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Printed 2000년 10월 9일

암반의 열-수리-역학 커플거동 해석을 위한 경계요소법

Boundary Element Method for Coupled Thermo-Hydro-Mechanical Behaviour of Rock Mass

By

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SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
Doctor of Philosophy
AT
한양대학교
성동구 행당동 17번지
2000년 12월

98. 10. 4.

"Derivation of cofactor of $B(\partial x, s)$"

Governing equation (complete set).

(1) $(\lambda+\mu)\,u_{j,ij} + \mu\,u_{i,jj} + \underline{\delta_{ij}\,\alpha}\,P_{,i} + \delta_{ij}(3\lambda+2\mu)\beta_T^s T_{,i} + b_i = 0$.

(2) $kT_{,jj} + \psi_T - \rho C_p \dot{T} + (3\lambda+2\mu)\beta_T^s T_0\,\dot{u}_{j,j} - \eta T_0 \dot{P} = 0$

(3) $kP_{,jj} + \psi_P - \dfrac{\alpha}{KB}\dot{P} + 3KM\,\dot{u}_{j,j} - \eta\dot{\theta} = 0$

T 와 θ 중 선택.

Laplace transformation of governing equation.

(1) $(\lambda+\mu)\,\tilde{u}_{j,ij} + \mu(\tilde{u}_{i,jj}) + \delta_{ij}\,\alpha\,\tilde{P}_{,i} + \delta_{ij}(3\lambda+2\mu)\beta_T^s\tilde{T}_{,i} + b_i = 0$

(2) $k\tilde{T}_{,jj} + \tilde{\psi}_T - \rho C_p s\tilde{T} + (3\lambda+2\mu)\beta_T^s T_0 s\,\tilde{u}_{j,j} - \eta T_0 s\tilde{p} = 0$

(3) $k\tilde{P}_{,jj} + \tilde{\psi}_P - \dfrac{\alpha s}{KB}\tilde{P} + \cancel{3}\alpha s\tilde{u}_{j,j} - \eta s\tilde{T} = 0.$

Laplace transformation domain fundamental solution.

$B(\partial x, s) = [B_{mn}(\partial x, s)]_{4,4}$.

★ All the components are considered.

$B_{ij}(\partial x; s) = (\lambda+\mu)\dfrac{\partial^2}{\partial x_i \partial x_j} + \delta_{ij}\mu\nabla^2$. $(i=1,2)$	$g_{11}, g_{12}, g_{21}, g_{22}$
$B_{33}(\partial x, s) = k\nabla^2 - \rho C_p s = k(\nabla^2 - \lambda_3^2)$	g_{33}
$B_{44}(\partial x, s) = k\nabla^2 - \dfrac{\alpha s}{KB} = k(\nabla^2 - \lambda_4^2)$.	g_{44}
$B_{i3}(\partial x, s) = (3\lambda+2\mu)\beta_T^s \dfrac{\partial}{\partial x_i}$	g_{13}, g_{23}
$B_{i4}(\partial x, s) = \alpha\dfrac{\partial}{\partial x_i}$	g_{14}, g_{24}
$B_{3j}(\partial x, s) = (3\lambda+2\mu)\beta_T^s T_0 s\dfrac{\partial}{\partial x_j}$	g_{31}, g_{32}
$B_{4j}(\partial x, s) = \alpha s\dfrac{\partial}{\partial x_j}$	g_{41}, g_{42}
$B_{34}(\partial x, s) = -\eta T_0 s$	g_{34}
$B_{43}(\partial x, s) = -\eta s$	g_{43}.

where $\lambda_3^2 = \dfrac{\rho C_p s}{k}$, $\lambda_4^2 = \dfrac{\alpha s}{kKB}$.

98.11.09

* Differential operator $B^*(\partial x, s)$ built from the cofactors of $B(\partial x, s)$:

$$
g = \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix}
$$

$$
B_{11}^*(\partial x, s) = (-1)^2 \begin{vmatrix} g_{22} & g_{23} & g_{24} \\ g_{32} & g_{33} & g_{34} \\ g_{42} & g_{43} & g_{44} \end{vmatrix}
$$

$$
= g_{22}(g_{33}g_{44} - g_{34}g_{43}) - g_{23}(g_{32}g_{44} - g_{34}g_{42})
$$

$$
+ g_{24}(g_{32}g_{43} - g_{42}g_{33})
$$

$$
= g_{22}g_{33}g_{44} - g_{22}g_{34}g_{43} - g_{23}g_{32}g_{44} + g_{23}g_{34}g_{42}
$$

$$
+ g_{24}g_{32}g_{43} - g_{24}g_{42}g_{33}
$$

$$
= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{\nabla^2 - \lambda_3^2\right\}\left\{\nabla^2 - \lambda_4^2\right\} kK
$$

$$
- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{\eta^2 s^2 T_0\right\}
$$

$$
- \left\{(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\right\}\left\{(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2}\right\}\left\{\nabla^2 - \lambda_4^2\right\} K
$$

$$
+ \left\{(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\right\}\left\{-\eta T_0 s\right\}\left\{\alpha s \frac{\partial}{\partial x_2}\right\}
$$

$$
+ \left\{\alpha \frac{\partial}{\partial x_2}\right\}\left\{(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2}\right\}\left\{-\eta s\right\}
$$

$$
- \left\{\alpha \frac{\partial}{\partial x_2}\right\}\left\{\alpha s \frac{\partial}{\partial x_2}\right\}\left\{\nabla^2 - \lambda_3^2\right\} K
$$

$$
= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{kK(\nabla^2 - \lambda_3^2)(\nabla^2 - \lambda_4^2) - \eta^2 s^2 T_0\right\}
$$

$$
- (3\lambda+2\mu)^2 (\beta_T^s)^2 T_0 s \left(\frac{\partial}{\partial x_2}\right)^2 \left\{\nabla^2 - \lambda_4^2\right\} K
$$

$$
- \alpha^2 s \left(\frac{\partial}{\partial x_2}\right)^2 \left\{\nabla^2 - \lambda_3^2\right\} K
$$

$$
- 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \left(\frac{\partial}{\partial x_2}\right)^2
$$

98. 11. 10.

$$B_{12}^{*}(\partial x, s) = (-1)^3 \begin{vmatrix} g_{21} & g_{23} & g_{24} \\ g_{31} & g_{33} & g_{34} \\ g_{41} & g_{43} & g_{44} \end{vmatrix}$$

$$= -g_{21}(g_{33}g_{44} - g_{34}g_{43}) + g_{23}(g_{31}g_{44} - g_{34}g_{41}) - g_{24}(g_{31}g_{43} - g_{33}g_{41})$$

$$= -g_{21}g_{33}g_{44} + g_{21}g_{34}g_{43} + g_{23}g_{31}g_{44} - \underline{g_{23}g_{34}g_{41} - g_{24}g_{31}g_{43}} + g_{24}g_{33}g_{41}$$

$$= -(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_3^2\}\{\nabla^2 - \lambda_4^2\} kK$$

$$+ (\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\eta^2 s^2 T_0$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_4^2\} K$$

$$+ (3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\eta T_0 s \alpha s \frac{\partial}{\partial x_1}$$

$$+ \alpha \frac{\partial}{\partial x_2}(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_1}\eta s$$

$$+ \alpha \frac{\partial}{\partial x_2}\alpha s \frac{\partial}{\partial x_1}\{\nabla^2 - \lambda_3^2\} k$$

$$= -(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\left(kK(\nabla^2 - \lambda_3^2)(\nabla^2 - \lambda_4^2) - \eta^2 s^2 T_0\right)$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_4^2\} K$$

$$+ \alpha^2 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_3^2\} k$$

$$+ 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \frac{\partial^2}{\partial x_1 \partial x_2}.$$

98.11.10.

$$B^{*}_{21}(\partial x, s) = (-1)^3 \begin{vmatrix} g_{12} & g_{13} & g_{14} \\ g_{32} & g_{33} & g_{34} \\ g_{42} & g_{43} & g_{44} \end{vmatrix}$$

$$= -g_{12}(g_{33}g_{44} - g_{34}g_{43}) + g_{13}(g_{32}g_{44} - g_{34}g_{42}) - g_{14}(g_{32}g_{43} - g_{33}g_{42})$$

$$= -g_{12}g_{33}g_{44} + g_{12}g_{34}g_{43} + g_{13}g_{32}g_{44} - \underline{g_{13}g_{34}g_{42}} - \underline{g_{14}g_{32}g_{43}} + g_{14}g_{33}g_{42}$$

$$= -(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_3^2\}\{\nabla^2 - \lambda_4^2\} kK$$

$$+ (\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\{\eta^2 s^2 T_0\}$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_4^2\} K$$

$$+ (3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1} \eta T_0 s \, \alpha s \frac{\partial}{\partial x_2}$$

$$+ \alpha \frac{\partial}{\partial x_1}(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2} \eta s$$

$$+ \alpha \frac{\partial}{\partial x_1} \alpha s \frac{\partial}{\partial x_2}\{\nabla^2 - \lambda_3^2\} k$$

$$= -(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\{kK(\nabla^2 - \lambda_3^2)(\nabla^2 - \lambda_4^2) - \eta^2 s^2 T_0\}$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_4^2\} K$$

$$+ \alpha^2 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2 - \lambda_3^2\} k$$

$$+ 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \frac{\partial^2}{\partial x_1 \partial x_2}$$ //

98.11.10.

$$B^*_{22}(\partial x, s) = (-1)^4 \begin{vmatrix} g_{11} & g_{13} & g_{14} \\ g_{31} & g_{33} & g_{34} \\ g_{41} & g_{43} & g_{44} \end{vmatrix}$$

$$= g_{11}(g_{33}g_{44} - g_{34}g_{43}) - g_{13}(g_{31}g_{44} - g_{34}g_{41}) + g_{14}(g_{31}g_{43} - g_{33}g_{41})$$

$$= g_{11}g_{33}g_{44} - g_{11}g_{34}g_{43} - g_{13}g_{31}g_{44} + g_{13}g_{34}g_{41} + g_{14}g_{31}g_{43} - g_{14}g_{33}g_{41}.$$

$$= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{\nabla^2 - \lambda_3^2\right\}\left\{\nabla^2 - \lambda_4^2\right\} kK$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\eta^2 s^2 T_0$$

$$- (3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1}(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_1}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$- (3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1}\eta T_0 s \alpha s \frac{\partial}{\partial x_1}$$

$$- \alpha\frac{\partial}{\partial x_1}(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_1}\eta s$$

$$- \alpha\frac{\partial}{\partial x_1}\alpha s \frac{\partial}{\partial x_1}\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{kK(\nabla^2 - \lambda_3^2)(\nabla^2 - \lambda_4^2) - \eta^2 s^2 T_0\right\}$$

$$- (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \left(\frac{\partial}{\partial x_1}\right)^2\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$- \alpha^2 s \left(\frac{\partial}{\partial x_1}\right)^2\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$- 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \left(\frac{\partial}{\partial x_1}\right)^2$$

98.11.16. #1

$$B_{13}^{*} = (-1)^{4}\begin{vmatrix} g_{21} & g_{22} & g_{24} \\ g_{31} & g_{32} & g_{34} \\ g_{41} & g_{42} & g_{44} \end{vmatrix}$$

$$= g_{21}(g_{32}g_{44} - g_{34}g_{42}) - g_{22}(g_{31}g_{44} - g_{34}g_{41}) + g_{24}(g_{31}g_{42} - g_{32}g_{41})$$

$$= g_{21}g_{32}g_{44} - g_{21}g_{34}g_{42} - g_{22}g_{31}g_{44} + g_{22}g_{34}g_{41} + g_{24}g_{31}g_{42} - g_{24}g_{32}g_{41}.$$

$$= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^S T_0 S \frac{\partial}{\partial x_2}\right\}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{-\eta T_0 S\right\}\left\{+\alpha S \frac{\partial}{\partial x_2}\right\}$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{+(3\lambda+2\mu)\beta_T^S T_0 S \frac{\partial}{\partial x_1}\right\}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{-\eta T_0 S\right\}\left\{+\alpha S \frac{\partial}{\partial x_1}\right\}$$

$$- \left\{+\alpha \frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^S T_0 S \frac{\partial}{\partial x_1}\right\}\left\{+\alpha S \frac{\partial}{\partial x_2}\right\}$$

$$+ \left\{+\alpha \frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^S T_0 S \frac{\partial}{\partial x_2}\right\}\left\{+\alpha S \frac{\partial}{\partial x_1}\right\}$$

$$= -\mu(3\lambda+2\mu)\beta_T^S T_0 S \frac{\partial}{\partial x_1}\nabla^2\left\{\nabla^2 - \lambda_4^2\right\}K - \mu\eta T_0 \alpha S^2 \nabla^2 \frac{\partial}{\partial x_1}$$

$$B_{13}^{*} = \left(-\mu(3\lambda+2\mu)\beta_T^S T_0 S \frac{\partial}{\partial x_1}\nabla^2\left\{\nabla^2 - \lambda_4^2\right\}K - \mu\eta T_0 \alpha S^2 \nabla^2 \frac{\partial}{\partial x_1}\right)\cancel{(-1)^{16}}.$$

98.11.16 #2

$$B^*_{31}(\partial x, s) = (-1)^4 \begin{vmatrix} g_{12} & g_{13} & g_{14} \\ g_{22} & g_{23} & g_{24} \\ g_{42} & g_{43} & g_{44} \end{vmatrix}$$

$$= g_{12}(g_{23}g_{44} - g_{24}g_{43}) - g_{13}(g_{22}g_{44} - g_{24}g_{42}) + g_{14}(g_{22}g_{43} - g_{23}g_{42})$$

$$= g_{12}g_{23}g_{44} - g_{12}g_{24}g_{43} - g_{13}g_{22}g_{44} + g_{13}g_{24}g_{42} + g_{14}g_{22}g_{43} - g_{14}g_{23}g_{42}.$$

$$= + \left\{ \cancel{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}} \right\} \left\{ +(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2} \right\} \left\{ \nabla^2 - \lambda_4^2 \right\} K$$

$$- \left\{ \cancel{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}} \right\} \left\{ +\alpha \frac{\partial}{\partial x_2} \right\} \left\{ -\eta s \right\}$$

$$- \left\{ +(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1} \right\} \left\{ \cancel{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2}} + \mu \nabla^2 \right\} \left\{ \nabla^2 - \lambda_4^2 \right\} K$$

$$+ \left\{ +(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1} \right\} \left\{ +\alpha \frac{\partial}{\partial x_2} \right\} \left\{ +\alpha s \frac{\partial}{\partial x_2} \right\}$$

$$+ \left\{ +\alpha \frac{\partial}{\partial x_1} \right\} \left\{ \cancel{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2}} + \mu \nabla^2 \right\} \left\{ -\eta s \right\}$$

$$- \left\{ +\alpha \frac{\partial}{\partial x_1} \right\} \left\{ +(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2} \right\} \left\{ +\alpha s \frac{\partial}{\partial x_2} \right\}$$

0

$$= -\mu(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1} \nabla^2 \{\nabla^2 - \lambda_4^2\} K - \mu\eta s \alpha \nabla^2 \frac{\partial}{\partial x_1} \quad = -B^*_{13}/T_0 s$$

$$B^*_{31} = -\left(\mu(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1} \nabla^2 \{\nabla^2 - \lambda_4^2\} K + \mu\eta s \alpha \frac{\partial}{\partial x_1} \nabla^2\right)$$

$$B^*_{i4}(\partial x, s) = (-1)^5 \begin{vmatrix} g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \\ g_{41} & g_{42} & g_{43} \end{vmatrix}$$

$$= -g_{21}(g_{32}g_{43} - g_{33}g_{42}) + g_{22}(g_{31}g_{43} - g_{33}g_{41}) - g_{23}(g_{31}g_{42} - g_{32}g_{41})$$

$$= -g_{21}g_{32}g_{43} + g_{21}g_{33}g_{42} + g_{22}g_{31}g_{43} - g_{22}g_{33}g_{41} - g_{23}g_{31}g_{42} + g_{23}g_{32}g_{41}$$

$$\begin{aligned}
= & - \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2}\right\}(-\eta s) \\
& + \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{\nabla^2 - \lambda_3^2\right\}k\left\{+\alpha s \frac{\partial}{\partial x_2}\right\} \\
& + \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_1}\right\}(-\eta s) \\
& - \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{\nabla^2 - \lambda_3^2\right\}k\left\{+\alpha s \frac{\partial}{\partial x_1}\right\} \\
& + \left\{+(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_1}\right\}\left\{+\alpha s \frac{\partial}{\partial x_2}\right\} \\
& - \left\{+(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2}\right\}\left\{+\alpha s \frac{\partial}{\partial x_1}\right\}
\end{aligned}$$

$$= \underline{-\mu\alpha s \frac{\partial}{\partial x_1}\nabla^2\left\{\nabla^2 - \lambda_3^2\right\}k - \mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 \nabla^2 \frac{\partial}{\partial x_1}}.$$

$$B^*_{i4} = -\left(\mu\alpha s \frac{\partial}{\partial x_i}\nabla^2\left\{\nabla^2 - \lambda_3^2\right\}k - \mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 \frac{\partial}{\partial x_i}\nabla^2\right)\cancel{(-1)^{i}}.$$

$$B^*_{4i}(\partial x, s) = (-1)^5 \begin{vmatrix} g_{12} & g_{13} & g_{14} \\ g_{22} & g_{23} & g_{24} \\ g_{32} & g_{33} & g_{34} \end{vmatrix}$$

$$= -g_{12}(g_{23}g_{34} - g_{24}g_{33}) + g_{13}(g_{22}g_{34} - g_{24}g_{32}) - g_{14}(g_{22}g_{33} - g_{23}g_{32})$$

$$= -g_{12}g_{23}g_{34} + g_{12}g_{24}g_{33} + g_{13}g_{22}g_{34} - g_{13}g_{24}g_{32} - g_{14}g_{22}g_{33} + g_{14}g_{23}g_{32}$$

$$= \cancel{-\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\right\}(-\eta T_0 s)}$$

$$\cancel{+\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{+\alpha\frac{\partial}{\partial x_2}\right\}\left\{\nabla^2 - \lambda_3^2\right\}k}$$

$$+\left\{+(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1}\right\}\left\{\cancel{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2}} + \mu\nabla^2\right\}\left\{-\eta T_0 s\right\}$$

$$\cancel{-\left\{+(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_1}\right\}\left\{+\alpha\frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2}\right\}}$$

$$-\left\{+\alpha\frac{\partial}{\partial x_1}\right\}\left\{\cancel{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2}} + \mu\nabla^2\right\}\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$\cancel{+\left\{+\alpha\frac{\partial}{\partial x_1}\right\}\left\{+(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_2}\right\}}$$

0

$$= -\mu\alpha\frac{\partial}{\partial x_1}\nabla^2\left\{\nabla^2 - \lambda_3^2\right\}k - \mu\eta(3\lambda+2\mu)\beta_T^s T_0 s \nabla^2 \frac{\partial}{\partial x_1} = B^*_{14}/s$$

$$B^*_{4i} = \left(-\mu\alpha\frac{\partial}{\partial x_i}\nabla^2\left\{\nabla^2 - \lambda_3^2\right\}k - \mu\eta(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_i}\nabla^2\right)\cancel{(-1)}$$

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$$B^{*}_{33}(\partial x, s) = (-1)^{6}\begin{vmatrix} g_{11} & g_{12} & g_{14} \\ g_{21} & g_{22} & g_{24} \\ g_{41} & g_{42} & g_{44} \end{vmatrix}$$

$$= g_{11}(g_{22}g_{44} - g_{24}g_{42}) - g_{12}(g_{21}g_{44} - g_{24}g_{41}) + g_{14}(g_{21}g_{42} - g_{22}g_{41})$$

$$= g_{11}g_{22}g_{44} - g_{11}g_{24}g_{42} - g_{12}g_{21}g_{44} + g_{12}g_{24}g_{41} + g_{14}g_{21}g_{42} - g_{14}g_{41}g_{22}$$

$$= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_1} + \mu\nabla^2\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2\partial x_2} + \mu\nabla^2\right\}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_1} + \mu\nabla^2\right\}\left\{+\alpha\frac{\partial}{\partial x_2}\right\}\left\{+\alpha s\frac{\partial}{\partial x_2}\right\}$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_2}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2\partial x_1}\right\}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_2}\right\}\left\{+\alpha\frac{\partial}{\partial x_2}\right\}\left\{+\alpha s\frac{\partial}{\partial x_1}\right\}$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2\partial x_1}\right\}\left\{+\alpha\frac{\partial}{\partial x_1}\right\}\left\{+\alpha s\frac{\partial}{\partial x_2}\right\}$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2\partial x_2} + \mu\nabla^2\right\}\left\{+\alpha\frac{\partial}{\partial x_1}\right\}\left\{+\alpha s\frac{\partial}{\partial x_1}\right\}$$

$$= \left\{(\lambda+\mu)^2\frac{\partial^4}{\partial x_1\partial x_1\partial x_2\partial x_2} + \mu\nabla^2\left((\lambda+\mu)\left[\frac{\partial^2}{\partial x_1\partial x_1} + \frac{\partial^2}{\partial x_2\partial x_2}\right]\right) + \mu^2\nabla^4\right\}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$- \left\{(\lambda+\mu)^2\frac{\partial^4}{\partial x_1\partial x_2\partial x_1\partial x_2}\right\}\left\{\nabla^2 - \lambda_4^2\right\}K$$

$$+ (\lambda+\mu)\alpha^2 s\frac{\partial^4}{\partial x_1\partial x_1\partial x_2\partial x_2} - \mu\nabla^2\alpha^2 s\frac{\partial^2}{\partial x_2\partial x_2}$$

$$- \cancel{+}\ (\lambda+\mu)\alpha^2 s\frac{\partial^4}{\partial x_1\partial x_1\partial x_2\partial x_2}$$

$$- \cancel{+}\ (\lambda+\mu)\alpha^2 s\frac{\partial^4}{\partial x_1\partial x_1\partial x_2\partial x_2}$$

$$+ (\lambda+\mu)\alpha^2 s\frac{\partial^4}{\partial x_1\partial x_1\partial x_2\partial x_2} - \mu\nabla^2\alpha^2 s\frac{\partial^2}{\partial x_1\partial x_1}$$

$$= \left\{\mu(\lambda+2\mu)\nabla^4\right\}\left\{\nabla^2 - \lambda_4^2\right\}K - \mu\alpha^2 s\nabla^4$$

98.11.23.

$$B_{44}^{*}(\partial x, s) = (-1)^{8} \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix}$$

$$= g_{11}(g_{22}g_{33} - g_{23}g_{32}) - g_{12}(g_{21}g_{33} - g_{23}g_{31}) + g_{13}(g_{21}g_{32} - g_{22}g_{31})$$

$$= g_{11}g_{22}g_{33} - g_{11}g_{23}g_{32} - g_{12}g_{21}g_{33} + g_{12}g_{23}g_{31} + g_{13}g_{21}g_{32} - g_{13}g_{22}g_{31}$$

$$= \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{+(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_2}\right\}$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}\left\{+(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_1}\right\}$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}\left\{+(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_1}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_2}\right\}$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{+(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_1}\right\}\left\{+(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_1}\right\}$$

$$= \left\{(\lambda+\mu)^2\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu\nabla^2(\lambda+\mu)\left(\frac{\partial^2}{\partial x_1 \partial x_1} + \frac{\partial^2}{\partial x_2 \partial x_2}\right) + \mu^2\nabla^4\right\}\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$- \left\{(\lambda+\mu)^2\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2}\right\}\left\{\nabla^2 - \lambda_3^2\right\}k$$

$$+ (\lambda+\mu)(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} - \mu\nabla^2(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\partial^2}{\partial x_2 \partial x_2}$$

$$- 2(\lambda+\mu)(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2}$$

$$+ (\lambda+\mu)(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\partial^2}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} - \mu\nabla^2(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\partial^2}{\partial x_1 \partial x_1}$$

$$= \underline{\underline{\left\{\mu(\lambda+2\mu)\nabla^4\right\}\left\{\nabla^2 - \lambda_3^2\right\}k - \mu(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\,\nabla^4}}$$

98.11.24.

$$B^{*}_{34}(\partial x, s) = (-1)^7 \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{41} & g_{42} & g_{43} \end{vmatrix}$$

$$= -g_{11}(g_{22}g_{43} - g_{23}g_{42}) + g_{12}(g_{21}g_{43} - g_{23}g_{41}) - g_{13}(g_{21}g_{42} - g_{22}g_{41})$$

$$= -g_{11}g_{22}g_{43} + g_{11}g_{23}g_{42} + g_{12}g_{21}g_{43} - g_{12}g_{23}g_{41} - g_{13}g_{21}g_{42} + g_{13}g_{22}g_{41}$$

$$= -\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu \nabla^2\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\{-\eta s\}$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{+(3\lambda+2\mu)\beta^s_T \frac{\partial}{\partial x_2}\right\}\left\{+\alpha s \frac{\partial}{\partial x_2}\right\}$$

$$+ \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}\{-\eta s\}$$

$$- \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta^s_T \frac{\partial}{\partial x_2}\right\}\left\{+\alpha s \frac{\partial}{\partial x_1}\right\}$$

$$- \left\{+(3\lambda+2\mu)\beta^s_T \frac{\partial}{\partial x_1}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}\left\{+\alpha s \frac{\partial}{\partial x_2}\right\}$$

$$+ \left\{+(3\lambda+2\mu)\beta^s_T \frac{\partial}{\partial x_1}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{+\alpha s \frac{\partial}{\partial x_1}\right\}$$

$$= -\left\{(\lambda+\mu)^2 \frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu(\lambda+2\mu)\nabla^4\right\}(-\eta s)$$

$$+ \left\{(\lambda+\mu)^2 \frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2}\right\}(-\eta s)$$

$$+ \left\{(\lambda+\mu)(3\lambda+2\mu)\beta^s_T \alpha s \frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu(3\lambda+2\mu)\beta^s_T \alpha s\, \nabla^2 \frac{\partial^2}{\partial x_2 \partial x_2}\right\}$$

$$+ \left\{(\lambda+\mu)(3\lambda+2\mu)\beta^s_T \alpha s \frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu(3\lambda+2\mu)\beta^s_T \alpha s\, \nabla^2 \frac{\partial^2}{\partial x_1 \partial x_1}\right\}$$

$$-2\left\{(\lambda+\mu)(3\lambda+2\mu)\beta^s_T \alpha s \frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2}\right\}$$

$$= +(3\lambda+2\mu)\mu\beta^s_T \alpha s \nabla^4 + (\lambda+2\mu)\mu\eta s \nabla^4.$$

98.11.24.

$$B^*_{43}(\partial x, s) = (-1)^7 \begin{vmatrix} g_{11} & g_{12} & g_{14} \\ g_{21} & g_{22} & g_{24} \\ g_{31} & g_{32} & g_{34} \end{vmatrix}$$

$$= -g_{11}(g_{22}g_{34} - g_{24}g_{32}) + g_{12}(g_{21}g_{34} - g_{24}g_{31}) - g_{14}(g_{21}g_{32} - g_{22}g_{31})$$

$$= -g_{11}g_{22}g_{34} + g_{11}g_{24}g_{32} + g_{12}g_{21}g_{34} - g_{12}g_{24}g_{31} - g_{14}g_{21}g_{32} + g_{14}g_{22}g_{31}$$

$$= -\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}(-\eta T_0 s)$$

$$+\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\right\}\left\{+\alpha\frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta^s_T T_0 s\frac{\partial}{\partial x_2}\right\}$$

$$+\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}(-\eta T_0 s)$$

$$-\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{+\alpha\frac{\partial}{\partial x_2}\right\}\left\{+(3\lambda+2\mu)\beta^s_T T_0 s\frac{\partial}{\partial x_1}\right\}$$

$$-\left\{+\alpha\frac{\partial}{\partial x_1}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_1}\right\}\left\{+(3\lambda+2\mu)\beta^s_T T_0 s\frac{\partial}{\partial x_2}\right\}$$

$$+\left\{+\alpha\frac{\partial}{\partial x_1}\right\}\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{+(3\lambda+2\mu)\beta^s_T T_0 s\frac{\partial}{\partial x_1}\right\}$$

$$= -\left\{(\lambda+\mu)^2\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu(\lambda+2\mu)\nabla^4\right\}(-\eta T_0 s)$$

$$+\left\{\cancel{(\lambda+\mu)^2}\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2}\right\}(-\eta T_0 s)$$

$$+\left\{(\lambda+\mu)(3\lambda+2\mu)\beta^s_T \alpha T_0 s\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu(3\lambda+2\mu)\beta^s_T \alpha T_0 s\nabla^2\frac{\partial^2}{\partial x_2 \partial x_2}\right\}$$

$$+\left\{(\lambda+\mu)(3\lambda+2\mu)\beta^s_T \alpha T_0 s\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2} + \mu(3\lambda+2\mu)\beta^s_T \alpha T_0 s\nabla^2\frac{\partial^2}{\partial x_1 \partial x_1}\right\}$$

$$-2\left\{(\lambda+\mu)(3\lambda+2\mu)\beta^s_T \alpha T_0 s\frac{\partial^4}{\partial x_1 \partial x_1 \partial x_2 \partial x_2}\right\}$$

$$= +(3\lambda+2\mu)\mu\beta^s_T \alpha T_0 s\nabla^4 + (\lambda+2\mu)\mu\eta T_0 s\nabla^4$$

98.11.25

General expression for B^*_{ij} $(i,j=1,2)$

$$B^*_{11}(\partial x, s) = \left\{(\lambda+\mu)\frac{\partial^2}{\partial x_2 \partial x_2} + \mu\nabla^2\right\}\left\{kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - \eta^2 s^2 T_0\right\}$$
$$- (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_2 \partial x_2}\{\nabla^2-\lambda_4^2\}K$$
$$- \alpha^2 s \frac{\partial^2}{\partial x_2 \partial x_2}\{\nabla^2-\lambda_3^2\}k$$
$$- 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \frac{\partial^2}{\partial x_2 \partial x_2}.$$

$$B^*_{12}(\partial x, s) = -\left\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_2}\right\}\left\{kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - \eta^2 s^2 T_0\right\}$$
$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2-\lambda_4^2\}K$$
$$+ \alpha^2 s \frac{\partial^2}{\partial x_1 \partial x_2}\{\nabla^2-\lambda_3^2\}k$$
$$+ 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \frac{\partial^2}{\partial x_1 \partial x_2}.$$

$$B^*_{ij}(\partial x, s) = -\frac{\partial^2}{\partial x_i \partial x_j}\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0$$
$$- (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\{\nabla^2-\lambda_4^2\}K$$
$$- \alpha^2 s\{\nabla^2-\lambda_3^2\}k$$
$$- 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2\Big]$$
$$+ \delta_{ij} (A)$$

$$(A) = \nabla^2\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0$$
$$- (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K$$
$$- \alpha^2 s(\nabla^2-\lambda_3^2)k$$
$$- 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2\Big]$$
$$+ \nabla^2\left[\mu kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - \mu\eta^2 s^2 T_0\right]$$

$$= (\lambda+2\mu)\left[kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - \eta^2 s^2 T_0\right]\nabla^2$$
$$+ \Big[-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K$$
$$- \alpha^2 s(\nabla^2-\lambda_3^2)k$$
$$- 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2\Big]\nabla^2$$

Assumption: $(A) = (\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)$.

where λ_1 & λ_2 are ↓

98.11.25.

$$
\begin{aligned}
A = {} & (\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2) \\
= {} & (\lambda+2\mu)kK\nabla^2(\nabla^4-(\lambda_1^2+\lambda_2^2)\nabla^2+\lambda_1^2\lambda_2^2) \\
= {} & (\lambda+2\mu)kK\nabla^2(\nabla^2-(\lambda_3^2+\lambda_4^2)\nabla^2+\lambda_3^2\lambda_4^2) \\
& -(\lambda+2\mu)\eta^2 s^2 T_0\nabla^2 \\
& -(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^4-\lambda_4^2\nabla^2)K \\
& -\alpha^2 s(\nabla^4-\lambda_3^2\nabla^2)k \\
& -2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\nabla^2 .
\end{aligned}
$$

$$\lambda_1^2+\lambda_2^2=\lambda_3^2+\lambda_4^2+\{(3\lambda+2\mu)^2(\beta_T^s)^2T_0sK+\alpha^2sk\}/(\lambda+2\mu)kK$$

$$
\begin{aligned}
\lambda_1^2\lambda_2^2=\lambda_3^2\lambda_4^2+\{ & (\lambda+2\mu)\eta^2s^2T_0+(3\lambda+2\mu)^2(\beta_T^s)^2T_0s\lambda_4^2K+\alpha^2s\lambda_3^2k \\
& -2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\}/(\lambda+2\mu)kK .
\end{aligned}
$$

98. 12. 2.

Determinant of B_{ij}

$$\det(B_{ij}) = \delta_{ij} B_{ik} B^*_{jk}.$$
$$= \delta_{ij}(B_{i1}B^*_{j1} + B_{i2}B^*_{j2} + B_{i3}B^*_{j3} + B_{i4}B^*_{j4}).$$

i 혹은 j 값과 무관하게 determinant는 일정하다.

