting $R(\eta) = w\log(1+\eta)$ in to (V.43), we have the following equivalent condition,

$$w(p_n^o, I) = g_{nn}(p_c + p_n^o) - (p_n^o g_{nn} + I + \sigma^2)\log(1 + \frac{p_n^o g_{nn}}{I+\sigma^2}) = 0. \tag{V.44}$$

Hence, $\frac{\partial p_n^o}{\partial I} = -\frac{\partial w}{\partial I} \Big/ \frac{\partial w}{\partial p_n^o} = \frac{p_n^o\gamma_n - \log(1+p_n^o\gamma_n)}{g_{nn}\log(1+p_n^o\gamma_n)}$. Since $x > \log(1+x)$ for all $x > 0$, we have $\frac{\partial p_n^o}{\partial I} > 0$. The monotonicity follows immediately. Furthermore,

$$\frac{\partial^2 p_n^o}{\partial I^2} = \frac{\partial \frac{\partial p_n^o}{\partial I}}{\partial I} = -\frac{p_n^o(-p_n^o\gamma_n + (1+p_n^o\gamma_n)\log(1+p_n^o\gamma_n))}{(I+\sigma^2)(I+\sigma^2+p_n^o g_{nn})\log(1+p_n^o\gamma_n)^2}. \tag{V.45}$$

We can easily show that $(1+x)\log(1+x) > x$ for all $x > 0$ since $(1+0)\log(1+0) = 0$ and $(1+x)\log(1+x) - x$ has positive first-order derivative when $x > 0$. Thus, $\frac{\partial^2 p_n^o}{\partial I^2} < 0$. Since I is a linear combination of $\mathbf{p}_{-n}$, $f_n(\mathbf{p}_{-n})$ is strictly concave in $\mathbf{p}_{-n}$. We get the scalability immediately by letting $F(\alpha) = \alpha f_n(\mathbf{p}_{-n}) - f_n(\alpha \mathbf{p}_{-n})$ and observing that $F(1) = 0$ and $\frac{\partial^2 F(\alpha)}{\partial \alpha^2} < 0$. ■

APPENDIX VI
PROOF OF AN EQUILIBRIUM

Proof: We need to show that one of the equilibrium has the form $\mathbf{p}_1^* = [p_a p_b]$ and $\mathbf{p}_2^* = [p_c 0]$, where p_a, p_b, and p_c are positive. We only need to verify that there exist p_a, p_b, and p_c that satisfy Theorem 1. Suppose $\mathbf{p}_2^* = [p_c 0]$. After some calculation, it is easy to see that $\sigma^2 \gg p_c g_{21}^{(1)}$ and $\eta_1^{(1)} \approx \frac{p_a g_{11}^{(1)}}{\sigma^2}$. Hence, both subchannels of User 1 have approximately the same SINR condition. Thus in the equilibrium, the transmit powers on the two subchannels of User 1 are almost the same. Besides, they cannot be zero. Hence, both are positive and satisfy the first condition of Theorem 1. Assume $\mathbf{p}_1^* = [p_a p_b]$. Now we verify $\mathbf{p}_2^*$. Since User 2 does not transmit on the second subchannel, $\frac{\sum_{j\neq 1} r_n^{(j)*}}{p_c + \sum_{j\neq 1} p_n^{(j)*}} = 0$ and the first condition of Theorem 1 should be satisfied. Hence, a positive power is allocated on the first subchannel in the equilibrium of User 2. Regarding the second subchannel, $\gamma_n^{(2)*} = \frac{g_{nn}^{(2)}}{p_b g_{12}^{(2)} + \sigma^2} = \frac{1}{p_b 1e^{10} + 1} \to 0$. Hence, $\frac{\sum_{j\neq 2} r_2^{(j)*}}{p_c + \sum_{j\neq 2} p_n^{(j)*}} > R'(0)\gamma_n^{(2)*} \to 0$ and condition 2 of Theorem 1 is satisfied. Hence, $\mathbf{p}_2^* = [p_c 0]$. Numerical methods can be used to determine the exact values of p_a, p_b, and p_c.

■

APPENDIX VII
PROOF OF AN EQUILIBRIUM

Proof: For any two power vectors $\mathbf{p}_{-n}$ and $\check{\mathbf{p}}_{-n}$, define the function $\mathfrak{F}_n(\theta) = F_n(\check{\mathbf{p}}_{-n} + \theta(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}))$. It is clear that $\mathfrak{F}_n(0) = F_n(\check{\mathbf{p}}_{-n})$ and $\mathfrak{F}_n(1) = F_n(\mathbf{p}_{-n})$; By the chain rule, we know that $\frac{\partial \mathfrak{F}_n}{\partial \theta} = (\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}) \frac{\partial F_n}{\partial(\check{\mathbf{p}}_{-n} + \theta(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}))}$. Hence, we have

$$F_n(\mathbf{p}_{-n}) - F_n(\check{\mathbf{p}}_{-n}) = \mathfrak{F}_n(1) - \mathfrak{F}_n(0) = \int_0^1 \mathfrak{F}_n'(\theta) d\theta = (\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}) \int_0^1 \frac{\partial F_n}{\partial(\check{\mathbf{p}}_{-n} + \theta(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}))} d\theta.$$

