$\Theta \quad \Omega \quad O \quad o \quad \omega$

$$f(n) \in \Theta(g(n)) \iff \{ \exists\, n_0, c_1, c_2$$

$$\forall\, n > n_0 \quad c_1 g(n) \leq f(n) \leq c_2 g(n) \}$$

$$f(n) \in \theta(g(n)) \iff \{ \exists\, n_0, c_1, c_2$$

$$\forall\, n > n_0 \quad c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$f(n) = 5n^2 - 20 \in \theta(n^2)$$

$$c_1 \leq 5 - \frac{20}{n^2}$$

$$c_1 n^2 \leq 5n^2 - 20 \leq c_2 n^2$$

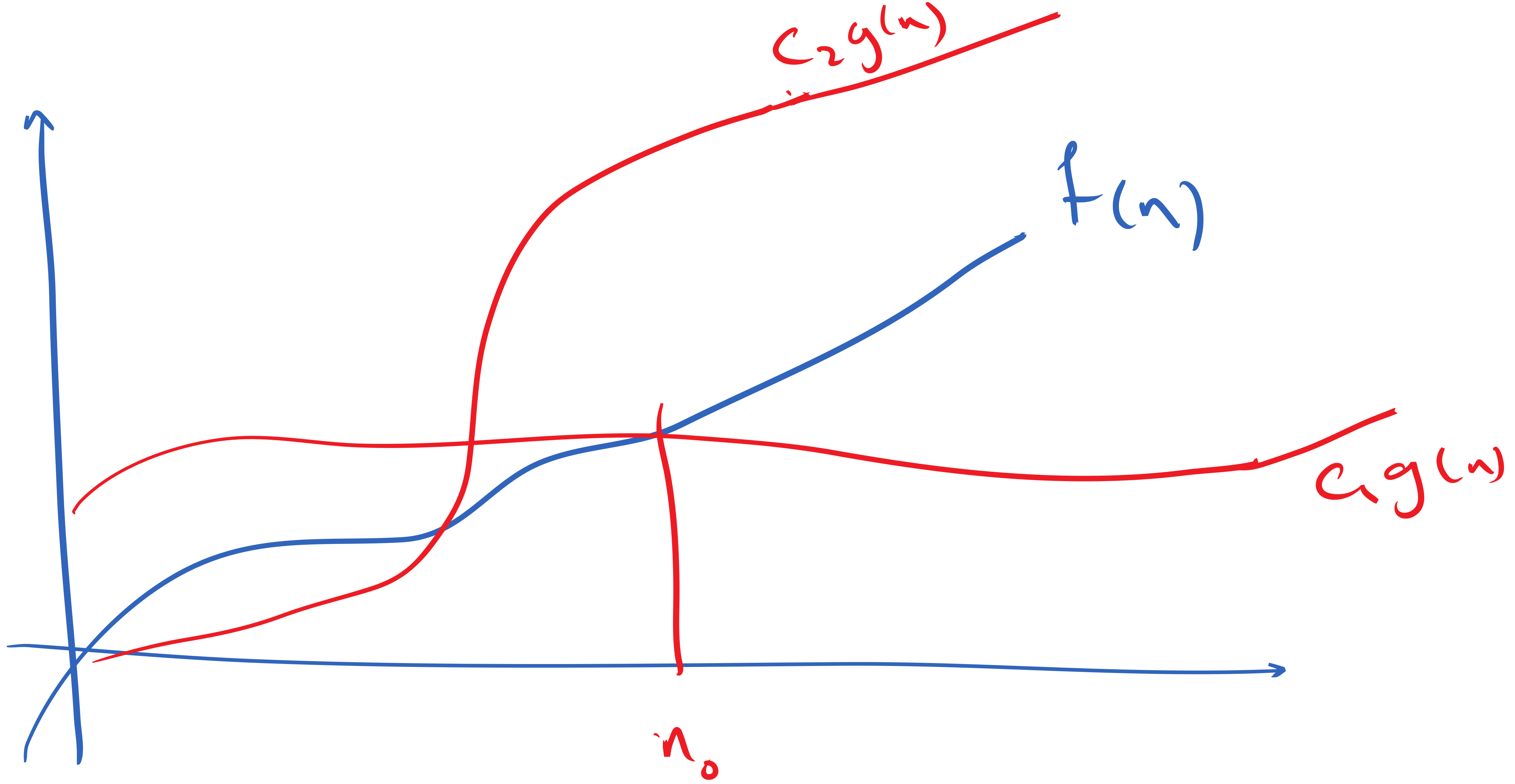
$$n_0 = \sqrt{20}$$

$$c_1 = 4$$

$$c_1 n^2 \leq 5n^2 - 20$$

$$5 - \frac{20}{n^2} \leq c_2 \qquad c_2 = 5$$

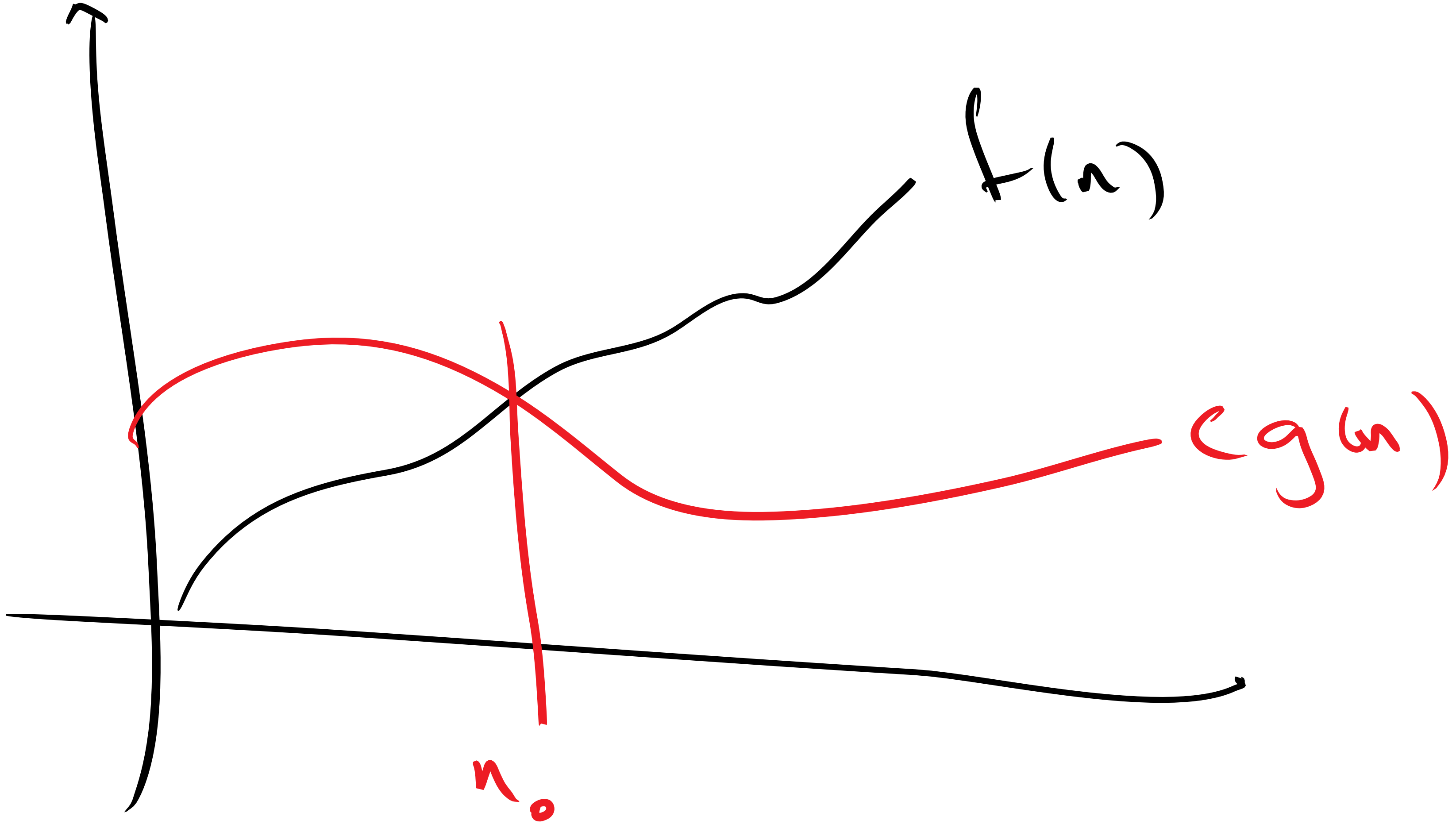