- $B_{11}B^*_{11} + B^*_{12}B_{12} + B_{13}B^*_{13} + B_{14}B^*_{14} = \det(B_{ij}).$

- $$B_{11}B^*_{11} = \Big\{(\lambda+\mu)\frac{\partial^2}{\partial x_1 \partial x_1} + \mu\nabla^2\Big\}\Big\{-\frac{\partial^2}{\partial x_1\partial x_1}\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0 - (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K - \alpha^2 s(\nabla^2-\lambda_3^2)k - 2(3\lambda+2\mu)(\beta_T^s)\eta T_0 \alpha s^2\Big] + (\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)\Big\}$$

- $$B_{12}B^*_{12} = \Big\{(\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_2}\Big\}\Big\{-\frac{\partial^2}{\partial x_1\partial x_2}\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0 - (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K - 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 - \alpha^2 s(\nabla^2-\lambda_3^2)k\Big]\Big\}$$

- $$B_{13}B^*_{13} = \Big\{+(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_1}\Big\}\Big\{-\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_1}\nabla^2(\nabla^2-\lambda_4^2)K - \mu\eta T_0\alpha s^2\nabla^2\frac{\partial}{\partial x_1}\Big\}$$

- $$B_{14}B^*_{14} = \Big\{+\alpha\frac{\partial}{\partial x_1}\Big\}\Big\{-\mu\alpha s\frac{\partial}{\partial x_1}\nabla^2(\nabla^2-\lambda_3^2)k - \mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2\frac{\partial}{\partial x_1}\nabla^2\Big\}$$

$$\mathrm{Det}(B_{ij}) = (\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_1}\Big(-\frac{\partial^2}{\partial x_1\partial x_1}\Big)\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0 - (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K - \alpha^2 s(\nabla^2-\lambda_3^2)k - 2(3\lambda+2\mu)(\beta_T^s)\eta T_0\alpha s^2\Big]$$
$$+ (\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_1}(\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)$$
$$- \mu\nabla^2\frac{\partial^2}{\partial x_1\partial x_1}\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0 - (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K - \alpha^2 s(\nabla^2-\lambda_3^2)k - 2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\Big]$$
$$+ (\lambda+2\mu)\mu kK\nabla^4(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)$$
$$- (\lambda+\mu)\frac{\partial^2}{\partial x_1\partial x_1}\frac{\partial^2}{\partial x_2\partial x_2}\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - (\lambda+\mu)\eta^2 s^2 T_0 - (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K - \alpha^2 s(\nabla^2-\lambda_3^2)k - 2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\Big]$$

98.12.2.

$$-\mu(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_1 \partial x_1}\nabla^2(\nabla^2-\lambda_4^2)k$$

$$-\mu(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \frac{\partial^2}{\partial x_1 \partial x_1}\nabla^2$$

$$-\mu \alpha^2 s \frac{\partial^2}{\partial x_1 \partial x_1}\nabla^2(\nabla^2-\lambda_3^2)k$$

$$-\mu(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2 \frac{\partial^2}{\partial x_1 \partial x_1}\nabla^2$$

$$\begin{aligned}
Det(B_{ij}) = &-(\lambda+2\mu)\frac{\partial^2}{\partial x_1 \partial x_1}\nabla^2\Big[(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2)-(\lambda+\mu)\eta^2 s^2 T_0 \\
&-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K-\alpha^2 s(\nabla^2-\lambda_3^2)k \\
&-2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2\Big] \\
&+(\lambda+2\mu)\frac{\partial^2}{\partial x_1 \partial x_1}(\lambda+\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2) \\
&+\mu\nabla^2\frac{\partial^2}{\partial x_1 \partial x_1}\Big[-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K-\alpha^2 s(\nabla^2-\lambda_3^2)k \\
&-2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2\Big] \\
&+(\lambda+2\mu)\mu kK\nabla^4(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)
\end{aligned}$$

$$\begin{aligned}
&(\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2) \\
&=(\lambda+2\mu)\nabla^2\left[kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2)-\eta^2 s^2 T_0\right] \\
&\quad+\nabla^2\left[-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K-\alpha^2 s(\nabla^2-\lambda_3^2)k-2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha s^2\right]
\end{aligned}$$

임으로 (이므로)

$$Det(B_{ij}) = (\lambda+2\mu)\mu kK\nabla^4(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2).$$

Back to Derivation of Laplace Domain Green Function Matrix.

$$\tilde{G}_{i3} = B^{*}_{3i}\cdot\varphi(r,s)$$

$$= (-)\left[\mu(3\lambda+2\mu)\beta_T^s \frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_4^2\}K + \mu\eta\alpha s\,\nabla^2\frac{\partial}{\partial x_i}\right]\cdot$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu kKs}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)-\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\right]$$

$$= +\mu(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_4^2\}K\cdot\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\cdot\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$-\mu(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_4^2\}K\cdot\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\cdot\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$+\mu\eta\alpha s\frac{\partial}{\partial x_i}\nabla^2\cdot\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\cdot\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$-\mu\eta\alpha s\frac{\partial}{\partial x_i}\nabla^2\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\cdot\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$= +\mu(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_i}\frac{K}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$-\mu(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_i}\frac{K}{\lambda_1^2-\lambda_2^2}\left(\frac{\lambda_4^2}{\lambda_1^2}K_0(\lambda_1 r)-\frac{\lambda_4^2}{\lambda_2^2}K_0(\lambda_2 r)\right)\cdot\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$+\mu(3\lambda+2\mu)\beta_T^s\frac{\partial}{\partial x_i}\lambda_4^2 K\frac{1}{\lambda_1^2\lambda_2^2}\ln r\,\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$+\mu\eta\alpha s\frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$-\mu\eta\alpha s\frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2\lambda_2^2}\ln r\,\frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kKs}\left[\frac{\mu(3\lambda+2\mu)\beta_T^s K}{\lambda_1^2-\lambda_2^2}\left(-\lambda_1 K_1(\lambda_1 r)+\lambda_2 K_1(\lambda_2 r)\right)\cdot\frac{x_i}{r}\right.$$

$$-\frac{\mu(3\lambda+2\mu)\beta_T^s K}{\lambda_1^2-\lambda_2^2}\left(-\frac{\lambda_4^2}{\lambda_1}K_1(\lambda_1 r)+\frac{\lambda_4^2}{\lambda_2}K_1(\lambda_2 r)\right)\cdot\frac{x_i}{r}$$

$$+\frac{\mu(3\lambda+2\mu)\beta_T^s\lambda_4^2 K}{\lambda_1^2\lambda_2^2}\frac{1}{r}\cdot\frac{x_i}{r}$$

$$+\frac{\mu\eta\alpha s}{\lambda_1^2-\lambda_2^2}\left(-\frac{1}{\lambda_1}K_1(\lambda_1 r)+\frac{1}{\lambda_2}K_1(\lambda_2 r)\right)\cdot\frac{x_i}{r}$$

$$\left.-\frac{\mu\eta\alpha s}{\lambda_1^2\lambda_2^2}\frac{1}{r}\frac{x_i}{r}\right]$$

$$= \frac{(+1)}{2\pi(\lambda+2\mu)\mu kKs}\left[\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)}\frac{1}{\lambda_1}\left\{+\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K\left(\overset{\lambda_1^2-\lambda_4^2}{\lambda_4^2-\lambda_1^2}\right)\right\}\right.$$

$$-\frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)}\frac{1}{\lambda_2}\left\{+\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K\left(\overset{\lambda_2^2-\lambda_4^2}{\cancel{\lambda_4^2-\lambda_2^2}}\right)\right\}$$

$$\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left\{+\mu\eta\alpha s-\mu(3\lambda+2\mu)\beta_T^s K\lambda_4^2\right\}\right]\cdot\frac{x_i}{r}$$

98.12.16.

$$\tilde{\sigma}_{T3i} = B^{*}_{i3}\cdot\varphi(r,s)$$

$$= (-1)\left[\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_4^2\}K + \mu\eta T_0\alpha s^2\nabla^2\frac{\partial}{\partial x_i}\right]\cdot \frac{1}{2\pi(\lambda+2\mu)\mu kKS}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)-\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\right]$$

$$\begin{aligned}= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kKS}\Big[&\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_4^2\}\frac{K}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\\ &-\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_4^2\}\frac{K}{4\lambda_1^2\lambda_2^2}r^2\ln r\\ &+\mu\eta T_0\alpha s^2\frac{\partial}{\partial x_i}\nabla^2\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\\ &-\mu\eta T_0\alpha s^2\frac{\partial}{\partial x_i}\nabla^2\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\Big]\end{aligned}$$

$$\begin{aligned}= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kKS}\Big[&\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\frac{K}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\\ &-\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\frac{K}{\lambda_1^2-\lambda_2^2}\left(\frac{\lambda_4^2}{\lambda_1^2}K_0(\lambda_1 r)-\frac{\lambda_4^2}{\lambda_2^2}K_0(\lambda_2 r)\right)\\ &+\mu(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\lambda_4^2K\frac{1}{\lambda_1^2\lambda_2^2}\ln r\\ &+\mu\eta T_0\alpha s^2\frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)\\ &-\mu\eta T_0\alpha s^2\frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2\lambda_2^2}\ln r\Big]\end{aligned}$$

중간과정 확인

$$\begin{aligned}= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kKS}\Big[&\frac{\mu(3\lambda+2\mu)\beta_T^s T_0 sK}{\lambda_1^2-\lambda_2^2}\left(-\lambda_1K_1(\lambda_1 r)+\lambda_2K_1(\lambda_2 r)\right)\\ &-\frac{\mu(3\lambda+2\mu)\beta_T^s T_0 sK}{\lambda_1^2-\lambda_2^2}\left(-\frac{\lambda_4^2}{\lambda_1}K_1(\lambda_1 r)+\frac{\lambda_4^2}{\lambda_2}K_1(\lambda_2 r)\right)\\ &+\frac{\mu(3\lambda+2\mu)\beta_T^s T_0 s\lambda_4^2K}{\lambda_1^2\lambda_2^2}\frac{1}{r}\\ &+\frac{\mu\eta T_0\alpha s^2}{\lambda_1^2-\lambda_2^2}\left(-\frac{1}{\lambda_1}K_1(\lambda_1 r)+\frac{1}{\lambda_2}K_1(\lambda_2 r)\right)\\ &-\frac{\mu\eta T_0\alpha s^2}{\lambda_1^2\lambda_2^2}\frac{1}{r}\Big]\cdot\frac{x_i}{r}\end{aligned}$$

$$\begin{aligned}= \frac{(+1)}{2\pi(\lambda+2\mu)\mu kKS}\Big[&\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\left\{+\mu\eta T_0\alpha s^2+\mu(3\lambda+2\mu)\beta_T^s T_0 sK\left(\overset{\lambda_1^2-\lambda_4^2}{\cancel{\lambda_4^2-\lambda_1^2}}\right)\right\}\\ &-\frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\left\{+\mu\eta T_0\alpha s^2+\mu(3\lambda+2\mu)\beta_T^s T_0 sK\left(\overset{\lambda_2^2-\lambda_4^2}{\cancel{\lambda_4^2-\lambda_2^2}}\right)\right\}\\ &+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left\{\mu\eta T_0\alpha s^2-\mu(3\lambda+2\mu)\beta_T^s T_0 sK\lambda_4^2\right\}\Big]\cdot\frac{x_i}{r}\end{aligned}$$

98.12.16.

$$\widetilde{G}_{i4} = B^{*}_{\cancel{4}i} \cdot \varphi(r,s)$$

$$= (-1)^{0}\left[+\mu\alpha \frac{\partial}{\partial x_i} \nabla^2\{\nabla^2-\lambda_3^2\}k + \mu\eta(3\lambda+2\mu)\beta_T^s T_0 s \frac{\partial}{\partial x_i}\nabla^2\right] \cdot$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu k K s}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right) - \frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r\right]$$

$$= \frac{(-1)^{0}}{2\pi(\lambda+2\mu)\mu k k s}\left[\mu\alpha\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_3^2\}k\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\right.$$

$$-\mu\alpha\frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_3^2\}\frac{k}{4\lambda_1^2\lambda_2^2} r^2 \ln r$$

$$+\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\nabla^2\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$\left.-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\nabla^2\frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r\right]$$

$$= \frac{(-1)^{0}}{2\pi(\lambda+2\mu)\mu k k s}\left[\mu\alpha\frac{\partial}{\partial x_i}\frac{k}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r) - K_0(\lambda_2 r)\right)\right.$$

$$-\mu\alpha\frac{\partial}{\partial x_i}\frac{k}{\lambda_1^2-\lambda_2^2}\left(\frac{\lambda_3^2}{\lambda_1^2}K_0(\lambda_1 r) - \frac{\lambda_3^2}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$+\mu\alpha\frac{\partial}{\partial x_i}\frac{\lambda_3^2 k}{\lambda_1^2\lambda_2^2}\ln r$$

$$+\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r) - \frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$\left.-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s\frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2\lambda_2^2}\ln r\right]$$

$$= \frac{(-1)^{0}}{2\pi(\lambda+2\mu)\mu k k s}\left[\frac{\mu\alpha k}{\lambda_1^2-\lambda_2^2}\left(-\lambda_1 K_1(\lambda_1 r) + \lambda_2 K_1(\lambda_2 r)\right)\right.$$

$$-\frac{\mu\alpha k}{\lambda_1^2-\lambda_2^2}\left(-\frac{\lambda_3^2}{\lambda_1}K_1(\lambda_1 r) + \frac{\lambda_3^2}{\lambda_2}K_1(\lambda_2 r)\right)$$

$$+\frac{\mu\alpha\lambda_3^2 k}{\lambda_1^2\lambda_2^2}\frac{1}{r}$$

$$+\frac{\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s}{\lambda_1^2-\lambda_2^2}\left(-\frac{1}{\lambda_1}K_1(\lambda_1 r) + \frac{1}{\lambda_2}K_1(\lambda_2 r)\right)$$

$$\left.-\frac{\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s}{\lambda_1^2\lambda_2^2}\frac{1}{r}\right]\cdot\frac{x_i}{r}$$

$$= \frac{(+1)^{0}}{2\pi(\lambda+2\mu)\mu k K s}\left[\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\left\{+\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s + \mu\alpha k\left(\overset{\lambda_1^2-\lambda_3^2}{\cancel{\lambda_3^2-\lambda_1^2}}\right)\right\}\right.$$

$$-\frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\left\{+\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s + \mu\alpha k\left(\overset{\lambda_2^2-\lambda_3^2}{\cancel{\lambda_3^2-\lambda_2^2}}\right)\right\}$$

$$\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left\{+\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s \mp \mu\alpha k\lambda_3^2\right\}\right]\cdot\frac{x_i}{r}$$

98.12.16.

$$\tilde{G}_{4i} = B_{i4}^{*} \cdot \varphi(r,s)$$

$$= (-1)\Big[\mu\alpha s \frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_3^2\}k + \mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 \frac{\partial}{\partial x_i}\nabla^2\Big]$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu k K s}\Big[\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big) - \frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r\Big]$$

$$\begin{aligned}
= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K s}\Big[& \mu\alpha s \frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_3^2\}k \frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big) \\
& - \mu\alpha s \frac{\partial}{\partial x_i}\nabla^2\{\nabla^2-\lambda_3^2\}\frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r \\
& + \mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 \frac{\partial}{\partial x_i}\nabla^2 \frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big) \\
& - \mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 \frac{\partial}{\partial x_i}\nabla^2 \frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r\Big]
\end{aligned}$$

$$\begin{aligned}
= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K s}\Big[& \mu\alpha s \frac{\partial}{\partial x_i}\frac{k}{\lambda_1^2-\lambda_2^2}\big(K_0(\lambda_1 r) - K_0(\lambda_2 r)\big) \\
& - \mu\alpha s \frac{\partial}{\partial x_i}\frac{k}{\lambda_1^2-\lambda_2^2}\Big(\frac{\lambda_3^2}{\lambda_1^2}K_0(\lambda_1 r) - \frac{\lambda_3^2}{\lambda_2^2}K_0(\lambda_2 r)\Big) \\
& + \mu\alpha s \frac{\partial}{\partial x_i}\frac{\lambda_3^2 k}{\lambda_1^2\lambda_2^2}\ln r \\
& + \mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 \frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^2}K_0(\lambda_1 r) - \frac{1}{\lambda_2^2}K_0(\lambda_2 r)\Big) \\
& - \mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 \frac{\partial}{\partial x_i}\frac{1}{\lambda_1^2\lambda_2^2}\ln r\Big]
\end{aligned}$$

$$\begin{aligned}
= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K s}\Big[& \frac{\mu\alpha s k}{\lambda_1^2-\lambda_2^2}\big(-\lambda_1 K_1(\lambda_1 r) + \lambda_2 K_1(\lambda_2 r)\big) \\
& - \frac{\mu\alpha s k}{\lambda_1^2-\lambda_2^2}\Big(-\frac{\lambda_3^2}{\lambda_1}K_1(\lambda_1 r) + \frac{\lambda_3^2}{\lambda_2}K_1(\lambda_2 r)\Big) \\
& + \frac{\mu\alpha s\lambda_3^2 k}{\lambda_1^2\lambda_2^2}\frac{1}{r} \\
& + \frac{\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2}{\lambda_1^2-\lambda_2^2}\Big(-\frac{1}{\lambda_1}K_1(\lambda_1 r) + \frac{1}{\lambda_2}K_1(\lambda_2 r)\Big) \\
& - \frac{\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2}{\lambda_1^2\lambda_2^2}\frac{1}{r}\Big\}\cdot\frac{x_i}{r}
\end{aligned}$$

$$\begin{aligned}
= \frac{(+1)}{2\pi(\lambda+2\mu)\mu k K s}\Big[& \frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\Big\{+\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 + \mu\alpha s k\big(\overset{\lambda_1^2-\lambda_3^2}{\cancel{\lambda_3^2-\lambda_1^2}}\big)\Big\} \\
& - \frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\Big\{+\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 + \mu\alpha s k\big(\overset{\lambda_2^2-\lambda_3^2}{\cancel{\lambda_3^2-\lambda_2^2}}\big)\Big\} \\
& + \frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\Big\{+\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s^2 - \mu\alpha s k\lambda_3^2\Big\}\Big]\cdot\frac{x_i}{r}
\end{aligned}$$

98.12.18.

$$\tilde{G}_{44} = B^{*}_{44}\cdot\varphi(r,s)$$

$$= \left[\mu(\lambda+2\mu)\nabla^4\{\nabla^2-\lambda_3^2\}k - \mu(3\lambda+2\mu)^2(\beta_T^s)^2 s\,\nabla^4\right]$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu kks}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)-\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\right]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kks}\Big[\mu(\lambda+2\mu)\nabla^4\{\nabla^2-\lambda_3^2\}k\cdot\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$-\mu(\lambda+2\mu)\nabla^4\{\nabla^2-\lambda_3^2\}k\,\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r$$

$$-\mu(3\lambda+2\mu)^2(\beta_T^s)^2 s\,\nabla^4\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$+\mu(3\lambda+2\mu)^2(\beta_T^s)^2 s\,\nabla^4\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\Big]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kks}\Big(\mu(\lambda+2\mu)\frac{k}{\lambda_1^2-\lambda_2^2}\left(\lambda_1^2K_0(\lambda_1 r)-\lambda_2^2K_0(\lambda_2 r)\right)$$

$$-\mu(\lambda+2\mu)\frac{k}{\lambda_1^2-\lambda_2^2}\left(\lambda_3^2K_0(\lambda_1 r)-\lambda_3^2K_0(\lambda_2 r)\right)$$

$$-\mu(3\lambda+2\mu)^2(\beta_T^s)^2 s\,\frac{1}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\Big]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kks}\Big[\frac{\mu K_0(\lambda_1 r)}{\lambda_1^2-\lambda_2^2}\left\{(\lambda+2\mu)k(\lambda_1^2-\lambda_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2 s\right\}$$

$$-\frac{\mu K_0(\lambda_1 r)}{\lambda_1^2-\lambda_2^2}\left\{(\lambda+2\mu)k(\lambda_2^2-\lambda_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2 s\right\}\Big]$$

$$\tilde{G}_{33} = B^{*}_{33}\cdot\varphi(r,s)$$

$$= \left[\mu(\lambda+2\mu)\nabla^4\{\nabla^2-\lambda_4^2\}k - \mu\alpha^2 s\,\nabla^4\right]\cdot$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu kks}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)-\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\right]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kks}\Big[\mu(\lambda+2\mu)\nabla^4\{\nabla^2-\lambda_4^2\}k\cdot\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$-\mu\alpha^2 s\,\nabla^4\cdot\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\Big]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kks}\Big[\mu(\lambda+2\mu)\frac{k}{\lambda_1^2-\lambda_2^2}\left(\lambda_1^2K_0(\lambda_1 r)-\lambda_2^2K_0(\lambda_2 r)\right)$$

$$-\mu(\lambda+2\mu)\frac{k}{\lambda_1^2-\lambda_2^2}\left(\lambda_4^2K_0(\lambda_1 r)-\lambda_4^2K_0(\lambda_2 r)\right)$$

$$-\frac{\mu\alpha^2 s}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\Big]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kks}\Big[\frac{\mu K_0(\lambda_1 r)}{\lambda_1^2-\lambda_2^2}\left\{(\lambda+2\mu)k(\lambda_1^2-\lambda_4^2)-\alpha^2 s\right\}$$

$$-\frac{\mu K_0(\lambda_2 r)}{\lambda_1^2-\lambda_2^2}\left\{(\lambda+2\mu)k(\lambda_2^2-\lambda_4^2)-\alpha^2 s\right\}\Big]$$

98.12.19.

$$\widetilde{G_{34}} = B_{43}^{*} \cdot \varphi(r, s)$$

$$= +\left[\mu(3\lambda+2\mu)(\beta_T^s) T_0 \alpha s \nabla^4 + (\lambda+2\mu)\mu\eta T_0 s \nabla^4\right] \cdot$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu k K s}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right) - \frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r\right]$$

$$= \frac{+T_0}{2\pi(\lambda+2\mu)\mu k K s}\left[\frac{\mu(3\lambda+2\mu)\beta_T^s \alpha s + (\lambda+2\mu)\mu\eta s}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r) - K_0(\lambda_2 r)\right)\right]$$

$$\widetilde{G_{43}} = B_{34}^{*} \cdot \varphi(r, s)$$

$$= +\left[\mu(3\lambda+2\mu)\beta_T^s \alpha \cancel{T_0} s \nabla^4 + (\lambda+2\mu)\mu\eta \cancel{T_0} s \nabla^4\right] \cdot$$

$$\frac{1}{2\pi(\lambda+2\mu)\mu k K s}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right) - \frac{1}{4\lambda_1^2\lambda_2^2} r^2 \ln r\right]$$

$$= \frac{+\mu(3\lambda+2\mu)\beta_T^s \alpha T_0 s + (\lambda+2\mu)\mu\eta T_0 s}{2\pi(\lambda+2\mu)\mu k K s(\lambda_1^2-\lambda_2^2)}\left(K_0(\lambda_1 r) - K_0(\lambda_2 r)\right)$$

$$\widetilde{G}_{ij} = B^*_{ji}\cdot\varphi(r,s)_{,,} = B^*_{ij}\,\varphi(r,s)$$

$$= \Big[-\Big\{(\lambda+\mu)\frac{\partial^2}{\partial x_i\partial x_j}\Big\}\Big\{ kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) - \eta^2 s^2 T_0\Big\}$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s \frac{\partial^2}{\partial x_i\partial x_j}\{\nabla^2-\lambda_4^2\}K$$

$$+ \alpha^2 s \frac{\partial^2}{\partial x_i\partial x_j}\{\nabla^2-\lambda_3^2\}k$$

$$+ 2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2 \frac{\partial^2}{\partial x_i\partial x_j}$$

$$+ \delta_{ij}(\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)\Big]$$

$$\times \frac{1}{2\pi(\lambda+2\mu)\mu kKs}\Big[\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big) - \frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r \quad - \frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^4\lambda_2^4}\ln r\Big]$$

$$= \Big\{\frac{\partial^2}{\partial x_i\partial x_j}\Big[-(\lambda+\mu)kK(\nabla^2-\lambda_3^2)(\nabla^2-\lambda_4^2) + (\lambda+\mu)\eta^2 s^2 T_0$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s(\nabla^2-\lambda_4^2)K + \alpha^2 s(\nabla^2-\lambda_3^2)k$$

$$+ 2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\Big]$$

$$+ \delta_{ij}(\lambda+2\mu)kK\nabla^2(\nabla^2-\lambda_1^2)(\nabla^2-\lambda_2^2)\Big\}$$

$$\times \frac{1}{2\pi(\lambda+2\mu)\mu kKs}\Big[\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big) - \frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r \quad - \frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^4\lambda_2^4}\ln r\Big]$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu kKs}\Big\{\frac{\partial^2}{\partial x_i\partial x_j}\Big[-(\lambda+\mu)kK\nabla^4\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$+ \cancel{(\lambda+\mu)kK\nabla^4\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r} \to 0 \quad + \cancel{(\lambda+\mu)kK\nabla^4\frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^4\lambda_2^4}\ln r} \to 0$$

$$+ (\lambda+\mu)kK(\lambda_3^2+\lambda_4^2)\nabla^2\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$- (\lambda+\mu)kK(\lambda_3^2+\lambda_4^2)\nabla^2\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r \quad \cancel{-(\lambda+\mu)kK(\lambda_3^2+\lambda_4^2)\nabla^2\frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^4\lambda_2^4}\ln r} \to 0$$

$$- (\lambda+\mu)kK\lambda_3^2\lambda_4^2\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$+ (\lambda+\mu)kK\lambda_3^2\lambda_4^2\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r \quad + (\lambda+\mu)kK\lambda_3^2\lambda_4^2\frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^4\lambda_2^4}\ln r$$

$$+ (\lambda+\mu)\eta^2 s^2 T_0\frac{1}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$- (\lambda+\mu)\eta^2 s^2 T_0\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r \quad - (\lambda+\mu)\eta^2 s^2 T_0\frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^4\lambda_2^4}\ln r$$

$$+ (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\nabla^2\frac{K}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$- (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\lambda_4^2\frac{K}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$+ \alpha^2 s\nabla^2\frac{k}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$- \alpha^2 s\lambda_3^2\frac{k}{\lambda_1^2-\lambda_2^2}\Big(\frac{1}{\lambda_1^4}K_0(\lambda_1 r) - \frac{1}{\lambda_2^4}K_0(\lambda_2 r)\Big)$$

$$\boxed{\begin{aligned}&+2(3\lambda+2\mu)\beta_T^S\eta T_0\alpha s^2\\ &\times\left\{\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)\right.\\ &\left.-\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\right\}\end{aligned}}$$

$$-\frac{1}{\lambda_1^2\lambda_2^2}\ln r$$

$$-(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\,\nabla^2\frac{k}{4\lambda_1^2\lambda_2^2}r^2\ln r\;\cancel{-(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\,\nabla^2\frac{k(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r}\to 0$$

$$+(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\,\lambda_4^2\frac{k}{4\lambda_1^2\lambda_2^2}r^2\ln r+(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\,\lambda_4^2\frac{k(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r$$

$$-\alpha^2 s\,\nabla^2\frac{k}{4\lambda_1^2\lambda_2^2}r^2\ln r\;\cancel{-\alpha^2 s\,\nabla^2\frac{k(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r}\to 0$$

$$+\alpha^2 s\,\lambda_3^2\frac{k}{4\lambda_1^2\lambda_2^2}r^2\ln r\;+\alpha^2 s\,\lambda_3^2\frac{k(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r\;\Big]$$

$$+\delta_{ij}(\lambda+2\mu)kK\,\nabla^6\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$-\delta_{ij}(\lambda+2\mu)kK\,\nabla^4\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$+\delta_{ij}(\lambda+2\mu)kK\,\nabla^2\frac{\lambda_1^2\lambda_2^2}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$-\delta_{ij}(\lambda+2\mu)kK\,\nabla^6\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\;\cancel{-\delta_{ij}(\lambda+2\mu)kK\,\nabla^6\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^4\lambda_2^4}\ln r}\to 0$$

$$+\delta_{ij}(\lambda+2\mu)kK\,\nabla^4\frac{\lambda_1^2+\lambda_2^2}{4\lambda_1^2\lambda_2^2}r^2\ln r\;\cancel{+\delta_{ij}(\lambda+2\mu)kK\,\nabla^4\frac{(\lambda_1^2+\lambda_2^2)^2}{\lambda_1^4\lambda_2^4}\ln r}\to 0$$

$$-\delta_{ij}(\lambda+2\mu)kK\,\nabla^2\frac{\lambda_1^2\lambda_2^2}{4\lambda_1^2\lambda_2^2}r^2\ln r\;\Big\}\;\cancel{-\delta_{ij}(\lambda+2\mu)kK\,\nabla^2\frac{\lambda_1^2\lambda_2^2(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r}\Big\}\to 0$$

$$=\frac{1}{2\pi(\lambda+2\mu)\mu kKS}\left\{\frac{\partial^2}{\partial x_i\partial x_j}\left[-(\lambda+\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\right.\right.$$

$$+(\lambda+\mu)kK(\lambda_3^2+\lambda_4^2)\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$-(\lambda+\mu)kK(\lambda_3^2+\lambda_4^2)\frac{1}{\lambda_1^2\lambda_2^2}\left(\ln r+\frac{1}{2}\right)$$

$$-(\lambda+\mu)kK\lambda_3^2\lambda_4^2\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$+(\lambda+\mu)kK\lambda_3^2\lambda_4^2\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\;+(\lambda+\mu)kK\lambda_3^2\lambda_4^2\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^4\lambda_2^4}\ln r$$

$$+(\lambda+\mu)\eta^2 s^2 T_0\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$\cancel{-(\lambda+\mu)\eta^2 s^2 T_0\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)}$$

$$-(\lambda+\mu)\eta^2 s^2 T_0\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\;-(\lambda+\mu)\eta^2 s^2 T_0\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^4\lambda_2^4}\ln r$$

$$+(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\frac{k}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$\cancel{-(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\frac{k}{\lambda_1^2\lambda_2^2}\left(\ln r+\frac{1}{2}\right)}$$

$$-(3\lambda+2\mu)^2(\beta_T^S)^2T_0 s\frac{\lambda_4^2 k}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$ ←99.2.15.

$$+\alpha^2 s\frac{k}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$-\alpha^2 s\frac{\lambda_3^2 k}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{K}{4\lambda_1^2\lambda_2^2}(4\ln r+2)$$

$$+(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\lambda_4^2 K}{4\lambda_1^2\lambda_2^2}r^2\ln r+(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\lambda_4^2 K(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r$$

$$-\alpha^2 s\frac{K}{4\lambda_1^2\lambda_2^2}(4\ln r+2)\ \cancel{+\alpha^2 s}$$

$$+\alpha^2 s\frac{\lambda_3^2 K}{4\lambda_1^2\lambda_2^2}r^2\ln r+\alpha^2 s\frac{\lambda_3^2 K(\lambda_1^2+\lambda_2^2)}{\lambda_1^4\lambda_2^4}\ln r$$

$$+2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^4}K_0(\lambda_1 r)-\frac{1}{\lambda_2^4}K_0(\lambda_2 r)\right)$$

$$-2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\frac{1}{4\lambda_1^2\lambda_2^2}r^2\ln r\Big\}-2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha s^2\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^4\lambda_2^4}\ln r\Big]$$

$$+\delta_{ij}(\lambda+2\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(\lambda_1^2K_0(\lambda_1 r)-\lambda_2^2K_0(\lambda_2 r)\right)$$

$$-\delta_{ij}(\lambda+2\mu)kK\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)$$

$$+\delta_{ij}(\lambda+2\mu)kK\frac{\lambda_1^2\lambda_2^2}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$-\delta_{ij}(\lambda+2\mu)kK\frac{\lambda_1^2\lambda_2^2}{\lambda_1^2\lambda_2^2}\left(\ln r+\frac{1}{2}\right)\Big\}$$

$$=\frac{1}{2\pi(\lambda+2\mu)\mu kKs}\Big\{-(\lambda+\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{\lambda_1K_1(\lambda_1 r)}{r}+\frac{x_ix_j}{r^2}\lambda_1^2K_0(\lambda_1 r)+\frac{2x_ix_j}{r^3}\lambda_1K_1(\lambda_1 r)\right)$$

$$+(\lambda+\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{\lambda_2K_1(\lambda_2 r)}{r}+\frac{x_ix_j}{r^2}\lambda_2^2K_0(\lambda_2 r)+\frac{2x_ix_j}{r^3}\lambda_2K_1(\lambda_2 r)\right)$$

$$+(\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1 r}+\frac{x_ix_j}{r^2}K_0(\lambda_1 r)+\frac{2x_ix_j}{r^3}\frac{1}{\lambda_1}K_1(\lambda_1 r)\right)$$

$$-(\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2 r}+\frac{x_ix_j}{r^2}K_0(\lambda_2 r)+\frac{2x_ix_j}{r^3}\frac{1}{\lambda_2}K_1(\lambda_2 r)\right)$$

$$-(\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_1^2\lambda_2^2}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_ix_j}{r^2}\left(\frac{2}{r^2}\right)\right)$$

$$-(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1^3 r}+\frac{x_ix_j}{\lambda_1^2r^2}K_0(\lambda_1 r)+\frac{2x_ix_j}{\lambda_1^3r^3}K_1(\lambda_1 r)\right)$$

$$+(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2^3 r}+\frac{x_ix_j}{\lambda_2^2r^2}K_0(\lambda_2 r)+\frac{2x_ix_j}{\lambda_2^3r^3}K_1(\lambda_2 r)\right)$$

$$+(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{4\lambda_1^2\lambda_2^2}\left(\delta_{ij}(2\ln r+1)+2\frac{x_ix_j}{r^2}\right)$$

$$+(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_1^4\lambda_2^4}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_ix_j}{r^2}\frac{2}{r^2}\right)\rightarrow$$

$$+(\lambda+\mu)\frac{\eta^2s^2T_0}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1^3 r}+\frac{x_ix_j}{\lambda_1^2r^2}K_0(\lambda_1 r)+\frac{2x_ix_j}{\lambda_1^3r^3}K_1(\lambda_1 r)\right)$$

$$-(\lambda+\mu)\frac{\eta^2s^2T_0}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2^3 r}+\frac{x_ix_j}{\lambda_2^2r^2}K_0(\lambda_2 r)+\frac{2x_ix_j}{\lambda_2^3r^3}K_1(\lambda_2 r)\right)$$

$$-(\lambda+\mu)\frac{\eta^2s^2T_0}{4\lambda_1^2\lambda_2^2}\left(\delta_{ij}(2\ln r+1)+2\frac{x_ix_j}{r^2}\right)$$

$$-(\lambda+\mu)\frac{\eta^2s^2T_0}{\lambda_1^2\lambda_2^2}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_ix_j}{r^2}\frac{2}{r^2}\right)\rightarrow$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0sK}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1 r}+\frac{x_ix_j}{r^2}K_0(\lambda_1 r)+\frac{2x_ix_j}{\lambda_1r^3}K_1(\lambda_1 r)\right)$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0sK}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2 r}+\frac{x_ix_j}{r^2}K_0(\lambda_2 r)+\frac{2x_ix_j}{\lambda_2r^3}K_1(\lambda_2 r)\right)$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0s\lambda_4^2K}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1^3 r}+\frac{x_ix_j}{\lambda_1^2r^2}K_0(\lambda_1 r)+\frac{2x_ix_j}{\lambda_1^3r^3}K_1(\lambda_1 r)\right)$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0s\lambda_4^2K}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2^3 r}+\frac{x_ix_j}{\lambda_2^2r^2}K_0(\lambda_2 r)+\frac{2x_ix_j}{\lambda_2^3r^3}K_1(\lambda_2 r)\right)$$

$$+\alpha^2\frac{sk}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1 r}+\frac{x_i x_j}{r^2}K_0(\lambda_1 r)+\frac{2x_i x_j}{\lambda_1 r^3}K_1(\lambda_1 r)\right)$$

$$-\alpha^2\frac{sk}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2 r}+\frac{x_i x_j}{r^2}K_0(\lambda_2 r)+\frac{2x_i x_j}{\lambda_2 r^3}K_1(\lambda_2 r)\right)$$

$$-\alpha^2\frac{s\lambda_3^2 k}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1^3 r}+\frac{x_i x_j}{\lambda_1^2 r^2}K_0(\lambda_1 r)+\frac{2x_i x_j}{\lambda_1^3 r^3}K_1(\lambda_1 r)\right)$$

$$+\alpha^2\frac{s\lambda_3^2 k}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2^3 r}+\frac{x_i x_j}{\lambda_2^2 r^2}K_0(\lambda_2 r)+\frac{2x_i x_j}{\lambda_2^3 r^3}K_1(\lambda_2 r)\right)$$

$$-(3\lambda+2\mu)^2(\beta_T^s)^2T_0 s\frac{K}{\lambda_1^2\lambda_2^2}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_i x_j}{r^2}\frac{2}{r^2}\right)$$

$$+(3\lambda+2\mu)^2(\beta_T^s)^2T_0 s\frac{\lambda_4^2 K}{4\lambda_1^2\lambda_2^2}\left(\delta_{ij}(2\ln r+1)+2\frac{x_i x_j}{r^2}\right)$$

$$+(3\lambda+2\mu)^2(\beta_T^s)^2T_0 s\frac{\lambda_4^2 K}{\lambda_1^4\lambda_2^4}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_i x_j}{r^2}\frac{2}{r^2}\right)\rightarrow \quad (\lambda_1^2+\lambda_2^2)$$

$$-\alpha^2 s\frac{k}{\lambda_1^2\lambda_2^2}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_i x_j}{r^2}\frac{2}{r^2}\right)$$

$$+\alpha^2 s\frac{\lambda_3^2 k}{4\lambda_1^2\lambda_2^2}\left(\delta_{ij}(2\ln r+1)+2\frac{x_i x_j}{r^2}\right)$$

$$+\alpha^2 s\frac{\lambda_3^2 k}{\lambda_1^4\lambda_2^4}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_i x_j}{r^2}\frac{2}{r^2}\right)\longrightarrow \quad (\lambda_1^2+\lambda_2^2)$$

$$+2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_1 r)}{\lambda_1^3 r}+\frac{x_i x_j}{\lambda_1^2 r^2}K_0(\lambda_1 r)+\frac{2x_i x_j}{\lambda_1^3 r^3}K_1(\lambda_1 r)\right)$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^2-\lambda_2^2}\left(-\delta_{ij}\frac{K_1(\lambda_2 r)}{\lambda_2^3 r}+\frac{x_i x_j}{\lambda_2^2 r^2}K_0(\lambda_2 r)+\frac{2x_i x_j}{\lambda_2^3 r^3}K_1(\lambda_2 r)\right)$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{4\lambda_1^2\lambda_2^2}\left(\delta_{ij}(2\ln r+1)+2\frac{x_i x_j}{r^2}\right)$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^4\lambda_2^4}\left(\delta_{ij}\frac{1}{r^2}-\frac{x_i x_j}{r^2}\frac{2}{r^2}\right) \quad (\lambda_1^2+\lambda_2^2)$$

$$+\delta_{ij}(\lambda+2\mu)\frac{kK}{\lambda_1^2-\lambda_2^2}\left(\lambda_1^2K_0(\lambda_1 r)-\lambda_2^2K_0(\lambda_2 r)\right)$$

$$-\delta_{ij}(\lambda+2\mu)\frac{kK}{\lambda_1^2-\lambda_2^2}\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)(\lambda_1^2+\lambda_2^2)$$

$$+\delta_{ij}(\lambda+2\mu)\frac{\lambda_1^2\lambda_2^2 kK}{\lambda_1^2-\lambda_2^2}\left(\frac{1}{\lambda_1^2}K_0(\lambda_1 r)-\frac{1}{\lambda_2^2}K_0(\lambda_2 r)\right)$$

$$-\delta_{ij}(\lambda+2\mu)\frac{\lambda_1^2\lambda_2^2 kK}{\lambda_1^2\lambda_2^2}\left(\ln r+\frac{1}{2}\right)\Bigg\}$$

$$=\frac{1}{2\pi(\lambda+2\mu)\mu kKs}\Bigg\{\delta_{ij}\Bigg[\frac{K_1(\lambda_1 r)}{r}\bigg((\lambda+\mu)kK\frac{\lambda_1}{\lambda_1^2-\lambda_2^2}-(\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_1(\lambda_1^2-\lambda_2^2)}$$

$$+(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_1^3(\lambda_1^2-\lambda_2^2)}-(\lambda+\mu)\frac{\eta^2s^2T_0}{\lambda_1^3(\lambda_1^2-\lambda_2^2)}$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 sK}{\lambda_1(\lambda_1^2-\lambda_2^2)}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 s\lambda_4^2K}{\lambda_1^3(\lambda_1^2-\lambda_2^2)}$$

$$-\alpha^2\frac{sk}{\lambda_1(\lambda_1^2-\lambda_2^2)}+\alpha^2\frac{s\lambda_3^2k}{\lambda_1^3(\lambda_1^2-\lambda_2^2)}$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^3(\lambda_1^2-\lambda_2^2)}\bigg)$$

$$-\frac{K_1(\lambda_2 r)}{r}\bigg((\lambda+\mu)kK\frac{\lambda_2}{\lambda_1^2-\lambda_2^2}-(\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_2(\lambda_1^2-\lambda_2^2)}$$

$$+(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_2^3(\lambda_1^2-\lambda_2^2)}-(\lambda+\mu)\frac{\eta^2s^2T_0}{\lambda_2^3(\lambda_1^2-\lambda_2^2)}$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 sK}{\lambda_2(\lambda_1^2-\lambda_2^2)}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 s\lambda_4^2K}{\lambda_2^3(\lambda_1^2-\lambda_2^2)}$$

$$-\alpha^2\frac{sk}{\lambda_2(\lambda_1^2-\lambda_2^2)}+\alpha^2\frac{s\lambda_3^2k}{\lambda_2^3(\lambda_1^2-\lambda_2^2)}$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_2^3(\lambda_1^2-\lambda_2^2)}\bigg)$$

$$\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^2\lambda_2^2}\times\left\{-(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_1^2\lambda_2^2}+(\lambda+\mu)\frac{\eta^2 s^2 T_0}{\lambda_1^2\lambda_2^2}-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 s\frac{\lambda_4^2 K}{\lambda_1^2\lambda_2^2}-\alpha^2 s\frac{\lambda_3^2 k}{\lambda_1^2\lambda_2^2}+2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^2\lambda_2^2}\right\}$$

