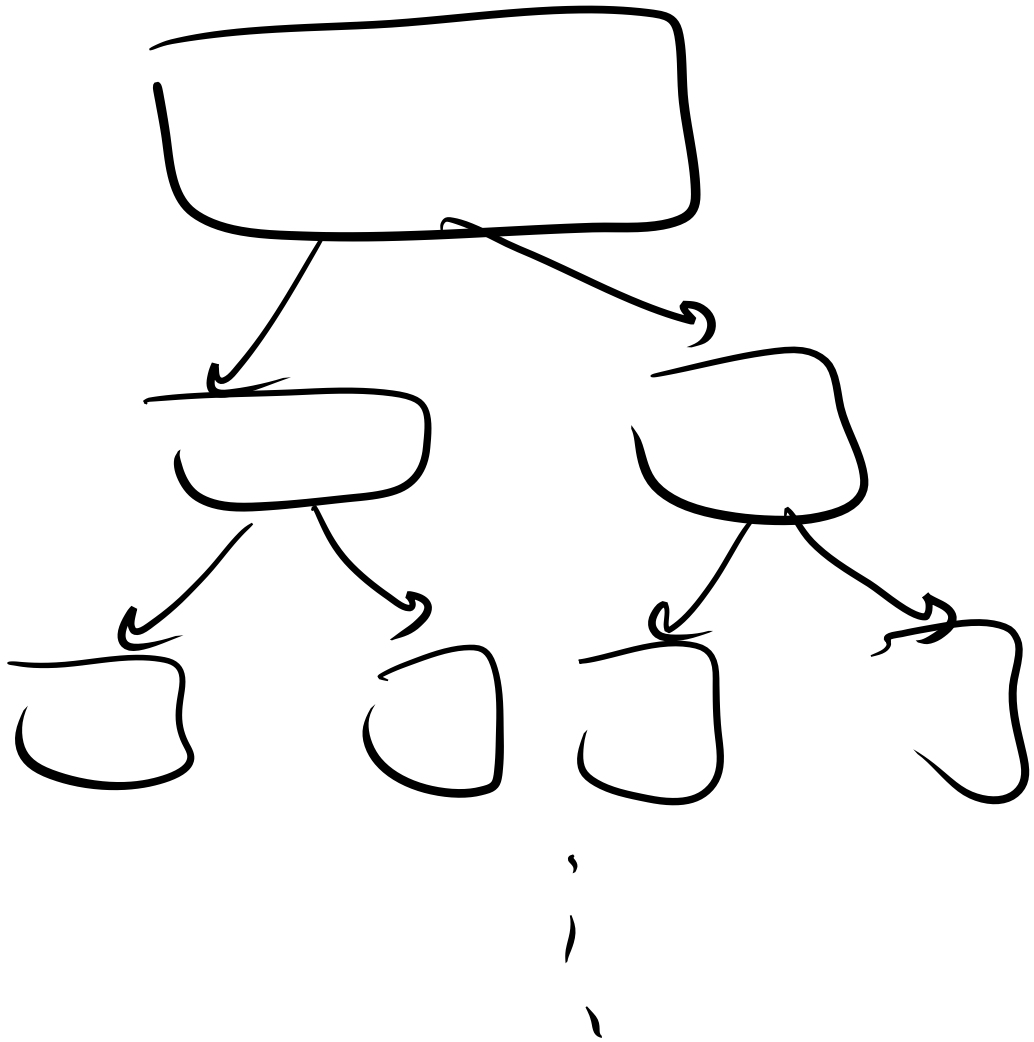
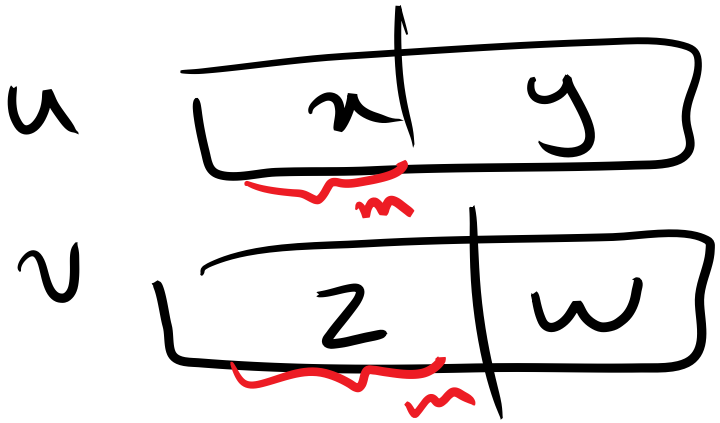
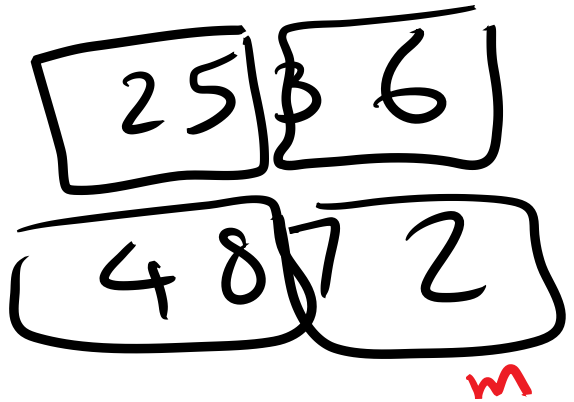






