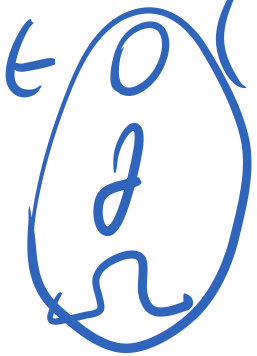
انحصری

$f(n) \in \Theta(f(n))$



$\propto$

تقریبی

$f(n) \in O(g(n))$ Ω θ

$g(n) \in O(f(n))$ Ω θ $\times$ $\times$ $\checkmark$

$f(n) \in O(g(n))$ یعنی $g(n) \notin O(f(n))$ ✗

$f(n) \in O(g(n))$ ، $g(n) \in O(h(n))$ ✓ نتیجه

$\rightarrow f(n) \in O(h(n))$

روش‌های تقسیم و غلبه

- ضرب اعداد بزرگ
- ضرب ماتریس‌ها
- Binary Search
- Merge Sort
- Quick Sort

```
def mergeSort(l,H):
    if (L<=H):
        m = (L+H)/2
        mergeSort(l, m)
        mergeSort(m+1, H)
        merge(l, m, H)
    else:
        return
```

$$B(n) = 2B(n/2) + n/2$$

$$w(n) = 2w(n/2) + n - 1$$

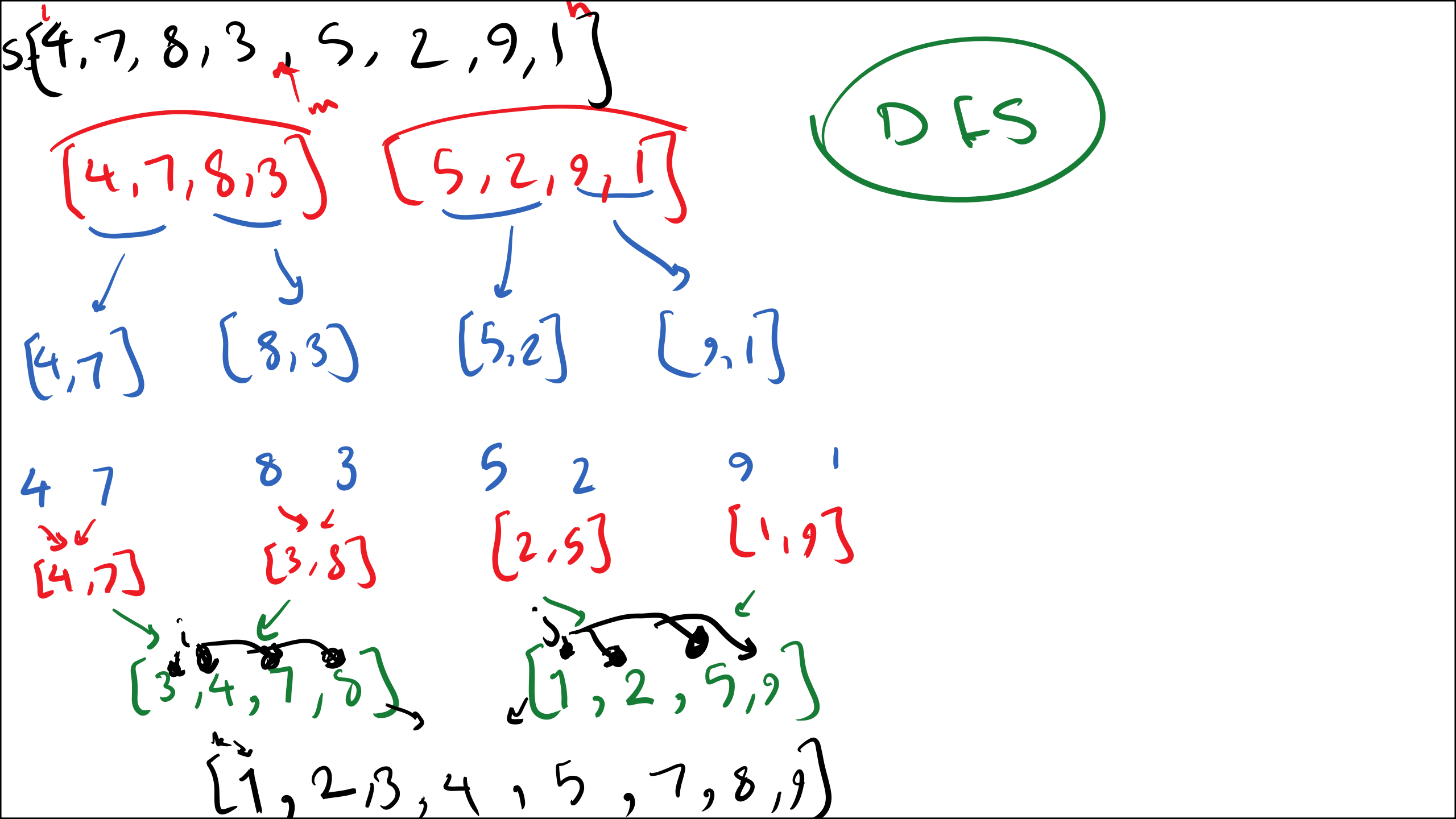
$$O(n \log(n))$$

```
def merge(L, m, H):
    i=L ; j=m+1 ; k=L
    U : new list ()
    while (i<=m and j<=H)
        if (S[i]<S[j])
            U[k++] = S[i++]
        else
            U[k++] = S[j++]
    while (i<=m)
        U[k++] = S[i++]
    while(j<=H)
        U[k++] = S[j++]
    S[L..H] = U[L..H]
```