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$$-\frac{1}{r^2}\left((\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_1^2\lambda_2^2}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 sK}{\lambda_1^2\lambda_2^2}+\alpha^2\frac{sk}{\lambda_1^2\lambda_2^2}\right)$$

$$+(2\ln r+1)\left((\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{4\lambda_1^2\lambda_2^2}-(\lambda+\mu)\frac{\eta^2 s^2 T_0}{4\lambda_1^2\lambda_2^2}\right.$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 s\lambda_4^2K}{4\lambda_1^2\lambda_2^2}+\alpha^2\frac{s\lambda_3^2k}{4\lambda_1^2\lambda_2^2}$$

$$\left.-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{4\lambda_1^2\lambda_2^2}-2(\lambda+2\mu)kK\frac{\lambda_1^2\lambda_2^2}{4\lambda_1^2\lambda_2^2}\right)$$

$$+K_0(\lambda_1 r)\cdot(\lambda+2\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(\lambda_1^2-(\lambda_1^2+\lambda_2^2)+\lambda_2^2\right)$$

$$-K_0(\lambda_2 r)(\lambda+2\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(\lambda_2^2-(\lambda_1^2+\lambda_2^2)+\lambda_1^2\right)\Big]$$

$$-\frac{x_i x_j}{r^2}\Big[\left(K_0(\lambda_1 r)+\frac{2}{\lambda_1}K_1(\lambda_1 r)\right)\left((\lambda+\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(\lambda_1^2-(\lambda_3^2+\lambda_4^2)+\lambda_3^2\lambda_4^2/\lambda_1^2\right)\right.$$

$$-(\lambda+\mu)\frac{\eta^2 s^2 T_0}{\lambda_1^2(\lambda_1^2-\lambda_2^2)}\overset{+}{-}(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 sK}{\lambda_1^2-\lambda_2^2}$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 s\lambda_4^2K}{\lambda_1^2(\lambda_1^2-\lambda_2^2)}\overset{+}{-}\alpha^2\frac{sk}{\lambda_1^2-\lambda_2^2}$$

$$\left.+\alpha^2\frac{s\lambda_3^2k}{\lambda_1^2(\lambda_1^2-\lambda_2^2)}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^2(\lambda_1^2-\lambda_2^2)}\right)$$

계산착오로 $K_1(\lambda r)$ 항을 추가하여 수정.

$$-\left(K_0(\lambda_2 r)+\frac{2}{\lambda_2}K_1(\lambda_2 r)\right)\left((\lambda+\mu)kK\frac{1}{\lambda_1^2-\lambda_2^2}\left(\lambda_2^2-(\lambda_3^2+\lambda_4^2)+\lambda_3^2\lambda_4^2/\lambda_2^2\right)\right.$$

$$-(\lambda+\mu)\frac{\eta^2 s^2 T_0}{\lambda_2^2(\lambda_1^2-\lambda_2^2)}\overset{+}{-}(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 sK}{\lambda_1^2-\lambda_2^2}$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 s\lambda_4^2K}{\lambda_2^2(\lambda_1^2-\lambda_2^2)}\overset{+}{-}\alpha^2\frac{sk}{\lambda_1^2-\lambda_2^2}$$

$$\left.+\alpha^2\frac{s\lambda_3^2k}{\lambda_2^2(\lambda_1^2-\lambda_2^2)}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_2^2(\lambda_1^2-\lambda_2^2)}\right)$$

$$-\frac{z}{r^2}\left((\lambda+\mu)kK\frac{\lambda_3^2+\lambda_4^2}{\lambda_1^2\lambda_2^2}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 sK}{\lambda_1^2\lambda_2^2}+\alpha^2\frac{sk}{\lambda_1^2\lambda_2^2}\right)$$

$$\left\{-(\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{\lambda_1^2\lambda_2^2}+(\lambda+\mu)\frac{\eta^2 s^2 T_0}{\lambda_1^2\lambda_2^2}-(3\lambda+2\mu)^2(\beta_T^s)^2T_0 s\frac{\lambda_4^2K}{\lambda_1^2\lambda_2^2}-\alpha^2 s\frac{\lambda_3^2k}{\lambda_1^2\lambda_2^2}+2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{\lambda_1^2\lambda_2^2}\right\}\times\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^2\lambda_2^2}$$

$$-z\left((\lambda+\mu)kK\frac{\lambda_3^2\lambda_4^2}{4\lambda_1^2\lambda_2^2}-(\lambda+\mu)\frac{\eta^2 s^2 T_0}{4\lambda_1^2\lambda_2^2}\right.$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0 s\lambda_4^2K}{4\lambda_1^2\lambda_2^2}+\alpha^2\frac{s\lambda_3^2k}{4\lambda_1^2\lambda_2^2}$$

$$\left.-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha s^2}{4\lambda_1^2\lambda_2^2}\right)\Big]\Big\}$$

$$G_{\lambda 3} = \mathcal{L}^{-1}\left\{\tilde{G}_{\lambda 3}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(-1)\cancel{x_\lambda/r}}{2\pi(\lambda+2\mu)\mu k K S}\left[\frac{K_1(\lambda_1 r)}{\lambda_1^2-\lambda_2^2}\frac{1}{\lambda_1}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_1^2)\right)\right.\right.$$

$$-\frac{K_1(\lambda_2 r)}{\lambda_1^2-\lambda_2^2}\frac{1}{\lambda_2}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_2^2)\right)$$

$$\left.\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K\lambda_4^2\right)\right]\right\}$$

$$= \cancel{\mathcal{L}^{-1}}\{$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K}\,\mathcal{L}^{-1}\left\{\frac{1}{\lambda_1(\lambda_1^2-\lambda_2^2)s}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_1^2)\right)\frac{x_\lambda}{r}K_1(\lambda_1 r)\right.$$

$$-\frac{1}{\lambda_2(\lambda_1^2-\lambda_2^2)s}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_2^2)\right)\frac{x_\lambda}{r}K_1(\lambda_2 r)$$

$$\left.+\frac{1}{\lambda_1^2\lambda_2^2 s}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K\lambda_4^2\right)\frac{x_\lambda}{r^2}\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K}\frac{x_\lambda}{r}\,\mathcal{L}^{-1}\left\{\frac{1}{\lambda_1(\lambda_1^2-\lambda_2^2)s}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_1^2)\right)K_1(\lambda_1 r)\right.$$

$$-\frac{1}{\lambda_2(\lambda_1^2-\lambda_2^2)s}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_2^2)\right)K_1(\lambda_2 r)$$

$$\left.+\frac{1}{\lambda_1^2\lambda_2^2 s}\left(-\mu\eta\alpha s+\mu(3\lambda+2\mu)\beta_T^s K\lambda_4^2\right)\frac{1}{r}\right\}.$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K}\frac{x_\lambda}{r}\,\mathcal{L}^{-1}\left\{\left(\frac{-\mu\eta\alpha}{\lambda_1(\lambda_1^2-\lambda_2^2)}+\frac{\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_1^2)}{\lambda_1(\lambda_1^2-\lambda_2^2)s}\right)K_1(\lambda_1 r)\right.$$

$$-\left(\frac{-\mu\eta\alpha}{\lambda_2(\lambda_1^2-\lambda_2^2)}+\frac{\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_2^2)}{\lambda_2(\lambda_1^2-\lambda_2^2)s}\right)K_1(\lambda_2 r)$$

$$\left.+\left(\frac{-\mu\eta\alpha}{\lambda_1^2\lambda_2^2}+\frac{\mu(3\lambda+2\mu)\beta_T^s K\lambda_4^2}{\lambda_1^2\lambda_2^2 s}\right)\frac{1}{r}\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K}\frac{x_\lambda}{r}\,\mathcal{L}^{-1}\left\{\frac{-\mu\eta\alpha}{\lambda_1(\lambda_1^2-\lambda_2^2)}K_1(\lambda_1 r)+\frac{\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_1^2)}{\lambda_1(\lambda_1^2-\lambda_2^2)s}K_1(\lambda_1 r)\right.$$

$$-\frac{-\mu\eta\alpha}{\lambda_2(\lambda_1^2-\lambda_2^2)}K_1(\lambda_2 r)-\frac{\mu(3\lambda+2\mu)\beta_T^s K(\lambda_4^2-\lambda_2^2)}{\lambda_2(\lambda_1^2-\lambda_2^2)s}K_1(\lambda_2 r)$$

$$\left.+\frac{-\mu\eta\alpha}{\lambda_1^2\lambda_2^2}\frac{1}{r}+\frac{\mu(3\lambda+2\mu)\beta_T^s K\lambda_4^2}{\lambda_1^2\lambda_2^2 s}\frac{1}{r}\right\}$$

$$\left\{\begin{aligned}\lambda_{\bar\lambda}^2 &= l_{\bar\lambda} s\\ \lambda_{\bar\lambda} &= \sqrt{l_{\bar\lambda} s}\\ &= b_{\bar\lambda}\sqrt{s}.\end{aligned}\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K}\frac{x_{\bar\lambda}}{r}\mathcal{L}^{-1}\left\{\frac{-\mu\eta\alpha}{b_1(b_1^2-b_2^2)}\frac{K_1(b_1\sqrt{s}r)}{\sqrt{s^3}} + \frac{\mu(3\lambda+2\mu)\beta_T^s K(b_4^2-b_1^2)}{b_1(b_1^2-b_2^2)}\frac{K_1(b_1\sqrt{s}r)}{\sqrt{s^3}}\right.$$

$$- \frac{-\mu\eta\alpha}{b_2(b_1^2-b_2^2)}\frac{K_1(b_2\sqrt{s}r)}{\sqrt{s^3}} - \frac{\mu(3\lambda+2\mu)\beta_T^s K(b_4^2-b_2^2)}{b_2(b_1^2-b_2^2)}\frac{K_1(b_2 r\sqrt{s})}{\sqrt{s^3}}$$

Inversion.

$$\left. + \frac{-\mu\eta\alpha}{b_1^2 b_2^2}\frac{1}{s^2 r} + \frac{\mu(3\lambda+2\mu)\beta_T^s K b_4^2}{b_1^2 b_2^2}\frac{1}{s^2 r}\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu k K}\frac{x_{\bar\lambda}}{r}\left\{\frac{-\mu\eta\alpha}{b_1(b_1^2-b_2^2)}\left[\frac{t}{b_1 r}\exp\left(-\frac{b_1^2 r^2}{4t}\right) - \frac{b_1 r}{4}E_1\left(\frac{b_1^2 r^2}{4t}\right)\right]\right.$$

$$+ \frac{\mu(3\lambda+2\mu)\beta_T^s K(b_4^2-b_1^2)}{b_1(b_1^2-b_2^2)}\left[\frac{t}{b_1 r}\exp\left(-\frac{b_1^2 r^2}{4t}\right) - \frac{b_1 r}{4}E_1\left(\frac{b_1^2 r^2}{4t}\right)\right]$$

$$- \frac{-\mu\eta\alpha}{b_2(b_1^2-b_2^2)}\left[\frac{t}{b_2 r}\exp\left(-\frac{b_2^2 r^2}{4t}\right) - \frac{b_2 r}{4}E_1\left(\frac{b_2^2 r^2}{4t}\right)\right]$$

$$- \frac{\mu(3\lambda+2\mu)\beta_T^s K(b_4^2-b_2^2)}{b_2(b_1^2-b_2^2)}\left[\frac{t}{b_2 r}\exp\left(-\frac{b_2^2 r^2}{4t}\right) - \frac{b_2 r}{4}E_1\left(\frac{b_2^2 r^2}{4t}\right)\right]$$

$$\left. + \frac{-\mu\eta\alpha}{b_1^2 b_2^2}\frac{t}{r} + \frac{\mu(3\lambda+2\mu)\beta_T^s K b_4^2}{b_1^2 b_2^2}\frac{t}{r}\right\}$$

$$\zeta_1 = \frac{b_1 r}{\sqrt{t}} = \frac{r}{\sqrt{c_1 t}}.$$

$$\zeta_2 = \frac{b_2 r}{\sqrt{t}} = \frac{r}{\sqrt{c_2 t}}.$$

C_1 & C_2 :

diffusivity?

$$C_1 = \frac{1}{b_1^2} = \frac{1}{l_1};\quad C_2 = \frac{1}{b_2^2} = \frac{1}{l_2}.$$

$$\left\{\begin{aligned}\lambda_1 &= \sqrt{l_1 s} = \sqrt{\frac{s}{C_1}}\\ \lambda_2 &= \sqrt{l_2 s} = \sqrt{\frac{s}{C_2}}\end{aligned}\right.$$

$$\lambda_1^2+\lambda_2^2 = \frac{\rho C_p \cancel{s}}{k} + \frac{\alpha\cancel{s}}{kKB} + \frac{(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 \cancel{s} K + \alpha^2 \cancel{s} k}{(\lambda+2\mu)kK}$$

$$= \frac{\cancel{s}}{C_1} + \frac{\cancel{s}}{C_2}.$$

$$\lambda_1^2\lambda_2^2 = \frac{\rho C_p \alpha \cancel{s^2}}{kKKB} + \frac{-(\lambda+2\mu)\eta^2 \cancel{s^2} T_0 + (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 \cancel{s}\frac{\alpha\cancel{s}}{\cancel{K}B}\cancel{K} + \alpha^2 \cancel{s}\frac{\rho C_p \cancel{s}}{\cancel{k}}\cancel{k}}{(\lambda+2\mu)kK}$$

$$- \frac{2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha \cancel{s^2}}{(\lambda+2\mu)kK}$$

$$= \frac{\cancel{s^2}}{C_1 C_2}$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{\rho C_p}{k} + \frac{\alpha}{kKB} + \frac{(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 K + \alpha^2 k}{(\lambda+2\mu)kK}$$

$$\frac{1}{C_1 C_2} = \frac{\rho C_p \alpha}{kKKB} + \frac{-(\lambda+2\mu)\eta^2 T_0 + (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 \alpha/KB + \alpha^2\rho C_p - 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha}{(\lambda+2\mu)kK}$$

$$= \frac{+(\lambda+2\mu)\rho C_p \alpha - (\lambda+2\mu)\eta^2 T_0 KB + (3\lambda+2\mu)^2(\beta_T^s)^2 T_0 \alpha + \alpha^2 \rho C_p - 2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha}{(\lambda+2\mu)kKKB}$$

 99.4.27~28.

$$\frac{C_1+C_2}{C_1C_2} = \frac{+(\lambda+2\mu)\rho C_p KB + (\lambda+2\mu)\alpha k + (3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB + \alpha^2 k KB}{(\lambda+2\mu) k \kappa KB}$$

$$C_1+C_2 = \frac{(\lambda+2\mu)\rho C_p KB + (\lambda+2\mu)\alpha k + (3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB + \alpha^2 k KB}{(\lambda+2\mu)\rho C_p \alpha - (\lambda+2\mu)\eta^2 T_0 KB + (3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \alpha + \alpha^2 \rho C_p - 2(3\lambda+2\mu)\beta_T^S \eta T_0 \alpha} = B$$

C_1 & C_2 구하기.

$$C_1C_2 = \frac{(\lambda+2\mu) k \kappa KB}{(\lambda+2\mu)\rho C_p \alpha - (\lambda+2\mu)\eta^2 T_0 KB + (3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \alpha + \alpha^2 \rho C_p - 2(3\lambda+2\mu)\beta_T^S \eta T_0 \alpha} = C$$

$$(C_1+C_2)^2 - 4C_1C_2 = (C_1-C_2)^2.$$

$$C_1 - C_2 = \sqrt{(C_1+C_2)^2 - 4C_1C_2}$$

$$= \frac{\sqrt{A - 4(\lambda+2\mu) k \kappa KB}}{\sqrt{(\lambda+2\mu)\rho C_p \alpha - (\lambda+2\mu)\eta^2 T_0 KB + (3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \alpha + \alpha^2 \rho C_p - 2(3\lambda+2\mu)\beta_T^S \eta T_0 \alpha}} = D$$

$$\boxed{\blacktriangle\ (a+b+c+d)^2 = a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd.}$$

$$\left\{(\lambda+2\mu)\rho C_p KB + (\lambda+2\mu)\alpha k + (3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB + \alpha^2 k KB\right\}^2$$
$$= \left((\lambda+2\mu)\rho C_p KB\right)^2 + \left((\lambda+2\mu)\alpha k\right)^2 + \left((3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB\right)^2 + \left(\alpha^2 k KB\right)^2$$
$$+ 2\left((\lambda+2\mu)\rho C_p KB\right)\left((\lambda+2\mu)\alpha k\right) + 2\left((\lambda+2\mu)\rho C_p KB\right)\left((3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB\right)$$
$$+ 2\left((\lambda+2\mu)\rho C_p KB\right)\left(\alpha^2 k KB\right) + 2\left((\lambda+2\mu)\alpha k\right)\left((3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB\right)$$
$$+ 2\left((\lambda+2\mu)\alpha k\right)\left(\alpha^2 k KB\right) + 2\left((3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \kappa KB\right)\left(\alpha^2 k KB\right) = A$$

$$\left.\begin{aligned} C_1 + C_2 &= B \\ C_1 - C_2 &= D \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} C_1 &= \frac{B+D}{2} \\ C_2 &= \frac{B-D}{2} \end{aligned}\right.$$

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$$G_{3\bar{i}} = \mathcal{L}^{-1}\left\{\tilde{G}_{3\bar{i}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(-1)}{2\pi(\lambda+2\mu)\mu kks}\left[\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\left(-\mu\eta T_0\alpha s^2+\mu(3\lambda+2\mu)\beta_T^S T_0 sk(\lambda_4^2-\lambda_1^2)\right)\right.\right.$$

$$-\frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\left(-\mu\eta T_0\alpha s^2+\mu(3\lambda+2\mu)\beta_T^S T_0 sk(\lambda_4^2-\lambda_2^2)\right)$$

$$\left.\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left(-\mu\eta T_0\alpha s^2+\mu(3\lambda+2\mu)\beta_T^S T_0 sk\lambda_4^2\right)\right]\frac{x_{\bar{i}}}{r}\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kk}\frac{x_{\bar{i}}}{r}\mathcal{L}^{-1}\left\{\frac{K_1(\lambda_1 r)}{\lambda_1(\lambda_1^2-\lambda_2^2)}\left(-\mu\eta T_0\alpha s+\mu(3\lambda+2\mu)\beta_T^S T_0 K(\lambda_4^2-\lambda_1^2)\right)\right.$$

$$-\frac{K_1(\lambda_2 r)}{\lambda_2(\lambda_1^2-\lambda_2^2)}\left(-\mu\eta T_0\alpha s+\mu(3\lambda+2\mu)\beta_T^S T_0 K(\lambda_4^2-\lambda_2^2)\right)$$

$$\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left(-\mu\eta T_0\alpha s+\mu(3\lambda+2\mu)\beta_T^S T_0 K\lambda_4^2\right)\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kk}\frac{x_{\bar{i}}}{r}\mathcal{L}^{-1}\left\{\frac{1}{b_1(b_1^2-b_2^2)}\left(-\mu\eta T_0\alpha+\mu(3\lambda+2\mu)\beta_T^S T_0 K(b_4^2-b_1^2)\right)\frac{K_1(b_1 r\sqrt{s})}{\sqrt{s}}\right.$$

$$-\frac{1}{b_2(b_1^2-b_2^2)}\left(-\mu\eta T_0\alpha+\mu(3\lambda+2\mu)\beta_T^S T_0 K(b_4^2-b_2^2)\right)\frac{K_1(b_2 r\sqrt{s})}{\sqrt{s}}$$

$$\left.+\frac{1}{b_1^2 b_2^2}\left(-\mu\eta T_0\alpha+\mu(3\lambda+2\mu)\beta_T^S T_0 K b_4^2\right)\frac{1}{rs}\right\}$$

$$= \frac{(-1)}{2\pi(\lambda+2\mu)\mu kk}\frac{x_{\bar{i}}}{r}\left\{\frac{1}{b_1(b_1^2-b_2^2)}\left(-\mu\eta T_0\alpha+\mu(3\lambda+2\mu)\beta_T^S T_0 K(b_4^2-b_1^2)\right)\frac{1}{b_1 r}\exp\left(-\frac{b_1^2 r^2}{4t}\right)\right.$$

$$-\frac{1}{b_2(b_1^2-b_2^2)}\left(-\mu\eta T_0\alpha+\mu(3\lambda+2\mu)\beta_T^S T_0 K(b_4^2-b_2^2)\right)\frac{1}{b_2 r}\exp\left(-\frac{b_2^2 r^2}{4t}\right)$$

$$\left.+\frac{1}{b_1^2 b_2^2}\left(-\mu\eta T_0\alpha+\mu(3\lambda+2\mu)\beta_T^S T_0 K b_4^2\right)\frac{1}{r}\right\}.$$

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$$G_{\bar{i}4} = \mathcal{L}^{-1}\left\{\widetilde{G_{\bar{i}4}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(-1)^{l}}{2\pi(\lambda+2\mu)\mu k k s}\left[\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s+\mu\alpha k(\lambda_3^2-\lambda_1^2)\right)\right.\right.$$

$$-\frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s+\mu\alpha k(\lambda_3^2-\lambda_2^2)\right)$$

$$\left.\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s+\mu\alpha k\lambda_3^2\right)\right]\frac{x_{\bar{i}}}{r}\right\}$$

$$= \frac{(-1)^{l}}{2\pi(\lambda+2\mu)\mu k k s}\frac{x_{\bar{i}}}{r}\mathcal{L}^{-1}\left\{\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s+\mu\alpha k(\lambda_3^2-\lambda_1^2)\right)\right.$$

$$-\frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s+\mu\alpha k(\lambda_3^2-\lambda_2^2)\right)$$

$$\left.+\frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0 s+\mu\alpha k\lambda_3^2\right)\right\}$$

$$= \frac{(-1)^{l}}{2\pi(\lambda+2\mu)\mu k k}\frac{x_{\bar{i}}}{r}\mathcal{L}^{-1}\left\{\frac{1}{(b_1^2-b_2^2)b_1}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0+\mu\alpha k(b_3^2-b_1^2)\right)\frac{K_1(b_1 r\sqrt{s})}{\sqrt{s^3}}\right.$$

$$-\frac{1}{(b_1^2-b_2^2)b_2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0+\mu\alpha k(b_3^2-b_2^2)\right)\frac{K_1(b_2 r\sqrt{s})}{\sqrt{s^3}}$$

$$\left.+\frac{1}{b_1^2 b_2^2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^S T_0+\mu\alpha k b_3^2\right)\frac{1}{r s^2}\right\}$$

$$= \frac{(-1)^{l}}{2\pi(\lambda+2\mu)\mu k k}\frac{x_{\bar{i}}}{r}\left\{\frac{-\mu\eta(3\lambda+2\mu)\beta_T^S T_0+\mu\alpha k(b_3^2-b_1^2)}{b_1(b_1^2-b_2^2)}\left[\frac{t}{b_1 r}\exp\left(-\frac{b_1^2 r^2}{4t}\right)-\frac{b_1 r}{4}E_1\left(\frac{b_1^2 r^2}{4t}\right)\right]\right.$$

$$-\frac{-\mu\eta(3\lambda+2\mu)\beta_T^S T_0+\mu\alpha k(b_3^2-b_2^2)}{b_2(b_1^2-b_2^2)}\left[\frac{t}{b_2 r}\exp\left(-\frac{b_2^2 r^2}{4t}\right)-\frac{b_2 r}{4}E_1\left(\frac{b_2^2 r^2}{4t}\right)\right]$$

$$\left.+\frac{-\mu\eta(3\lambda+2\mu)\beta_T^S T_0+\mu\alpha k b_3^2}{b_1^2 b_2^2}\frac{t}{r}\right\}$$

$$G_{44} = \mathcal{L}^{-1}\left\{\tilde{G}_{44}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2\pi(\lambda+2\mu)\mu kKs}\left[\frac{\mu K_0(\lambda_1 r)}{\lambda_1^2-\lambda_2^2}\left((\lambda+2\mu)k(\lambda_1^2-\lambda_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2 s\right)\right.\right.$$

$$\left.\left.-\frac{\mu K_0(\lambda_2 r)}{\lambda_1^2-\lambda_2^2}\left((\lambda+2\mu)k(\lambda_2^2-\lambda_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2 s\right)\right]\right\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)kK}\mathcal{L}^{-1}\left\{\frac{K_0(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)s}\left((\lambda+2\mu)k(\lambda_1^2-\lambda_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2 s\right)\right.$$

$$\left.-\frac{K_0(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)s}\left((\lambda+2\mu)k(\lambda_2^2-\lambda_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2 s\right)\right\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)kK}\mathcal{L}^{-1}\left\{\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)k(b_1^2-b_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2\right)\frac{K_0(b_1 r\sqrt{s})}{s}\right.$$

$$\left.-\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)k(b_2^2-b_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2\right)\frac{K_0(b_2 r\sqrt{s})}{s}\right\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)kK}\left\{\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)k(b_1^2-b_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2\right)\frac{1}{2}E_1\left(\frac{b_1^2 r^2}{4t}\right)\right.$$

$$\left.-\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)k(b_2^2-b_3^2)-(3\lambda+2\mu)^2(\beta_T^s)^2\right)\frac{1}{2}E_1\left(\frac{b_2^2 r^2}{4t}\right)\right\}.$$

$$G_{33} = \mathcal{L}^{-1}\left\{\tilde{G}_{33}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{\mu}{2\pi(\lambda+2\mu)\mu kKs}\left[\frac{1}{\lambda_1^2-\lambda_2^2}\left((\lambda+2\mu)K(\lambda_1^2-\lambda_4^2)-\alpha^2 s\right)K_0(\lambda_1 r)\right.\right.$$

$$\left.\left.-\frac{1}{\lambda_1^2-\lambda_2^2}\left((\lambda+2\mu)K(\lambda_2^2-\lambda_4^2)-\alpha^2 s\right)K_0(\lambda_2 r)\right]\right\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)kK}\mathcal{L}^{-1}\left\{\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)K(b_1^2-b_4^2)-\alpha^2\right)\frac{K_0(b_1 r\sqrt{s})}{s}\right.$$

$$\left.-\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)K(b_2^2-b_4^2)-\alpha^2\right)\frac{K_0(b_2 r\sqrt{s})}{s}\right\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)kK}\left\{\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)K(b_1^2-b_4^2)-\alpha^2\right)\frac{1}{2}E_1\left(\frac{b_1^2 r^2}{4t}\right)\right.$$

$$\left.-\frac{1}{b_1^2-b_2^2}\left((\lambda+2\mu)K(b_2^2-b_4^2)-\alpha^2\right)\frac{1}{2}E_1\left(\frac{b_2^2 r^2}{4t}\right)\right\}$$

$$G_{4i} = \mathcal{L}^{-1}\{\bar{G}_{4i}\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(-1)^{i}}{2\pi(\lambda+2\mu)\mu R K s}\left[\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 + \mu\alpha s k(\lambda_3^2-\lambda_1^2)\right)\right.\right.$$

$$- \frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 + \mu\alpha s k(\lambda_3^2-\lambda_2^2)\right)$$

$$\left.\left. + \frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 + \mu\alpha s k\lambda_3^2\right)\right]\frac{x_i}{r}\right\}$$

$$= \frac{(-1)^{i}}{2\pi(\lambda+2\mu)\mu R K}\frac{x_i}{r}\mathcal{L}^{-1}\left\{\frac{K_1(\lambda_1 r)}{(\lambda_1^2-\lambda_2^2)\lambda_1 s}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 + \mu\alpha s k(\lambda_3^2-\lambda_1^2)\right)\right.$$

$$- \frac{K_1(\lambda_2 r)}{(\lambda_1^2-\lambda_2^2)\lambda_2 s}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 + \mu\alpha s k(\lambda_3^2-\lambda_2^2)\right)$$

$$\left. + \frac{1}{r}\frac{1}{\lambda_1^2\lambda_2^2 s}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 s^2 + \mu\alpha s k\lambda_3^2\right)\right\}$$

$$= \frac{(-1)^{i}}{2\pi(\lambda+2\mu)\mu R K}\frac{x_i}{r}\mathcal{L}^{-1}\left\{\frac{1}{(b_1^2-b_2^2)b_1}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 + \mu\alpha k(b_3^2-b_1^2)\right)\frac{K_1(b_1 r\sqrt{s})}{\sqrt{s}}\right.$$

$$- \frac{1}{(b_1^2-b_2^2)b_2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 + \mu\alpha k(b_3^2-b_2^2)\right)\frac{K_1(b_2 r\sqrt{s})}{\sqrt{s}}$$

$$\left. + \frac{1}{b_1^2 b_2^2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 + \mu\alpha k\, b_3^2\right)\frac{1}{rs}\right\}$$

$$= \frac{(-1)^{i}}{2\pi(\lambda+2\mu)\mu R K}\frac{x_i}{r}\left\{\frac{1}{(b_1^2-b_2^2)b_1}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 + \mu\alpha k(b_3^2-b_1^2)\right)\frac{1}{b_1 r}\exp\left(-\frac{b_1^2 r^2}{4t}\right)\right.$$

$$- \frac{1}{(b_1^2-b_2^2)b_2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 + \mu\alpha k(b_3^2-b_2^2)\right)\frac{1}{b_2 r}\exp\left(-\frac{b_2^2 r^2}{4t}\right)$$

$$\left. + \frac{1}{b_1^2 b_2^2}\left(-\mu\eta(3\lambda+2\mu)\beta_T^s T_0 + \mu\alpha k b_3^2\right)\frac{1}{r}\right\}$$

$$G_{34} = \mathcal{L}^{-1}\left\{\widetilde{G}_{34}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{T_0}{2\pi(\lambda+2\mu)\mu kKs}\left[\frac{\mu(3\lambda+2\mu)\beta_T^s\alpha s+(\lambda+2\mu)\mu\eta s}{\lambda_1^2-\lambda_2^2}\right]\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\right\}$$

$$= \frac{T_0}{2\pi(\lambda+2\mu)kK}\mathcal{L}^{-1}\left\{\frac{(3\lambda+2\mu)\beta_T^s\alpha+(\lambda+2\mu)\eta}{b_1^2-b_2^2}\left(\frac{K_0(b_1 r\sqrt{s})-K_0(b_2 r\sqrt{s})}{s}\right)\right\}$$

$$= \frac{T_0}{2\pi(\lambda+2\mu)kK}\left\{\frac{(3\lambda+2\mu)\beta_T^s\alpha+(\lambda+2\mu)\eta}{b_1^2-b_2^2}\right\}\frac{1}{2}\left(E_1\left(\frac{b_1^2r^2}{4t}\right)-E_1\left(\frac{b_2^2r^2}{4t}\right)\right)$$

$$G_{43} = \mathcal{L}^{-1}\left\{\widetilde{G}_{43}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2\pi(\lambda+2\mu)kKs}\left[\frac{\mu(3\lambda+2\mu)\beta_T^s\alpha\cancel{T_0}s+(\lambda+2\mu)\mu\cancel{T_0}\eta s}{\lambda_1^2-\lambda_2^2}\right]\left(K_0(\lambda_1 r)-K_0(\lambda_2 r)\right)\right\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)kK}\mathcal{L}^{-1}\left\{\frac{(3\lambda+2\mu)\beta_T^s\alpha\cancel{T_0}+(\lambda+2\mu)\cancel{T_0}\eta}{b_1^2-b_2^2}\left(\frac{K_0(b_1 r\sqrt{s})-K_0(b_2 r\sqrt{s})}{s}\right)\right\}$$

$$= \frac{1}{4\pi(\lambda+2\mu)kK}\left\{\frac{(3\lambda+2\mu)\beta_T^s\alpha\cancel{T_0}+(\lambda+2\mu)\cancel{T_0}\eta}{b_1^2-b_2^2}\right\}\left(E_1\left(\frac{b_1^2r^2}{4t}\right)-E_1\left(\frac{b_2^2r^2}{4t}\right)\right)$$

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$$G_{ij} = \mathcal{L}^{-1}\{\widetilde{G_{ij}}\}$$

$$= \frac{1}{2\pi(\lambda+2\mu)\mu k k}\,\mathcal{L}^{-1}\Big\{\delta_{ij}\Big[\frac{K_1(b_1 r\sqrt{s})}{r}\Big((\lambda+\mu)kk\frac{b_1}{b_1^2-b_2^2}\frac{\sqrt{s}}{s^2} - (\lambda+\mu)kk\frac{b_3^2+b_4^2}{b_1(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$+(\lambda+\mu)kk\frac{b_3^2 b_4^2}{b_1^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2} - (\lambda+\mu)\frac{\eta^2 T_0}{b_1^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^S)^2 T_0 K}{b_1(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2} + (3\lambda+2\mu)^2\frac{(\beta_T^S)^2 T_0 b_4^2 K}{b_1^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$-\alpha^2\frac{k}{b_1(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2} + \alpha^2\frac{b_3^2 k}{b_1^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$-2(3\lambda+2\mu)\frac{\beta_T^S\eta T_0\alpha}{b_1^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}\Big)$$

$$-\frac{K_1(b_2 r\sqrt{s})}{r}\Big((\lambda+\mu)kk\frac{b_2}{b_1^2-b_2^2}\frac{\sqrt{s}}{s^2} - (\lambda+\mu)kk\frac{b_3^2+b_4^2}{b_2(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$+(\lambda+\mu)kk\frac{b_3^2 b_4^2}{b_2^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2} - (\lambda+\mu)\frac{\eta^2 T_0}{b_2^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^S)^2 T_0 K}{b_2(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2} + (3\lambda+2\mu)^2\frac{(\beta_T^S)^2 T_0 b_4^2 K}{b_2^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$-\alpha^2\frac{k}{b_2(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2} + \alpha^2\frac{b_3^2 k}{b_2^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}$$

$$-2(3\lambda+2\mu)\frac{\beta_T^S\eta T_0\alpha}{b_2^3(b_1^2-b_2^2)}\frac{\sqrt{s}}{s^2}\Big)$$

$$-\frac{1}{r^2}\Big((\lambda+\mu)kk\frac{b_3^2+b_4^2}{b_1^2 b_2^2}\frac{1}{s^2} + (3\lambda+2\mu)^2\frac{(\beta_T^S)^2 T_0 K}{b_1^2 b_2^2}\frac{1}{s^2} + \alpha^2\frac{k}{b_1^2 b_2^2}\frac{1}{s^2}\Big)$$

$$\left\{\begin{aligned} &-(\lambda+\mu)kk\frac{b_3^2 b_4^2}{b_1^2 b_2^2} + (\lambda+\mu)\frac{\eta^2 \cancel{s} T_0}{b_1^2 b_2^2} \\ &-(3\lambda+2\mu)^2(\beta_T^S)^2 T_0 \cancel{s}\frac{b_4^2 K}{b_1^2 b_2^2} \\ &-\alpha^2 s\frac{b_3^2 k}{b_1^2 b_2^2} + 2(3\lambda+2\mu)\beta_T^S\eta T_0\alpha\cancel{s}\frac{1}{b_1^2 b_2^2}\end{aligned}\right\}\frac{b_1^2+b_2^2}{b_1^2+b_2^2}\cdot\frac{1}{s^2}$$

$$+(2\ln r+1)\Big((\lambda+\mu)kk\frac{b_3^2 b_4^2}{4b_1^2 b_2^2}\frac{1}{s} - (\lambda+\mu)\frac{\eta^2 T_0}{4b_1^2 b_2^2}\frac{1}{s}$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^S)^2 T_0 b_4^2 K}{4b_1^2 b_2^2\cdot s} + \alpha^2\frac{b_3^2 k}{4b_1^2 b_2^2}\frac{1}{s}$$

$$-2(3\lambda+2\mu)\frac{\beta_T^S\eta T_0\alpha}{4b_1^2 b_2^2}\frac{1}{s} - 2(\lambda+2\mu)\frac{kk b_1^2 b_2^2 s^2}{4b_1^2 b_2^2 s^3}\Big)$$

$$\cancel{+K_0(b_1 r\sqrt{s})(\lambda+2\mu)kk\frac{1}{b_1^2-b_2^2}\Big(\frac{b_1^2}{s} - \frac{(b_1^2+b_2^2)s}{s^2} + \frac{b_1^2 b_2^2 s^2}{b_1^2 s^3}\Big)} \to \underset{\sim}{0}$$

$$\cancel{-K_0(b_2 r\sqrt{s})(\lambda+2\mu)kk\frac{1}{b_1^2-b_2^2}\Big(\frac{b_2^2}{s} - \frac{(b_1^2+b_2^2)s}{s^2} + \frac{b_1^2 b_2^2 s^2}{b_2^2 s^3}\Big)} \to \underset{\sim}{0}$$

$$
-\frac{x_i x_j}{r^2}\Bigg[\left(K_0(b_1 r\sqrt{s}) + \frac{2}{b_1\sqrt{s}}K_1(b_1 r\sqrt{s})\right)\Bigg((\lambda+\mu)\frac{kK}{b_1^2-b_2^2}\left(\frac{b_1^2}{s}-\frac{b_3^2+b_4^2}{s^2}+\frac{b_3^2 b_4^2}{b_1^2 s^3}\right)
$$

$$
-(\lambda+\mu)\frac{\eta^2 T_0}{b_1^2(b_1^2-b_2^2)}\frac{1}{s}-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 K}{b_1^2-b_2^2}\frac{1}{s}
$$

$$
+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 b_4^2 K}{b_1^2(b_1^2-b_2^2)}\frac{1}{s}-\alpha^2\frac{k}{b_1^2-b_2^2}\frac{1}{s}
$$

$$
+\alpha^2\frac{b_3^2 k}{b_1^2(b_1^2-b_2^2)}\frac{1}{s}-2(3\lambda+2\mu)\frac{\beta_T^s \eta T_0 \alpha}{b_1^2(b_1^2-b_2^2)}\frac{1}{s}\Bigg)
$$

$$
-\left(K_0(b_2 r\sqrt{s}) + \frac{2}{b_2\sqrt{s}}K_1(b_2 r\sqrt{s})\right)\Bigg((\lambda+\mu)\frac{kK}{b_1^2-b_2^2}\left(\frac{b_2^2}{s}-\frac{b_3^2+b_4^2}{s^2}+\frac{b_3^2 b_4^2}{b_2^2 s^3}\right)
$$

$$
-(\lambda+\mu)\frac{\eta^2 T_0}{b_2^2(b_1^2-b_2^2)}\frac{1}{s}-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 K}{b_1^2-b_2^2}\frac{1}{s}
$$

$$
+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 b_4^2 K}{b_2^2(b_1^2-b_2^2)}\frac{1}{s}-\alpha^2\frac{k}{b_1^2-b_2^2}\frac{1}{s}
$$

$$
+\alpha^2\frac{b_3^2 k}{b_2^2(b_1^2-b_2^2)}\frac{1}{s}-2(3\lambda+2\mu)\frac{\beta_T^s \eta T_0 \alpha}{b_2^2(b_1^2-b_2^2)}\frac{1}{s}\Bigg)
$$

$$
-\frac{z_i}{r^2}\left((\lambda+\mu)kK\frac{b_3^2+b_4^2}{b_1^2 b_2^2}\frac{1}{s^2}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 K}{b_1^2 b_2^2}\frac{1}{s^2}+\alpha^2\frac{k}{b_1^2 b_2^2}\frac{1}{s^2}\right)
$$

$$
\left\{\begin{array}{l}
-(\lambda+\mu)kK\frac{b_3^2 b_4^2}{b_1^2 b_2^2}+(\lambda+\mu)\frac{\eta^2 \cancel{s} T_0}{b_1^2 b_2^2} \\
-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0 \cancel{s}\frac{b_4^2 K}{b_1^2 b_2^2} \\
-\alpha^2 s\frac{b_3^2 k}{b_1^2 b_2^2}+2(3\lambda+2\mu)\beta_T^s \eta T_0 \alpha \cancel{s}\frac{1}{b_1^2 b_2^2}
\end{array}\right\}\times\frac{b_1^2+b_2^2}{b_1^2 b_2^2}\cdot\frac{1}{s^2}
$$

$$
-2\Bigg((\lambda+\mu)kK\frac{b_3^2 b_4^2}{4 b_1^2 b_2^2}\frac{1}{s}-(\lambda+\mu)\frac{\eta^2 T_0}{4 b_1^2 b_2^2}\frac{1}{s}
$$

$$
+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 \eta T_0 b_4^2 K}{4 b_1^2 b_2^2 s}+\alpha^2\frac{b_3^2 k}{4 b_1^2 b_2^2}\frac{1}{s}-2(3\lambda+2\mu)\frac{\beta_T^s \eta T_0 \alpha}{4 b_1^2 b_2^2}\frac{1}{s}\Bigg)\Bigg]\Bigg\}
$$