ضرب اعداد بزرگ

$x = u // 10^m$

$y = u \% 10^m$

$z = v // 10^m$

$w = v \% 10^m$

$$(u.v) = (x * 10^m + y)(z * 10^m + w)$$

$$(u.v) = (xz) * 10^{2m} + (xw + zy) * 10^m + yw$$

```
def prod(u, v)
    if ( شرط پایه ) :
```

تعداد ارقام

$m = n/2$

$x = u // 10^m$ $z = v // 10^m$

$y = u \% 10^m$ $w = v \% 10^m$

return $prod(x, z) * 10^{2m} + prod(x, w) * 10^m$
$+ prod(z, y) * 10^m + prod(z, w)$

$T(n) = 4T(n/2) + Cn \in O(n^2)$

$$r = (x+y)(z+w)$$

$$r = xz + xw + yz + yw$$

p $\qquad$ q

$$r - (xz + yw)$$

$$r - (p+q)$$

```
def prod(u,v)
    if(  ):
        m = n//2
        x = u//10**m
        y = u%10**m
        z = v//10**m
        w = v%10**m
        p = prod(x,z)
        q = prod(y,w)
        r = prod((x+y),(z+w))
        return p*10**(2m) + (r-(p+q))*10**m + q
```

$T(n) = 3T(n/2) + Cn \longrightarrow O(n^{\log_2 3})$

$w(n)$

$$A_{n \times n} \times B_{n \times n} = C_{n \times n}$$

$$C_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} A_{ik} B_{kj} \qquad O(n^3)$$

$$A_{n \times m} \times B_{m \times p} = C_{n \times p}$$

$$O(n \cdot m \cdot p)$$

Matrix multiplication $\mathbf{C} = \mathbf{A}\ \mathbf{B}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} \\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} \end{bmatrix}$$

ضرب الماتريكس

$$\mathbf{C}_{1,1} = \mathbf{A}_{1,1}\mathbf{B}_{1,1} + \mathbf{A}_{1,2}\mathbf{B}_{2,1}$$
$$\mathbf{C}_{1,2} = \mathbf{A}_{1,1}\mathbf{B}_{1,2} + \mathbf{A}_{1,2}\mathbf{B}_{2,2}$$
$$\mathbf{C}_{2,1} = \mathbf{A}_{2,1}\mathbf{B}_{1,1} + \mathbf{A}_{2,2}\mathbf{B}_{2,1}$$
$$\mathbf{C}_{2,2} = \mathbf{A}_{2,1}\mathbf{B}_{1,2} + \mathbf{A}_{2,2}\mathbf{B}_{2,2}$$

Classic: $O(n^3)$ multiplications.

$$\mathbf{M}_1 := (\mathbf{A}_{1,1} + \mathbf{A}_{2,2})(\mathbf{B}_{1,1} + \mathbf{B}_{2,2})$$
$$\mathbf{M}_2 := (\mathbf{A}_{2,1} + \mathbf{A}_{2,2})\mathbf{B}_{1,1}$$
$$\mathbf{M}_3 := \mathbf{A}_{1,1}(\mathbf{B}_{1,2} - \mathbf{B}_{2,2})$$
$$\mathbf{M}_4 := \mathbf{A}_{2,2}(\mathbf{B}_{2,1} - \mathbf{B}_{1,1})$$
$$\mathbf{M}_5 := (\mathbf{A}_{1,1} + \mathbf{A}_{1,2})\mathbf{B}_{2,2}$$
$$\mathbf{M}_6 := (\mathbf{A}_{2,1} - \mathbf{A}_{1,1})(\mathbf{B}_{1,1} + \mathbf{B}_{1,2})$$
$$\mathbf{M}_7 := (\mathbf{A}_{1,2} - \mathbf{A}_{2,2})(\mathbf{B}_{2,1} + \mathbf{B}_{2,2})$$