$$c_1 \leq 5 - \frac{20}{n^2}$$



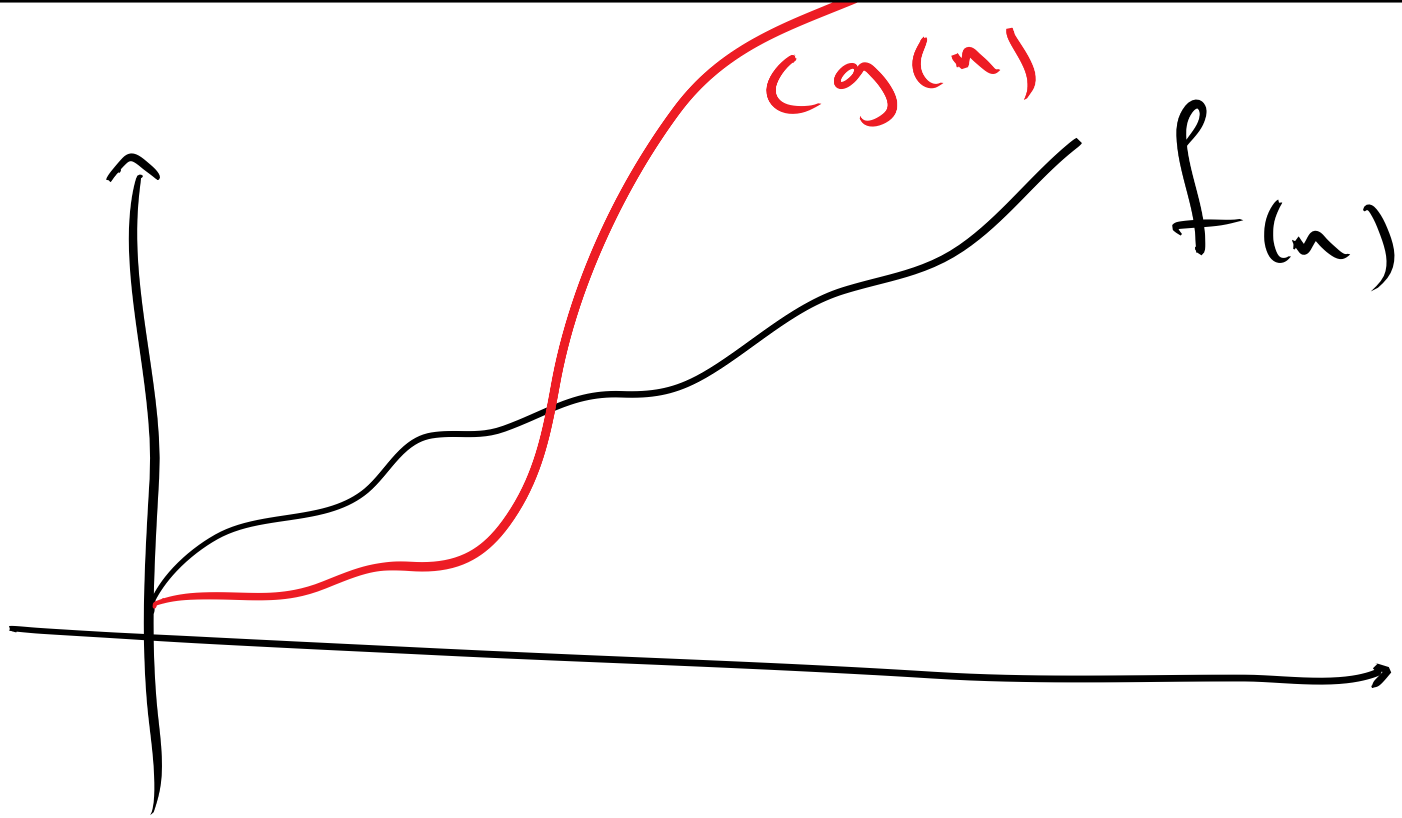
$c_2 g(n)$
$f(n)$
$c_1 g(n)$
n_0

$$f(n) \in O(g(n)) : \{ \exists\, n_0, c \quad \forall\, n > n_0 : f(n) \leq c\,g(n) \}$$

$$f(n) \in \Omega(g(n)) : \{ \exists\, n_0, c \quad \forall\, n > n_0 : c\,g(n) \leq f(n) \}$$