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$$= \frac{1}{2\pi(\lambda+2\mu)\mu kK}\mathcal{L}^{-1}\Bigg\{\delta_{ij}\Bigg[\frac{K_1(b_1 r\sqrt{s})}{r\sqrt{s^3}}\Bigg((\lambda+\mu)kK\frac{b_1}{b_1^2-b_2^2} - (\lambda+\mu)kK\frac{b_3^2+b_4^2}{b_1(b_1^2-b_2^2)}$$

$$+(\lambda+\mu)kK\frac{b_3^2 b_4^2}{b_1^3(b_1^2-b_2^2)} - (\lambda+\mu)\frac{\eta^2 T_0}{b_1^3(b_1^2-b_2^2)}$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 K}{b_1(b_1^2-b_2^2)} + (3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 b_4^2 K}{b_1^3(b_1^2-b_2^2)}$$

$$-\alpha^2\frac{k}{b_1(b_1^2-b_2^2)} + \alpha^2\frac{b_3^2 k}{b_1^3(b_1^2-b_2^2)}$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{b_1^3(b_1^2-b_2^2)}\Bigg)$$

$$-\frac{K_1(b_2 r\sqrt{s})}{r\sqrt{s^3}}\Bigg((\lambda+\mu)kK\frac{b_2}{b_1^2-b_2^2} - (\lambda+\mu)kK\frac{b_3^2+b_4^2}{b_2(b_1^2-b_2^2)}$$

$$+(\lambda+\mu)kK\frac{b_3^2 b_4^2}{b_2^3(b_1^2-b_2^2)} - (\lambda+\mu)\frac{\eta^2 T_0}{b_2^3(b_1^2-b_2^2)}$$

$$-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 K}{b_2(b_1^2-b_2^2)} + (3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 b_4^2 K}{b_2^3(b_1^2-b_2^2)}$$

$$-\alpha^2\frac{k}{b_2(b_1^2-b_2^2)} + \alpha^2\frac{b_3^2 k}{b_2^3(b_1^2-b_2^2)} \longrightarrow$$

$$-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{b_2^3(b_1^2-b_2^2)}\Bigg)$$

$$-\frac{1}{r^2 s^2}\Bigg((\lambda+\mu)kK\frac{b_3^2+b_4^2}{b_1^2 b_2^2} + (3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 K}{b_1^2 b_2^2} + \alpha^2\frac{k}{b_1^2 b_2^2}\Bigg)$$

$$\left\{\begin{aligned} &-(\lambda+\mu)kK\frac{b_3^2 b_4^2}{b_1^2 b_2^2} + (\lambda+\mu)\frac{\eta^2 T_0}{b_1^2 b_2^2} \\ &-(3\lambda+2\mu)^2(\beta_T^s)^2 T_0\frac{b_4^2 K}{b_1^2 b_2^2} \\ &-\alpha^2\frac{b_3^2 k}{b_1^2 b_2^2} + 2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha\frac{1}{b_1^2 b_2^2}\end{aligned}\right\}\times\frac{b_1^2+b_2^2}{b_1^2 b_2^2}$$

$$+\frac{(2\ln r+1)}{s}\Bigg((\lambda+\mu)\frac{kK b_3^2 b_4^2}{4b_1^2 b_2^2} - (\lambda+\mu)\frac{\eta^2 T_0}{4b_1^2 b_2^2} + (3\lambda+2\mu)^2\frac{(\beta_T^s)^2 T_0 b_4^2 K}{4b_1^2 b_2^2}$$

$$+\alpha^2\frac{b_3^2 k}{4b_1^2 b_2^2} - 2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{4b_1^2 b_2^2} - 2(\lambda+2\mu)\frac{kK}{4}\Bigg)\Bigg]$$

$$-\frac{x_i x_j}{r^2}\left[\left(\frac{K_0(b_1 r\sqrt{s})}{s}+\frac{2K_1(b_1 r\sqrt{s})}{b_1\sqrt{s^3}}\right)\left((\lambda+\mu)\frac{kK}{b_1^2-b_2^2}\left(b_1^2-(b_3^2+b_4^2)+b_3^2b_4^2/b_1^2\right)\right.\right.$$

$$-(\lambda+\mu)\frac{\eta^2T_0}{b_1^2(b_1^2-b_2^2)}-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0K}{b_1^2-b_2^2}$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0b_4^2K}{b_1^2(b_1^2-b_2^2)}-\alpha^2\frac{k}{b_1^2-b_2^2}$$

$$\left.+\alpha^2\frac{b_3^2k}{b_1^2(b_1^2-b_2^2)}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{b_1^2(b_1^2-b_2^2)}\right)$$

$$-\left(\frac{K_0(b_2 r\sqrt{s})}{s}+\frac{2K_1(b_2 r\sqrt{s})}{b_2\sqrt{s^3}}\right)\left((\lambda+\mu)\frac{kK}{b_1^2-b_2^2}\left(b_2^2-(b_3^2+b_4^2)+b_3^2b_4^2/b_2^2\right)\right.$$

$$-(\lambda+\mu)\frac{\eta^2T_0}{b_1^2(b_1^2-b_2^2)}-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0K}{b_1^2-b_2^2}$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0b_4^2K}{b_2^2(b_1^2-b_2^2)}-\alpha^2\frac{k}{b_1^2-b_2^2}$$

$$\left.+\alpha^2\frac{b_3^2k}{b_2^2(b_1^2-b_2^2)}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{b_2^2(b_1^2-b_2^2)}\right)$$

$$-\frac{2}{r^2s^2}\left((\lambda+\mu)kK\frac{b_3^2+b_4^2}{b_1^2b_2^2}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0K}{b_1^2b_2^2}+\alpha^2\frac{k}{b_1^2b_2^2}\right)$$

$$-(\lambda+\mu)kK\frac{b_3^2b_4^2}{b_1^2b_2^2}+(\lambda+\mu)\frac{\eta^2T_0}{b_1^2b_2^2}-(3\lambda+2\mu)^2(\beta_T^s)^2T_0\frac{b_4^2K}{b_1^2b_2^2}-\alpha^2\frac{b_3^2k}{b_1^2b_2^2}+2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha\frac{1}{b_1^2b_2^2}\times\frac{b_1^2+b_2^2}{b_1^2b_2^2}$$

$$-\frac{2}{s}\left((\lambda+\mu)kK\frac{b_3^2b_4^2}{4b_1^2b_2^2}-(\lambda+\mu)\frac{\eta^2T_0}{4b_1^2b_2^2}\right.$$

$$+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2\eta T_0b_4^2K}{4b_1^2b_2^2}+\alpha^2\frac{b_3^2k}{4b_1^2b_2^2}$$

$$\left.\left.-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{4b_1^2b_2^2}\right)\right]\Bigg\}$$

$$
\begin{aligned}
= \frac{1}{2\pi(\lambda+2\mu)\mu kK} \Bigg\{ \delta_{ij} \Bigg[\Bigg(& (\lambda+\mu) kK \frac{b_1}{b_1^2-b_2^2} - (\lambda+\mu) kK \frac{b_3^2+b_4^2}{b_1(b_1^2-b_2^2)} \\
& + (\lambda+\mu) kK \frac{b_3^2 b_4^2}{b_1^3(b_1^2-b_2^2)} - (\lambda+\mu) \frac{\eta^2 T_0}{b_1^3(b_1^2-b_2^2)} \\
& - (3\lambda+2\mu)^2 \frac{(\beta_T^s)^2 T_0 k}{b_1(b_1^2-b_2^2)} + (3\lambda+2\mu)^2 \frac{(\beta_T^s)^2 T_0 b_4^2 k}{b_1^3(b_1^2-b_2^2)} \\
& - \alpha^2 \frac{k}{b_1(b_1^2-b_2^2)} + \alpha^2 \frac{b_3^2 k}{b_1^3(b_1^2-b_2^2)} \\
& - 2(3\lambda+2\mu) \frac{\beta_T^s \eta T_0 \alpha}{b_1^3(b_1^2-b_2^2)} \Bigg) \left(\frac{t}{b_1 r^2} \exp\left(-\frac{b_1^2 r^2}{4t}\right) - \frac{b_1}{4} E_1\left(\frac{b_1^2 r^2}{4t}\right) \right) \\
- \Bigg(& (\lambda+\mu) kK \frac{b_2}{b_1^2-b_2^2} - (\lambda+\mu) kK \frac{b_3^2+b_4^2}{b_2(b_1^2-b_2^2)} \\
& + (\lambda+\mu) kK \frac{b_3^2 b_4^2}{b_2^3(b_1^2-b_2^2)} - (\lambda+\mu) \frac{\eta^2 T_0}{b_2^3(b_1^2-b_2^2)} \\
& - (3\lambda+2\mu)^2 \frac{(\beta_T^s)^2 T_0 K}{b_2(b_1^2-b_2^2)} + (3\lambda+2\mu)^2 \frac{(\beta_T^s)^2 T_0 b_4^2 K}{b_2^3(b_1^2-b_2^2)} \\
& - \alpha^2 \frac{k}{b_2(b_1^2-b_2^2)} + \alpha^2 \frac{b_3^2 k}{b_2^3(b_1^2-b_2^2)} \\
& - 2(3\lambda+2\mu) \frac{\beta_T^s \eta T_0 \alpha}{b_2^3(b_1^2-b_2^2)} \Bigg) \left(\frac{t}{b_2 r^2} \exp\left(-\frac{b_2^2 r^2}{4t}\right) - \frac{b_2}{4} E_1\left(\frac{b_2^2 r^2}{4t}\right) \right) \\
- \Bigg(& (\lambda+\mu) \cancel{kK} \frac{b_3^2+b_4^2}{b_1^2 b_2^2} + (3\lambda+2\mu)^2 \frac{(\beta_T^s)^2 T_0 K}{b_1^2 b_2^2} + \alpha^2 \frac{k}{b_1^2 b_2^2} \Bigg) \frac{t}{r^2} \\
+ \Bigg(& (\lambda+\mu) kK \frac{b_3^2 b_4^2}{4 b_1^2 b_2^2} - (\lambda+\mu) \frac{\eta^2 T_0}{4 b_1^2 b_2^2} + (3\lambda+2\mu)^2 \frac{(\beta_T^s)^2 T_0 b_4^2 k}{4 b_1^2 b_2^2} \\
& + \alpha^2 \frac{b_3^2 k}{4 b_1^2 b_2^2} - 2(3\lambda+2\mu) \frac{\beta_T^s \eta T_0 \alpha}{4 b_1^2 b_2^2} - 2(\lambda+2\mu) kK \frac{b_1^2 b_2^2}{4 b_1^2 b_2^2} \Bigg) (2 \ln r + 1) \Bigg]
\end{aligned}
$$

$$
\left.
\begin{aligned}
& -(\lambda+\mu) kK \frac{b_3^2 b_4^2}{b_1^2 b_2^2} + (\lambda+\mu) \frac{\eta^2 T_0}{b_1^2 b_2^2} \\
& -(3\lambda+2\mu)^2 (\beta_T^s)^2 T_0 \frac{b_4^2 K}{b_1^2 b_2^2} \\
& -\alpha^2 \frac{b_3^2 k}{b_1^2 b_2^2} + 2(3\lambda+2\mu) \beta_T^s \eta T_0 \alpha \frac{1}{b_1^2 b_2^2}
\end{aligned}
\right\} \times \frac{b_1^2+b_2^2}{b_1^2 b_2^2}
$$

$$
-\frac{x_i x_j}{r^2}\Bigg[\Bigg(\bigg((\lambda+\mu)\frac{kK}{b_1^2-b_2^2}\left(b_1^2-(b_3^2+b_4^2)+b_3^2b_4^2/b_1^2\right)
$$

$$
-(\lambda+\mu)\frac{\eta^2T_0}{b_1^2(b_1^2-b_2^2)}-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0K}{b_1^2-b_2^2}
$$

$$
+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0b_4^2K}{b_1^2(b_1^2-b_2^2)}-\alpha^2\frac{k}{b_1^2-b_2^2}
$$

$$
+\alpha^2\frac{b_3^2k}{b_1^2(b_1^2-b_2^2)}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{b_1^2(b_1^2-b_2^2)}\bigg)\left(\cancel{\frac{1}{2}E_1\left(\frac{b_1^2r^2}{4t}\right)}+\frac{2t}{b_1^2r^2}e^{-\frac{b_1^2r^2}{4t}}-\cancel{\frac{1}{2}E_1\left(\frac{b_1^2r^2}{4t}\right)}\right)
$$

$$
-\bigg((\lambda+\mu)\frac{kK}{b_1^2-b_2^2}\left(b_2^2-(b_3^2+b_4^2)+b_3^2b_4^2/b_2^2\right)
$$

$$
-(\lambda+\mu)\frac{\eta^2T_0}{b_2^2(b_1^2-b_2^2)}-(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0K}{b_1^2-b_2^2}
$$

$$
+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0b_4^2K}{b_2^2(b_1^2-b_2^2)}-\alpha^2\frac{k}{b_1^2-b_2^2}
$$

$$
+\alpha^2\frac{b_3^2k}{b_2^2(b_1^2-b_2^2)}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{b_2^2(b_1^2-b_2^2)}\bigg)\left(\cancel{\frac{1}{2}E_1\left(\frac{b_2^2r^2}{4t}\right)}+\frac{2t}{b_2^2r^2}e^{-\frac{b_1^2r^2}{4t}}-\cancel{\frac{1}{2}E_1\left(\frac{b_2^2r^2}{4t}\right)}\right)
$$

$$
-\left((\lambda+\mu)kK\frac{b_3^2+b_4^2}{2b_1^2b_2^2}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2T_0K}{2b_1^2b_2^2}+\alpha^2\frac{k}{2b_1^2b_2^2}\right)\frac{t}{r^2}
$$

$$
\left\{\begin{aligned}&-(\lambda+\mu)kK\frac{b_3^2b_4^2}{2b_1^2b_2^2}+(\lambda+\mu)\frac{\eta^2T_0}{2b_1^2b_2^2}\\&-(3\lambda+2\mu)^2(\beta_T^s)^2T_0\frac{b_4^2K}{2b_1^2b_2^2}\\&-\alpha^2\frac{b_3^2k}{2b_1^2b_2^2}+2(3\lambda+2\mu)\beta_T^s\eta T_0\alpha\frac{1}{2b_1^2b_2^2}\end{aligned}\right\}\times\frac{b_1^2+b_2^2}{b_1^2b_2^2}
$$

$$
-z\bigg((\lambda+\mu)kK\frac{b_3^2b_4^2}{4b_1^2b_2^2}-(\lambda+\mu)\frac{\eta^2T_0}{4b_1^2b_2^2}+(3\lambda+2\mu)^2\frac{(\beta_T^s)^2\eta T_0b_4^2K}{4b_1^2b_2^2}
$$

$$
+\alpha^2\frac{b_3^2k}{4b_1^2b_2^2}-2(3\lambda+2\mu)\frac{\beta_T^s\eta T_0\alpha}{4b_1^2b_2^2}\bigg)\Bigg)\Bigg]\Bigg\}
$$

Reciprocal theorem.

$$\int_V \dot\sigma_{ij}^{(1)}\varepsilon_{ij}^{(2)} - \dot\sigma_{ij}^{(2)}\varepsilon_{ij}^{(1)}\,dV - \int_V N_{ij}\left(\dot\theta^{(1)}\varepsilon_{ij}^{(2)} - \dot\theta^{(2)}\varepsilon_{ij}^{(1)}\right)dV - \int_V M_{ij}\left(\dot p^{(1)}\varepsilon_{ij}^{(2)} - \dot p^{(2)}\varepsilon_{ij}^{(1)}\right)dV = 0$$

$$K p_{,jj} + \Psi_p = \frac{\alpha}{KB}\dot p - 3KM\dot u_{j,j} + \eta\dot\theta$$ ← pore pressure dissipation equation

$$\kappa\int_V \delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,ij}\,dV = \kappa\int_V\left(\delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,i}\right)_{,j}dV - \kappa\int_V\delta_{ij}\tilde p^{(1)}_{,j}\tilde p^{(2)}_{,i}\,dV$$

$$= \kappa\int_S\left(\delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,i}\right)n_j\,dS - \kappa\int_V \delta_{ij}\tilde p^{(1)}_{,j}\tilde p^{(2)}_{,i}\,dV$$

from pore pressure dissipation equation

$$\kappa\int_V\delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,ij}\,dV = \frac{\alpha s}{KB}\int_V \tilde p^{(1)}\tilde p^{(2)}dV - \frac{3KMs}{\alpha}\int_V \tilde p^{(1)}\tilde\varepsilon^{(2)}_{ij}\delta_{ij}\,dV + \eta s\int_V\tilde p^{(1)}\tilde\theta^{(2)}dV - \int_V\tilde p^{(1)}\tilde\Psi_p^{(2)}dV$$

from above two equation

$$\kappa\int_S\left(\delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,i}\right)n_j\,dS - \kappa\int_V\delta_{ij}\tilde p^{(1)}_{,j}\tilde p^{(2)}_{,i}\,dV = \frac{\alpha s}{KB}\int_V\tilde p^{(1)}\tilde p^{(2)}dV - 3KMs\int_V\delta_{ij}\tilde p^{(1)}\tilde\varepsilon^{(2)}_{ij}\,dV + \eta s\int_V \tilde p^{(1)}\tilde\theta^{(2)}dV - \int_V\tilde p^{(1)}\tilde\Psi_p^{(2)}dV$$

$$3KMs\int_V\delta_{ij}\tilde p^{(1)}\tilde\varepsilon^{(2)}_{ij}\,dV = -\kappa\int_S\left(\delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,i}\right)n_j\,dS + \kappa\int_V\delta_{ij}\tilde p^{(1)}_{,j}\tilde p^{(2)}_{,i}\,dV + \frac{\alpha s}{KB}\int_V \tilde p^{(1)}\tilde p^{(2)}dV + \eta s\int_V\tilde p^{(1)}\tilde\theta^{(2)}dV - \int_V\tilde p^{(1)}\tilde\Psi_p^{(2)}dV$$

$$\frac{k}{T_0}\theta_{,jj} + \Psi_T = \frac{C_p}{T_0}\dot\theta + \eta\dot p - (3\lambda+2\mu)\beta_T^s\dot\varepsilon_{jj}$$ ← thermal diffusion equation.

$$\frac{k}{T_0}\int_V\delta_{ij}\tilde\theta^{(1)}\tilde\theta^{(2)}_{,ij}\,dV = \frac{k}{T_0}\int_V\left(\delta_{ij}\tilde\theta^{(1)}\tilde\theta^{(2)}_{,i}\right)_{,j}dV - \frac{k}{T_0}\int_V\delta_{ij}\tilde\theta^{(1)}_{,j}\tilde\theta^{(2)}_{,i}\,dV$$

$$= \frac{k}{T_0}\int_S\left(\delta_{ij}\tilde\theta^{(1)}\tilde\theta^{(2)}_{,i}\right)n_j\,dV - \frac{k}{T_0}\int_V\delta_{ij}\tilde\theta^{(1)}_{,j}\tilde\theta^{(2)}_{,i}\,dV$$

from thermal diffusion equation.

$$\frac{k}{T_0}\int_V\delta_{ij}\tilde\theta^{(1)}\tilde\theta^{(2)}_{,ij}\,dV = \frac{C_p}{T_0}s\int_V\tilde\theta^{(1)}\tilde\theta^{(2)}dV + \eta s\int_V\tilde\theta^{(1)}\tilde p^{(2)}dV - (3\lambda+2\mu)\beta_T^s s\int_V\tilde\theta^{(1)}\tilde\varepsilon^{(2)}_{ij}\delta_{ij}\,dV - \int_V\tilde\theta^{(1)}\tilde\Psi_T^{(2)}dV$$

$$(3\lambda+2\mu)\beta_T^s s\int_V\tilde\theta^{(1)}\tilde\varepsilon^{(2)}_{ij}\delta_{ij}\,dV = -\frac{k}{T_0}\int_S\left(\delta_{ij}\tilde\theta^{(1)}\tilde\theta^{(2)}_{,i}\right)n_j\,dS + \frac{k}{T_0}\int_V\delta_{ij}\tilde\theta^{(1)}_{,j}\tilde\theta^{(2)}_{,i}\,dV + \frac{C_p s}{T_0}\int_V\tilde\theta^{(1)}\tilde\theta^{(2)}dV + \eta s\int_V\tilde\theta^{(1)}\tilde p^{(2)}dV - \int_V\tilde\theta^{(1)}\tilde\Psi_T^{(2)}dV$$

$$3KM s\int_V \delta_{ij}(\tilde p^{(1)}\tilde\varepsilon_{ij}^{(2)} - \tilde p^{(2)}\tilde\varepsilon_{ij}^{(1)})dV = -\kappa\int_S(\delta_{ij}\tilde p^{(1)}\tilde p_{,i}^{(2)})n_j - (\delta_{ij}\tilde p^{(2)}\tilde p_{,i}^{(1)})n_j\,dS$$
$$+\eta s\int_V \tilde p^{(1)}\tilde\theta^{(2)} - \tilde p^{(2)}\tilde\theta^{(1)}\,dV - \int_V \tilde p^{(1)}\tilde\Psi_p^{(2)} - \tilde p^{(2)}\tilde\Psi_p^{(1)}\,dV$$

$$(3\lambda+2\mu)\beta_T^s s\int_V \delta_{ij}(\tilde\theta^{(1)}\tilde\varepsilon_{ij}^{(2)} - \tilde\theta^{(2)}\tilde\varepsilon_{ij}^{(1)})dV = -\frac{k}{T_0}\int_S(\delta_{ij}\tilde\theta^{(1)}\tilde\theta_{,i}^{(2)})n_j - (\delta_{ij}\tilde\theta^{(2)}\tilde\theta_{,i}^{(1)})n_j\,dS$$
$$+\eta s\int_V \tilde\theta^{(1)}\tilde p^{(2)} - \tilde\theta^{(2)}\tilde p^{(1)}\,dV - \int_V \tilde\theta^{(1)}\tilde\Psi_T^{(2)} - \tilde\theta^{(2)}\tilde\Psi_T^{(1)}\,dV.$$

From the extended Betti's reciprocity equation.

$$s\int_V \tilde\sigma_{ij}^{(1)}\tilde\varepsilon_{ij}^{(2)} - \tilde\sigma_{ij}^{(2)}\tilde\varepsilon_{ij}^{(1)}\,dV - \frac{k}{T_0}\int_S(\delta_{ij}\tilde\theta^{(1)}\tilde\theta_{,i}^{(2)})n_j - (\delta_{ij}\tilde\theta^{(2)}\tilde\theta_{,i}^{(1)})n_j\,dS$$
$$-\int_V \tilde\theta^{(1)}\tilde\Psi_T^{(2)} - \tilde\theta^{(2)}\tilde\Psi_T^{(1)}\,dV - \kappa\int_S(\delta_{ij}\tilde p^{(1)}\tilde p_{,i}^{(2)})n_j - (\delta_{ij}\tilde p^{(2)}p_{,i}^{(1)})n_j\,dS$$
$$-\int_V \tilde p^{(1)}\tilde\Psi_p^{(2)} - \tilde p^{(2)}\tilde\Psi_p^{(1)}\,dV = 0.$$

$$\int_V \tilde\sigma_{ij}^{(1)}\tilde\varepsilon_{ij}^{(2)}\,dV = \int_V(\sigma_{ij}^{(1)}u_i^{(2)})_{,j}\,dV - \int_V \sigma_{ij,j}^{(1)}u_i^{(2)}\,dV$$
$$= \int_S(\sigma_{ij}^{(1)}u_i^{(2)})n_j\,dS - \int_V \sigma_{ij,j}^{(1)}u_i^{(2)}\,dV$$
$$= \int_S t_i^{(1)}u_i^{(2)}\,dS + \int_V b_i^{(1)}u_i^{(2)}\,dV.$$

$$\int_V \tilde\sigma_{ij}^{(2)}\tilde\varepsilon_{ij}^{(1)}\,dV = \int_S t_i^{(2)}u_i^{(1)}\,dS + \int_V \tilde b_i^{(2)}\tilde u_i^{(1)}\,dV.$$

$$\int(\tilde\sigma_{ij}^{(1)}\tilde\varepsilon_{ij}^{(2)} - \tilde\sigma_{ij}^{(2)}\tilde\varepsilon_{ij}^{(1)})dV = \int_S \tilde t_i^{(1)}\tilde u_i^{(2)} - \tilde t_i^{(2)}\tilde u_i^{(1)}\,dS + \int_V \tilde b_i^{(1)}\tilde u_i^{(2)} - \tilde b_i^{(2)}\tilde u_i^{(1)}\,dV.$$

Finally reciprocal equation is

$$s\int_S \tilde t_i^{(1)}\tilde u_i^{(2)} - \tilde t_i^{(2)}\tilde u_i^{(1)}\,dS + \frac{k}{T_0}\int_S(\delta_{ij}\tilde\theta^{(1)}\tilde\theta_{,i}^{(2)})n_j - (\delta_{ij}\tilde\theta^{(2)}\tilde\theta_{,i}^{(1)})n_j\,dS + \kappa\int_S(\delta_{ij}\tilde p^{(1)}\tilde p_{,i}^{(2)})n_j - (\delta_{ij}\tilde p^{(2)}p_{,i}^{(1)})n_j\,dS$$
$$+s\int_V \tilde b_i^{(1)}\tilde u_i^{(2)} - \tilde b_i^{(2)}\tilde u_i^{(1)}\,dV + \int_V \tilde\theta^{(1)}\tilde\Psi_T^{(2)} - \tilde\theta^{(2)}\tilde\Psi_T^{(1)}\,dV + \int_V \tilde p^{(1)}\tilde\Psi_p^{(2)} - \tilde p^{(2)}\tilde\Psi_p^{(1)}\,dV = 0.$$

"S" 가 곱해지는 것은 구성방정식

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} + N_{ij}\theta + M_{ij}P$$

의 시간 미분을 취한공식

$\dot\sigma_{ij} = \lambda\dot\varepsilon_{kk}\delta_{ij} + 2\mu\dot\varepsilon_{ij} + N_{ij}\dot\theta + M_{ij}\dot P$ 를 Reciprocal theorem 에 도입했기 때문이다.

$$s\int_V \tilde\sigma^{(1)}_{ij}\tilde\varepsilon^{(2)}_{ij} - \tilde\sigma^{(2)}_{ij}\tilde\varepsilon^{(1)}_{ij}\,dV - N_{ij}\overset{?}{s}\int_V \tilde\theta^{(1)}\tilde\varepsilon^{(2)}_{ij} - \tilde\theta^{(2)}\tilde\varepsilon^{(1)}_{ij}\,dV - M_{ij}\overset{?}{s}\int_V \tilde p^{(1)}\tilde\varepsilon^{(2)}_{ij} - \tilde p^{(2)}\tilde\varepsilon^{(1)}_{ij}\,dV$$

$$-s\int_V \tilde\sigma^{(1)}_{ij}\tilde\varepsilon^{a(2)}_{ij} - \tilde\sigma^{(2)}_{ij}\tilde\varepsilon^{a(1)}_{ij}\,dV + N_{ij}\overset{?}{s}\int_V \tilde\theta^{(1)}\tilde\varepsilon^{a(2)}_{ij} - \tilde\theta^{(2)}\tilde\varepsilon^{a(1)}_{ij}\,dV + M_{ij}\overset{?}{s}\int_V \tilde p^{(1)}\tilde\varepsilon^{a(2)}_{ij} - \tilde p^{(2)}\tilde\varepsilon^{a(1)}_{ij}\,dV = 0.$$

(1) → variable, (2) → fundamental solution.

$$s\int_S \tilde t^{(1)}_i\tilde u^{(2)}_i - \tilde t^{(2)}_i\tilde u^{(1)}_i\,dS - \frac{k\overset{?}{\cancel{s}}}{T_0}\int_S (\delta_{ij}\tilde\theta^{(1)}\tilde\theta^{(2)}_{,i})n_j - (\delta_{ij}\tilde\theta^{(2)}\tilde\theta^{(1)}_{,i})n_j\,dS$$

$$-k\cancel{s}\int_S (\delta_{ij}\tilde p^{(1)}\tilde p^{(2)}_{,i})n_j - (\delta_{ij}\tilde p^{(2)}\tilde p^{(1)}_{,i})n_j\,dS + s\int_V \tilde b^{(1)}_i\tilde u^{(2)}_i - \tilde b^{(2)}_i\tilde u^{(1)}_i\,dV$$

$$-\cancel{s}\int_V \tilde\theta^{(1)}\tilde\Psi^{(2)}_T - \tilde\theta^{(2)}\tilde\Psi^{(1)}_T\,dV - \cancel{s}\int_V \tilde p^{(1)}\tilde\Psi^{(2)}_P - \tilde p^{(2)}\tilde\Psi^{(1)}_P\,dV$$

$$-s\int_V \tilde\sigma^{(1)}_{ij}\tilde\varepsilon^{a(2)}_{ij} - \tilde\sigma^{(2)}_{ij}\tilde\varepsilon^{a(1)}_{ij}\,dV + N_{ij}s\int_V \tilde\theta^{(1)}\tilde\varepsilon^{a(2)}_{ij} - \tilde\theta^{(2)}\tilde\varepsilon^{a(1)}_{ij}\,dV + M_{ij}s\int_V \tilde p^{(1)}\tilde\varepsilon^{a(2)}_{ij} - \tilde p^{(2)}\tilde\varepsilon^{a(1)}_{ij}\,dV = 0.$$

$$\Rightarrow s\int_S \tilde t_i\tilde u^*_i - \tilde t^*_i u_i\,dS - \frac{k\cancel{s}}{T_0}\int_S (\delta_{ij}\tilde\theta\,\tilde\theta^*_{,i})n_j - (\delta_{ij}\tilde\theta^*\theta_{,i})n_j\,dS$$

$$-k\cancel{s}\int_S (\delta_{ij}\tilde p\,\tilde p^*_{,i})n_j - (\delta_{ij}\tilde p^* p_{,i})n_j\,dS + s\int_V \tilde b_i\tilde u^*_i - \tilde b^*_i\tilde u_i\,dV$$

$$-\cancel{s}\int_V \tilde\theta\tilde\Psi^*_T - \tilde\theta^*\tilde\Psi_T\,dV - \cancel{s}\int_V \tilde p\tilde\Psi^*_P - \tilde p^*\tilde\Psi_P\,dV$$

$$+s\int_V (\tilde\sigma^*_{ij} - \overset{N_{ij}}{\tilde\theta^*} - \overset{M_{ij}}{\tilde p^*})\,\underline{\underline{\varepsilon^a_{ij}}}\,dV.$$

$$\begin{cases} \tilde q_{T,i} = -k\tilde\theta_{,i}n_i. \\ \tilde q_{P,i} = -k\tilde p_{,i}n_i \end{cases}$$

$$\Rightarrow s\int_S \tilde t_i\tilde u^*_i - \tilde t^*_i\tilde u_i\,dS + \frac{\cancel{s}}{T_0}\int_S \tilde\theta\,\tilde q^*_{T,i} - \tilde\theta^*\tilde q_{T,i}\,dS + \cancel{s}\int_S \tilde p\,\tilde q^*_{P,i} - \tilde p^*\tilde q_{P,i}\,dS$$

$$+s\int_V \tilde b_i\tilde u^*_i - \tilde b^*_i\tilde u_i\,dV + \cancel{s}\int_V \tilde\theta\tilde\Psi^*_T - \tilde\theta^*\tilde\Psi_T\,dV + \cancel{s}\int_V \tilde p\tilde\Psi^*_P - \tilde p^*\tilde\Psi_P\,dV$$

$$+s\int_V (\sigma^*_{ij} - \overset{N_{ij}}{\tilde\theta^*} - \overset{M_{ij}}{\tilde p^*})\,\varepsilon^a_{ij}\,dV.$$

$$\Rightarrow s\int_S \tilde t_i\tilde u^*_i - \tilde t^*_i\tilde u_i\,dS + \frac{1}{T_0}\int_S \tilde\theta\,\tilde q^*_{T,i} - \tilde\theta^*\tilde q_{T,i}\,dS + \int_S \tilde p\,\tilde q^*_{P,i} - \tilde p^*\tilde q_{P,i}\,dS$$

$$+s\int_V \tilde b_i\tilde u^*_i - \tilde b^*_i\tilde u_i\,dV + \int_V \tilde\theta\tilde\Psi^*_T - \tilde\theta^*\tilde\Psi_T\,dV + \int_V \tilde p\tilde\Psi^*_P - \tilde p^*\tilde\Psi_P\,dV$$

$$+s\int_V (\sigma^*_{ij} - N_{ij}\tilde\theta^* - M_{ij}\tilde p^*)\,\varepsilon^a_{ij}\,dV$$

$$\int_S \dot{t}_i^{(1)} * u_i^{(2)} - t_i^{(2)} * \dot{u}_i^{(1)} + \frac{1}{T_0}\theta^{(1)} * q_T^{(2)} - \frac{1}{T_0}\theta^{(2)} * q_T^{(1)} + p^{(1)} * q_p^{(2)} - p^{(2)} * q_p^{(1)}\, ds$$

$$+ \int_V \dot{b}_i^{(1)} * u_i^{(2)} - b_i^{(2)} * \dot{u}_i^{(1)} - \theta^{(1)} * \psi_T^{(2)} + \theta^{(2)} * \psi_T^{(1)} - p^{(1)} * \psi_p^{(2)} + p^{(2)} * \psi_p^{(1)}\, dV = 0$$

Case (1)

$$b_i^{(1)} = \delta(x-\xi) H(t) \delta_{ij} e_j, \quad \psi_T^{(1)} = 0, \quad \psi_p^{(1)} = 0, \quad b_i^{(2)} = 0, \quad \psi_T^{(2)} = 0, \quad \psi_p^{(2)} = 0$$

$$u_i^{(1)} = G_{ij} e_j, \quad \theta^{(1)} = G_{\theta j} e_j, \quad p^{(1)} = G_{pj} e_j, \quad t_i^{(1)} = F_{ij} e_j, \quad q_T^{(1)} = F_{\theta j} e_j, \quad q_p^{(1)} = F_{pj} e_j$$

$$\Rightarrow -C_{ij}(\xi) u_i(\xi, t) = \int_S \dot{F}_{ij} * u_i(x,t) - \dot{G}_{ij} * t_i(x,t) + \frac{1}{T_0} G_{\theta j} * q_T(x,t)$$
$$- \frac{1}{T_0} F_{\theta j} * \theta(x,t) + G_{pj} * q_p(x,t) - F_{pj} * p(x,t)\, ds$$

Case (2)

$$b_i^{(1)} = 0, \quad \psi_T^{(1)} = 0, \quad \psi_p^{(1)} = \delta(x-\xi)\delta(t), \quad b_i^{(2)} = 0, \quad \psi_T^{(2)} = 0, \quad \psi_p^{(2)} = 0$$

$$u_i^{(1)} = G_{ip}, \quad \theta^{(1)} = G_{\theta p}, \quad p^{(1)} = G_{pp}, \quad t_i^{(1)} = F_{ip}, \quad q_T = F_{\theta p}, \quad q_p = F_{pp}$$

$$\Rightarrow -C(\xi) p(\xi, t) = \int_S \dot{F}_{ip} * u_i(x,t) - \dot{G}_{ip} * t_i(x,t) + \frac{1}{T_0} G_{\theta p} * q_T(x,t) - \frac{1}{T_0} F_{\theta p} * \theta(x,t)$$
$$+ G_{pp} * q_p(x,t) - F_{pp} * p(x,t)\, ds$$

Case (3)

$$b_i^{(1)} = 0, \quad \psi_T^{(1)} = \delta(x-\xi)\delta(t), \quad \psi_p^{(1)} = 0, \quad b_i^{(2)} = 0, \quad \psi_T^{(2)} = 0, \quad \psi_p^{(2)} = 0$$

$$u_i^{(1)} = G_{i\theta}, \quad \theta^{(1)} = G_{\theta\theta}, \quad p^{(1)} = G_{p\theta}, \quad t_i^{(1)} = F_{i\theta}, \quad q_T = F_{\theta\theta}, \quad q_p = F_{p\theta}$$

$$\Rightarrow -C(\xi) \theta(\xi, t) = \int_S \dot{F}_{i\theta} * u_i(x,t) - \dot{G}_{i\theta} * t_i(x,t) + \frac{1}{T_0} G_{\theta\theta} * q_T(x,t) - \frac{1}{T_0} F_{\theta\theta} * \theta(x,t)$$
$$+ G_{p\theta} * q_p(x,t) - F_{p\theta} * p(x,t)\, ds$$