$O(n^{\log_2 7})$

$$\mathbf{C}_{1,1} = \mathbf{M}_1 + \mathbf{M}_4 - \mathbf{M}_5 + \mathbf{M}_7$$
$$\mathbf{C}_{1,2} = \mathbf{M}_3 + \mathbf{M}_5$$
$$\mathbf{C}_{2,1} = \mathbf{M}_2 + \mathbf{M}_4$$
$$\mathbf{C}_{2,2} = \mathbf{M}_1 - \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_6$$

$T(n) = 7T(n/2) + C_n$

$O(n^{2.8074})$ multiplications.

Matrix multiplication $\mathbf{C} = \mathbf{A}\ \mathbf{B}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} \\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} \end{bmatrix}$$

ضرب استراسن

$$\mathbf{C}_{1,1} = \mathbf{A}_{1,1}\mathbf{B}_{1,1} + \mathbf{A}_{1,2}\mathbf{B}_{2,1}$$
$$\mathbf{C}_{1,2} = \mathbf{A}_{1,1}\mathbf{B}_{1,2} + \mathbf{A}_{1,2}\mathbf{B}_{2,2}$$
$$\mathbf{C}_{2,1} = \mathbf{A}_{2,1}\mathbf{B}_{1,1} + \mathbf{A}_{2,2}\mathbf{B}_{2,1}$$
$$\mathbf{C}_{2,2} = \mathbf{A}_{2,1}\mathbf{B}_{1,2} + \mathbf{A}_{2,2}\mathbf{B}_{2,2}$$

Classic: $O(n^3)$ multiplications.

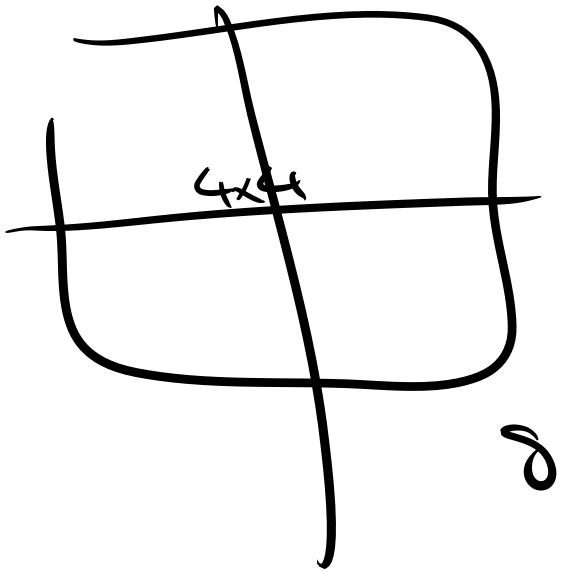
$$\mathbf{M}_1 := (\mathbf{A}_{1,1} + \mathbf{A}_{2,2})(\mathbf{B}_{1,1} + \mathbf{B}_{2,2})$$
$$\mathbf{M}_2 := (\mathbf{A}_{2,1} + \mathbf{A}_{2,2})\mathbf{B}_{1,1}$$
$$\mathbf{M}_3 := \mathbf{A}_{1,1}(\mathbf{B}_{1,2} - \mathbf{B}_{2,2})$$
$$\mathbf{M}_4 := \mathbf{A}_{2,2}(\mathbf{B}_{2,1} - \mathbf{B}_{1,1})$$
$$\mathbf{M}_5 := (\mathbf{A}_{1,1} + \mathbf{A}_{1,2})\mathbf{B}_{2,2}$$
$$\mathbf{M}_6 := (\mathbf{A}_{2,1} - \mathbf{A}_{1,1})(\mathbf{B}_{1,1} + \mathbf{B}_{1,2})$$
$$\mathbf{M}_7 := (\mathbf{A}_{1,2} - \mathbf{A}_{2,2})(\mathbf{B}_{2,1} + \mathbf{B}_{2,2})$$

$$\mathbf{C}_{1,1} = \mathbf{M}_1 + \mathbf{M}_4 - \mathbf{M}_5 + \mathbf{M}_7$$
$$\mathbf{C}_{1,2} = \mathbf{M}_3 + \mathbf{M}_5$$
$$\mathbf{C}_{2,1} = \mathbf{M}_2 + \mathbf{M}_4$$
$$\mathbf{C}_{2,2} = \mathbf{M}_1 - \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_6$$

$O(n^{2.8074})$ multiplications.