Ω



O

جست وجوی ترتیبی :

$B(n) = 1$

$W(n) = n$

$$A(n) = \sum_{i=1}^{n} \frac{1}{n} \times i = \frac{1}{\cancel{n}} \sum_{i=1}^{n} i = \frac{1}{\cancel{n}} \times \frac{\cancel{n}(n+1)}{2} = \frac{n+1}{2}$$

احتمال حضور عنصر در لیست P

$$A(n) = \sum_{i=1}^{n} \frac{P}{n} \times i + (1-P) \times n = \frac{P}{\cancel{n}} \times \frac{\cancel{n}(n+1)}{2} + (1-P) \times n$$

$f(n) \in \Theta(g(n))$

O

Ω

$\exists\, c_1, c_2, n_0 \quad \forall\, n \geq n_0 \quad c\, g(n) \leq f(n) \leq c_2\, g(n)$

$$f(n) \in \underset{\text{small}}{o}(g(n))$$

$$\forall c>0 \;\exists n_0>0 \quad \forall n > n_0 \qquad 0 < f(n) \leq c\,g(n)$$

$$n^2+2n \in o(n^2)$$

$$n^2+2n \leq c n^2$$

$$1+\frac{2}{n} \leq c$$

$$\overbrace{n^2+4n}^{f} \in \Omega(\overbrace{n^3}^{g})$$

$f(n) \in \Omega(g(n))$

$\forall n > n_0$

$\exists c, n_0 > 0 :$

$$c\,g(n) \le f(n)$$

$$c n^3 \le n^2 + 4n$$

$$c \le \frac{1}{n} + \frac{4}{n^2}$$

$$\cancel{c = 0}$$

$$4n^2 + 3n - 5 \in \theta(n^2)$$

$\forall n > n_0$

$\exists n_0, c_1, c_2 > 0 \quad : \quad c_1 n^2 \le 4n^2 + 3n - 5 \le c_2 n^2$

$$c_1 \le 4 + \frac{3}{n} - \frac{5}{n^2} \le c_2$$

$n_0 = \sqrt{5}$ $\qquad c_1 \le 3$ ✓ $\qquad\qquad n_0 = 3$ $\qquad c_2 \ge 5$

$\log_a^n \in \Theta(\log_b^n)$ $\quad$ $a < b$

$c_1 \log_b^n \le \log_a^n \le c_2 \log_b^n$

$\log_b^n$ $\quad$ $c_1 \le \dfrac{\log_a^n}{\log_b^n} \le c_2$ $\rightarrow$ عدد ثابت

$f(n)$ $\quad \begin{matrix}\theta \\ \Omega \\ \omega \\ o\end{matrix} \quad$ $g(n)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \begin{cases} \infty \\ c \\ 0 \end{cases}$$

∞: رشد f از g بیشتر است

$f(n) \in \Omega(g(n))$, $f(n) \in \omega(g(n))$

$g(n) \in O(f(n))$ $\quad$ $g(n) \in o(f(n))$

c:

$f(n) \in \theta(g(n))$ $\quad$ $f(n) \in \Omega(g(n))$ ،و

$f(n) \in O(g(n))$ $\quad$ $g(n) \in \theta(f(n))$ هم رشد

0: رشد f از g کمتر است

$f(n) \in O(g(n))$ $\quad$ $f(n) \in o(g(n))$

$g(n) \in \Omega(f(n))$ $\quad$ $g(n) \in \omega(f(n))$

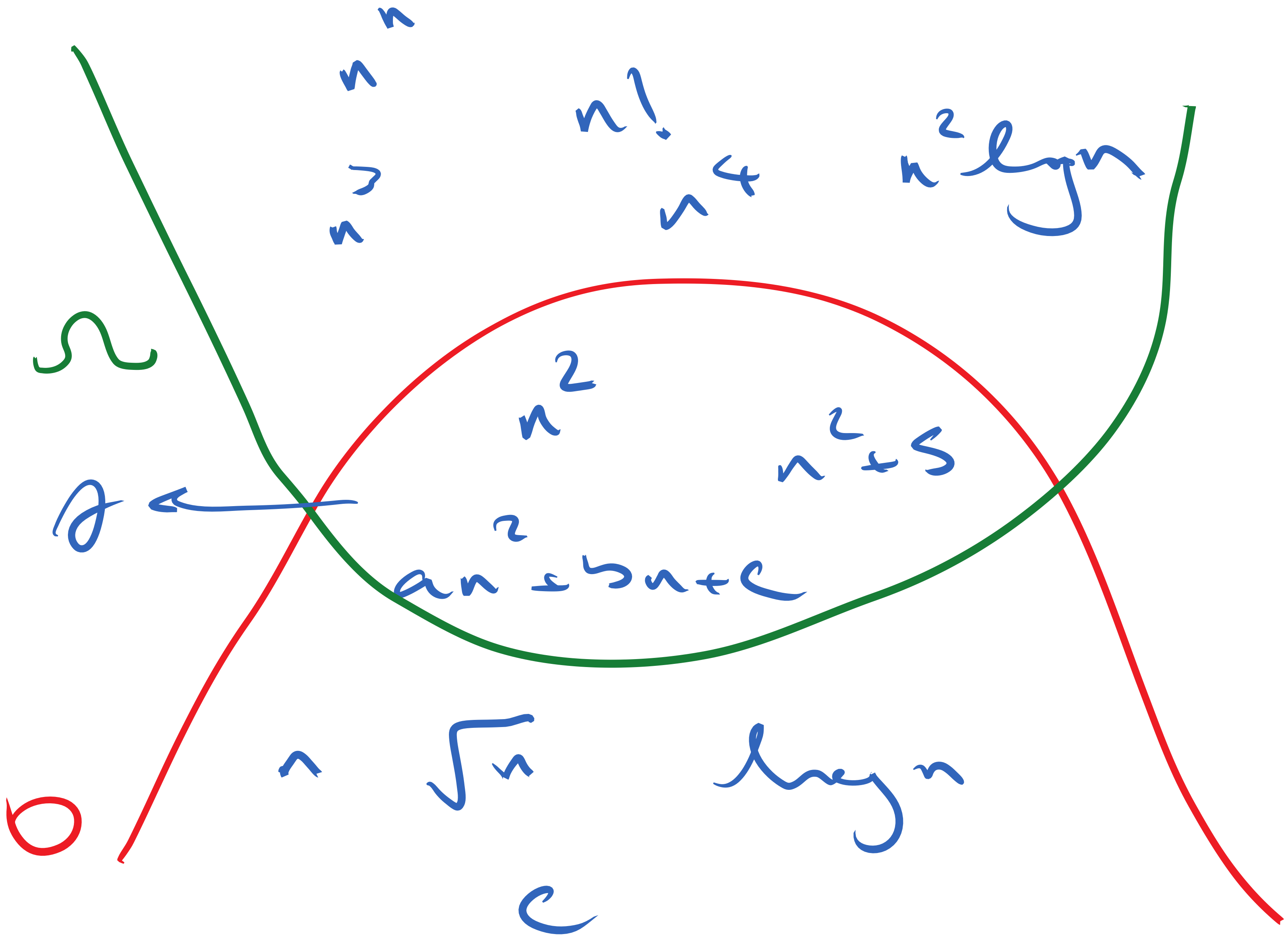
$$\sqrt{n} \in \Theta(\log_a(n))$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\log_a n} \overset{\text{hop}}{=\!=\!\Rightarrow} \lim \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n\ln a}} = \lim \frac{n\ln a}{2\sqrt{n}} =$$