$$\text{Thus, } ||F_n(\mathbf{p}_{-n}) - F_n(\check{\mathbf{p}}_{-n})|| = \left|\left|(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}) \int_0^1 \frac{\partial F_n}{\partial(\check{\mathbf{p}}_{-n} + \theta(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}))} d\theta\right|\right|$$
$$\leq ||(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n})|| \left|\left|\int_0^1 \frac{\partial F_n}{\partial(\check{\mathbf{p}}_{-n} + \theta(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}))} d\theta\right|\right| \leq ||(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n})|| \int_0^1 \left|\left|\frac{\partial F_n}{\partial(\check{\mathbf{p}}_{-n} + \theta(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}))}\right|\right| d\theta$$
$$\leq ||(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n})|| \int_0^1 \left|\left|\sup_{\mathbf{p}_{-n}} \frac{\partial F_n}{\partial \mathbf{p}_{-n}}\right|\right| d\theta = ||(\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n})|| \left|\left|\sup_{\mathbf{p}_{-n}} \frac{\partial F_n}{\partial \mathbf{p}_{-n}}\right|\right|.$$

Besides, according to the chain rule, $\frac{\partial F_n}{\partial \mathbf{p}_{-n}} = \frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}} \frac{\partial \tilde{F}_n}{\partial \mathbf{I}_{-n}}$; Hence, we have

$$\frac{||F_n(\mathbf{p}_{-n}) - F_n(\check{\mathbf{p}}_{-n})||}{||\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}||} \leq \sup_{\mathbf{p}_{-n}} \left|\left| \frac{\partial F_n}{\partial \mathbf{p}_{-n}} \right|\right| = \sup_{\mathbf{p}_{-n}} \left|\left| \frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}} \frac{\partial \tilde{F}_n}{\partial \mathbf{I}_{-n}} \right|\right| \leq \left|\left| \frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}} \right|\right| \sup_{\mathbf{I}_{-n}} \left|\left| \frac{\partial \tilde{F}_n}{\partial \mathbf{I}_{-n}} \right|\right|;$$

When $\left|\left| \frac{\partial \mathbf{I}_n}{\partial \mathbf{p}_{-n}} \right|\right| < \frac{1}{\sup_{\mathbf{I}_n} \left|\left| \frac{\partial \tilde{F}_n}{\partial \mathbf{I}_n} \right|\right|}$, $\frac{||F_n(\mathbf{p}_{-n}) - F_n(\check{\mathbf{p}}_{-n})||}{||\mathbf{p}_{-n} - \check{\mathbf{p}}_{-n}||} < 1$. The uniqueness of equilibrium follows immediately from Theorem 3. ■

REFERENCES

[1] Draft IEEE std 802.16e/D9, "IEEE standard for local and metropolitan area networkspart 16: air interface for fixed and mobile broadband wireless access systems," June 2005.

[2] D. Gesbert, S. G. Kiani, A. Gjendemsj, G. E. Ien , "Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks," *Proc. of the IEEE*, vol. 95, no. 12, pp. 2393–2409, Dec. 2007.

[3] A. GjendemsjØ, D. Gesbert, G. E. Øien, and S. G. Kiani, "Optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. RAWNET (WiOpt)*, Apr. 2006.

[4] K. Lahiri, A. Raghunathan, S. Dey, and D. Panigrahi, "Battery-driven system design: A new frontier in low power design," in *Proc. Intl. Conf. on VLSI Design*, Bangalore, India, Jan. 2002, pp. 261–267.

[5] F. Meshkati, H. V. Poor, and S. C. Schwartz, "Energy-efficient resource allocation in wireless networks," *IEEE Commun. Magazine*, pp. 58–68, May 2007.

[6] G. W. Miao, N. Himayat, Y. Li, and D. Bormann, "Energy-efficient design in wireless ofdma," in *Proc. IEEE ICC 2008*, May 2008, pp. 3307–3312.

[7] G. Miao, N. Himayat, and Y. Li, "Energy-efficient transmission in frequency-selective channels," in *Proc. IEEE Globecom 2008*, Nov. 2008, pp. 1–5.

[8] ——, "Energy-efficient link adaptation in frequency-selective channels," submitted to *IEEE Trans. Commun.*

[9] R. G. Gallager, *Information Theory and Relaible Communication*. John Wiley & Sons, Inc., 1968.

[10] M. Pischella and J.-C. Belfiore,, "Power control in distributed cooperative OFDMA cellular networks," vol. 7, no. 5, pp. 1900–1906, May 2008.

[11] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[12] D. Goodman and N. Mandayam, "Power control for wireless data," *IEEE Pers. Commun. Mag.*, vol. 7, no. 2, pp. 48–54, Apr. 2000.

[13] E. Wolfstetter, *Topics in Microeconomics: Industrial Organization, Auctions, and Incentives*. Cambridge University Press, 1999.

[14] R. D. Yates, "A framework for uplink power control in cellular radio systems," pp. 1341–1347, Sept. 1995.

[15] R. G. Bartle, *The elements of real analysis*. New York: John Wiley, 1964.

[16] J. W. Friedman, *Oligopoly and the Theory of Games*. North-Holland Pub. Co., 1977.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[18] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory.*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[19] A. T. Koc, S. Talwar, A. Papathanassiou, R. Yang, N. Himayat, and H. Yin, "IEEE c802.16m-08/666r2: Uplink power control recommendations for IEEE 802.16m." [Online]. Available: http://wirelessman.org/tgm/index.html

[20] K. Feher, *Advanced Digital Communications*. Englewood Cliffs, NJ: Prentice-Hall, 1987.