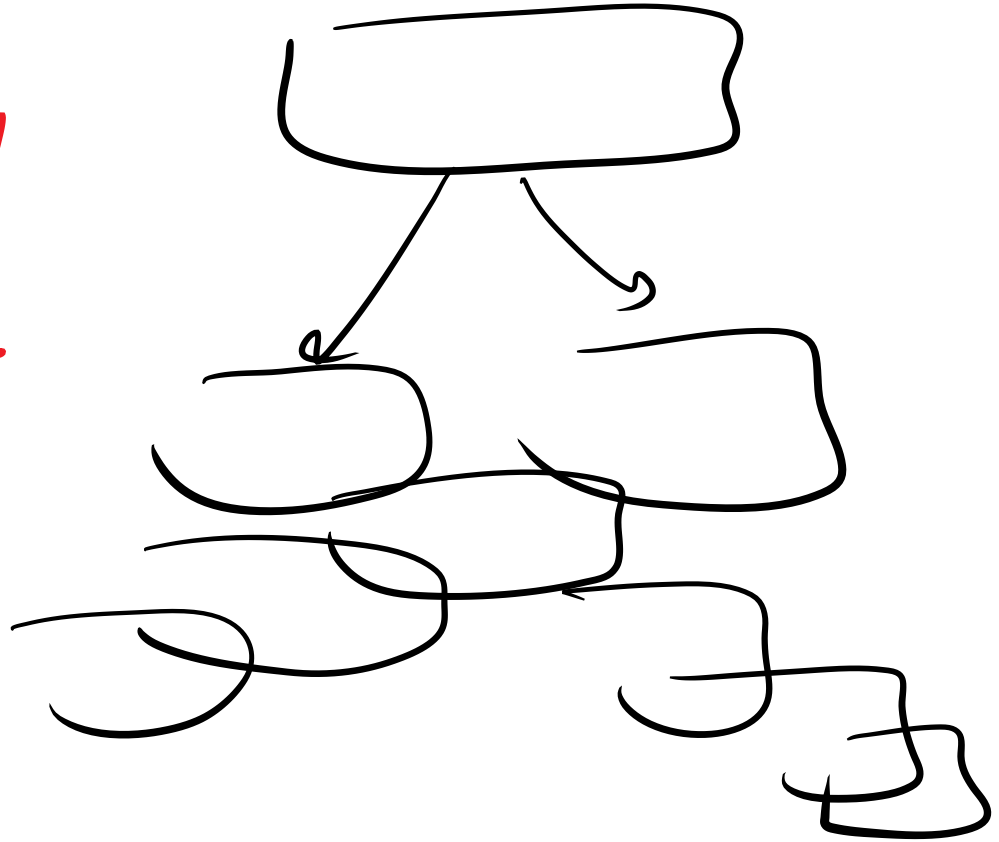
$O(n^{\log_2 7})$

$T(n) = 7T(n/2) + 18n^2$


4x4
8x8

برنامه‌نویسی چیست؟

حافظه



اصل بهینگی

fibonacci:

```
L = [1, 1]
    for i in range(2, n):
        L.append(L[i-1], L[i-2])

    return L[-1]
```

حرف حاصل $O(n)$

پیچیدگی زمان $O(n)$

```
a = 1
b = 1
for i in range(n-2):
    c = a + b
    a = b
    b = c
```

$O(1)$ حجم ثابت

$O(n)$ [illegible]

d[10|5|7|20|8]