G^*_{ij} =의 다항의 계수를 구하는 과정.

$$\frac{1}{2\pi(\lambda+2\mu)\mu k k_s}\Bigg\{\delta_{ij}\Bigg[\frac{K_1(\lambda_1 r)}{r(\lambda_1^2-\lambda_2^2)\lambda_1^3}\Big(\ \mathrm{A}\ \Big)$$

$$-\frac{K_1(\lambda_2 r)}{r(\lambda_1^2-\lambda_2^2)\lambda_2^3}\Big(\ \mathrm{B}\ \Big)$$

$$-\frac{1}{r^2\lambda_1^2\lambda_2^2}\Big(\ \mathrm{C}\ \Big)$$

$$+\frac{2\ln r+1}{4\lambda_1^2\lambda_2^2}\Big(\ \mathrm{d}\ \Big)\Bigg]$$

$$-\frac{x_i x_j}{r^2}\Bigg[\frac{K_0(\lambda_1 r)}{\lambda_1^2(\lambda_1^2-\lambda_2^2)}\Big(\ \mathrm{e}\ \Big)$$

$$-\frac{K_0(\lambda_2 r)}{\lambda_2^2(\lambda_1^2-\lambda_2^2)}\Big(\ \mathrm{f}\ \Big)$$

$$-\frac{1}{2r^2\lambda_1^2\lambda_2^2}\Big(\ \mathrm{g}\ \Big)$$

$$-\frac{1}{2\lambda_1^2\lambda_2^2}\Big(\ \mathrm{h}.\ \Big)$$

e = A

f = B

g = C

Derivation of reciprocal theorem for initial value problem.

$$\mathcal{L}\{\dot{f}\} = sF(s) - f(0). \quad ; \text{ Laplace transformation.}$$

$$\mathcal{L}\{\dot{\sigma}_{ij}\} = \mathcal{L}\{\lambda \dot{\varepsilon}_{kk}\delta_{ij} + 2\mu\dot{\varepsilon}_{ij} + N_{ij}\dot{\theta} + M_{ij}\dot{P}\} \quad ; \text{ equilibrium equation.}$$

$$s\tilde{\sigma}_{ij} - \sigma^{\circ}_{ij} = s\lambda\tilde{\varepsilon}_{kk}\delta_{ij} + 2s\mu\tilde{\varepsilon}_{ij} + sN_{ij}\tilde{\theta} + M_{ij}\tilde{P}$$
$$- \lambda\varepsilon^{\circ}_{kk}\delta_{ij} - 2\mu\varepsilon^{\circ}_{ij} - N_{ij}\theta^{\circ} - M_{ij}P^{\circ} \quad \cdots (1).$$

$$\Rightarrow s\tilde{\sigma}_{ij} = s\lambda\tilde{\varepsilon}_{kk}\delta_{ij} + 2s\mu\tilde{\varepsilon}_{ij} + sN_{ij}\tilde{\theta} + M_{ij}\tilde{P}$$
$$+ (\sigma^{\circ}_{ij} - \lambda\varepsilon^{\circ}_{kk}\delta_{ij} - 2\mu\varepsilon^{\circ}_{ij} - N_{ij}\theta^{\circ} - M_{ij}P^{\circ})$$

Laplace transformation of thermal diffusion equation.

$$\kappa\tilde{P}_{,jj} + \tilde{\Psi}_P = \frac{s\alpha}{KB}\tilde{P} - M_{ij}\tilde{u}_{j,ij} + \eta s\tilde{\theta} - \left(\frac{\alpha}{KB}P^{\circ} - M_{ij}u^{\circ}_{j,ij} + \eta\theta^{\circ}\right) \quad \cdots (2)$$

Laplace transformation of hydraulic diffusion equation

$$\frac{k}{T_0}\tilde{\theta}_{,jj} + \tilde{\Psi}_T = \frac{sC_p}{T_0}\tilde{\theta} - N_{ij}\tilde{u}_{j,ij} + \eta s\tilde{P} - \left(\frac{C_p}{T_0}\theta^{\circ} - N_{ij}u^{\circ}_{j,ij} + \eta P^{\circ}\right) \quad \cdots (3).$$

Apply reciprocal theorem to (1).

$$\int_V (s\tilde{\sigma}^{(1)}_{ij} - \sigma^{\circ(1)}_{ij})\varepsilon^{(2)}_{ij} - (s\tilde{\sigma}^{(2)}_{ij} - \sigma^{\circ(2)}_{ij})\varepsilon^{(1)}_{ij}\, dv$$
$$- \int_V N_{ij}(s\tilde{\theta}^{(1)} - \theta^{\circ(1)})\varepsilon^{(2)}_{ij} - N_{ij}(s\tilde{\theta}^{(2)} - \theta^{\circ(2)})\varepsilon^{(1)}_{ij}\, dv$$
$$- \int_V M_{ij}(s\tilde{P}^{(1)} - P^{\circ(1)})\varepsilon^{(2)}_{ij} - M_{ij}(s\tilde{P}^{(2)} - P^{\circ(2)})\varepsilon^{(1)}_{ij}\, dv = 0. \quad \cdots (4)$$

From the Green theorem,

$$\kappa\int_V \delta_{ij}\tilde{P}^{(1)}\tilde{P}^{(2)}_{,ij}\, dv = \kappa\int_S (\delta_{ij}\tilde{P}^{(1)}\tilde{P}^{(2)}_{,i})n_j\, ds - \kappa\int_V \delta_{ij}\tilde{P}^{(1)}_{,j}\tilde{P}^{(2)}_{,i}\, dv \quad \cdots (5)$$

From the equation (2),

$$\kappa\int_V \delta_{ij}\tilde{P}^{(1)}\tilde{P}^{(2)}_{,ij}\, dv = \frac{\alpha s}{KB}\int_V \tilde{P}^{(1)}\tilde{P}^{(2)} dv - M_{ij}\int_V \tilde{P}^{(1)}\tilde{\varepsilon}^{(2)}_{ij}\delta_{ij}\, dv + \eta s\int_V \tilde{P}^{(1)}\tilde{\theta}^{(2)} dv$$
$$- \int_V \tilde{P}^{(1)}\tilde{\Psi}^{(2)}_P dv - \int_V \frac{\alpha}{KB}\tilde{P}^{(1)}P^{\circ(2)} - M_{ij}\tilde{P}^{(1)}\varepsilon^{\circ(2)}_{ij} + \eta\tilde{P}^{(1)}\theta^{\circ(2)} dv \quad \cdots (6)$$

From the equation (5) and (6),

$$\kappa\int_S (\delta_{ij}\tilde{P}^{(1)}\tilde{P}^{(2)}_{,i})n_j\, ds - \kappa\int_V \delta_{ij}\tilde{P}^{(1)}_{,j}\tilde{P}^{(2)}_{,i}\, dv$$
$$= \frac{\alpha s}{KB}\int_V \tilde{P}^{(1)}\tilde{P}^{(2)} dv - M_{ij}s\int_V \tilde{P}^{(1)}\tilde{\varepsilon}^{(2)}_{ij}\delta_{ij}\, dv + \eta s\int_V \tilde{P}^{(1)}\tilde{\theta}^{(2)} dv - \int_V \tilde{P}^{(1)}\tilde{\Psi}^{(2)}_P dv$$
$$- \int_V \frac{\alpha}{KB}\tilde{P}^{(1)}P^{\circ(2)} - M_{ij}\tilde{P}^{(1)}\varepsilon^{\circ(2)}_{ij} + \eta\tilde{P}^{(1)}\theta^{\circ(2)} dv$$

$$M_{\bar{A}J}\, s\int_V \tilde{p}^{(1)}\tilde{\varepsilon}_{\bar{A}J}^{(2)} - \tilde{p}^{(2)}\tilde{\varepsilon}_{\bar{A}J}^{(1)}\, dV$$

$$= -\kappa\int_S \left(\delta_{\bar{A}J}\tilde{p}^{(1)}\tilde{p}_{,\bar{A}}^{(2)} - \delta_{\bar{A}J}\tilde{p}^{(2)}\tilde{p}_{,\bar{A}}^{(1)}\right)n_J\, ds + \kappa\int_V \delta_{\bar{A}J}\tilde{p}_{,J}^{(1)}\tilde{p}_{,\bar{A}}^{(2)} - \delta_{\bar{A}J}\tilde{p}_{,J}^{(2)}\tilde{p}_{,\bar{A}}^{(1)}\, dV$$

$$+ \frac{\alpha s}{KB}\int_V \tilde{p}^{(1)}\tilde{p}^{(2)} - \tilde{p}^{(2)}\tilde{p}^{(1)}\, dV + \eta s\int_V \tilde{p}^{(1)}\tilde{\theta}^{(2)} - \tilde{p}^{(2)}\tilde{\theta}^{(1)}\, dV - \int_V \tilde{p}^{(1)}\tilde{\Psi}_P^{(2)} - \tilde{p}^{(2)}\tilde{\Psi}_P^{(1)}\, dV$$

$$- \frac{\alpha}{KB}\int_V \tilde{p}^{(1)}p^{0(2)} - \tilde{p}^{(2)}p^{0(1)}\, dV + M_{\bar{A}J}\int_V \tilde{p}^{(1)}\varepsilon_{\bar{A}J}^{0(2)} - \tilde{p}^{(2)}\varepsilon_{\bar{A}J}^{0(1)}\, dV - \eta\int_V \tilde{p}^{(1)}\theta^{0(2)} - \tilde{p}^{(2)}\theta^{0(1)}\, dV. \quad \cdots (7)$$

By the same reason,

$$N_{\bar{A}J}\, s\int_V \tilde{\theta}^{(1)}\tilde{\varepsilon}_{\bar{A}J}^{(2)} - \tilde{\theta}^{(2)}\tilde{\varepsilon}_{\bar{A}J}^{(1)}\, dV$$

$$= -\frac{k}{T_0}\int_S \left(\delta_{\bar{A}J}\tilde{\theta}^{(1)}\tilde{\theta}_{,\bar{A}}^{(2)} - \delta_{\bar{A}J}\tilde{\theta}^{(2)}\tilde{\theta}_{,\bar{A}}^{(1)}\right)n_J\, ds + \frac{k}{T_0}\int_V \delta_{\bar{A}J}\tilde{\theta}_{,J}^{(1)}\tilde{\theta}_{,\bar{A}}^{(2)} - \delta_{\bar{A}J}\tilde{\theta}_{,J}^{(2)}\tilde{\theta}_{,\bar{A}}^{(1)}\, dV$$

$$+ \frac{sC_p}{T_0}\int_V \tilde{\theta}^{(1)}\tilde{\theta}^{(2)} - \tilde{\theta}^{(2)}\tilde{\theta}^{(1)}\, dV + \eta s\int_V \tilde{\theta}^{(1)}\tilde{p}^{(2)} - \tilde{\theta}^{(2)}\tilde{p}^{(1)}\, dV - \int_V \tilde{\theta}^{(1)}\tilde{\Psi}_T^{(2)} - \tilde{\theta}^{(2)}\tilde{\Psi}_T^{(1)}\, dV$$

$$- \frac{C_p}{T_0}\int_V \tilde{\theta}^{(1)}\theta^{0(2)} - \tilde{\theta}^{(2)}\theta^{0(1)}\, dV + N_{\bar{A}J}\int_V \tilde{\theta}^{(1)}\varepsilon_{\bar{A}J}^{0(2)} - \tilde{\theta}^{(2)}\varepsilon_{\bar{A}J}^{0(1)}\, dV - \eta\int_V \tilde{\theta}^{(1)}p^{0(2)} - \tilde{\theta}^{(2)}p^{0(1)}\, dV \quad \cdots (8).$$

Apply eq. (7) & (8) to eq. (4),

$$s\int_V \tilde{\sigma}_{\bar{A}J}^{(1)}\tilde{\varepsilon}_{\bar{A}J}^{(2)} - \tilde{\sigma}_{\bar{A}J}^{(2)}\tilde{\varepsilon}_{\bar{A}J}^{(1)}\, dV - sM_{\bar{A}J}\int_V \tilde{p}^{(1)}\tilde{\varepsilon}_{\bar{A}J}^{(2)} - \tilde{p}^{(2)}\tilde{\varepsilon}_{\bar{A}J}^{(1)}\, dV - sN_{\bar{A}J}\int_V \tilde{\theta}^{(1)}\tilde{\varepsilon}_{\bar{A}J}^{(2)} - \tilde{\theta}^{(2)}\tilde{\varepsilon}_{\bar{A}J}^{(1)}\, dV$$

$$= s\int_S \tilde{t}_{\bar{A}}^{(1)}\tilde{u}_{\bar{A}}^{(2)} - \tilde{t}_{\bar{A}}^{(2)}\tilde{u}_{\bar{A}}^{(1)}\, ds + \frac{k}{T_0}\int_S \left(\delta_{\bar{A}J}\tilde{\theta}^{(1)}\tilde{\theta}_{,\bar{A}}^{(2)}\right)n_J - \left(\delta_{\bar{A}J}\tilde{\theta}^{(2)}\tilde{\theta}_{,\bar{A}}^{(1)}\right)n_J\, ds$$

$$+ \kappa\int_S \left(\delta_{\bar{A}J}\tilde{p}^{(1)}\tilde{p}_{,\bar{A}}^{(2)}\right)n_J - \left(\delta_{\bar{A}J}\tilde{p}^{(2)}\tilde{p}_{,\bar{A}}^{(1)}\right)n_J\, ds + s\int_V \tilde{b}_{\bar{A}}^{(1)}\tilde{u}_{\bar{A}}^{(2)} - \tilde{b}_{\bar{A}}^{(2)}\tilde{u}_{\bar{A}}^{(1)}\, dV$$

$$+ \int_V \tilde{\theta}^{(1)}\tilde{\Psi}_T^{(2)} - \tilde{\theta}^{(2)}\tilde{\Psi}_T^{(1)}\, dV + \int_V \tilde{p}^{(1)}\tilde{\Psi}_P^{(2)} - \tilde{p}^{(2)}\tilde{\Psi}_P^{(1)}\, dV$$

$$- \int_V \tilde{\varepsilon}_{\bar{A}J}^{(1)}\left(\sigma_{\bar{A}J}^{0(2)} - M_{\bar{A}J}p^{0(2)} - N_{\bar{A}J}\theta^{0(2)}\right) - \tilde{\varepsilon}_{\bar{A}J}^{(2)}\left(\sigma_{\bar{A}J}^{0(1)} - M_{\bar{A}J}p^{0(1)} - N_{\bar{A}J}\theta^{0(1)}\right) dV$$

$$+ \int_V \tilde{p}^{(1)}\left(\frac{\alpha}{KB}p^{0(2)} - M_{\bar{A}J}\varepsilon_{\bar{A}J}^{0(2)} + \eta\theta^{0(2)}\right) - \tilde{p}^{(2)}\left(\frac{\alpha}{KB}p^{0(1)} - M_{\bar{A}J}\varepsilon_{\bar{A}J}^{0(1)} + \eta\theta^{0(1)}\right) dV$$

$$+ \int_V \tilde{\theta}^{(1)}\left(\frac{C_p}{T_0}\theta^{0(2)} - N_{\bar{A}J}\varepsilon_{\bar{A}J}^{0(2)} + \eta p^{0(2)}\right) - \tilde{\theta}^{(2)}\left(\frac{C_p}{T_0}\theta^{0(1)} - N_{\bar{A}J}\varepsilon_{\bar{A}J}^{0(1)} + \eta p^{0(1)}\right) dV.$$

2000.7.29. #1.

$$\frac{\partial G_{ij}}{\partial x_k} = \frac{1}{4\pi(1-\nu)\mu RK}\frac{\partial}{\partial x_k}\left\{\delta_{ij}\left[a_1\left(\frac{1}{4\eta_1^2}e^{-\eta_1^2}-\frac{1}{4}E_1(\eta_1^2)\right)-a_2\left(\frac{1}{4\eta_2^2}e^{-\eta_2^2}-\frac{1}{4}E_1(\eta_2^2)\right)-\frac{a_3}{4\eta_1\eta_2}-a_4(2\ln r+1)\right]\right.$$
$$\left.-\frac{x_ix_j}{r^2}\left[\frac{a_1}{2\eta_1^2}e^{-\eta_1^2}-\frac{a_2}{2\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2\eta_1\eta_2}-a_5\right]\right\}.$$

$$\frac{\partial}{\partial x_k}E_1(\eta_1^2) = -\frac{\partial r}{\partial x_k}\frac{2\eta_1^2}{r}\frac{e^{-\eta_1^2}}{\eta_1^2} = -\frac{\partial r}{\partial x_k}\frac{2}{r}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}E_1(\eta_2^2) = -\frac{\partial r}{\partial x_k}\frac{2}{r}e^{-\eta_2^2}.$$

$$\frac{\partial}{\partial x_k}\frac{1}{\eta_1^2}e^{-\eta_1^2} = -\frac{\partial r}{\partial x_k}\frac{2(1+\eta_1^2)}{r\eta_1^2}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}\frac{1}{\eta_2^2}e^{-\eta_2^2} = -\frac{\partial r}{\partial x_k}\frac{2(1+\eta_2^2)}{r\eta_2^2}e^{-\eta_2^2}.$$

$$\frac{\partial}{\partial x_k}\frac{1}{\eta_1\eta_2} = -\frac{\partial r}{\partial x_k}\frac{2}{r\eta_1\eta_2}.$$

← Hand book of mathmatical function with ~ I.A. Stegun (229~230).

• $\frac{\partial G_{ij}}{\partial x_k}$ 식의 2번째 줄을 먼저 푼다.

$$\frac{\partial}{\partial x_k}\frac{x_ix_j}{r^2}\left[\frac{a_1}{2\eta_1^2}e^{-\eta_1^2}-\frac{a_2}{2\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2\eta_1\eta_2}-a_5\right]$$

$$=\frac{x_j}{r^2}\frac{\partial x_i}{\partial x_k}\left[\frac{a_1}{2\eta_1^2}e^{-\eta_1^2}-\frac{a_2}{2\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2\eta_1\eta_2}-a_5\right]+\frac{x_i}{r^2}\frac{\partial x_j}{\partial x_k}\left[\frac{1}{2\eta_1^2}e^{-\eta_1^2}-\frac{1}{2\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2\eta_1\eta_2}-a_5\right]$$

$$+x_ix_j\frac{\partial r}{\partial x_k}\left(-2\frac{1}{r^3}\right)\left[\frac{a_1}{2\eta_1^2}e^{-\eta_1^2}-\frac{a_2}{2\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2\eta_1\eta_2}-a_5\right]$$

$$+\frac{x_ix_j}{r^2}\left[\frac{a_1}{2}\times-\frac{\partial r}{\partial x_k}\frac{2(1+\eta_1^2)}{r\eta_1^2}e^{-\eta_1^2}-\frac{a_2}{2}\times-\frac{\partial r}{\partial x_k}\frac{2(1+\eta_2^2)}{r\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2}\times-\frac{\partial r}{\partial x_k}\frac{2}{r\eta_1\eta_2}\right]$$

$$=\left[\frac{a_1}{2\eta_1^2}e^{-\eta_1^2}-\frac{a_2}{2\eta_2^2}e^{-\eta_2^2}-\frac{a_3}{2\eta_1\eta_2}-a_5\right]\left(\frac{\delta_{ik}}{r}\frac{x_j}{r}+\frac{\delta_{jk}}{r}\frac{x_i}{r}\right)$$

$$+\left[-a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{2a_3}{4\eta_1\eta_2}+2a_5\right]\frac{1}{r}\frac{x_ix_jx_k}{r^3}$$

$$\frac{\partial G_{ij}}{\partial x_k}=\frac{1}{4\pi(1-\nu)\mu RK}\left\{\frac{\delta_{ij}}{r}\frac{x_k}{r}\left[-\frac{a_1}{2}\left(\frac{1}{\eta_1^2}e^{-\eta_1^2}\right)+\frac{a_2}{2}\left(\frac{1}{\eta_2^2}e^{-\eta_2^2}\right)+\frac{a_3}{2}\frac{1}{\eta_1\eta_2}-2a_4\right]\right.$$
$$+\left(\frac{\delta_{ik}}{r}\frac{x_j}{r}+\frac{\delta_{jk}}{r}\frac{x_i}{r}\right)\left[-\frac{a_1}{2\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{2\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{2\eta_1\eta_2}+a_5\right]$$
$$\left.+\frac{1}{r}\frac{x_ix_jx_k}{r^3}\left[a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{4\eta_1\eta_2}-2a_5\right]\right\}.$$

$$\overline{\varepsilon_{ijk}^{q_{ij}}}=\frac{1}{2}\left(\frac{\partial G_{ij}}{\partial x_k}+\frac{\partial G_{ik}}{\partial x_j}\right)$$

$$=\frac{1}{8\pi(1-\nu)\mu RKr}\left\{(\delta_{ij}r_{,k}+\delta_{ik}r_{,j})\left[-a_1\frac{1}{\eta_1^2}e^{-\eta_1^2}+a_2\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{8\eta_1\eta_2}-2a_4+a_5\right]\right.$$
$$+\delta_{jk}r_{,i}\left[-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}+2a_5\right]$$
$$\left.+2r_{,i}r_{,j}r_{,k}\left[a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}-2a_5\right]\right\}$$

2000. 7. 29. #2.

$$\sigma^{G_{\bar{\lambda}\bar{j}}}_{\bar{\lambda}\bar{j}\underline{k}} = \frac{2\mu\nu}{1-2\nu}\delta_{\bar{j}k}\,\varepsilon_{\bar{\lambda}mm} + 2\mu\,\varepsilon_{\bar{\lambda}\bar{j}k} \;\rightarrow\; \sigma_{\bar{\lambda}\bar{j}\underline{k}} = \frac{2\mu\nu}{1-2\nu}\delta_{\bar{\lambda}\bar{j}}\,\varepsilon_{mm\underline{k}} + 2\mu\,\varepsilon_{\bar{\lambda}\bar{j}\underline{k}}$$

$$\varepsilon_{\bar{\lambda}mm} = \frac{1}{8\pi(1-\nu)\mu kkr}\Big\{(\delta_{\bar{\lambda}m}r_{,m} + \delta_{\bar{\lambda}m}r_{,m})\Big[-a_1\frac{1}{\eta_1^2}e^{-\eta_1^2} + a_2\frac{1}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\Big]$$

$$+\,\delta_{mm}r_{,\bar{\lambda}}\Big[-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} + 2a_5\Big]$$

$$+\,2r_{,\bar{\lambda}}r_{,m}r_{,m}\Big[a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} - 2a_5\Big]\Big\}.$$

$$= \frac{2r_{,\bar{\lambda}}}{8\pi(1-\nu)\mu kkr}\Big\{\frac{a_3}{4\eta_1\eta_2} - 2a_4 + a_5 + a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\Big\}$$

$$\sigma^{G_{\bar{\lambda}\bar{j}}}_{\bar{\lambda}\bar{j}k} = \frac{1}{4\pi(1-\nu)kkr}\Big\{(\delta_{\bar{j}}r_{,k} + \delta_{\bar{\lambda}k}r_{,\bar{j}})\Big[-a_1\frac{1}{\eta_1^2}e^{-\eta_1^2} + a_2\frac{1}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\Big]$$

$$+\,\delta_{\bar{j}k}r_{,\bar{\lambda}}\Big[-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - \frac{4\nu}{(1-2\nu)}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5\Big] + \frac{2\nu}{1-2\nu}\big(a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\big)$$

$$+\,2r_{,\bar{\lambda}}r_{,\bar{j}}r_{,k}\Big[a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} - 2a_5\Big]\Big\}.$$

Reciprocal theorem에 적용하기 위해서 $\bar{\lambda}$와 k를 transpose 해주어야 한다.

$$F_{\bar{\lambda}\bar{j}} = \sigma^{G_{\bar{\lambda}\bar{j}}}_{\bar{\lambda}\bar{j}k}\,n_k$$

$$= \frac{1}{4\pi(1-\nu)kkr}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{\bar{\lambda}\bar{j}}\Big(-a_1\frac{1}{\eta_1^2}e^{-\eta_1^2} + a_2\frac{1}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\Big)$$

$$+\,2r_{,\bar{\lambda}}r_{,\bar{j}}\Big(a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} - 2a_5\Big)\Big\}$$

$$+\,r_{,\bar{\lambda}}n_{\bar{j}}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - \frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}\big(a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\big)\Big)$$

$$+\,r_{,\bar{j}}n_{\bar{\lambda}}\Big(-a_1\frac{1}{\eta_1^2}e^{-\eta_1^2} + a_2\frac{1}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\Big)\Big]$$

$$G_{\bar{\lambda}\bar{j}} = \frac{1}{4\pi(1-\nu)\mu kk}\Big\{\delta_{\bar{\lambda}\bar{j}}\Big[a_1^{\bar{\lambda}\bar{j}}\Big(\frac{1}{4\eta_1^2}e^{-\eta_1^2} - \frac{1}{4}E_1(\eta_1^2)\Big) - a_2^{\bar{\lambda}\bar{j}}\Big(\frac{1}{4\eta_2^2}e^{-\eta_2^2} - \frac{1}{4}E_1(\eta_2^2)\Big) - \frac{a_3^{\bar{\lambda}\bar{j}}}{4\eta_1\eta_2} - a_4^{\bar{\lambda}\bar{j}}(2\ln r + 1)\Big]$$

$$-\,\frac{x_{\bar{\lambda}}x_{\bar{j}}}{r^2}\Big[\frac{a_1^{\bar{\lambda}\bar{j}}}{2\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\bar{\lambda}\bar{j}}}{2\eta_2^2}e^{-\eta_2^2} - \frac{a_3^{\bar{\lambda}\bar{j}}}{2\eta_1\eta_2} - a_5^{\bar{\lambda}\bar{j}}\Big]\Big\}$$

$$a_1^{\bar{\lambda}\bar{j}} = \frac{-1}{b_1^2(b_1^2-b_2^2)}\frac{\mu}{(\lambda+\mu)(\lambda+2\mu)}\Big[\alpha_T^2T_0k(b_1^2-b_4^2) + \alpha^2k(b_1^2-b_3^2) - 2\alpha_T\eta T_0\alpha\Big]$$

$$a_2^{\bar{\lambda}\bar{j}} = \frac{-1}{b_2^2(b_1^2-b_2^2)}\frac{\mu}{(\lambda+\mu)(\lambda+2\mu)}\Big[\alpha_T^2T_0k(b_2^2-b_4^2) + \alpha^2k(b_2^2-b_3^2) - 2\alpha_T\eta T_0\alpha\Big]$$

$$\cancel{a_3^{\bar{\lambda}\bar{j}} = \frac{kk(b_1^2+b_2^2)}{b_1b_2} + \frac{\mu}{(\lambda+\mu)(\lambda+2\mu)b_1b_2}(\alpha_T^2T_0k + \alpha^2k) - \frac{\mu}{(\lambda+\mu)(\lambda+2\mu)b_1^3b_2^3}(\alpha_T^2T_0kb_4^2 + \alpha^2kb_3^2)}$$

$$a_4^{\bar{\lambda}\bar{j}} = +\frac{(3-4\nu)kk}{4} - \frac{\mu}{4(\lambda+\mu)(\lambda+2\mu)b_1^2b_2^2}\big(\alpha_T^2T_0kb_4^2 + \alpha^2kb_3^2 + 2\alpha_T\eta T_0\alpha\big)$$

$$a_5^{\bar{\lambda}\bar{j}} = \frac{kk}{2} + \frac{\mu}{2(\lambda+\mu)(\lambda+2\mu)b_1^2b_2^2}\big(\alpha_T^2T_0kb_4^2 + \alpha^2kb_3^2 + 2\alpha_T\eta T_0\alpha\big)$$

$$a_3^{\bar{\lambda}\bar{j}} = \frac{\mu}{(\lambda+\mu)(\lambda+2\mu)b_1^3b_2^3}\Big[\alpha_T^2T_0k\big(b_1^2b_2^2 - (b_1^2+b_2^2)b_4^2\big) + \alpha^2k\big(b_1^2b_2^2 - (b_1^2+b_2^2)b_3^2\big) - 2\alpha_T\eta T_0\alpha(b_1^2+b_2^2)\Big]$$

$$\Big(\frac{a_1^{\bar{\lambda}\bar{j}}}{b_1^2} - \frac{a_2^{\bar{\lambda}\bar{j}}}{b_2^2} - \frac{a_3^{\bar{\lambda}\bar{j}}}{b_1b_2}\Big) = 0. \leftarrow$$ 필수조건. ☆

2000.7.30. #3.

$r_{,j} n_j = \frac{\partial r}{\partial n}$

$$P^*_{ij} = \sigma^*_{ij,k} n_k$$

$$= \frac{1}{4\pi(1-\nu)krkr}\Bigg[\frac{\partial r}{\partial n}\Big\{\delta_{ij}\Big(-\frac{1}{2}a_1\Big(\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+\eta_1^2E_1'(\eta_1^2)+E_1(\eta_1^2)\Big)$$

$$+\frac{1}{2}a_2\Big(\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\eta_2^2E_1'(\eta_2^2)+E_1(\eta_2^2)\Big)$$

$$+\frac{3}{4}\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)$$

$$+2r_{,i}r_{,j}\Big(a_1\eta_1^2E_1'(\eta_1^2)-a_1E_1(\eta_1^2)-a_2\eta_2^2E_1'(\eta_2^2)+a_2E_1(\eta_2^2)+\frac{a_3}{\eta_1\eta_2}+2a_5\Big)\Big\}$$

$$+r_{,i}n_j\Big\{\frac{\nu}{1-2\nu}\Big(-a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+a_1\eta_1^2E_1'(\eta_1^2)-a_2\eta_2^2E_1'(\eta_2^2)$$

$$-5a_1E_1(\eta_1^2)+5a_2E_1(\eta_2^2)+\frac{9}{2}\frac{a_3}{\eta_1\eta_2}-a_4+10a_5\Big)$$

$$-a_1E_1(\eta_1^2)+a_2E_1(\eta_2^2)+\frac{a_3}{2\eta_1\eta_2}+2a_5\Big\}$$

$$+r_{,j}n_i\Big\{-\frac{a_1}{2}\Big(\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+\eta_1^2E_1'(\eta_1^2)+E_1(\eta_1^2)\Big)$$

$$+\frac{a_2}{2}\Big(\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\eta_2^2E_1'(\eta_2^2)+E_1(\eta_2^2)\Big)$$

$$+\frac{3}{4}\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big\}\Bigg]$$

2000. 1. 3.

$$G_{\theta j} = \frac{-\cancel{k\eta_j} T_0}{2\pi(\lambda+2\mu)kK}\left(\frac{x_j}{r}\right)\left(\frac{r}{4t}\right)\left\{\frac{-\eta\alpha+\alpha_T K(b_4^2-b_1^2)}{b_1^2-b_2^2}\frac{4t}{b_1^2r^2}e^{-\frac{b_1^2r^2}{4t}}\right.$$

$$-\frac{-\eta\alpha+\alpha_T K(b_4^2-b_2^2)}{b_1^2-b_2^2}\frac{4t}{b_2^2r^2}e^{-\frac{b_2^2r^2}{4t}}$$

$$\left.+\frac{-\eta\alpha+\alpha_T K b_4^2}{b_1b_2}\frac{4t}{b_1b_2r^2}\right\}$$

$$= \frac{+r\,n_j}{2\pi(\lambda+2\mu)kK\,4t}\left\{a_1^{\theta j}\eta_1^{-2}e^{-\eta_1^2} - a_2^{\theta j}\eta_2^{-2}e^{-\eta_2^2} + a_3^{\theta j}\frac{1}{\eta_1\eta_2}\right\}$$

$$a_1^{\theta j} = \frac{+\eta T_0\alpha \mp \alpha_T T_0K(b_1^2-b_4^2)}{b_1^2-b_2^2},\quad a_2^{\theta j} = \frac{\pm\eta T_0\alpha \mp \alpha_T T_0K(b_2^2-b_4^2)}{b_1^2-b_2^2},\quad a_3^{\theta j} = \frac{+\eta T_0\alpha \mp \alpha_T T_0 K b_4^2}{b_1b_2}$$

$$\frac{\partial G_{\theta j}}{\partial x_i} = \frac{1}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_i}\left\{(x_j)\left(a_1^{\theta j}\eta_1^{-2}e^{-\eta_1^2} - a_2^{\theta j}\eta_2^{-2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{1}{8\pi(\lambda+2\mu)kKt}\frac{\partial x_j}{\partial x_i}\left\{a_1^{\theta j}\eta_1^{-2}e^{-\eta_1^2} - a_2^{\theta j}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right\} + \frac{x_j}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_i}\left\{a_1^{\theta j}\eta_1^{-2}e^{-\eta_1^2} - a_2^{\theta j}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right\}$$

$$= \frac{1}{8\pi(\lambda+2\mu)kKt}\left\{\delta_{ij}\left(a_1^{\theta j}\eta_1^{-2}e^{-\eta_1^2} - a_2^{\theta j}\eta_2^{-2}e^{-\eta_2^2} + a_3^{\theta j}\frac{1}{\eta_1\eta_2}\right) - 2r_{,i}r_{,j}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + a_3^{\theta j}\frac{1}{\eta_1\eta_2}\right)\right\}$$

$$F_{\theta j} = -k\frac{\partial G_{\theta j}}{\partial x_i}n_i$$

$$= -\frac{1}{8\pi(\lambda+2\mu)Kt}\left\{n_j\left(a_1^{\theta j}\eta_1^{-2}e^{-\eta_1^2} - a_2^{\theta j}\eta_2^{-2}e^{-\eta_2^2} + a_3^{\theta j}\frac{1}{\eta_1\eta_2}\right) - 2r_{,j}\frac{\partial n}{\partial r}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + a_3^{\theta j}\frac{1}{\eta_1\eta_2}\right)\right\}.$$

$$G_{Pj} = \frac{-r}{2\pi(\lambda+2\mu)kK}\frac{x_j}{r}\left\{\left(-\eta\alpha_T T_0 + \alpha k(b_3^2-b_1^2)\right)\frac{1}{b_1^2-b_2^2}\frac{4t}{b_1^2r^2}e^{-\frac{b_1^2r^2}{4t}}\right.$$

$$-\left(-\eta\alpha_T T_0 + \alpha k(b_3^2-b_2^2)\right)\frac{1}{b_1^2-b_2^2}\frac{4t}{b_2^2r^2}e^{-\frac{b_2^2r^2}{4t}}$$

$$\left.+\frac{-\eta\alpha_T T_0+\alpha k b_3^2}{b_1b_2}\frac{4t}{b_1b_2r^2}\right\}$$

$$= \frac{+r\,r_{,j}}{8\pi(\lambda+2\mu)kKt}\left\{a_1^{Pj}\eta_1^{-2}e^{-\eta_1^2} - a_2^{Pj}\eta_2^{-2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right\}$$

$$a_1^{Pj} = \frac{\eta T_0\alpha_T \mp \alpha k(b_1^2-b_3^2)}{b_1^2-b_2^2},\quad a_2^{Pj} = \frac{\eta T_0\alpha_T \mp \alpha k(b_2^2-b_3^2)}{b_1^2-b_2^2},\quad a_3^{Pj} = \frac{\eta T_0\alpha_T \mp \alpha k b_3^2}{b_1b_2}$$

$$\frac{\partial G_{Pj}}{\partial x_i} = \frac{1}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_i}\left\{(x_j)\left(a_1^{Pj}\eta_1^{-2}e^{-\eta_1^2} - a_2^{Pj}\eta_2^{-2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{1}{8\pi(\lambda+2\mu)kKt}\frac{\partial x_j}{\partial x_i}\left\{a_1^{Pj}\eta_1^{-2}e^{-\eta_1^2} - a_2^{Pj}\eta_2^{-2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right\}$$

$$+ \frac{x_i}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_i}\left\{a_1^{Pj}\eta_1^{-2}e^{-\eta_1^2} - a_2^{Pj}\eta_2^{-2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right\}$$

$$= \frac{1}{8\pi(\lambda+2\mu)kKt}\left\{\delta_{ij}\left(a_1^{Pj}\eta_1^{-2}e^{-\eta_1^2} - a_2^{Pj}\eta_2^{-2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right) - 2r_{,i}r_{,j}\left(a_1^{Pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{Pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right)\right\}$$

$$F_{Pj} = -k\frac{\partial G_{Pj}}{\partial x_i}n_i$$

$$= -\frac{1}{8\pi(\lambda+2\mu)Kt}\left\{n_j\left(a_1^{Pj}\eta_1^{-2}e^{-\eta_1^2} - a_2^{Pj}\eta_2^{-2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right) - 2\frac{\partial n}{\partial r}r_{,j}\left(a_1^{Pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{Pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + a_3^{Pj}\frac{1}{\eta_1\eta_2}\right)\right\}$$

2000 . 1 . 4.

