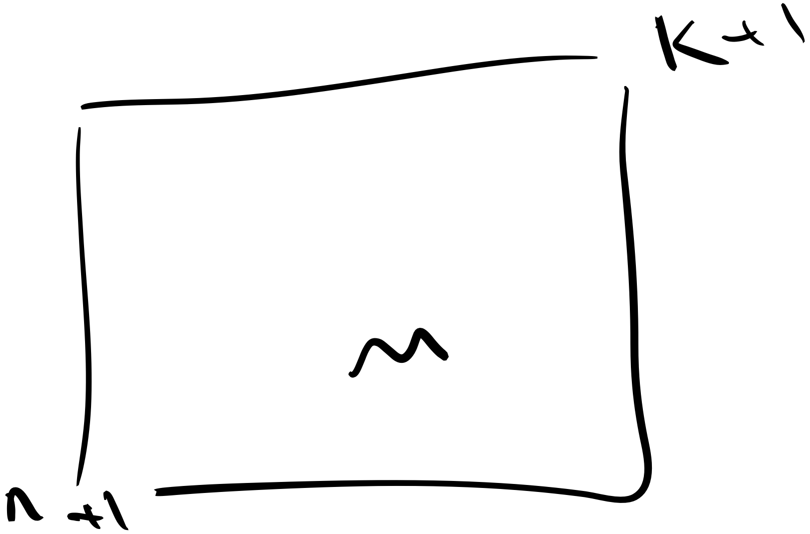
ضریب دوجمله‌ای

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{5}{3} = \binom{4}{2} + \binom{4}{3}$$

```
def f(k,n):
    if k==0 or k==n:
        return 1

    return f(k-1,n-1)
           + f(k,n-1)
```




```
def f(n,k):
    for i=0 -> n:
        for j=0 -> k:
            if j==0 or
               j==k:
                m[i,j]=1
            else:
                m[i,j]=m[i-1,j-1]+m[i-1,j]
    return m[i,j]
```

سرعت زمانی $O(nk)$

حافظه $O(nk)$

$$\binom{5}{2}$$

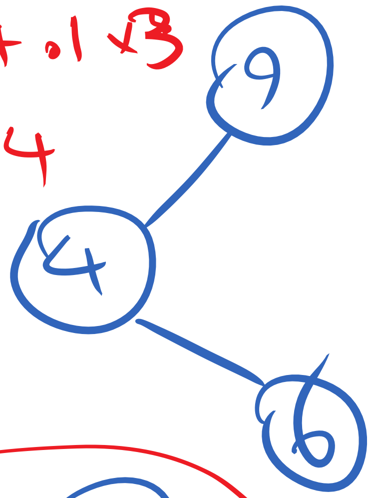
n \ k	0	1	2
0	1		
1	1	1	
2	1	2	1
3	1	3	3
4	1	4	6
5	1	5	10 ...

Optimal Binary Search Tree

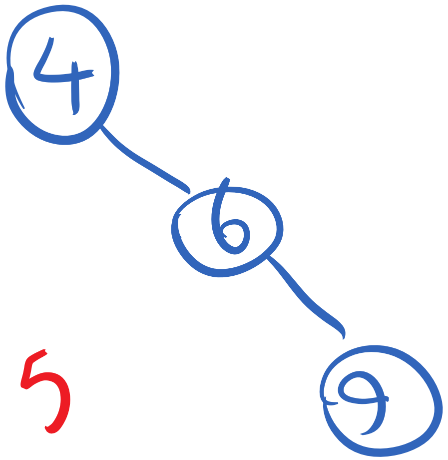
BST

.2	.1	.7
↑	↑	↑
4,	6,	9

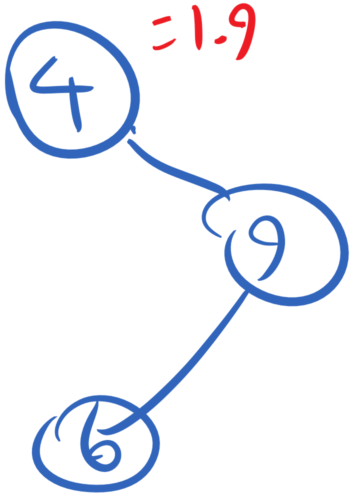
$.7 + .2 \times 2$
$+ .1 \times 3$
$= 1.4$