A B C D

10×5 5×7 7×20 20×8

ضرب زنجیره ای ماتریس ها

(AB) (CD) → 10×5×7 + 7×20×8

AB 10×7

CD 7×8

مرتبه: 2016

10×7×8

(A(BC))D

BC 5×20

: 3300

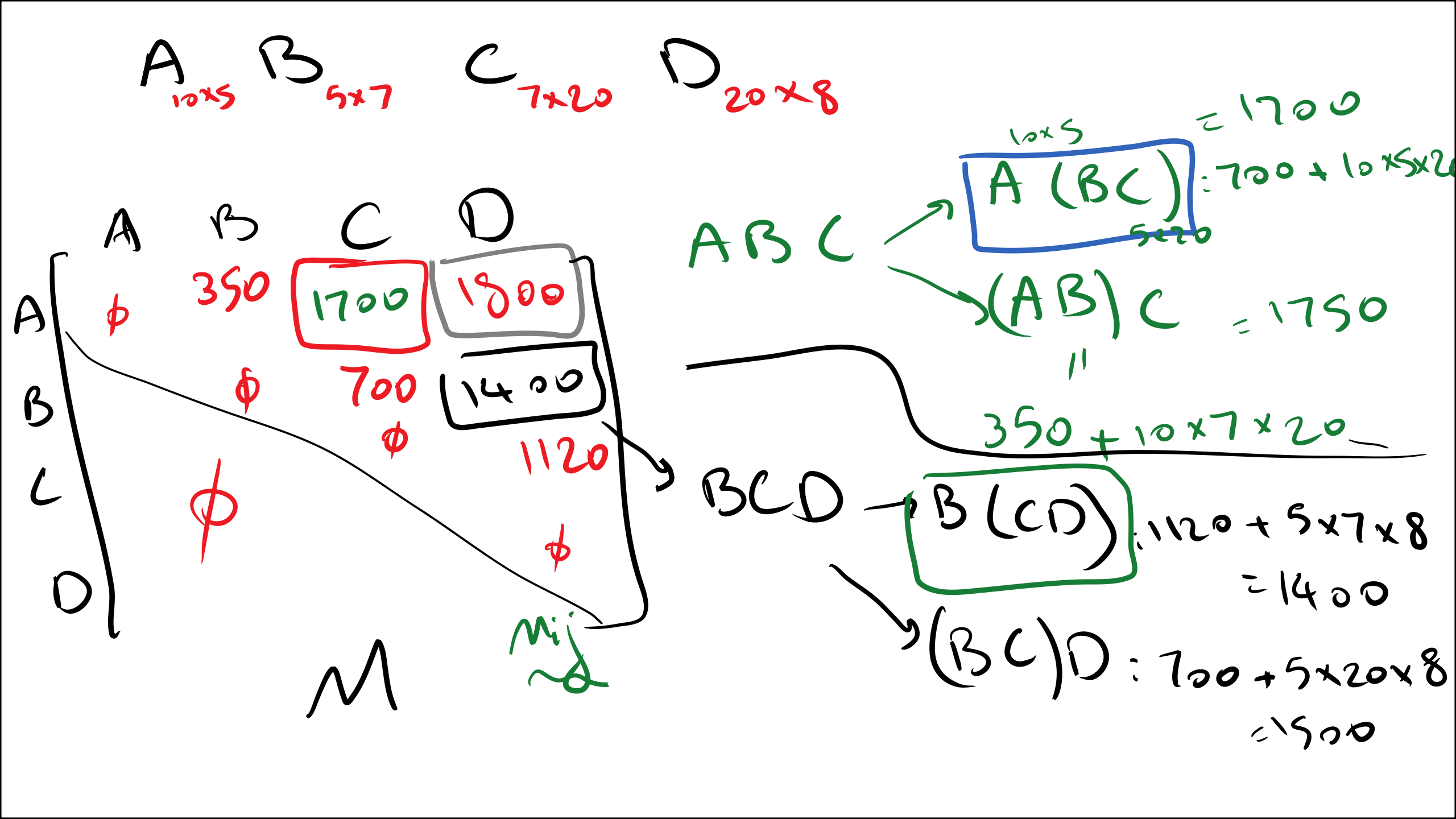
5×7×20

+10×5×20

+10×20×8

عدد کاتالان

$\frac{1}{n}\binom{2(n-1)}{n-1}$


A 10×5
B 5×7
C 7×20
D 20×8
A
B
C
D
A
B
C
D
0
350
1700
1800
0
700
1400
0
1120
0
Min
M
ABC
A(BC) : 700 + 10×5×20
= 1700
(AB)C = 1750
350 + 10×7×20
BCD
B(CD) : 1120 + 5×7×8
= 1400
(BC)D : 700 + 5×20×8
= 1500

ABCD

M14

A(B(CD)) : 1400 + 10×5×8 = 1800 ✓

(AB)(CD) = 350 + 1120 + 10×7×8 = 2030

(AB: 10×7, CD: 7×8)

(ABC)D = 1700 + 10×20×8 = 3300

(ABC: 10×20)

def MultMatrix ($d_{(n+1)}$ (ابعاد ماتریس‌ها)): n تعداد ماتریس‌ها

M = Zeros(n,n) T = Zeros(n,n)

for S=1 to n-1:

for i=1 to n-S:

j = i+S

$M_{ij} = \min_{k: i \to j-1}(M_{ik} + M_{k+1,j} + d_{i-1} \times d_k \times d_j)$

$T_{ij} = k$