$$G_{i\theta} = \frac{1}{2\pi(\lambda+2\mu)kK}(x_i)\left\{ \frac{\eta\alpha-\alpha_T K(b_4^2-b_1^2)}{b_1^2-b_2^2}\left[\frac{t}{b_1^2r^2}\exp\left(-\frac{b_1^2r^2}{4t}\right)-\frac{1}{4}E_1\left(\frac{b_1^2r^2}{4t}\right)\right]\right.$$
$$-\frac{\eta\alpha-\alpha_T K(b_4^2-b_2^2)}{b_1^2-b_2^2}\left[\frac{t}{b_2^2r^2}\exp\left(-\frac{b_2^2r^2}{4t}\right)-\frac{1}{4}E_1\left(\frac{b_2^2r^2}{4t}\right)\right]$$
$$\left.+\frac{\eta\alpha-\alpha_T K b_4^2}{\cancel{b_1^2 b_2^2}}\frac{t}{b_1b_2r^2}\right\}$$
$$=\frac{r r_{,i}}{8\pi(\lambda+2\mu)kK}\left\{a_1^{i\theta}\left[\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right]-a_2^{i\theta}\left[\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right]+a_3^{i\theta}\frac{1}{\eta_1\eta_2}\right\}$$

$$a_1^{i\theta}=\frac{\eta\alpha-\alpha_T K(b_4^2-b_1^2)}{b_1^2-b_2^2},\quad a_2^{i\theta}=\frac{\eta\alpha-\alpha_T K(b_4^2-b_2^2)}{b_1^2-b_2^2},\quad a_3^{i\theta}=\frac{\eta\alpha-\alpha_T K b_4^2}{b_1 b_2}$$

$$\frac{\partial G_{i\theta}}{\partial x_j}=\frac{1}{8\pi(\lambda+2\mu)kK}\frac{\partial}{\partial x_j}\left[(x_i)\left\{a_1^{i\theta}\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+a_3^{i\theta}\frac{1}{\eta_1\eta_2}\right\}\right]$$
$$=\frac{1}{8\pi(\lambda+2\mu)kK}\frac{\partial x_i}{\partial x_j}\left\{a_1^{i\theta}\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+a_3^{i\theta}\frac{1}{\eta_1\eta_2}\right\}$$
$$+\frac{x_i}{8\pi(\lambda+2\mu)kK}\left\{a_1^{i\theta}\left(-r_{,j}\frac{2(1+\eta_1^2)}{r\eta_1^2}e^{-\eta_1^2}+\frac{2r_{,j}}{r}e^{-\eta_1^2}\right)-a_2^{i\theta}\left(-r_{,j}\frac{2(1+\eta_2^2)}{r\eta_2^2}e^{-\eta_2^2}+\frac{2r_{,j}}{r}e^{-\eta_2^2}\right)+a_3^{i\theta}\left(-r_{,j}\frac{2}{r\eta_1\eta_2}\right)\right\}$$
$$=\frac{1}{8\pi(\lambda+2\mu)kK}\left\{\delta_{ij}\left[a_1^{i\theta}\left(\frac{1}{\eta_1^2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\frac{1}{\eta_2^2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right]\right.$$
$$\left.-2r_{,i}r_{,j}\left[a_1^{i\theta}\left(\frac{1}{\eta_1^2}\right)e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right]\right\}$$

$$\frac{\partial G_{j\theta}}{\partial x_i}=\frac{\partial G_{i\theta}}{\partial x_j}.$$

$$\varepsilon_{ij\theta}=\frac{1}{2}\left(\frac{\partial G_{i\theta}}{\partial x_j}+\frac{\partial G_{j\theta}}{\partial x_i}\right)=\frac{\partial G_{i\theta}}{\partial x_j}$$

$$\varepsilon_{mm\theta}=\frac{\partial G_{m\theta}}{\partial x_m}$$
$$=\frac{1}{8\pi(\lambda+2\mu)kK}\left\{-2a_1^{i\theta}\left(\quad E_1(\eta_1^2)\right)+2a_2^{i\theta}\left(\quad E_1(\eta_2^2)\right)\right\}$$
$$=-\frac{1}{4\pi(\lambda+2\mu)kK}\left\{a_1^{i\theta}\left(\quad E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\quad E_1(\eta_2^2)\right)\right\}$$

$$\sigma_{ij\theta}=\frac{2\mu\nu}{1-2\nu}\delta_{ij}\varepsilon_{mm\theta}+2\mu\varepsilon_{ij\theta}$$
$$=\frac{2\mu\nu}{1-2\nu}\frac{-\delta_{ij}}{4\pi(\lambda+2\mu)kK}\left\{a_1^{i\theta}\left(\quad E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\quad E_1(\eta_2^2)\right)\right\}$$
$$+\frac{2\mu}{8\pi(\lambda+2\mu)kK}\left\{\delta_{ij}\left[a_1^{i\theta}\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right]\right.$$
$$\left.-2r_{,i}r_{,j}\left[a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right]\right\}$$
$$=\frac{\mu}{4\pi(\lambda+2\mu)kK}\left\{\delta_{ij}\left(a_1^{i\theta}\left[\frac{-2\nu}{1-2\nu}\left(\quad E_1(\eta_1^2)\right)+\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)\right]\right.\right.$$
$$\left.-a_2^{i\theta}\left[\frac{-2\nu}{1-2\nu}\left(\quad E_1(\eta_2^2)\right)+\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)\right]+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right)$$
$$\left.-2r_{,i}r_{,j}\left[a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right]\right\}$$

$$F_{i\theta}=\sigma_{ij\theta}n_j$$
$$=\frac{\mu}{4\pi(\lambda+2\mu)kK}\left\{n_j\left(a_1^{i\theta}\left[\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-\frac{2\nu}{1-2\nu}\left(\quad E_1(\eta_1^2)\right)\right]\right.\right.$$
$$\left.-a_2^{i\theta}\left[\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)-\frac{2\nu}{1-2\nu}\left(\quad E_1(\eta_2^2)\right)\right]+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right)$$
$$\left.-2\frac{\partial n}{\partial r}r_{,i}\left[a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right]\right\}.$$

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$$G_{ip} = \frac{1}{2\pi(\lambda+2\mu)kK}(x_i)\left\{\frac{\eta\alpha_T T_0 - \alpha k(b_3^2-b_1^2)}{b_1^2-b_2^2}\left[\frac{t}{b_1^2r^2}\exp\left(-\frac{b_1^2r^2}{4t}\right)-\frac{1}{4}E_1\left(\frac{b_1^2r^2}{4t}\right)\right]\right.$$
$$-\frac{\eta\alpha_T T_0 - \alpha k(b_3^2-b_2^2)}{b_1^2-b_2^2}\left[\frac{t}{b_2^2r^2}\exp\left(-\frac{b_2^2r^2}{4t}\right)-\frac{1}{4}E_1\left(\frac{b_2^2r^2}{4t}\right)\right]$$
$$\left.+\frac{\eta\alpha_T T_0 - \alpha k b_3^2}{\cancel{b_1^2-b_2^2}}\frac{t}{b_1b_2r^2}\right\}$$
$$= \frac{r r_{,i}}{8\pi(\lambda+2\mu)kK}\left\{a_1^{ip}\left[\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right]-a_2^{ip}\left[\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right]+a_3^{ip}\frac{1}{\eta_1\eta_2}\right\}$$

$$a_1^{ip} = \frac{\eta\alpha_T T_0 - \alpha k(b_3^2-b_1^2)}{b_1^2-b_2^2},\quad a_2^{ip} = \frac{\eta\alpha_T T_0 - \alpha k(b_3^2-b_2^2)}{b_1^2-b_2^2},\quad a_3^{ip} = \frac{\eta\alpha_T T_0 - \alpha k b_3^2}{b_1 \times b_2}$$

$$\frac{\partial G_{ip}}{\partial x_j} = \frac{1}{8\pi(\lambda+2\mu)kK}\frac{\partial}{\partial x_j}\left[(x_i)\left\{a_1^{ip}\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{ip}\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+a_3^{ip}\frac{1}{\eta_1\eta_2}\right\}\right]$$
$$= \frac{1}{8\pi(\lambda+2\mu)kK}\frac{\partial x_i}{\partial x_j}\left\{a_1^{ip}\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{ip}\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+a_3^{ip}\frac{1}{\eta_1\eta_2}\right\}$$
$$+\frac{x_i}{8\pi(\lambda+2\mu)kK}\left\{a_1^{ip}\left(-r_{,j}\frac{2(1+\eta_1^2)}{r\eta_1^2}e^{-\eta_1^2}+\frac{2r_{,j}}{r}e^{-\eta_1^2}\right)-a_2^{ip}\left(-r_{,j}\frac{2(1+\eta_2^2)}{r\eta_2^2}e^{-\eta_2^2}+\frac{2r_{,j}}{r}e^{-\eta_2^2}\right)+a_3^{ip}\left(-r_{,j}\frac{2}{r\eta_1\eta_2}\right)\right\}$$
$$= \frac{1}{8\pi(\lambda+2\mu)kK}\left\{\delta_{ij}\left[a_1^{ip}\left(\frac{1}{\eta_1^2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{ip}\left(\frac{1}{\eta_2^2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+\frac{a_3^{ip}}{\eta_1\eta_2}\right]\right.$$
$$\left.-2r_{,i}r_{,j}\left[a_1^{ip}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{ip}}{\eta_1\eta_2}\right]\right\}$$

$$\frac{\partial G_{jp}}{\partial x_i} = \frac{\partial G_{ip}}{\partial x_j}$$
$$\varepsilon_{ijp} = \frac{1}{2}\left(\frac{\partial G_{ip}}{\partial x_j}+\frac{\partial G_{jp}}{\partial x_i}\right) = \frac{\partial G_{ip}}{\partial x_j}$$
$$\varepsilon_{mmp} = \frac{\partial G_{mp}}{\partial x_m}$$
$$= \frac{1}{8\pi(\lambda+2\mu)kK}\left\{-2a_1^{ip}\left(\qquad E_1(\eta_1^2)\right)+2a_2^{ip}\left(\qquad E_1(\eta_2^2)\right)\right\}$$
$$= -\frac{1}{4\pi(\lambda+2\mu)kK}\left\{a_1^{ip}\left(\qquad E_1(\eta_1^2)\right)-a_2^{ip}\left(\qquad E_1(\eta_2^2)\right)\right\}$$

$$\sigma_{ijp} = \frac{2\mu\nu}{1-2\nu}\delta_{ij}\varepsilon_{mmp}+2\mu\varepsilon_{ijp}$$
$$= \frac{2\mu\nu}{1-2\nu}\frac{-\delta_{ij}}{4\pi(\lambda+2\mu)kK}\left\{a_1^{ip}\left(\qquad E_1(\eta_1^2)\right)-a_2^{ip}\left(\qquad E_1(\eta_2^2)\right)\right\}$$
$$+\frac{2\mu}{8\pi(\lambda+2\mu)kK}\left[\delta_{ij}\left\{a_1^{ip}\left(\frac{1}{\eta_1^2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{ip}\left(\frac{1}{\eta_2^2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)+\frac{a_3^{ip}}{\eta_1\eta_2}\right\}\right.$$
$$\left.-2r_{,i}r_{,j}\left\{a_1^{ip}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{ip}}{\eta_1\eta_2}\right\}\right]$$
$$= \frac{\mu}{4\pi(\lambda+2\mu)kK}\left\{\delta_{ij}\left(a_1^{ip}\left[-\frac{2\nu}{1-2\nu}\left(\qquad E_1(\eta_1^2)\right)+\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)\right]\right.\right.$$
$$\left.-a_2^{ip}\left[-\frac{2\nu}{1-2\nu}\left(\qquad E_1(\eta_2^2)\right)+\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)\right]+\frac{a_3^{ip}}{\eta_1\eta_2}\right)$$
$$\left.-2r_{,i}r_{,j}\left[a_1^{ip}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{1}{\eta_2^2}\qquad+\frac{a_3^{ip}}{\eta_1\eta_2}\right]\right\}$$

$$F_{ip} = \sigma_{ijp}n_j$$
$$= \frac{\mu}{4\pi(\lambda+2\mu)kK}\left\{n_j\left(a_1^{ip}\left[-\frac{2\nu}{1-2\nu}\left(\qquad E_1(\eta_1^2)\right)+\left(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\right)\right]\right.\right.$$
$$\left.-a_2^{ip}\left[-\frac{2\nu}{1-2\nu}\left(\qquad+E_1(\eta_2^2)\right)+\left(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\right)\right]+\frac{a_3^{ip}}{\eta_1\eta_2}\right)$$
$$\left.-2r_{,i}\frac{\partial r}{\partial n}\left[a_1^{ip}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{1}{\eta_2^2}\qquad+\frac{a_3^{ip}}{\eta_1\eta_2}\right]\right\}$$

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$$G_{\theta P} = \frac{1}{2\pi(\lambda+2\mu)k\kappa}\left\{\frac{\alpha_T T_0 \alpha + (\lambda+2\mu)\eta T_0}{2(b_1^2-b_2^2)}\left(E_1\left(\frac{b_1^2 r^2}{4t}\right) - E_1\left(\frac{b_2^2 r^2}{4t}\right)\right)\right\}$$

$$= \frac{a_1^{\theta P}}{4\pi(\lambda+2\mu)k\kappa}\left\{E_1(\eta_1^2) - E_1(\eta_2^2)\right\} \quad \text{where } a_1^{\theta P} = \frac{\alpha_T T_0 \alpha + (\lambda+2\mu)\eta T_0}{b_1^2 - b_2^2}$$

$$\frac{\partial G_{\theta P}}{\partial x_j} = \frac{a_1^{\theta P}}{4\pi(\lambda+2\mu)k\kappa}\frac{\partial}{\partial x_j}\left\{E_1(\eta_1^2) - E_1(\eta_2^2)\right\}$$

$$= \frac{-a_1^{\theta P}}{2\pi(\lambda+2\mu)k\kappa r}\frac{\partial r}{\partial x_j}\left\{e^{-\eta_1^2} - e^{-\eta_2^2}\right\}$$

- $F_{\theta P} = -k\dfrac{\partial G_{\theta P}}{\partial x_j} n_j$

$$= \frac{a_1^{\theta P}}{2\pi(\lambda+2\mu)\kappa r}\frac{\partial r}{\partial n}\left\{e^{-\eta_1^2} - e^{-\eta_2^2}\right\}$$

$$G_{P\theta} = \frac{a_1^{P\theta}}{4\pi(\lambda+2\mu)k\kappa}\left\{E_1(\eta_1^2) - E_1(\eta_2^2)\right\} \quad \text{where } a_1^{P\theta} = \frac{\alpha_T \alpha + (\lambda+2\mu)\eta}{b_1^2 - b_2^2}$$

$$\frac{\partial G_{P\theta}}{\partial x_j} = \frac{-a_1^{P\theta}}{2\pi(\lambda+2\mu)k\kappa r}\left\{e^{-\eta_1^2} - e^{-\eta_2^2}\right\}$$

- $F_{P\theta} = -k\dfrac{\partial G_{P\theta}}{\partial x_j} n_j = \dfrac{a_1^{P\theta}}{2\pi(\lambda+2\mu)\kappa r}\dfrac{\partial r}{\partial n}\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right)$

$$G_{\theta\theta} = \frac{1}{4\pi(\lambda+2\mu)k\kappa}\left\{a_1^{\theta\theta}E_1(\eta_1^2) - a_2^{\theta\theta}E_1(\eta_2^2)\right\}$$

where $a_1^{\theta\theta} = \dfrac{(\lambda+2\mu)k(b_1^2-b_4^2) - \alpha^2}{b_1^2 - b_2^2}$, $a_2^{\theta\theta} = \dfrac{(\lambda+2\mu)k(b_2^2-b_4^2) - \alpha^2}{b_1^2 - b_2^2}$

$$\frac{\partial G_{\theta\theta}}{\partial x_j} = \frac{-1}{2\pi(\lambda+2\mu)k\kappa r}\frac{\partial r}{\partial x_j}\left\{a_1^{\theta\theta}e^{-\eta_1^2} - a_2^{\theta\theta}e^{-\eta_2^2}\right\}$$

- $F_{\theta\theta} = -k\dfrac{\partial G_{\theta\theta}}{\partial x_j} n_j = \dfrac{1}{2\pi(\lambda+2\mu)\kappa r}\dfrac{\partial r}{\partial n}\left\{a_1^{\theta\theta}e^{-\eta_1^2} - a_2^{\theta\theta}e^{-\eta_2^2}\right\}$

$$G_{PP} = \frac{1}{4\pi(\lambda+2\mu)k\kappa}\left\{a_1^{PP}E_1(\eta_1^2) - a_2^{PP}E_1(\eta_2^2)\right\}$$

where $a_1^{PP} = \dfrac{(\lambda+2\mu)k(b_1^2-b_3^2) - \alpha_T^2}{b_1^2 - b_2^2}$, $a_2^{PP} = \dfrac{(\lambda+2\mu)k(b_2^2-b_3^2) - \alpha_T^2}{b_1^2 - b_2^2}$

$$\frac{\partial G_{PP}}{\partial x_j} = \frac{-1}{2\pi(\lambda+2\mu)k\kappa r}\frac{\partial r}{\partial x_j}\left\{a_1^{PP}e^{-\eta_1^2} - a_2^{PP}e^{-\eta_2^2}\right\}$$

$$F_{PP} = -k\frac{\partial G_{PP}}{\partial x_j} n_j = \frac{1}{2\pi(\lambda+2\mu)\kappa r}\frac{\partial r}{\partial n}\left\{a_1^{PP}e^{-\eta_1^2} - a_2^{PP}e^{-\eta_2^2}\right\}$$

$$P_{ij} = \frac{-1}{4\pi(1-\nu)r}\left\{\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right]\frac{\partial r}{\partial n} + (1-2\nu)(r_{,i}n_j - r_{,j}n_i)\right\}$$

$$\frac{\partial P_{ij}}{\partial x_k} = -\frac{1}{4\pi(1-\nu)}\frac{\partial}{\partial x_k}\left(\left(\frac{1}{r}\right)\left\{\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right]\frac{\partial r}{\partial n} + (1-2\nu)(r_{,i}n_j - r_{,j}n_i)\right\}\right)$$

$$= -\frac{1}{4\pi(1-\nu)}\left\{\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right]\frac{\partial r}{\partial n} + (1-2\nu)(r_{,i}n_j - r_{,j}n_i)\right\}\frac{\partial r}{\partial x_k}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)$$

$$- \frac{1}{4\pi(1-\nu)r}\left\{\frac{\partial r}{\partial n}\frac{\partial}{\partial x_k}\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right] + (1-2\nu)\frac{\partial}{\partial x_k}(r_{,i}n_j - r_{,j}n_i)\right\}$$

$$- \frac{1}{4\pi(1-\nu)r}\left\{\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right]\frac{\partial}{\partial x_k}\left(\frac{\partial r}{\partial n}\right)\right\}.$$

$$\frac{\partial}{\partial x_k}(r_{,j}n_i) = \frac{1}{r}(n_i\delta_{jk} - n_i r_{,j}r_{,k}) \qquad \frac{\partial}{\partial x_k}\left(r_{,j}\frac{\partial r}{\partial n}\right) = \frac{1}{r}\left\{\frac{\partial r}{\partial n}(\delta_{jk} - 2r_{,j}r_{,k}) + r_{,j}n_k\right\}.$$

$$\frac{\partial}{\partial x_k}(r_{,i}n_j) = \frac{1}{r}(n_j\delta_{ik} - n_j r_{,i}r_{,k})$$

$$\frac{\partial}{\partial x_k}\underbrace{(r_{,i}n_i)}_{\left(=\frac{\partial r}{\partial n}\right)} = \frac{1}{r}\left(n_k - r_{,k}\frac{\partial r}{\partial n}\right)$$

$$\frac{\partial}{\partial x_k}(r_{,i}r_{,j}) = \frac{1}{r}(\delta_{ik}r_{,j} + \delta_{jk}r_{,i} - 2r_{,i}r_{,j}r_{,k})$$

$$\frac{\partial P_{ij}}{\partial x_k} = \frac{1}{4\pi(1-\nu)r^2}\left\{\left[(1-2\nu)r_{,k}\delta_{ij} + 2r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n} + (1-2\nu)(n_j r_{,i}r_{,k} - n_i r_{,j}r_{,k})\right\}$$

$$- \frac{1}{4\pi(1-\nu)r^2}\left\{2\left[\delta_{ik}r_{,j} + \delta_{jk}r_{,i} - 2r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n} + (1-2\nu)\left[n_j\delta_{ik} - n_i\delta_{jk} + n_i r_{,j}r_{,k} - n_j r_{,i}r_{,k}\right]\right\}$$

$$- \frac{1}{4\pi(1-\nu)r^2}\left\{\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right]\left(n_k - r_{,k}\frac{\partial r}{\partial n}\right)\right\}$$

$$= \frac{1}{4\pi(1-\nu)r^2}\left\{\left[(1-2\nu)r_{,k}\delta_{ij} + 2r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n} + (1-2\nu)(n_j r_{,i}r_{,k} - n_i r_{,j}r_{,k})\right.$$

$$-2\left[\delta_{ik}r_{,j} + \delta_{jk}r_{,i} - 2r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n} - (1-2\nu)\left[n_j\delta_{ik} - n_i\delta_{jk} + n_i r_{,j}r_{,k} - n_j r_{,i}r_{,k}\right]$$

$$\left.-\left[(1-2\nu)\delta_{ij}n_k - (1-2\nu)\delta_{ij}r_{,k}\frac{\partial r}{\partial n} + 2n_k r_{,i}r_{,j} - 2r_{,i}r_{,j}r_{,k}\frac{\partial r}{\partial n}\right]\right\}$$

$$= \frac{1}{4\pi(1-\nu)r^2}\left\{\left[(1-2\nu)r_{,k}\delta_{ij} + 2r_{,i}r_{,j}r_{,k} - 2r_{,j}\delta_{ik} - 2r_{,i}\delta_{jk} + 4r_{,i}r_{,j}r_{,k} + (1-2\nu)r_{,k}\delta_{ij} + 2r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n}\right.$$

$$+ (1-2\nu)(n_j r_{,i}r_{,k} - n_i r_{,j}r_{,k}) - (1-2\nu)(n_j\delta_{ik} - n_i\delta_{jk})$$

$$\left.+ (1-2\nu)(n_j r_{,i}r_{,k} - n_i r_{,j}r_{,k}) - (1-2\nu)n_k\delta_{ij} - 2n_k r_{,i}r_{,j}\right\}$$

$$= \frac{1}{4\pi(1-\nu)r^2}\left\{2\left[(1-2\nu)r_{,k}\delta_{ij} - r_{,j}\delta_{ik} - r_{,i}\delta_{jk} + 4r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n}\right.$$

$$\left.+2(1-2\nu)(n_j r_{,i}r_{,k} - n_i r_{,j}r_{,k}) - (1-2\nu)(n_j\delta_{ik} - n_i\delta_{jk} + n_k\delta_{ij}) - 2n_k r_{,i}r_{,j}\right\}.$$

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$$\frac{\partial P_{ik}}{\partial x_j} = \frac{1}{4\pi(1-\nu)r^2}\Big\{2\big[(1-2\nu)r_{,j}\delta_{ik} - r_{,k}\delta_{ij} - r_{,i}\delta_{jk} + 4r_{,i}r_{,j}r_{,k}\big]\frac{\partial r}{\partial n}$$
$$+2(1-2\nu)(n_k r_{,i} r_{,j} - n_i r_{,j} r_{,k}) - (1-2\nu)(n_k\delta_{ij} - n_i\delta_{jk} + n_j\delta_{ik}) - 2n_j r_{,i} r_{,k}\Big\}$$

$$\varepsilon_{ijk} = \frac{1}{2}\left(\frac{\partial P_{ij}}{\partial x_k} + \frac{\partial P_{ik}}{\partial x_j}\right)$$

$$= \frac{1}{8\pi(1-\nu)r^2}\Big\{2\big[(1-2\nu)r_{,k}\delta_{ij} - r_{,j}\delta_{ik} - r_{,i}\delta_{jk} + 4r_{,i}r_{,j}r_{,k}\big]\frac{\partial r}{\partial n}$$
$$+2(1-2\nu)(n_j r_{,i} r_{,k} - n_i r_{,j} r_{,k}) - (1-2\nu)(n_j\delta_{ik} - n_i\delta_{jk} + n_k\delta_{ij}) - 2n_k r_{,i} r_{,j}$$
$$+2\big[(1-2\nu)r_{,j}\delta_{ik} - r_{,k}\delta_{ij} + r_{,i}\delta_{jk} + 4r_{,i}r_{,j}r_{,k}\big]\frac{\partial r}{\partial n}$$
$$+2(1-2\nu)(n_k r_{,i} r_{,j} - n_i r_{,j} r_{,k}) - (1-2\nu)(n_k\delta_{ij} - n_i\delta_{jk} + n_j\delta_{ik}) - 2n_j r_{,i} r_{,k}\Big\}.$$

$$= \frac{1}{8\pi(1-\nu)r^2}\Big\{2\big[(1-2\nu)(r_{,k}\delta_{ij} + r_{,j}\delta_{ik}) - (r_{,k}\delta_{ij} + r_{,j}\delta_{ik}) - 2r_{,i}\delta_{jk} + 8r_{,i}r_{,j}r_{,k}\big]\frac{\partial r}{\partial n}$$
$$+2(1-2\nu)(n_j r_{,i} r_{,k} - n_i r_{,j} r_{,k} + n_k r_{,i} r_{,j} - n_i r_{,j} r_{,k})$$
$$-(1-2\nu)(n_j\delta_{ik} - n_i\delta_{jk} + n_k\delta_{ij} + n_k\delta_{ij} - n_i\delta_{jk} + n_j\delta_{ik})$$
$$-2(n_k r_{,i} r_{,j} + n_j r_{,i} r_{,k})\Big\}$$

$$= \frac{1}{8\pi(1-\nu)r^2}\Big\{2\big[-2\nu(r_{,k}\delta_{ij} + r_{,j}\delta_{ik}) - 2r_{,i}\delta_{jk} + 8r_{,i}r_{,j}r_{,k}\big]\frac{\partial r}{\partial n}$$
$$-2\big[2(1-2\nu)n_i r_{,j} r_{,k}\big]$$
$$+2\big[(1-2\nu)(n_j r_{,i} r_{,k} + n_k r_{,i} r_{,j})\big]$$
$$+2\big[(1-2\nu)n_i\delta_{jk}\big]$$
$$-2\big[n_k r_{,i} r_{,j} + n_j r_{,i} r_{,k}\big]\Big\}$$

$$= \frac{1}{4\pi(1-\nu)r^2}\Big\{\big[-2\nu(r_{,k}\delta_{ij} + r_{,j}\delta_{ik}) - 2r_{,i}\delta_{jk} + 8r_{,i}r_{,j}r_{,k}\big]\frac{\partial r}{\partial n}$$
$$-2(1-2\nu)n_i r_{,j} r_{,k} - 2\nu(n_j r_{,i} r_{,k} + n_k r_{,i} r_{,j}) + (1-2\nu)(n_i\delta_{jk} - n_j\delta_{ik} - n_k\delta_{ij})\Big\}$$

$$\varepsilon_{imm} = \frac{1}{4\pi(1-\nu)r^2}\Big\{\big[-2\nu(r_{,i} + r_{,i}) - 4r_{,i} + 8r_{,i}\big]\frac{\partial r}{\partial n}$$
$$-2(1-2\nu)n_i - 2\nu\left(r_{,i}\frac{\partial r}{\partial n} + r_{,i}\frac{\partial r}{\partial n}\right) + 2(1-2\nu)n_i - 2(1-2\nu)n_i\Big\}$$

$$= \frac{1}{4\pi(1-\nu)r^2}\Big\{4(1-2\nu)r_{,i}\frac{\partial r}{\partial n} - 2(1-2\nu)n_i\Big\}$$

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$$\sigma_{ijk} = \left(\frac{2\mu\nu}{1-2\nu}\right)\varepsilon_{imm}\delta_{jk} + 2\mu\varepsilon_{ijk}$$

$$= \frac{2\mu\nu}{4\pi(1-\nu)r^2}\left\{4r_{,i}\delta_{jk}\frac{\partial r}{\partial n} - 2n_i\delta_{jk}\right\}$$

$$+ \frac{2\mu}{4\pi(1-\nu)r^2}\left\{\left[-2\nu(r_{,k}\delta_{ij} + r_{,j}\delta_{ik}) - 2r_{,i}\delta_{jk} + 8r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n}\right.$$

$$-2(1-2\nu)n_i r_{,j}r_{,k} - 2\nu(n_j r_{,i}r_{,k} + n_k r_{,i}r_{,j})$$

$$\left. + (1-2\nu)(n_i\delta_{jk} - n_j\delta_{ik} - n_k\delta_{ij})\right\}$$

$$= \frac{\mu}{2\pi(1-\nu)r^2}\left\{\left[-2\nu(r_{,k}\delta_{ij} + r_{,j}\delta_{ik}) - 2(1-2\nu)r_{,i}\delta_{jk} + 8r_{,i}r_{,j}r_{,k}\right]\frac{\partial r}{\partial n}\right.$$

$$-2\nu(n_j r_{,i}r_{,k} + n_k r_{,i}r_{,j}) - (1-2\nu)(2n_i r_{,j}r_{,k} + n_j\delta_{ik} + n_k\delta_{ij})$$

$$\left. + (1-4\nu)n_i\delta_{jk}\right\}$$

$$\frac{\partial}{\partial x_k}(r_{,j}n_i) = \frac{1}{r}(n_i\delta_{jk} - n_i r_{,j} r_{,k})$$

$$\frac{\partial}{\partial x_k}\left(r_{,j}\frac{\partial r}{\partial n}\right) = \frac{1}{r}\left\{\frac{\partial r}{\partial n}(\delta_{jk} - 2r_{,j}r_{,k}) + r_{,j}n_k\right\}.$$

$$\frac{\partial}{\partial x_k}(r_{,i}n_j) = \frac{n_j}{r}(\delta_{ik} - r_{,i}r_{,k})$$

$$\frac{\partial}{\partial x_k}(r_{,i}n_{,i}) = \frac{1}{r}\left(n_k - r_{,k}\frac{\partial r}{\partial n}\right).$$

$$\frac{\partial}{\partial x_k}(r_{,i}r_{,j}) = \frac{1}{r}(\delta_{ik}r_{,j} + \delta_{jk}r_{,i} - 2r_{,i}r_{,j}r_{,k})$$

$$\frac{\partial}{\partial x_k}E_1(\eta_1^2) = \cancel{\frac{2r_{,k}}{r}\eta_1^2 E_1'(\eta_1^2)} = -\frac{2r_{,k}}{r}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}E_1(\eta_2^2) = \cancel{\frac{2r_{,k}}{r}\eta_2^2 E_1'(\eta_2^2)} = -\frac{2r_{,k}}{r}e^{-\eta_2^2}$$

$$\frac{\partial}{\partial x_k}\frac{1}{\eta_1^2}e^{-\eta_1^2} = -\frac{2r_{,k}}{r}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}\eta_1^2 e^{-\eta_1^2} = \frac{2r_{,k}}{r}\eta_1^2(1-\eta_1^2)e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}\frac{1}{\eta_2^2}e^{-\eta_2^2} = -\frac{2r_{,k}}{r}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}$$

$$\frac{\partial}{\partial x_k}\eta_2^2 e^{-\eta_2^2} = \frac{2r_{,k}}{r}\eta_2^2(1-\eta_2^2)e^{-\eta_2^2}.$$

$$\frac{\partial}{\partial x_k}\left(\frac{1}{\eta_1\eta_2}\right) = -\frac{2r_{,k}}{r}\frac{1}{\eta_1\eta_2}$$

$$\frac{\partial}{\partial x_k}e^{-\eta_1^2} = -\frac{2r_{,k}}{r}\eta_1^2 e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}e^{-\eta_2^2} = -\frac{2r_{,k}}{r}\eta_2^2 e^{-\eta_2^2}.$$

$$\frac{\partial}{\partial x_k}\eta_1^2E_1'(\eta_1^2) = \frac{2r_{,k}}{r}\left(\eta_1^2E_1'(\eta_1^2) + \eta_1^4E_1''(\eta_1^2)\right) \rightarrow \frac{\partial}{\partial x_k}e^{-\eta_1^2} = -\frac{r_{,k}}{r}2\eta_1^2e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}\eta_2^2E_1'(\eta_2^2) = \frac{2r_{,k}}{r}\left(\eta_2^2E_1'(\eta_2^2) + \eta_2^4E_1''(\eta_2^2)\right) \rightarrow \frac{\partial}{\partial x_k}e^{-\eta_2^2} = -\frac{r_{,k}}{r}2\eta_2^2e^{-\eta_2^2}$$

$$\frac{\partial}{\partial x_k}\left(\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}\right) = -\frac{2r_{,k}}{r}\frac{1+\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}\left(\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}\right) = -\frac{2r_{,k}}{r}\frac{1+\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}$$

$$\frac{\partial}{\partial x_j}\left(\frac{\partial r}{\partial n}\right) = \frac{1}{r}\left(n_j - r_{,j}\frac{\partial r}{\partial n}\right)$$

$$\frac{\partial}{\partial x_k}\left\{\left(\frac{2+\eta_1^2}{\eta_1^2}\right)e^{-\eta_1^2}\right\} = -\frac{2r_{,k}}{r}\frac{2+2\eta_1^2+\eta_2^4}{\eta_1^2}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_k}\left(\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}\right) = -\frac{2r_{,k}}{r}\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}$$

$$\frac{\partial}{\partial x_j}\left(\frac{1}{r}\frac{\partial r}{\partial n}\right) = \frac{1}{r^2}\left(n_j - 2r_{,j}\frac{\partial r}{\partial n}\right)$$

$$\frac{\partial}{\partial x_k}\left\{\left(\frac{1+2\eta_1^2}{\eta_1^2}\right)e^{-\eta_1^2}\right\} = -\frac{2r_{,k}}{r}\frac{1+\eta_1^2+2\eta_1^4}{\eta_1^2}e^{-\eta_1^2}$$

$$\frac{\partial}{\partial x_j}\left(\frac{1}{r^2}\frac{\partial r}{\partial n}\right) = \frac{1}{r^3}\left(n_j - 3r_{,j}\frac{\partial r}{\partial n}\right)$$

$$\frac{\partial}{\partial x_k}\left\{\left(\frac{1+2\eta_2^2}{\eta_2^2}\right)e^{-\eta_2^2}\right\} = -\frac{2r_{,k}}{r}\frac{1+\eta_2^2+2\eta_2^4}{\eta_2^2}e^{-\eta_2^2}$$

$$\begin{bmatrix} \underline{g_{\lambda j}} & g_{\theta j} & g_{pj} \\ g_{\lambda\theta} & \underline{g_{\theta\theta}} & g_{p\theta} \\ g_{\lambda p} & g_{\theta p} & \underline{g_{pp}} \end{bmatrix} \qquad \begin{aligned} g_{\lambda j} &= F_{\lambda j} \\ g_{\theta\theta} &= F_{\theta\theta} \\ g_{pp} &= F_{pp}. \end{aligned}$$

$$\begin{bmatrix} f_{\lambda j} & f_{\theta j} & f_{pj} \\ f_{\lambda\theta} & f_{\theta\theta} & f_{p\theta} \\ f_{\lambda p} & f_{\theta p} & f_{pp} \end{bmatrix}$$

$$g_{\theta jk} = \sigma^{G\theta j}_{\theta jk}.$$
$$\sigma_{\theta jk} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\,\varepsilon^{G\theta j}_{\theta mm} + 2\mu\,\varepsilon^{G\theta j}_{\theta jk}$$
$$\varepsilon^{G\theta j}_{\theta jk} = \frac{1}{2}\left(\frac{\partial G_{\theta j}}{\partial x_k} + \frac{\partial G_{\theta k}}{\partial x_j}\right).$$

$$g_{pjk} = \sigma_{pjk}$$
$$\sigma_{pjk} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\,\varepsilon^{Gpj}_{pmm} + 2\mu\,\varepsilon^{Gpj}_{pjk}$$
$$\varepsilon^{Gpj}_{pjk} = \frac{1}{2}\left(\frac{\partial G_{pj}}{\partial x_k} + \frac{\partial G_{pj}}{\partial x_j}\right).$$

$$g_{\lambda pj} = -k\frac{\partial G_{\lambda p}}{\partial x_j}$$

$$g_{\lambda\theta j} = -k\frac{\partial G_{\lambda\theta}}{\partial x_j}$$

$$g_{\theta pj} = -k\frac{\partial G_{\theta p}}{\partial x_j}$$

$$g_{p\theta j} = -k\frac{\partial G_{p\theta}}{\partial x_j}.$$

$$f_{\theta jk} = \sigma^{F\theta j}_{\theta jk}, \quad \sigma^{F\theta j}_{\theta jk} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\,\varepsilon^{F\theta j}_{\theta mm} + 2\mu\,\varepsilon^{F\theta j}_{\theta jk}, \quad \varepsilon^{F\theta j}_{\theta jk} = \frac{1}{2}\left(\frac{\partial F_{\theta j}}{\partial x_k} + \frac{\partial F_{\theta k}}{\partial x_j}\right).$$

$$f_{pjk} = \sigma^{Fpj}_{pjk}, \quad \sigma^{Fpj}_{pjk} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\,\varepsilon^{Fpj}_{pmm} + 2\mu\,\varepsilon^{Fpj}_{pjk}, \quad \varepsilon^{Fpj}_{pjk} = \frac{1}{2}\left(\frac{\partial F_{pj}}{\partial x_k} + \frac{\partial F_{pk}}{\partial x_j}\right).$$

$$f_{\lambda\theta j} = -k\frac{\partial F_{\lambda\theta}}{\partial x_j}, \quad f_{\theta\theta j} = -k\frac{\partial F_{\theta\theta}}{\partial x_j}, \quad f_{p\theta j} = -k\frac{\partial F_{p\theta}}{\partial x_j}$$

$$f_{\lambda pj} = -k\frac{\partial F_{\lambda p}}{\partial x_j}, \quad f_{\theta pj} = -k\frac{\partial F_{\theta p}}{\partial x_j}, \quad f_{ppj} = -k\frac{\partial F_{pp}}{\partial x_j}.$$