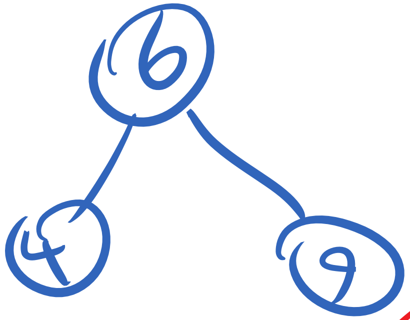


$.2 + 2 \times .1$
$+ .7 \times 3$
$= 2.5$

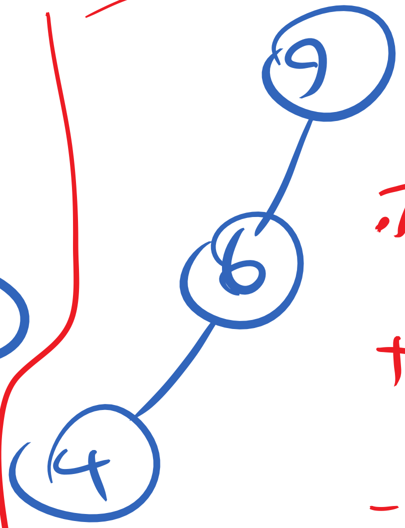


$.2 + .7 \times 2 + 3 \times .1$
$= 1.9$





$.1 + .2 \times 2$
$+ .7 \times 2$
$= 1.9$



$.7 + .1 \times 2$
$+ .2 \times 3$
$= 1.5$

$P_1 \quad P_2 \quad P_3$

$.2 \quad .1 \quad .7$

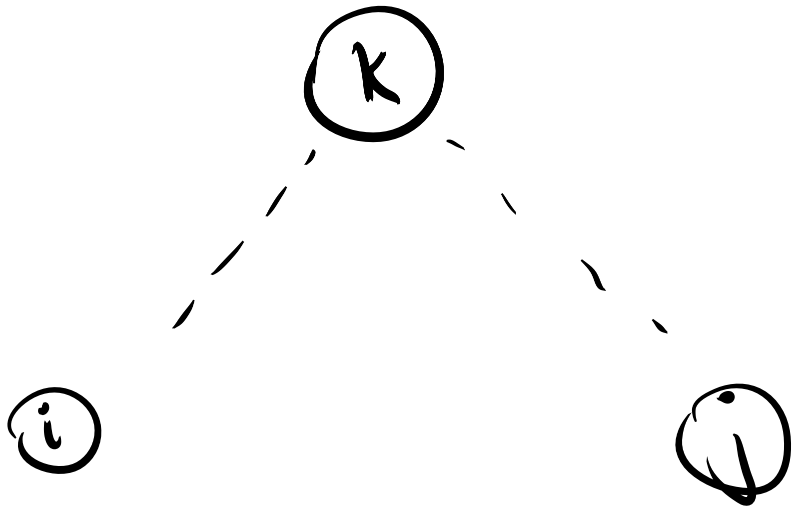
$4, \quad 6, \quad 9$

M_{ij}

$i \leq k \leq j$

$M_{ii} = P_i$

$M_{i\,i+1}$



P_1 P_2 P_3

.2 .1 .7

4, 6, 9

$M_{ij} = \min_k \{ M_{i,k-1} + M_{k+1,j} \} + \sum_{l=i}^{j} P_l$

$i \le k \le j$

$k=1$

M_{12} : 4 — 6 : $.2 + 2 \times .1 = .4$

$0 + .1 + .1 + .2 = .4$

6 — 4 : $.1 + 2 \times .2 = .5$

$k=2$

$M_{11} + M_{3,2} + P_1 + P_2$

$.2 + 0 + .2 + .1$

$$
\begin{array}{c|ccc}
 & 1 & 2 & 3 \\
\hline
1 & .2 & \boxed{.4} & \\
2 & & .1 & \\
3 & \emptyset & & .7
\end{array}
$$

$$\begin{array}{ccc} p_1 & p_2 & p_3 \\ .2 & .1 & .7 \\ \uparrow & \uparrow & \uparrow \\ 4, & 6, & 9 \end{array}$$

$$M_{ij} = \min_{k} \{ M_{i\,k-1} + M_{k+1\,j} \} + \sum_{l=i}^{j} p_l$$

$$i \le k \le j$$

$$M_{23} \begin{cases} k=2: \cancel{M_{21}} + M_{33} + p_2 + p_3 \\ \quad\ \ 0 \qquad .7 \quad + .1 + .7 = 1.5 \\ k=3: \cancel{M_{22}} + \cancel{M_{43}} + \cancel{p_2 + p_3} = .9 \\ \quad\ \ .1 \qquad 0 \qquad .8 \end{cases}$$

6

9

9

6

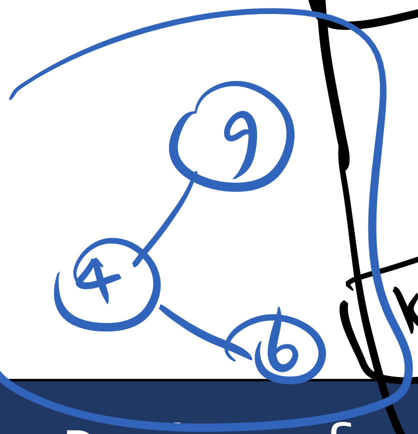
$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & .2 & \boxed{.4} & \\ 2 & & .1 & \boxed{.9} \\ 3 & \emptyset & & .7 \end{array}$$

$$\begin{array}{ccc} p_1 & p_2 & p_3 \\ .2 & .1 & .7 \\ \uparrow & \uparrow & \uparrow \\ 4, & 6, & 9 \end{array}$$

$$M_{ij} : \min_{k} \{ M_{i,k-1} + M_{k+1,j} \} + \sum_{l=i}^{j} p_l$$

$$i \le k \le j$$

$$M_{13} \begin{cases} \boxed{k=1} : \cancel{M_{10}}^{0} + M_{23} + 1 = 1.9 \\ \boxed{k=2} \; \cancel{M_{11}}^{.2} + \cancel{M_{33}}^{.7} + 1 = 1.9 \\ \boxed{k=3} : \cancel{M_{12}}^{.4} + \cancel{M_{43}}^{0} + 1 = 1.4 \end{cases}$$



$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & .2 & \boxed{.4} & \boxed{1.4} \\ 2 & & .1 & \boxed{.9} \\ 3 & \emptyset & & .7 \end{array}$$

def OBST(P[1..n]):

$M = \text{zeros}(n, n)$

for i : $1 \rightarrow n$:

$M_{ii} = P_i$

for $s = 1$ to $n-1$:

for $i = 1$ to $n-s$:

$j = i + s$

$$M_{ij} = \min_{i \le k \le j} \{ M_{i,k-1} + M_{k+1,j} \} + \sum_{l=i}^{j} p_l$$

Save

$M_{ij} \rightarrow \phi$, $i > j$

$T(n) = O(n^3)$: پیچیدگی زمانی

$O(n^2)$ پیچیدگی حافظه