n طول لیست

$B(n) = n/2$

$w(n) = n-1$


[4,7,8,3,5,2,9,1]
DFS
[4,7,8,3]
[5,2,9,1]
[4,7]
[8,3]
[5,2]
[9,1]
4 7
8 3
5 2
9 1
[4,7]
[3,8]
[2,5]
[1,9]
[3,4,7,8]
[1,2,5,9]
[1,2,3,4,5,7,8,9]

```
QuickSort(L,H)
if(L>=H): return
p = partition(L,H)
QuickSort(L,p-1)
QuickSort(p+1,H)
```

$W(n) = W(n-1) + n-1$

$O(n^2)$

$B(n) = 2B(n/2) + n-1$

$O(n \log n)$

```
def partition(L, H)
    j = L        pivot = S[L]

    for i : L+1 → H :
        if S[i] < pivot :
            j++ ;
            swap(S[i], S[j])
    swap(S[L], S[j])
    return j
```

$\omega(n) = n-1$

وسط Range اگر X ی لیست بهترین حالت

طول

[illegible] pivot عنصر وسط لیست

generate شود.

$$A(n) = n-1 + \frac{1}{n}\left(\sum_{p=1}^{n} A(p-1) + \sum_{p=1}^{n} A(n-p)\right)$$

$$A(n) = n-1 + \frac{2}{n}\sum_{p=1}^{n} A(p-1)$$

$\times n$

$$nA(n) = n(n-1) + 2\left(\sum_{p=1}^{n} A(p-1)\right)$$

$$(n-1)A(n-1) = (n-1)(n-2) + 2\sum_{p=1}^{n-1} A(p-1)$$

$$nA(n) = n(n-1) + 2\left(\sum_{p=1}^{n} A(p-1)\right) \quad \text{(I)}$$

$$(n-1)A(n-1) = (n-1)(n-2) + 2\sum_{p=1}^{n-1} A(p-1) \quad \text{(II)}$$

I − II $\Rightarrow$

$$nA(n) - (n-1)A(n-1) = 2(n-1) + 2A(n-1)$$

$$nA(n) = 2(n-1) + (n+1)A(n-1) \quad / \, n(n+1)$$

$$\Rightarrow \frac{A(n)}{n+1} = \frac{2(n-1)}{n(n+1)} + \frac{A(n-1)}{n}$$

$$a_n = \frac{2(n-1)}{n(n+1)} + a_{n-1}$$

$$a_n = \frac{A(n)}{n+1}$$

$$a_n = 2\ln n$$

$$A_n = n\ln n$$