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$$F_{ij}=\frac{1}{4\pi(1-\nu)\kappa k r}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}\left(-a_1\frac{1}{\eta_1^2}e^{-\eta_1^2}+a_2\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)\right.\right.$$
$$\left.+2r_{,i}r_{,j}\left(a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2\frac{2+\eta_2^2}{\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}-2a_5\right)\right\}$$
$$+r_{,i}n_j\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)$$
$$\left.+r_{,j}n_i\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)\right]$$

$$A=-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5$$

$$B=a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}-2a_5$$

$$C=-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)$$

$$\frac{\partial A}{\partial x_k}=\frac{\partial}{\partial x_k}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)=A'\frac{r_{,k}}{r}$$
$$=\frac{r_{,k}}{r}\left(2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}\right)$$

$$A'=2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}$$

$$\frac{\partial B}{\partial x_k}=\frac{\partial}{\partial x_k}\left(a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}-2a_5\right)=B'\frac{r_{,k}}{r}$$
$$=\frac{r_{,k}}{r}\left(-2a_1\frac{2+\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}\right)$$

$$B'=-2a_1\frac{2+\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}$$

$$\frac{\partial C}{\partial x_k}=\frac{\partial}{\partial x_k}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}+\frac{2(1-\nu)}{1-2\nu}a_5-\frac{4\nu}{1-2\nu}a_4+\frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)$$
$$=\frac{r_{,k}}{r}\left(2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}\left(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2}\right)\right)$$

$$C'=2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{2a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}\left(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2}\right).$$

$$F_{ij}=\frac{1}{4\pi(1-\nu)\kappa k r}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}A+2r_{,i}r_{,j}B\right\}+r_{,i}n_jC+r_{,j}n_iA\right]$$

$$\frac{\partial F_{ij}}{\partial x_k}=-\frac{1}{4\pi(1-\nu)\kappa k r^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}r_{,k}A+2r_{,i}r_{,j}r_{,k}B\right\}+r_{,i}n_jr_{,k}C+r_{,j}n_ir_{,k}A\right]$$
$$+\frac{1}{4\pi(1-\nu)\kappa k r^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}r_{,k}A'+2(\delta_{ik}r_{,j}B+\delta_{jk}r_{,i}B-2r_{,i}r_{,j}r_{,k}B)+2r_{,i}r_{,j}r_{,k}B'\right\}\right.$$
$$+(n_j\delta_{ik}-n_jr_{,i}r_{,k})C+(n_i\delta_{jk}-n_ir_{,j}r_{,k})A$$
$$\left.+r_{,i}n_jr_{,k}C'+r_{,j}n_ir_{,k}A'\right]$$
$$+\frac{1}{4\pi(1-\nu)\kappa k r^2}\left[\frac{\partial r}{\partial n}\left\{-\delta_{ij}r_{,k}A-2r_{,i}r_{,j}r_{,k}B\right\}+\delta_{ij}n_kA+2r_{,i}r_{,j}n_kB\right]$$
$$=-\frac{1}{4\pi(1-\nu)\kappa k r^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}r_{,k}(2A-A')-2\delta_{ik}r_{,j}B-2\delta_{jk}r_{,i}B+2r_{,i}r_{,j}r_{,k}(4B-B')\right\}\right.$$
$$\left.+r_{,i}n_jr_{,k}(2C-C')+n_ir_{,j}r_{,k}(2A-A')-\delta_{ik}n_jC-\delta_{jk}n_iA-\delta_{ij}n_kA-2r_{,i}r_{,j}n_kB\right]$$

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$$2A-A' = -2a_1\frac{1}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{1}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3}{\eta_1\eta_2} - 4a_4 + 2a_5$$
$$-2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3}{\eta_1\eta_2}$$
$$= -2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5$$

$$4B-B' = 4a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - 4a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{8a_3}{\eta_1\eta_2} - 8a_5$$
$$+2a_1\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2} - \frac{4a_3}{\eta_1\eta_2}$$
$$= 2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2} - 2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2} - \frac{12a_3}{\eta_1\eta_2} - 8a_5$$

$$2C-C' = -2\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + 2\frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3}{\eta_1\eta_2} - \frac{8\nu}{1-2\nu}a_4 + \frac{4(1-\nu)}{1-2\nu}a_5 + \frac{4\nu}{1-2\nu}\left(a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\right)$$
$$-2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3}{\eta_1\eta_2} - \frac{4\nu}{1-2\nu}\left(a_1\eta_1^2e^{-\eta_1^2} - a_2\eta_2^2e^{-\eta_2^2}\right)$$
$$= -2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - \frac{8\nu}{1-2\nu}a_4 + \frac{4(1-\nu)}{1-2\nu}a_5 + \frac{4\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2} - a_2(1-\eta_2^2)e^{-\eta_2^2}\right)$$

$$\frac{\partial F_{ij}}{\partial x_k} = -\frac{1}{4\pi(1-\nu)kkr^2}\Bigg[\frac{\partial r}{\partial n}\Bigg\{\delta_{ij}r_{,k}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\right)$$
$$+\delta_{ik}r_{,j}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} + 4a_5\right)$$
$$+\delta_{jk}r_{,i}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} + 4a_5\right)$$
$$+2r_{,i}r_{,j}r_{,k}\left(2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} - \frac{12a_3}{\eta_1\eta_2} - 8a_5\right)\Bigg\}$$
$$+r_{,i}n_jr_{,k}\Bigg(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - \frac{8\nu}{1-2\nu}a_4 + \frac{4(1-\nu)}{1-2\nu}a_5 + \frac{4\nu}{1-2\nu}\big(a_1(1-\eta_1^2)e^{-\eta_1^2} - a_2(1-\eta_2^2)e^{-\eta_2^2}\big)\Bigg)$$
$$+r_{,j}n_ir_{,k}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\right)$$
$$-\delta_{ik}n_j\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - \frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}\big(a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\big)\right)$$
$$-\delta_{jk}n_i\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\right)$$
$$-\delta_{ij}n_k\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\right)$$
$$-2r_{,i}r_{,j}n_k\left(a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} - 2a_5\right)\Bigg]$$

$$
\begin{aligned}
\frac{\partial F_{kj}}{\partial x_i} = -\frac{1}{4\pi(1-\nu)kkr^2}\Big[\frac{\partial r}{\partial n}\Big\{ & \delta_{kj} r_{,i}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-4a_4+2a_5\Big)\\
&+\delta_{ik} r_{,j}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}+4a_5\Big)\\
&+\delta_{ji} r_{,k}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}+4a_5\Big)\\
&+2r_{,i}r_{,j}r_{,k}\Big(2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{12a_3}{\eta_1\eta_2}-8a_5\Big)\Big\}\\
&+n_j r_{,i}r_{,k}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-\frac{8\nu}{1-2\nu}a_4+\frac{4(1-\nu)}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\big(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\big)\Big)\\
&+n_k r_{,i}r_{,j}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-4a_4+2a_5\Big)\\
&+n_i r_{,i}r_{,k}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}+4a_5\Big)\\
&-n_j\delta_{ik}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}\big(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\big)\Big)\\
&-n_i\delta_{jk}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)\\
&-n_k\delta_{ij}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)\Big]
\end{aligned}
$$

$$
\varepsilon^{F_{ij}}_{ik}=\frac{1}{2}\left(\frac{\partial F_{ij}}{\partial x_k}+\frac{\partial F_{kj}}{\partial x_i}\right)\cong\frac{1}{2}\left(\frac{\partial F_{ik}}{\partial x_j}+\frac{\partial F_{jk}}{\partial x_i}\right).\neq\frac{1}{2}\left(\frac{\partial F_{ik}}{\partial x_j}+\frac{\partial F_{ij}}{\partial x_k}\right).
$$

$$
\begin{aligned}
=-\frac{1}{8\pi(1-\nu)kkr^2}\Big[\frac{\partial r}{\partial n}\Big\{ & \delta_{ij} r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)\\
&+\delta_{ik} r_{,j}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}+8a_5\Big)\\
&+\delta_{jk} r_{,i}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)\\
&+4r_{,i}r_{,j}r_{,k}\Big(2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{12a_3}{\eta_1\eta_2}-8a_5\Big)\Big\}\\
&+n_i r_{,j}r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)\\
&+2n_j r_{,i}r_{,k}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-\frac{8\nu}{1-2\nu}a_4+\frac{4(1-\nu)}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\big(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\big)\Big)\\
&+n_k r_{,i}r_{,j}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)\\
&-2n_i\delta_{jk}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)\\
&-2n_j\delta_{ik}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}\big(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\big)\Big)\\
&-2n_k\delta_{ij}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)\Big]
\end{aligned}
$$

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$$\frac{\partial F_{ik}}{\partial x_j} = -\frac{1}{4\pi(1-\nu)kkr^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{ik} r_{,j}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\right)\right.\right.$$

$$+\delta_{ij} r_{,k}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} + 4a_5\right)$$

$$+\delta_{jk} r_{,i}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} + 4a_5\right)$$

$$\left.+2r_{,i}r_{,j}r_{,k}\left(2a_1\frac{6+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{6+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} - \frac{12a_3}{\eta_1\eta_2} - 8a_5\right)\right\}$$

$$+r_{,i}n_k r_{,j}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - \frac{8\nu}{1-2\nu}a_4 + \frac{4(1-\nu)}{1-2\nu}a_5 + \frac{4\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2} - a_2(1-\eta_2^2)e^{-\eta_2^2}\right)\right)$$

$$+r_{,k}n_i r_{,j}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\right)$$

$$-\delta_{ij}n_k\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - \frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\right)\right)$$

$$-\delta_{jk}n_i\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\right)$$

$$-\delta_{ik}n_j\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\right)$$

$$\left.-2r_{,i}n_j r_{,k}\left(a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} - 2a_5\right)\right]$$

$$\varepsilon^{F_{ij}}_{ij\,k} = \frac{1}{2}\left(\frac{\partial F_{ij}}{\partial x_k} + \frac{\partial F_{ik}}{\partial x_j}\right)$$

$$= -\frac{1}{8\pi(1-\nu)kkr^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}r_{,k}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - 4a_4 + 6a_5\right)\right.\right.$$

$$+\delta_{ik}r_{,j}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - 4a_4 + 6a_5\right)$$

$$+\delta_{jk}r_{,i}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} + 8a_5\right)$$

$$\left.+4r_{,i}r_{,j}r_{,k}\left(2a_1\frac{6+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{6+3\eta_2^2+\eta_2^4}{\eta_2^2} - \frac{12a_3}{\eta_1\eta_2} - 8a_5\right)\right\}$$

$$+2n_i r_{,j}r_{,k}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\right)$$

$$+(n_j r_{,i}r_{,k} + n_k r_{,i}r_{,j})\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - \frac{8\nu a_4}{1-2\nu} + \frac{8-12\nu}{1-2\nu}a_5\right.$$

$$\left.+\frac{4\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2} - a_2(1-\eta_2^2)e^{-\eta_2^2}\right)\right)$$

$$-2n_i\delta_{jk}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\right)$$

$$\left.-(n_j\delta_{ik} + n_k\delta_{ij})\left(-\frac{2a_1}{\eta_1^2}e^{-\eta_1^2} + \frac{2a_2}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3}{\eta_1\eta_2} - \frac{2a_4}{1-2\nu} + \frac{(3-4\nu)a_5}{1-2\nu} + \frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\right)\right)\right]$$

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$$\varepsilon^{F_{ij}}_{mj,m} = -\frac{1}{8\pi(1-\nu)kKr^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{jm} r_{,m}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\right)\right.\right.$$

$$+\delta_{mm} r_{,j}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}+8a_5\right)$$

$$+\delta_{jm} r_{,m}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\right)$$

$$\left.+4r_{,m}r_{,j}r_{,m}\left(2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{12a_3}{\eta_1\eta_2}-8a_5\right)\right\}$$

$$+n_m r_{,j} r_{,m}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\right)$$

$$+2n_j r_{,m} r_{,m}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-\frac{8\nu}{1-2\nu}a_4+\frac{4(1-\nu)}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\left(a_1(1+\eta_1^2)e^{-\eta_1^2}-a_2(1+\eta_2^2)e^{-\eta_2^2}\right)\right)$$

$$+n_m r_{,j} r_{,m}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\right)$$

$$-2n_m\delta_{jm}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)$$

$$-2n_j\delta_{mm}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)$$

$$\left.-2n_m\delta_{jm}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)\right]$$

$$= -\frac{1}{8\pi(1-\nu)kKr^2}\left[\frac{\partial r}{\partial n}\left\{r_{,j}\left(-4a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+4a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{16a_3}{\eta_1\eta_2}-8a_4+12a_5\right.\right.\right.$$

$$-4a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+4a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{16a_3}{\eta_1\eta_2}+16a_5$$

$$+8a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-8a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{48a_3}{\eta_1\eta_2}-32a_5$$

$$\left.\left.-4a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+4a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{16a_3}{\eta_1\eta_2}-8a_4+12a_5\right)\right\}$$

$$+n_j\left(-4a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+4a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-\frac{16\nu}{1-2\nu}a_4+\frac{8(1-\nu)}{1-2\nu}a_5+\frac{8\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\right)\right.$$

$$+4\frac{a_1}{\eta_1^2}e^{-\eta_1^2}-4\frac{a_2}{\eta_2^2}e^{-\eta_2^2}-\frac{4a_3}{\eta_1\eta_2}+8a_4-4a_5$$

$$\left.\left.+4\frac{a_1}{\eta_1^2}e^{-\eta_1^2}-4\frac{a_2}{\eta_2^2}e^{-\eta_2^2}-\frac{4a_3}{\eta_1\eta_2}+\frac{16\nu}{1-2\nu}a_4-\frac{8(1-\nu)}{1-2\nu}a_5-\frac{8\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)\right]$$

$$= -\frac{1}{8\pi(1-\nu)kKr^2}\left[r_{,j}\frac{\partial r}{\partial n}\left(8\left(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2}-2a_4+a_5+a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right.\right.$$

$$\left.+n_j\left(-4\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)+8a_4-4a_5-\frac{8\nu}{1-2\nu}\left(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2}\right)\right)\right]$$

$$\varepsilon^{F\hat{i}j}_{\hat{i}mm} = -\frac{1}{8\pi(1-\nu)kkr^2}\left[\frac{\partial r}{\partial n}\left\{\delta_{\hat{i}m} r_{,m}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\right)\right.\right.$$

$$+\delta_{\hat{i}m} r_{,m}\left(\qquad // \qquad\right)$$

$$+\delta_{mm} r_{,\hat{i}}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}+8a_5\right)$$

$$\left.+4r_{,\hat{i}}r_{,m}r_{,m}\left(2a_1\frac{6+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{6+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{12a_3}{\eta_1\eta_2}-8a_5\right)\right\}$$

$$+2n_{\hat{i}}r_{,m}r_{,m}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-4a_4+2a_5\right)$$

$$+r_{,\hat{i}}n_m r_{,m}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-\frac{8\nu a_4}{1-2\nu}+\frac{8-12\nu}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\right)\right)$$

$$+r_{,\hat{i}}n_m r_{,m}\left(\qquad // \qquad\right)$$

$$-2n_{\hat{i}}\delta_{mm}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)$$

$$-n_m\delta_{\hat{i}m}\left(-\frac{2a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{2a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{2a_3}{\eta_1\eta_2}-\frac{2a_4}{1-2\nu}+\frac{\overset{(3-4\nu)}{a_5}}{1-2\nu}+\frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)$$

$$\left.-n_m\delta_{\hat{i}m}\left(\qquad // \qquad\right)\right]$$

$$= -\frac{1}{8\pi(1-\nu)kkr^2}\left[r_{,\hat{i}}\frac{\partial r}{\partial n}\left(-4a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-4a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+8a_1\frac{6+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}\right.\right.$$

$$+4a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+4a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+8a_2\frac{6+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}$$

$$\left.+\frac{16a_3}{\eta_1\eta_2}+\frac{16a_3}{\eta_1\eta_2}-\frac{48a_3}{\eta_1\eta_2}-8a_4+12a_5+16a_5-32a_5\right)$$

$$+2n_{\hat{i}}\left(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-4a_4+2a_5\right)$$

$$+2r_{,\hat{i}}\frac{\partial r}{\partial n}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-\frac{8\nu a_4}{1-2\nu}+\frac{8-12\nu}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\right)\right)$$

$$-4n_{\hat{i}}\left(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\right)$$

$$\left.-2n_{\hat{i}}\left(-\frac{2a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{2a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{2a_3}{\eta_1\eta_2}-\frac{2a_4}{1-2\nu}+\frac{\overset{(3-4\nu)}{a_5}}{1-2\nu}+\frac{2\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)\right]$$

$$= -\frac{1}{8\pi(1-\nu)kkr^2}\left[r_{,\hat{i}}\frac{\partial r}{\partial n}\left(a_1\frac{16+8\eta_1^2+8\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-a_2\frac{16+8\eta_2^2+8\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{16a_3}{\eta_1\eta_2}-8a_4-4a_5\right)\right.$$

$$+2r_{,\hat{i}}\frac{\partial r}{\partial n}\left(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-\frac{8\nu a_4}{1-2\nu}+\frac{8-12\nu}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\left(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\right)\right)$$

$$+n_{\hat{i}}\left(-4a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+\frac{4a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{4a_1}{\eta_1^2}e^{-\eta_1^2}\right.$$

$$+4a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-\frac{4a_2}{\eta_2^2}e^{-\eta_2^2}-\frac{4a_2}{\eta_2^2}e^{-\eta_2^2}$$

$$+\frac{1}{\eta_1\eta_2}\left(8a_3-4a_3-4a_3\right)$$

$$\left.\left.+8a_4-8a_4+\frac{4a_4}{1-2\nu}+4a_5-4a_5-\frac{\overset{(3-4\nu)}{2a_5}}{1-2\nu}-\frac{4\nu}{1-2\nu}\left(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\right)\right)\right]$$

$$\sigma^{F\lambda j}_{\lambda j,k} = \frac{2\mu\nu}{1-2\nu}\delta_{\lambda k}\varepsilon^{F\lambda j}_{mj,m} + 2\mu\,\varepsilon^{F\lambda j}_{\lambda j,k}$$

$$= \frac{2\mu\nu}{1-2\nu}\times\frac{-\delta_{\lambda k}}{8\pi(1-\nu)kkr^2}\Big\{ r_{,j}\frac{\partial r}{\partial n}\Big(8\big(a_1\eta_1^2e^{-\eta_1^2} - a_2\eta_2^2e^{-\eta_2^2} - 2a_4 + a_5 + a_1e^{-\eta_1^2} - a_2e^{-\eta_2^2}\big)\Big)$$

$$-4n_j\Big(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}-2a_4+a_5+\frac{2\nu}{1-2\nu}\big(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2}\big)\Big)\Big]$$

$$+2\mu\times\frac{-1}{8\pi(1-\nu)kkr^2}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{\lambda j}r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$+\delta_{\lambda k}r_{,j}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}+8a_5\Big)$$

$$+\delta_{jk}r_{,\lambda}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$+4r_{,\lambda}r_{,j}r_{,k}\Big(2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{12a_3}{\eta_1\eta_2}-8a_5\Big)\Big\}$$

$$+n_\lambda r_{,j}r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$+2n_j r_{,\lambda}r_{,k}\Big(-2a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{4a_3}{\eta_1\eta_2}-\frac{8\nu a_4}{1-2\nu}+\frac{4(1-\nu)}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\big(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\big)\Big)$$

$$+n_k r_{,\lambda}r_{,j}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$-2n_\lambda\delta_{jk}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)$$

$$-2n_j\delta_{\lambda k}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}\big(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\big)\Big)$$

$$-2n_k\delta_{\lambda j}\Big(-\frac{a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3}{\eta_1\eta_2}-2a_4+a_5\Big)\Big]$$

$\sigma^{F\lambda j}_{\lambda jk}$ $\downarrow$ & $\sigma^{F\lambda j}_{\lambda j,\underline{\underline{k}}}$ (notation을 통일하기위해서 변화).

λj에 대해서 symmetry
λk or jk에 대해서는
asymetric (?).

$$\sigma^{F\lambda j}_{\lambda jk} = \frac{-\mu}{4\pi(1-\nu)kkr^2}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{\lambda k}r_{,j}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$+\delta_{\lambda j}r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}+8a_5$$

$$+\frac{8\nu}{1-2\nu}\big(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2}-2a_4+a_5+a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\big)\Big)$$

$$+\delta_{jk}r_{,\lambda}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$+4r_{,\lambda}r_{,j}r_{,k}\Big(2a_1\frac{6+4\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2}-2a_2\frac{6+4\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2}-\frac{12a_3}{\eta_1\eta_2}-8a_5\Big)\Big\}$$

$$+n_\lambda r_{,j}r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$+n_{,k}r_{,\lambda}r_{,j}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-\frac{16\nu}{1-2\nu}a_4+\frac{8(1-\nu)}{1-2\nu}a_5+\frac{8\nu}{1-2\nu}\big(a_1(1-\eta_1^2)e^{-\eta_1^2}-a_2(1-\eta_2^2)e^{-\eta_2^2}\big)\Big)$$

$$+n_j n_{,\lambda}r_{,k}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{8a_3}{\eta_1\eta_2}-4a_4+6a_5\Big)$$

$$-(n_\lambda\delta_{jk}+n_j\delta_{\lambda k})\Big(-\frac{2a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{2a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{2a_3}{\eta_1\eta_2}-4a_4+2a_5\Big)$$

$$-n_k\delta_{\lambda j}\Big(-\frac{2a_1}{\eta_1^2}e^{-\eta_1^2}+\frac{2a_2}{\eta_2^2}e^{-\eta_2^2}+\frac{2a_3}{\eta_1\eta_2}-\frac{8\nu}{1-2\nu}a_4+\frac{4(1-\nu)}{1-2\nu}a_5+\frac{4\nu}{1-2\nu}\big(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}\big)$$

$$+\frac{4\nu}{1-2\nu}\big(a_1e^{-\eta_1^2}-a_2e^{-\eta_2^2}-2a_4+a_5+\frac{2\nu}{1-2\nu}(a_1\eta_1^2e^{-\eta_1^2}-a_2\eta_2^2e^{-\eta_2^2})\big)\Big)\Big]$$

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$$\varepsilon^{F_{ij}}_{imm} = +\frac{1}{8(1-\nu)kkr^2}\Bigg[r_{,i}\frac{\partial r}{\partial n}\Big(a_1\frac{+10\eta_1^2 - 4\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{+10\eta_2^2 - 4\eta_2^4}{\eta_2^2}e^{-\eta_2^2} - \cancel{\frac{4\nu a_3}{(1-2\nu)\eta_1\eta_2}} + \frac{8(1-3\nu)a_4}{1-2\nu} + \frac{2(3-10\nu)}{1-2\nu}a_5\Big)$$

$$+ n_i\Big(a_1\frac{+2\eta_1^2+4\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{+2\eta_2^2+4\eta_2^4}{\eta_2^2}e^{-\eta_2^2} - \cancel{\frac{3a_3}{4(1-\nu)\eta_1\eta_2}} + \frac{4a_4}{1-2\nu} + \frac{2(5-6\nu)}{1-2\nu}a_5\Big)\Bigg]$$

계산하였음.

φ이 되는게 아니라 없어지는 항.

$$\sigma^{F_{ij}}_{ijk} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\varepsilon^{F_{ij}}_{imm} + 2\mu\,\varepsilon^{F_{ij}}_{ijk}$$

$$= \frac{2\mu\nu}{1-2\nu}\times\frac{\delta_{jk}}{8(1-\nu)kkr^2}\Bigg\{r_{,i}\frac{\partial r}{\partial n}\Big(a_1(10-4\eta_1^2)e^{-\eta_1^2} - a_2(10-4\eta_2^2)e^{-\eta_2^2} - \cancel{\frac{4\nu a_3}{(1-2\nu)\eta_1\eta_2}} + \frac{8(1-3\nu)a_4}{1-2\nu} + \frac{2(3-10\nu)a_5}{1-2\nu}\Big)$$

$$+ n_i\Big(a_1(2+4\eta_1^2)e^{-\eta_1^2} - a_2(2+4\eta_2^2)e^{-\eta_2^2} - \cancel{\frac{3a_3}{4(1-\nu)\eta_1\eta_2}} + \frac{4a_4}{1-2\nu} + \frac{2(5-6\nu)a_5}{1-2\nu}\Big)\Bigg]$$

$$-2\mu\times\frac{\cancel{\delta_{jk}}\ 1}{8(1-\nu)kkr^2}\Bigg[\frac{\partial r}{\partial n}\begin{pmatrix}\delta_{ij}r_{,k}\\+\\\delta_{ik}r_{,j}\end{pmatrix}\Big(-2a_1\frac{4+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - 4a_4 + 4a_5\Big)$$

$$+ \delta_{jk}r_{,i}\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} + 4a_5\Big)$$

$$+ 4r_{,i}r_{,j}r_{,k}\Big(2a_1\frac{6+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{6+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} - \frac{12a_3}{\eta_1\eta_2} - 8a_5\Big)\Big\}$$

$$+ 2n_i r_{,j}r_{,k}\Big(-2a_1\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\Big)$$

$$+ (n_j r_{,i}r_{,k} + n_k r_{,i}r_{,j})\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - \frac{8\nu a_4}{1-2\nu} + \frac{10-12\nu}{1-2\nu}a_5\Big)$$

$$-2n_i\delta_{jk}\Big(-a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3}{\eta_1\eta_2} - 2a_4 + a_5\Big)$$

$$-(n_j\delta_{ik} + n_k\delta_{ij})\Big(-a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3}{\eta_1\eta_2} - \frac{2a_4}{1-2\nu} + \frac{5-6\nu}{1-2\nu}a_5\Big)\Big]$$

$$= \frac{\mu}{4\pi(1-\nu)kkr^2}\Bigg\{\delta_{jk}r_{,i}\frac{\partial r}{\partial n}\Big[\frac{\nu}{1-2\nu}\Big(a_1(10-4\eta_1^2)e^{-\eta_1^2} - a_2(10-4\eta_2^2)e^{-\eta_2^2} - \cancel{\frac{4\nu a_3}{(1-2\nu)\eta_1\eta_2}} + \frac{8(1-3\nu)a_4}{1-2\nu} + \frac{2(3-10\nu)a_5}{1-2\nu}\Big)$$

$$-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} + 4a_5\Big]$$

$$+ (\delta_{ij}r_{,k} + \delta_{ik}r_{,j})\frac{\partial r}{\partial n}\Big[-2a_1\frac{4+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - 4a_4 + 4a_5\Big]$$

$$+ 4r_{,i}r_{,j}r_{,k}\frac{\partial r}{\partial n}\Big(2a_1\frac{6+3\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{6+3\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} - \frac{12a_3}{\eta_1\eta_2} - 8a_5\Big)$$

$$+ 2n_i r_{,j}r_{,k}\Big(-2a_1\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{4a_3}{\eta_1\eta_2} - 4a_4 + 2a_5\Big)$$

$$+ (n_j r_{,i}r_{,k} + n_k r_{,i}r_{,j})\Big(-2a_1\frac{4+2\eta_1^2}{\eta_1^2}e^{-\eta_1^2} + 2a_2\frac{4+2\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{8a_3}{\eta_1\eta_2} - \frac{8\nu a_4}{1-2\nu} + \frac{10-12\nu}{1-2\nu}a_5\Big)$$

$$+ n_i\delta_{jk}\Big[\frac{\nu}{1-2\nu}\Big(a_1(2+4\eta_1^2)e^{-\eta_1^2} - a_2(2+4\eta_2^2)e^{-\eta_2^2} - \cancel{\frac{3a_3}{4(1-\nu)\eta_1\eta_2}} + \frac{4a_4}{1-2\nu} + \frac{2(5-6\nu)a_5}{1-2\nu}\Big)$$

$$+ 2a_1\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - 2a_2\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} + 4a_4 - 2a_5\Big]$$

$$+ (n_j\delta_{ik} + n_k\delta_{ij})\Big(a_1\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} - \frac{2a_3}{\eta_1\eta_2} + \frac{2a_4}{1-2\nu} - \frac{5-6\nu}{1-2\nu}a_5\Big)\Bigg\}$$

2001.3.7. #2.

$\boxed{g_{\theta jk}}$

$$\frac{\partial G_{T\theta j}}{\partial x_k} = \frac{\cancel{T_\theta}}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_k}\left\{(x_j)\left(\frac{a_1^{\theta j}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\theta j}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{\cancel{T_\theta}}{8\pi(\lambda+2\mu)kKt}\delta_{jk}\left\{\frac{a_1^{\theta j}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\theta j}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right\}$$

$$+ \frac{\cancel{T_\theta}\, x_j}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_k}\left\{\frac{a_1^{\theta j}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\theta j}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right\}$$

$$= \frac{\cancel{T_\theta}}{8\pi(\lambda+2\mu)kKt}\left\{\delta_{jk}\left(\frac{a_1^{\theta j}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\theta j}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right) - 2r_{,j}r_{,k}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}$$

$$\varepsilon_{\theta jk}^{G_{\theta j}} = \frac{1}{2}\left(\frac{\partial G_{\theta j}}{\partial x_k} + \frac{\partial G_{\theta k}}{\partial x_j}\right) = \frac{\partial G_{\theta j}}{\partial x_k}.$$

$$\varepsilon_{\theta mm}^{G_{\theta j}} = \frac{\cancel{T_\theta}}{8\pi(\lambda+2\mu)kKt}\left\{\overset{\overset{2}{\shortparallel}}{\delta_{mm}}\left(\cancel{\frac{a_1^{\theta j}}{\eta_1^2}}e^{-\eta_1^2} - \cancel{\frac{a_2^{\theta j}}{\eta_2^2}}e^{-\eta_2^2} + \cancel{\frac{a_3^{\theta j}}{\eta_1\eta_2}}\right) - 2\overset{\overset{1}{\shortparallel}}{r_{,m}r_{,m}}\left(a_1^{\theta j}\cancel{\frac{1+\eta_1^2}{\eta_1^2}}e^{-\eta_1^2} - a_2^{\theta j}\cancel{\frac{1+\eta_2^2}{\eta_2^2}}e^{-\eta_2^2} + \cancel{\frac{a_3^{\theta j}}{\eta_1\eta_2}}\right)\right\}$$

$$= \frac{-2\cancel{T_\theta}}{8\pi(\lambda+2\mu)kKt}\left\{a_1^{\theta j}e^{-\eta_1^2} - a_2^{\theta j}e^{-\eta_2^2}\right\}.$$

$$= -\frac{\cancel{T_\theta}}{4\pi(\lambda+2\mu)kKt}\left(a_1^{\theta j}e^{-\eta_1^2} - a_2^{\theta j}e^{-\eta_2^2}\right)$$

$$\sigma_{\theta jk}^{G_{\theta j}} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\,\varepsilon_{\theta mm}^{G_{\theta j}} + 2\mu\,\varepsilon_{\theta jk}^{G_{\theta j}} = \boxed{g_{\theta jk}}.$$

$$= \frac{2\mu\cancel{T_\theta}}{8\pi(\lambda+2\mu)kKt}\left\{-\frac{2\nu}{1-2\nu}\delta_{jk}\left(a_1^{\theta j}e^{-\eta_1^2} - a_2^{\theta j}e^{-\eta_2^2}\right) + \delta_{jk}\left(\frac{a_1^{\theta j}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\theta j}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right) - 2r_{,j}r_{,k}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{\mu\cancel{T_\theta}}{4\pi(\lambda+2\mu)kKt}\left[\delta_{jk}\left\{-\frac{2\nu}{1-2\nu}\left(a_1^{\theta j}e^{-\eta_1^2} - a_2^{\theta j}e^{-\eta_2^2}\right) + \frac{a_1^{\theta j}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{\theta j}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right\} - 2r_{,j}r_{,k}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right].$$

$\boxed{g_{pjk}}$

$$\frac{\partial G_{pj}}{\partial x_k} = \frac{1}{8\pi(\lambda+2\mu)kKt}\frac{\partial}{\partial x_k}\left\{(x_j)\left(\frac{a_1^{pj}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{pj}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{1}{8\pi(\lambda+2\mu)kKt}\left\{\delta_{jk}\left(\frac{a_1^{pj}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{pj}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right) - 2r_{,k}r_{,j}\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right\}$$

$$\varepsilon_{pjk}^{G_{pj}} = \frac{1}{2}\left(\frac{\partial G_{pj}}{\partial x_k} + \frac{\partial G_{pk}}{\partial x_j}\right) = \frac{\partial G_{pj}}{\partial x_k}.$$

$$\varepsilon_{pmm}^{G_{pj}} = -\frac{1}{4\pi(\lambda+2\mu)kKt}\left(a_1^{pj}e^{-\eta_1^2} - a_2^{pj}e^{-\eta_2^2}\right)$$

$$\sigma_{pjk}^{G_{pj}} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\,\varepsilon_{pmm}^{G_{pj}} + 2\mu\,\varepsilon_{pjk}^{G_{pj}} = g_{pjk}$$

$$= \frac{2\mu}{8\pi(\lambda+2\mu)kKt}\left\{-\frac{2\nu}{1-2\nu}\delta_{jk}\left(a_1^{pj}e^{-\eta_1^2} - a_2^{pj}e^{-\eta_2^2}\right) + \delta_{jk}\left(\frac{a_1^{pj}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{pj}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right) - 2r_{,j}r_{,k}\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{\mu}{4\pi(\lambda+2\mu)kKt}\left\{\delta_{jk}\left[-\frac{2\nu}{1-2\nu}\left(a_1^{pj}e^{-\eta_1^2} - a_2^{pj}e^{-\eta_2^2}\right) + \frac{a_1^{pj}}{\eta_1^2}e^{-\eta_1^2} - \frac{a_2^{pj}}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right] - 2r_{,k}r_{,j}\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right\}.$$

2001.3.7. #3.