$$\lim_{n\to\infty} \frac{\ln a}{\frac{1}{2\sqrt{n}}} = \frac{\sqrt{n}}{\ln a}$$

$$= \infty$$

$f(n) = n^2 + 5n - 6$



$$T(n) = T(n-1) + T(n-2)$$

$$T(n) = T(n/2) + 2$$

$$n = 2^k \longrightarrow k = \log_2^n$$

$$T(2^k) = T(2^{k-1}) + 2$$

$$T(k) = T(k-1) + 2$$

$$r - 1 = 0 \longrightarrow r = 1$$

$$T(n) = 2T(n/2) + n - 1$$

$$T(k) = \underbrace{\alpha 1^k}_{\times} + \underbrace{\alpha_2 k 1^k}$$

$$T(n) \in \theta(\alpha \log_2^n)$$

$$\theta T(n) = \alpha T(n-1) + \beta(T-2) + \gamma T(n-3)$$

$$\theta(T(n)) - \alpha(T(n-1)) - \beta(T(n-2)) - \gamma(T(n-3))$$

$$\theta r^3 - \alpha r^2 - \beta r - \gamma = 0$$

- r_1
- r_2
- r_3

$$T(n) = t_1 r_1^n + t_2 r_2^n + t_3 r_3^n$$

$r_1 = r_2$, $r_1 \neq r_3$ $\Rightarrow T(n) = t_1 r_1^n + t_2 n r_1^n + t_3 r_3^n$

$r_1 = r_2$

$r_1 = r_2 = r_3 \Rightarrow T(n) = t_1 r_1^n + t_2 n r_1^n + t_3 n^2 r_1^n$

$$T(n) = T(n-1) + c2^n$$

$r - 1 = 0$

$r = 1$

$$T(n) = t_1 1^n + t_2 2^n$$

$$T(n) = 2T(n-1) + 2^n$$

$r - 2 = 0$

$r = 2$

$$T(n) = t_1 2^n + t_2 n 2^n$$

$$T(n) = T(n-1) + c1^n$$

$$T(n) = 2T(n/2) + n$$

$$n = 2^k \rightarrow k = \log_2 n$$

$$T(2^k) = 2T(2^{k-1}) + 2^k$$

$$T(k) = 2T(k-1) + 2^k$$

$$T(k) = \alpha_1 2^k + \alpha_2 k\, 2^k$$

$r - 2 = 0 \quad r = 2$

$$T(n) = \underbrace{\alpha_1 n} + \alpha_2 \cdot \underbrace{\log_2 n \cdot n}_{n \log n}$$

$$T(n) = 2T(n/2) + 3T(n/4) + 4^n + 8$$

$$n = 2^k \longrightarrow 4^{(2^k)}$$

$$T(2^k) = 2T(2^{k-1}) + 3T(2^{k-2}) + 4^{2^k} + 8$$

$$r^2 - 2r - 3 = 0$$

$r_1 : 3$

$r_2 : -1$

$$\Rightarrow T(k) = \underbrace{\alpha_1 3^k + \alpha_2 (-1)^k + \alpha_3 4^{2^k} + \alpha_4 1^k}$$

$k = \log_2^n$

$T(n) = O(4^n)$

نظرية Master

$$T(n) = aT(n/b) + f(n)$$

① $\left(n^{\log_b^{a-\epsilon}}\right) \in O(f(n))$ ① $T(n) \in O(f(n))$

② $\left(n^{\log_b^{a}}\right) \in \Theta(f(n))$ ② $T(n) \in \Theta(\log n \times f(n))$

③ $\left(n^{\log_b^{a+\epsilon}}\right) \in \Omega(f(n))$ ③ $\rightarrow T(n) \in \Theta\left(n^{\log_b^{a}}\right)$

$$T(n) = 2T(n/2) + n$$

$$n^{\log_2 2} = n \qquad f(n) = n \qquad \Rightarrow T(n) = n \log n$$

$$T(n) = T(n/2) + 2$$

$$n^{\log_2 1} = 1 \qquad f(n) = c \qquad \Rightarrow T(n) = \log n$$

$T(n) = 4T(n/2) + n$ $\qquad$ $T(n) = O(n^2)$

$n^{\log_2 4}$ $\qquad$ n

$\underset{\sim}{n^2}$ $\qquad$ $\sim$