$$g_{i\theta j} = -k\frac{\partial G_{i\theta}}{\partial x_j} = -\frac{\cancel{r\,r_{,i}}\ 1}{8\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left\{(r\,r_{,i})\left(a_1^{i\theta}\left(\frac{e^{-\eta_1^2}}{\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\frac{e^{-\eta_2^2}}{\eta_2^2}-E_1(\eta_2^2)\right)+a_3^{i\theta}\frac{1}{\eta_1\eta_2}\right)\right\}.$$

$$= -\frac{1}{8\pi(\lambda+2\mu)k}\left\{\frac{\partial x_i}{\partial x_j}\left(a_1^{i\theta}\left(\frac{e^{-\eta_1^2}}{\eta_1^2}-E_1(\eta_1^2)\right)-a_2^{i\theta}\left(\frac{e^{-\eta_2^2}}{\eta_2^2}-E_1(\eta_2^2)\right)+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right)\right.$$

$$\left.-\frac{\partial r_{,j}\,x_i}{r}\left(a_1^{i\theta}\left(\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-e^{-\eta_1^2}\right)-a_2^{i\theta}\left(\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}-e^{-\eta_2^2}\right)+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right)\right\}.$$

$$= -\frac{1}{8\pi(\lambda+2\mu)k}\left\{\delta_{ij}\left(a_1^{i\theta}\left[\frac{e^{-\eta_1^2}}{\eta_1^2}-E_1(\eta_1^2)\right]-a_2^{i\theta}\left[\frac{e^{-\eta_2^2}}{\eta_2^2}-E_1(\eta_2^2)\right]+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right)\right.$$

$$\left.-2r_{,i}r_{,j}\left(a_1^{i\theta}\frac{e^{-\eta_1^2}}{\eta_1^2}-a_2^{i\theta}\frac{e^{-\eta_2^2}}{\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\right)\right\}$$

$$g_{p\theta j} = -k\frac{\partial G_{p\theta}}{\partial x_j} = \frac{a_1^{p\theta}}{4\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left(E_1(\eta_1^2)-E_1(\eta_2^2)\right)$$

$$= \frac{a_1^{p\theta}r_{,j}}{2\pi(\lambda+2\mu)kr}\left(e^{-\eta_1^2}-e^{-\eta_2^2}\right)$$

$$g_{ipj} = -k\frac{\partial G_{ip}}{\partial x_j} = -\frac{1}{8\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left\{(r\,r_{,j})\left(a_1^{ip}\left[\frac{e^{-\eta_1^2}}{\eta_1^2}-E_1(\eta_1^2)\right]-a_2^{ip}\left[\frac{e^{-\eta_2^2}}{\eta_2^2}-E_1(\eta_2^2)\right]+\frac{a_3^{ip}}{\eta_1\eta_2}\right)\right\}$$

$$= -\frac{1}{8\pi(\lambda+2\mu)k}\left\{\delta_{ij}\left(a_1^{ip}\left[\frac{e^{-\eta_1^2}}{\eta_1^2}-E_1(\eta_1^2)\right]-a_2^{ip}\left[\frac{e^{-\eta_2^2}}{\eta_2^2}-E_1(\eta_2^2)\right]+\frac{a_3^{ip}}{\eta_1\eta_2}\right)\right.$$

$$\left.-2r_{,i}r_{,j}\left(a_1^{ip}\frac{e^{-\eta_1^2}}{\eta_1^2}-a_2^{ip}\frac{e^{-\eta_2^2}}{\eta_2^2}+\frac{a_3^{ip}}{\eta_1\eta_2}\right)\right\}.$$

$$g_{\theta pj} = -k\frac{\partial G_{\theta p}}{\partial x_j} = \frac{-a_1^{\theta p}}{4\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left(E_1(\eta_1^2)-E_1(\eta_2^2)\right)$$

$$= \frac{a_1^{\theta p}r_{,j}}{2\pi(\lambda+2\mu)kr}\left(e^{-\eta_1^2}-e^{-\eta_2^2}\right)$$

$$F_{\theta j} = -\frac{1}{8\pi(\lambda+2\mu)kt}\left\{n_j\left(a_1^{\theta j}\frac{e^{-\eta_1^2}}{\eta_1^2} - a_2^{\theta j}\frac{e^{-\eta_2^2}}{\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}$$

$$\frac{\partial F_{\theta j}}{\partial x_k} = \frac{1}{8\pi(\lambda+2\mu)kt}\left\{\frac{2n_j r_{,k}}{r}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right.$$

$$+\frac{2}{r}\left(\frac{\partial r}{\partial n}(\delta_{jk} - 2r_{,j}r_{,k}) + r_{,j}n_k\right)\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)$$

$$\left. -\frac{4r_{,j}r_{,k}}{r}\frac{\partial r}{\partial n}\left(a_1^{\theta j}\frac{1+\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}$$

$$= \frac{1}{4\pi(\lambda+2\mu)ktr}\left[\frac{\partial r}{\partial n}\left\{\delta_{jk}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right) - 2r_{,j}r_{,k}\left(a_1^{\theta j}\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}\right.$$

$$\left. +(n_j r_{,k} + n_k r_{,j})\left\{a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right\}\right]$$

$$\frac{\partial F_{\theta j}}{\partial x_k} = \frac{\partial F_{\theta k}}{\partial x_j} = \varepsilon_{\theta jk}^{F\theta j}.$$

$$\varepsilon_{\theta mm}^{F\theta j} = \frac{1}{4\pi(\lambda+2\mu)ktr}\left[\frac{\partial r}{\partial n}\left\{\overset{2}{\delta_{mm}}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right) - 2\overset{1}{r_{,m}r_{,m}}\left(a_1^{\theta j}\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}\right.$$

$$\left. + 2\overbrace{n_m r_{,m}}^{\frac{\partial r}{\partial n}}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right\}.$$

$$= -\frac{1}{2\pi(\lambda+2\mu)ktr}\frac{\partial r}{\partial n}\left(a_1^{\theta j}\eta_1^2 e^{-\eta_1^2} - a_2^{\theta j}\eta_2^2 e^{-\eta_2^2}\right).$$

$$\sigma_{\theta jk}^{F\theta j} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\varepsilon_{\theta mm}^{F\theta j} + 2\mu\,\varepsilon_{\theta jk}^{F\theta j} = f_{\theta jk}$$

$$= \frac{2\mu}{4\pi(\lambda+2\mu)ktr}\left[\frac{-2\nu}{1-2\nu}\delta_{jk}\frac{\partial r}{\partial n}\left(a_1^{\theta j}\eta_1^2 e^{-\eta_1^2} - a_2^{\theta j}\eta_2^2 e^{-\eta_2^2}\right) + \delta_{jk}\frac{\partial r}{\partial n}\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right.$$

$$-2r_{,j}r_{,k}\left(a_1^{\theta j}\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3^{\theta j}}{\eta_1\eta_2}\right)$$

$$\left. +(n_j r_{,k} + n_k r_{,j})\left(a_1^{\theta j}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{\theta j}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{\theta j}}{\eta_1\eta_2}\right)\right]$$

$$F_{pj} = \frac{-1}{8\pi(\lambda+2\mu)kt}\left\{n_j\left(a_1^{pj}\frac{e^{-\eta_1^2}}{\eta_1^2} - a_2^{pj}\frac{e^{-\eta_2^2}}{\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right\}$$

$$\frac{\partial F_{pj}}{\partial x_k} = \frac{1}{4\pi(\lambda+2\mu)ktr}\left[\frac{\partial r}{\partial n}\left\{\delta_{jk}\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right) - 2r_{,j}r_{,k}\left(a_1^{pj}\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right\}\right.$$

$$\left. +(n_j r_{,k} + n_k r_{,j})\left\{a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right\}\right]$$

$$\frac{\partial F_{pj}}{\partial x_k} = \frac{\partial F_{pk}}{\partial x_j} = \varepsilon_{pjk}^{Fpj}$$

$$\sigma_{pjk}^{Fpj} = \frac{2\mu\nu}{1-2\nu}\delta_{jk}\varepsilon_{pmm}^{Fpj} + 2\mu\,\varepsilon_{pjk}^{Fpj} = f_{pjk}$$

$$= \frac{1}{2\pi(\lambda+2\mu)ktr}\left(-\frac{2\nu}{1-2\nu}\delta_{jk}\frac{\partial r}{\partial n}\left(a_1^{pj}\eta_1^2 e^{-\eta_1^2} - a_2^{pj}\eta_2^2 e^{-\eta_2^2}\right) + \delta_{jk}\frac{\partial r}{\partial n}\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right.$$

$$-2r_{,j}r_{,k}\left(a_1^{pj}\frac{2+2\eta_1^2+\eta_1^4}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{2+2\eta_2^2+\eta_2^4}{\eta_2^2}e^{-\eta_2^2} + \frac{2a_3^{pj}}{\eta_1\eta_2}\right)$$

$$\left. +(n_j r_{,k} + n_k r_{,j})\left(a_1^{pj}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2} - a_2^{pj}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2} + \frac{a_3^{pj}}{\eta_1\eta_2}\right)\right].$$

$$F_{i\theta}=\frac{\mu}{4\pi(\lambda+2\mu)kk}\Big\{n_i\Big(a_1^{i\theta}\Big[\big(\eta_1^{-2}e^{-\eta_1^2}-E_1(\eta_1^2)\big)-\frac{2\nu}{1-2\nu}\big(\qquad E_1(\eta_1^2)\big)\Big]$$

$$-a_2^{i\theta}\Big[\big(\eta_2^{-2}e^{-\eta_2^2}-E_1(\eta_2^2)\big)-\frac{2\nu}{1-2\nu}\big(\qquad E_1(\eta_2^2)\big)\Big]+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big)$$

$$-2\frac{\partial r}{\partial n}r_{,i}\Big[a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big]\Big\}$$

$$\frac{\partial F_{i\theta}}{\partial x_k}=\frac{\mu}{4\pi(\lambda+2\mu)kk}\frac{\partial}{\partial x_k}\Big\{n_i(\cdots)-2\frac{\partial r}{\partial n}r_{,i}(\cdots)\Big\}$$

$$=\frac{\mu}{4\pi(\lambda+2\mu)kk}\Big\{n_i\Big(a_1^{i\theta}\Big[\Big(-\frac{2r_{,k}}{r}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}+\cancel{\frac{2r_{,k}}{r}e^{-\eta_1^2}}\Big)-\frac{2\nu}{1-2\nu}\Big(\qquad -\frac{2r_{,k}}{r}e^{-\eta_1^2}\Big)\Big]$$

$$-a_2^{i\theta}\Big[\Big(-\frac{2r_{,k}}{r}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\cancel{\frac{2r_{,k}}{r}e^{-\eta_2^2}}\Big)-\frac{2\nu}{1-2\nu}\Big(\qquad -\frac{2r_{,k}}{r}e^{-\eta_2^2}\Big)-a_3^{i\theta}\frac{2r_{,k}}{r}\frac{1}{\eta_1\eta_2}\Big)$$

$$-2\frac{\partial r}{\partial n}r_{,i}\Big[a_1^{i\theta}\Big(-\frac{2r_{,k}}{r}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}\Big)-a_2^{i\theta}\Big(-\frac{2r_{,k}}{r}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}\Big)-a_3^{i\theta}\frac{2r_{,k}}{r}\frac{1}{\eta_1\eta_2}\Big]$$

$$-2\frac{1}{r}\Big(\frac{\partial r}{\partial n}(\delta_{ik}-2r_{,i}r_{,k})+r_{,i}n_k\Big)\Big[a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big]\Big\}$$

$$=\frac{\mu}{4\pi(\lambda+2\mu)kk}\Big\{\frac{2n_ir_{,k}}{r}\Big(a_1^{i\theta}\Big[-\frac{1}{\eta_1^2}e^{-\eta_1^2}+\frac{2\nu}{1-2\nu}(\quad -1)e^{-\eta_1^2}\Big]-a_2^{i\theta}\Big[-\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{2\nu}{1-2\nu}(\quad -1)e^{-\eta_2^2}\Big]-a_3^{i\theta}\frac{1}{\eta_1\eta_2}\Big)$$

$$+\frac{4r_{,i}r_{,k}}{r}\frac{\partial r}{\partial n}\Big[a_1^{i\theta}\frac{1+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+a_3^{i\theta}\frac{1}{\eta_1\eta_2}\Big]$$

$$-\frac{2r_{,i}n_k}{r}\Big(a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big)$$

$$-\frac{2\delta_{ik}}{r}\frac{\partial r}{\partial n}\Big(a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big)$$

$$+\frac{4r_{,i}r_{,k}}{r}\frac{\partial r}{\partial n}\Big(a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big)\Big\}$$

$$f_{i\theta j}=-k\frac{\partial F_{i\theta}}{\partial x_j}=-\frac{\mu}{2\pi(\lambda+2\mu)kr}\Big\{2r_{,i}r_{,j}\frac{\partial r}{\partial n}\Big(a_1^{i\theta}\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}+\frac{2a_3^{i\theta}}{\eta_1\eta_2}\Big)$$

$$-n_ir_{,j}\Big(a_1^{i\theta}\Big[\frac{2}{\eta_1^2}e^{-\eta_1^2}+\frac{2\nu}{1-2\nu}\times e^{-\eta_1^2}\Big]-a_2^{i\theta}\Big[\frac{2}{\eta_2^2}e^{-\eta_2^2}+\frac{2\nu}{1-2\nu}\times e^{-\eta_2^2}\Big]+\frac{2a_3^{i\theta}}{\eta_1\eta_2}\Big)$$

$$-n_jr_{,i}\Big(a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big)$$

$$-\delta_{ij}\frac{\partial r}{\partial n}\Big(a_1^{i\theta}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{i\theta}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{i\theta}}{\eta_1\eta_2}\Big)\Big\}.$$

$$f_{ipj}=-k\frac{\partial F_{ip}}{\partial x_j}=-\frac{\mu}{2\pi(\lambda+2\mu)kr}\Big\{2r_{,i}r_{,j}\frac{\partial r}{\partial n}\Big(a_1^{ip}\frac{2+\eta_1^2}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{2+\eta_2^2}{\eta_2^2}e^{-\eta_2^2}\Big)$$

$$-n_ir_{,j}\Big(a_1^{ip}\Big[\frac{2}{\eta_1^2}e^{-\eta_1^2}-\frac{2\nu}{1-2\nu}\qquad e^{-\eta_1^2}\Big]-a_2^{ip}\Big[\frac{2}{\eta_2^2}e^{-\eta_2^2}-\frac{2\nu}{1-2\nu}\qquad e^{-\eta_2^2}\Big]+\frac{a_3^{ip}}{\eta_1\eta_2}\Big)$$

$$-n_jr_{,i}\Big(a_1^{ip}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{ip}}{\eta_1\eta_2}\Big)$$

$$-\delta_{ij}\frac{\partial r}{\partial n}\Big(a_1^{ip}\frac{1}{\eta_1^2}e^{-\eta_1^2}-a_2^{ip}\frac{1}{\eta_2^2}e^{-\eta_2^2}+\frac{a_3^{ip}}{\eta_1\eta_2}\Big)\Big\}.$$

$$F_{p\theta} = \frac{a_1^{p\theta}}{2\pi(\lambda+2\mu)kr}\frac{\partial r}{\partial n}\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right)$$

$$\frac{\partial F_{p\theta}}{\partial x_j} = \frac{a_1^{p\theta}}{2\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left\{\left(\frac{1}{r}\frac{\partial r}{\partial n}\right)\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right)\right\}$$

$$= \frac{a_1^{p\theta}}{2\pi(\lambda+2\mu)k}\left\{\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right)\frac{\partial}{\partial x_j}\left(\frac{1}{r}\frac{\partial r}{\partial n}\right) + \frac{1}{r}\frac{\partial r}{\partial n}\frac{\partial}{\partial x_j}\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right)\right\}$$

$$= \frac{a_1^{p\theta}}{2\pi(\lambda+2\mu)k}\left\{\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right)\frac{1}{r^2}\left(n_j - 2r_{,j}\frac{\partial r}{\partial n}\right) + \frac{1}{r}\frac{\partial r}{\partial n}\frac{-2r_{,j}}{r}\left(\eta_1^2 e^{-\eta_1^2} - \eta_2^2 e^{-\eta_2^2}\right)\right\}$$

$$= \frac{a_1^{p\theta}}{2\pi(\lambda+2\mu)kr^2}\left\{n_j\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left((1+\eta_1^2)e^{-\eta_1^2} - (1+\eta_2^2)e^{-\eta_2^2}\right)\right\}$$

$$f_{p\theta j} = -k\frac{\partial F_{p\theta}}{\partial x_j} = \frac{-a_1^{p\theta}}{2\pi(\lambda+2\mu)r^2}\left\{n_j\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left((1+\eta_1^2)e^{-\eta_1^2} - (1+\eta_2^2)e^{-\eta_2^2}\right)\right\}$$

$$f_{\theta p j} = -k\frac{\partial F_{\theta p}}{\partial x_j} = \frac{-a_1^{\theta p}}{2\pi(\lambda+2\mu)r^2}\left\{n_j\left(e^{-\eta_1^2} - e^{-\eta_2^2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left((1+\eta_1^2)e^{-\eta_1^2} - (1+\eta_2^2)e^{-\eta_2^2}\right)\right\}$$

$$F_{\theta\theta} = \frac{1}{2\pi(\lambda+2\mu)kr}\frac{\partial r}{\partial n}\left\{a_1^{\theta\theta}e^{-\eta_1^2} - a_2^{\theta\theta}e^{-\eta_2^2}\right\}$$

$$f_{\theta\theta j} = -k\frac{\partial F_{\theta\theta}}{\partial x_j} = \frac{-k}{2\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left\{\left(\frac{1}{r}\frac{\partial r}{\partial n}\right)\left(a_1^{\theta\theta}e^{-\eta_1^2} - a_2^{\theta\theta}e^{-\eta_2^2}\right)\right\}$$

$$= \frac{-k}{2\pi(\lambda+2\mu)kr^2}\left\{n_j\left(a_1^{\theta\theta}e^{-\eta_1^2} - a_2^{\theta\theta}e^{-\eta_2^2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left(a_1^{\theta\theta}(1+\eta_1^2)e^{-\eta_1^2} - a_2^{\theta\theta}(1+\eta_2^2)e^{-\eta_2^2}\right)\right\}$$

$$f_{pp j} = -k\frac{\partial F_{pp}}{\partial x_j} = \frac{-k}{2\pi(\lambda+2\mu)k}\frac{\partial}{\partial x_j}\left\{\left(\frac{1}{r}\frac{\partial r}{\partial n}\right)\left(a_1^{pp}e^{-\eta_1^2} - a_2^{pp}e^{-\eta_2^2}\right)\right\}$$

$$= \frac{-k}{2\pi(\lambda+2\mu)kr^2}\left\{n_j\left(a_1^{pp}e^{-\eta_1^2} - a_2^{pp}e^{-\eta_2^2}\right) - 2r_{,j}\frac{\partial r}{\partial n}\left(a_1^{pp}(1+\eta_1^2)e^{-\eta_1^2} - a_2^{pp}(1+\eta_1^2)e^{-\eta_2^2}\right)\right\}.$$

2001.4.4. #1.

$$\oint_{\Gamma_\varepsilon} \dot{F}_{ij} * u_i \, d\Gamma .$$

F_{ij} is singular at $r=0$. Integration in Cauchy-principal value sense.

Transient term in F_{ij} will be vanishing if r reaches zero (not singular)

$$\lim_{\varepsilon\to 0}\oint_{\Gamma_\varepsilon} \dot{F}_{ij} * u_i \, d\Gamma = \lim_{\varepsilon\to 0}\int_{\Gamma_\varepsilon} F_{ij} u_i \, d\Gamma .$$

$$\lim_{\varepsilon\to 0} F_{ij} = \frac{1}{4\pi(1-\nu)kK\varepsilon}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}(-2a_4+a_5)+2r_i r_j(a_1-a_2-2a_5)\right\}+r_i n_j(-2\nu kK+2a_5) + r_j n_i(-2a_4+a_5)\right]$$

In brief, $Q = \dfrac{\mu}{(\lambda+\mu)(\lambda+2\mu)b_1^2 b_2^2}\left(\alpha_T^2 T_0 K b_4^2 + \alpha^2 k b_3^2 + 2\alpha_T \eta T_0 \alpha\right)$.

$$-2a_4+a_5 = -\frac{(3-4\nu)kK}{2}+\frac{kK}{2}+Q$$

$$a_1-a_2 = -Q$$

$$a_1-a_2-2a_5 = -Q-kK-Q = -kK-2Q$$

$$-2\nu kK+2a_5 = (1-2\nu)kK+Q$$

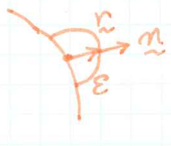
$$-2a_4+a_5 = -(1-2\nu)kK+Q .$$

$$\lim_{\varepsilon\to 0} F_{ij} = -\frac{1}{4\pi(1-\nu)kK\varepsilon}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}(1-2\nu)kK+2r_i r_j kK\right\}+(1-2\nu)(r_j n_i - r_i n_j)\right]$$

$$+\frac{1}{4\pi(1-\nu)kK\varepsilon}\left[\frac{\partial r}{\partial n}\left\{\delta_{ij}Q-4r_i r_j Q\right\}+Q(r_i n_j + r_j n_i)\right]$$

$$\frac{\partial r}{\partial n} = 1.$$

$$n_i r_j - n_j r_i = \frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j} - \frac{\partial r}{\partial x_j}\cdot\frac{\partial r}{\partial x_i} = 0. \qquad \left(\frac{r_i}{r} = r_{,i} = n_i\right).$$



$$\lim_{\varepsilon\to 0}\oint_{\Gamma_\varepsilon} u_i F_{ij}\, d\Gamma = \lim_{\varepsilon\to 0}\oint_{\Gamma_\varepsilon} -\frac{u_i}{4\pi(1-\nu)kK\varepsilon}\left[\delta_{ij}(1-2\nu)kK + 2r_i r_j kK\right]$$ ← not summation

$$+\frac{u_i}{4\pi(1-\nu)kK\varepsilon}\left[\delta_{ij}Q - 4r_i r_j Q + (r_i n_j + r_j n_i)Q\right]\varepsilon\, d\theta .$$

$$\oint_{\Gamma_\varepsilon} d\Gamma_\varepsilon = \int_0^{\pi}\varepsilon\, d\theta.$$; semi circle.

$$\oint_{\Gamma_\varepsilon} r_i r_j \, d\Gamma_\varepsilon \begin{cases} = \int_0^{\pi}\varepsilon \sin^2\theta\, d\theta \text{ or } \int_0^{\pi}\varepsilon\cos^2\theta\, d\theta = \frac{1}{2} \\ = \int_0^{\pi}\varepsilon\sin\theta\cos\theta\, d\theta = 0. \end{cases}$$

$$\lim_{\varepsilon \to 0} \int_0^{\pi} - \left[u_i (1-2\nu) kk + 2 u_i \underbrace{r_{,i} r_{,j}}_{\frac{1}{2}} kk \right] \frac{\varepsilon \, d\theta}{4\pi (1-\nu) kk \varepsilon}$$

$$+ \lim_{\varepsilon \to 0} \int_0^{\pi} \left[u_i Q - \overbrace{4 \underbrace{r_{,i} r_{,j}}_{\frac{1}{2}} Q + \underbrace{r_{,i} n_j}_{\frac{1}{2}} Q + \underbrace{r_{,j} n_i}_{\frac{1}{2}} Q}^{\to \emptyset} \right] \frac{\varepsilon \, d\theta}{4\pi (1-\nu) kk \varepsilon}$$

$$= - \frac{2\pi (1-\nu) kk \varepsilon}{4\pi (1-\nu) kk \varepsilon} u_i = - u_i / 2$$

Derivation of g_{ij}. 위에서는 3차원의 경우에 대해서 알아보았지만 본절에서는 2차원 elastostatistic 의 경우에 대하여 internal stress를 구하기 위한 식을 유도하도록 한다.

$$g_{ij} = \mu\left(\frac{\partial W_i}{\partial x_j} + \frac{\partial W_j}{\partial x_i}\right) + \frac{2\nu\mu}{1-2\nu}\frac{\partial W_k}{\partial x_k}\delta_{ij} - \dot{\sigma}^a_{ij} \tag{6.11}$$

여기서

$$\frac{\partial W_i}{\partial x_m} = -\dot{\sigma}^a_{jk}\int_{\Gamma_1}\varepsilon^*_{jki}r_{,m}d\Gamma \tag{6.12}$$

여기서 Γ_1은 단위길이를 반지름으로 하는 원이다.

앞절에서 구한 방식과 같은 방법으로 2차원의 변형율해를 구할 수 있으며 다음과 같다.

$$\varepsilon^*_{jki} = -\frac{1}{8\pi\mu(1-\nu)r}[(1-2\nu)(\delta_{ij}r_{,k}+\delta_{ik}r_{,j}) - \delta_{jk}r_{,i} + 2r_{,i}r_{,j}r_{,k}] \tag{6.13}$$

(6.14)

$$\varepsilon^*_{jki}r_{,m} = -\frac{1}{8\pi\mu(1-\nu)r}[(1-2\nu)(\delta_{ij}r_{,k}r_{,m}+\delta_{ik}r_{,j}r_{,m}) - \delta_{jk}r_{,i}r_{,m} + 2r_{,i}r_{,j}r_{,k}r_{,m}]$$

(6.15)

$$\begin{aligned}\int_{\Gamma_1}\varepsilon^*_{jki}r_{,m}d\Gamma &= -\frac{1}{8\pi\mu(1-\nu)}\int_{\Gamma_1}(1-2\nu)(\delta_{ij}r_{,k}r_{,m}+\delta_{ik}r_{,j}r_{,m}) - \delta_{jk}r_{,i}r_{,m} + 2r_{,i}r_{,j}r_{,k}r_{,m}d\Gamma \\ &= -\frac{1}{8\pi\mu(1-\nu)}\left[(1-2\nu)(\delta_{ij}\delta_{km}+\delta_{ik}\delta_{jm}) - \delta_{jk}\delta_{im} + \frac{1}{2}(\delta_{ij}\delta_{km}+\delta_{ik}\delta_{jm}+\delta_{im}\delta_{jk})\right]\end{aligned}$$

(6.16)

$$\begin{aligned}\frac{\partial W_i}{\partial x_m} &= \dot{\sigma}^a_{jk}\int_{\Gamma_1}\varepsilon^*_{jki}r_{,m}d\Gamma \\ &= -\frac{\dot{\sigma}^a_{jk}}{8\pi\mu(1-\nu)}\left[(1-2\nu)(\delta_{ij}\delta_{km}+\delta_{ik}\delta_{jm}) - \delta_{jk}\delta_{im} + \frac{1}{2}(\delta_{ij}\delta_{km}+\delta_{ik}\delta_{jm}+\delta_{im}\delta_{jk})\right] \\ &= -\frac{\dot{\sigma}^a_{jk}}{8\pi\mu(1-\nu)}\left[\delta_{ij}\delta_{km}\left(\frac{3-4\nu}{2}\right) + \delta_{ik}\delta_{jm}\left(\frac{3-4\nu}{2}\right) - \frac{1}{2}\delta_{jk}\delta_{im}\right] \\ &= -\frac{1}{8\pi\mu(1-\nu)}\left[(3-4\nu)\dot{\sigma}^a_{im} - \frac{1}{2}\dot{\sigma}^a_{pp}\delta_{im}\right]\end{aligned}$$

여기서 유도과정에서 실수하기 쉬운 부분을 집고 넘어가고자 한다. 다음의 두번째 항이 $\frac{1}{2}$에서 1로 바뀐것을 주목하기 바란다.

$$\frac{\partial W_k}{\partial x_k} = -\frac{1}{8\pi\mu(1-\nu)}[(3-4\nu)\dot{\sigma}^a_{kk} - \dot{\sigma}^a_{kk}] \tag{6.17}$$

6.16를 6.11에 입력하여 정리하면

(6.18)

$$
\begin{aligned}
g_{ij} &= \frac{1}{8(1-\nu)}[(6-8\nu)\dot{\sigma}^a_{ij} - \dot{\sigma}^a_{pp}\delta_{ij}] + \frac{\nu}{8(1-\nu)(1-2\nu)}[(4-8\nu)\dot{\sigma}^a_{pp}\delta_{ij}] - \dot{\sigma}^a_{ij} \\
&= -\frac{1}{8(1-\nu)}[2\dot{\sigma}^a_{ij} + (1-4\nu)\dot{\sigma}^a_{pp}\delta_{ij}]
\end{aligned}
$$

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$$\lim_{t\to 0} F_{ij} = \frac{1}{4\pi(1-\nu)kkr}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}(a_1-a_2-2a_5)\Big\} + r_{,i}n_j\Big(-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)$$
$$+ r_{,j}n_i(-2a_4+a_5)\Big]$$

$$\lim_{t\to 0}\frac{\partial F_{ij}}{\partial x_k} = \frac{1}{4\pi(1-\nu)kk}\frac{\partial}{\partial x_k}\Big(\frac{1}{r}\Big)\Big[\frac{\partial r}{\partial n}\Big\{\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}(a_1-a_2-2a_5)\Big\}$$
$$+r_{,i}n_j\Big(-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)+r_{,j}n_i(-2a_4+a_5)\Big]$$

$$= \frac{1}{4\pi(1-\nu)kk}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}(a_1-a_2-2a_5)\Big\}$$
$$+r_{,i}n_j\Big(-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)+r_{,j}n_i(-2a_4+a_5)\Big]\frac{\partial r}{\partial x_k}\frac{\partial}{\partial r}\Big(\frac{1}{r}\Big)$$

$$+\frac{1}{4\pi(1-\nu)kkr}\Big[\frac{\partial r}{\partial n}\frac{\partial}{\partial x_k}\Big\{\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}(a_1-a_2-2a_5)\Big\}$$
$$+\frac{\partial}{\partial x_k}(r_{,i}n_j)\Big(-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)$$
$$+\frac{\partial}{\partial x_k}(r_{,j}n_i)(-2a_4+a_5)\Big]$$

$$+\frac{1}{4\pi(1-\nu)kkr}\Big[\Big\{\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}(a_1-a_2-2a_5)\Big\}\frac{\partial}{\partial x_k}\Big(\frac{\partial r}{\partial n}\Big)\Big]$$

$$\lim_{t\to 0}\frac{\partial F_{ij}}{\partial x_k} = +\frac{1}{4\pi(1-\nu)kkr^2}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{ij}r_{,k}(+2a_4\mp a_5)\mp 2r_{,i}r_{,j}r_{,k}(a_1-a_2-2a_5)\Big\}$$
$$+r_{,i}n_jr_{,k}\Big(+\frac{4\nu}{1-2\nu}a_4\mp\frac{2(1-\nu)}{1-2\nu}a_5\mp\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)+r_{,j}n_ir_{,k}(+2a_4\mp a_5)\Big]$$

$$+\frac{1}{4\pi(1-\nu)kkr^2}\Big[2(\delta_{ik}r_{,j}+\delta_{jk}r_{,i}-2r_{,i}r_{,j}r_{,k})(a_1-a_2-2a_5)\frac{\partial r}{\partial n}$$
$$+(n_j\delta_{ik}-n_jr_{,i}r_{,k})\Big(-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)$$
$$+(n_i\delta_{jk}-n_in_jr_{,k})(-2a_4+a_5)\Big]$$

$$+\frac{1}{4\pi(1-\nu)kkr^2}\Big[\Big\{\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}(a_1-a_2-2a_5)\Big\}\Big(n_k-r_{,k}\frac{\partial r}{\partial n}\Big)\Big]$$

$$=\frac{1}{4\pi(1-\nu)kkr^2}\Big[\frac{\partial r}{\partial n}\Big\{\delta_{ij}r_{,k}(2a_4-a_5)-2r_{,i}r_{,j}r_{,k}(a_1-a_2-2a_5)$$
$$+2(\delta_{ik}r_{,j}+\delta_{jk}r_{,i}-2r_{,i}r_{,j}r_{,k})(a_1-a_2-2a_5)$$
$$+\delta_{ij}r_{,k}(2a_4-a_5)-2r_{,i}r_{,j}r_{,k}(a_1-a_2-2a_5)\Big\}$$
$$+r_{,i}n_jr_{,k}\Big(\frac{4\nu}{(1-2\nu)}a_4-\frac{2(1-\nu)}{(1-2\nu)}a_5-\frac{2\nu}{(1-2\nu)}(a_1-a_2)\Big)+r_{,j}n_ir_{,k}(2a_4-a_5)$$
$$+(n_j\delta_{ik}-n_jr_{,i}r_{,k})\Big(-\frac{4\nu}{1-2\nu}a_4+\frac{2(1-\nu)}{1-2\nu}a_5+\frac{2\nu}{1-2\nu}(a_1-a_2)\Big)$$
$$+(n_i\delta_{jk}-n_ir_{,j}r_{,k})(-2a_4+a_5)$$
$$+n_k\delta_{ij}(-2a_4+a_5)+2r_{,i}r_{,j}n_k(a_1-a_2-2a_5)\Big]$$

$$\lim_{t\to 0}\frac{\partial F_{ij}}{\partial x_k} = \frac{1}{4\pi(1-\nu)kKr^2}\Bigg[\frac{\partial r}{\partial n}\Big\{2\delta_{ij}r_{,k}(2a_4-a_5) + 2(\delta_{ik}r_{,j} + \delta_{jk}r_{,i})(a_1-a_2-2a_5) - 8r_{,i}r_{,j}r_{,k}(a_1-a_2-2a_5)\Big\}$$

$$+2n_j r_{,i}r_{,k}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2)\right)$$

$$+2n_i r_{,j}r_{,k}(2a_4-a_5)$$

$$+2n_k r_{,i}r_{,j}(a_1-a_2-2a_5)$$

$$+n_j\delta_{ik}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2)\right)$$

$$+n_i\delta_{jk}(-2a_4+a_5) + n_k\delta_{ij}(-2a_4+a_5)\Bigg]$$

$$\lim_{t\to 0}\frac{\partial F_{ik}}{\partial x_j} = \frac{1}{4\pi(1-\nu)kKr^2}\Bigg[\frac{\partial r}{\partial n}\Big\{2\delta_{ik}r_{,j}(2a_4-a_5) + 2(\delta_{ij}r_{,k} + \delta_{jk}r_{,i})(a_1-a_2-2a_5) - 8r_{,i}r_{,j}r_{,k}(a_1-a_2-2a_5)\Big\}$$

$$+2n_k r_{,i}r_{,j}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2)\right)$$

$$+2n_i r_{,j}r_{,k}(2a_4-a_5)$$

$$+2n_j r_{,i}r_{,k}(a_1-a_2-2a_5)$$

$$+n_k\delta_{ij}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2)\right)$$

$$+n_i\delta_{jk}(-2a_4+a_5) + n_j\delta_{ik}(-2a_4+a_5)\Bigg]$$

$$\lim_{t\to 0}\varepsilon_{ijk} = \frac{1}{2}\left(\lim_{t\to 0}\frac{\partial F_{ij}}{\partial x_k} + \lim_{t\to 0}\frac{\partial F_{ik}}{\partial x_j}\right)$$

$$= \frac{1}{8\pi(1-\nu)kKr^2}\Bigg[\frac{\partial r}{\partial n}\Big\{2\delta_{ij}r_{,k}(2a_4-a_5+a_1-a_2-2a_5) + 2\delta_{ik}r_{,j}(2a_4-a_5+a_1-a_2-2a_5)$$

$$+4\delta_{jk}r_{,i}(a_1-a_2-2a_5) - 16r_{,i}r_{,j}r_{,k}(a_1-a_2-2a_5)\Big\}$$

$$+2n_j r_{,i}r_{,k}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2) + a_1-a_2-2a_5\right)$$

$$+4n_i r_{,j}r_{,k}(2a_4-a_5)$$

$$+2n_k r_{,i}r_{,j}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2) + a_1-a_2-2a_5\right)$$

$$+n_j\delta_{ik}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2) - 2a_4+a_5\right)$$

$$+2n_i\delta_{jk}(-2a_4+a_5)$$

$$+n_k\delta_{ij}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2) - 2a_4+a_5\right)\Bigg]$$

$$\lim_{t\to 0} \varepsilon_{\hat{i}mm} = \frac{1}{8\pi(1-\nu)kkr^2}\left[\frac{\partial r}{\partial n}\left\{2\delta_{\hat{i}m} r_{,m}(2a_4-a_5+a_1-a_2-2a_5) + 2\delta_{\hat{i}m} r_{,m}(2a_4-a_5+a_1-a_2-2a_5)\right.\right.$$

$$\left. + 4\delta_{mm} r_{,\hat{i}}(a_1-a_2-2a_5) - 16 r_{,\hat{i}} r_{,m} r_{,m}(a_1-a_2-2a_5)\right\}$$

$$+2r_{,\hat{i}} n_m r_{,m}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2) + a_1-a_2-2a_5\right)$$

$$+4n_{\hat{i}} r_{,m} r_{,m}(2a_4-a_5)$$

$$+2r_{,\hat{i}} n_m r_{,m}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2) + a_1-a_2-2a_5\right)$$

$$+2n_{\hat{i}}\delta_{mm}(-2a_4+a_5)$$

$$\left. +2n_m\delta_{\hat{i}m}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2) - 2a_4+a_5\right)\right]$$

$$= \frac{1}{8\pi(1-\nu)kkr^2}\left[\frac{\partial r}{\partial n}\left\{2r_{,\hat{i}}(2a_4-a_5+a_1-a_2-2a_5) + 2r_{,\hat{i}}(2a_4-a_5+a_1-a_2-2a_5)\right.\right.$$

$$\left. + 8r_{,\hat{i}}(a_1-a_2-2a_5) - 16r_{,\hat{i}}(a_1-a_2-2a_5)\right\}$$

$$+4\frac{\partial r}{\partial n}\cdot r_{,\hat{i}}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2) + a_1-a_2-2a_5\right)$$

$$+4n_{\hat{i}}(2a_4-a_5) + 4n_{\hat{i}}(-2a_4+a_5)$$

$$\left. +2n_{\hat{i}}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2) - 2a_4+a_5\right)\right]$$

$$= \frac{1}{8\pi(1-\nu)kkr^2}\left[r_{,\hat{i}}\frac{\partial r}{\partial n}\left\{4(2a_4-a_5) \overset{4}{+} \cancel{(a_1-a_2-\overset{2}{a_5})} + 8\cancel{(a_1-a_2-2a_5)} - 16\cancel{(a_1-a_2-2a_5)}\right.\right.$$

$$\left. +4\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2)\right) + 4\cancel{(a_1-a_2-2a_5)}\right\}$$

$$\left. +2n_{\hat{i}}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2) - 2a_4+a_5\right)\right]$$

$$= \frac{1}{8\pi(1-\nu)kkr^2}\left[4r_{,\hat{i}}\frac{\partial r}{\partial n}\left(\frac{4\nu}{1-2\nu}a_4 - \frac{2(1-\nu)}{1-2\nu}a_5 - \frac{2\nu}{1-2\nu}(a_1-a_2) + 2a_4-a_5\right)\right.$$

$$\left. +2n_{\hat{i}}\left(-\frac{4\nu}{1-2\nu}a_4 + \frac{2(1-\nu)}{1-2\nu}a_5 + \frac{2\nu}{1-2\nu}(a_1-a_2) - 2a_4+a_5\right)\right].$$

$\cancel{u_\lambda}$

$$-u_j = \int_S F_{\lambda j} u_\lambda + \frac{1}{T_0} F_{\theta j}\theta + F_{pj} p \, ds$$

$$- \int_S G_{\lambda j} t_\lambda + \frac{1}{T_0} G_{\theta j} q_T + G_{pj} q_p \, ds$$

$$+ \int_V \frac{1}{T_0} G_{\theta j} \cdot \delta(x-\xi_\theta)\Lambda_\theta \, dV$$

$$+ \int_V G_{pj} \, \delta(x-\xi_p)\Lambda_p \, dV$$

$\rightarrow$ $\boxed{0.5}$

$$u_j = \int_S + F_{\lambda j} u_\lambda + \frac{1}{T_0} F_{\theta j}\theta + F_{pj} p \, ds$$

$$+ \int_S G_{\lambda j} t_\lambda + \frac{1}{T_0} G_{\theta j}\theta + G_{pj} p \, ds$$

$$- \int_V \frac{1}{T_0} G_{\theta j}\,\delta(x-\xi_\theta)\Lambda_\theta + G_{pj}\,\delta(x-\xi_p)\Lambda_p \, dV$$

$$\theta = \int_S F_{\lambda\theta} u_\lambda + \frac{1}{T_0} F_{\theta\theta}\theta + F_{p\theta} p \, ds$$

$$- \int_S G_{\lambda\theta} t_\lambda + \frac{1}{T_0} G_{\theta\theta} q_\theta + G_{p\theta} q_p \, ds$$

$$+ \int_V \frac{1}{T_0} G_{\theta\theta}\,\delta(x-\xi_\theta)\Lambda_\theta \, dV$$

$$+ \int_V G_{p\theta}\,\delta(x-\xi_p)\Lambda_p \, dV$$

$$p = \int_S F_{\lambda p} u_\lambda + \frac{1}{T_0} F_{\theta p}\theta + F_{pp} p \, ds$$

$$- \int_S G_{\lambda p} t_\lambda + \frac{1}{T_0} G_{\theta p}\theta + G_{pp} p \, ds$$

$$+ \int_V \frac{1}{T_0} G_{\theta p}\,\delta(x-\xi_\theta)\Lambda_\theta \, dV$$

$$+ \int_V G_{pp}\,\delta(x-\xi_p)\Lambda_p \, dV.$$

$$u_j + \int_S F_{\lambda j} u_\lambda + \frac{1}{T_0} F_{\theta j}\theta + F_{pj} p \, ds + \int_V \frac{1}{T_0} G_{\theta j}\,\delta(x-\xi_\theta)\Lambda_\theta + G_{pj}\,\delta(x-\xi_p)\Lambda_p \, dV$$

$$= \int_S G_{\lambda j} t_\lambda + \frac{1}{T_0} G_{\theta j}\theta + G_{pj} p \, ds.$$

$$\theta - \int_S F_{\lambda\theta} u_\lambda + \frac{1}{T_0} F_{\theta\theta}\theta + F_{p\theta} p \, ds - \int_V \frac{1}{T_0} G_{\theta\theta}\,\delta(x-\xi_\theta)\Lambda_\theta + G_{p\theta}\,\delta(x-\xi_p)\Lambda_p \, dV$$

$$= -\int_S G_{\lambda\theta} t_\lambda + \frac{1}{T_0} G_{\theta\theta}\theta + G_{p\theta} p \, ds.$$

$$p - \int_S F_{\lambda p} u_\lambda + \frac{1}{T_0} F_{\theta p}\theta + F_{pp} p \, ds - \int_V \frac{1}{T_0} G_{\theta p}\,\delta(x-\xi_\theta)\Lambda_\theta + G_{pp}\,\delta(x-\xi_p)\Lambda_p \, dV$$

$$= -\int_S G_{\lambda p} u_\lambda + \frac{1}{T_0} G_{\theta p}\theta + G_{pp} p \, ds.$$

"non-dimensional", "nondimensional"