Approximation of Circle by Cubic Bezier Curves

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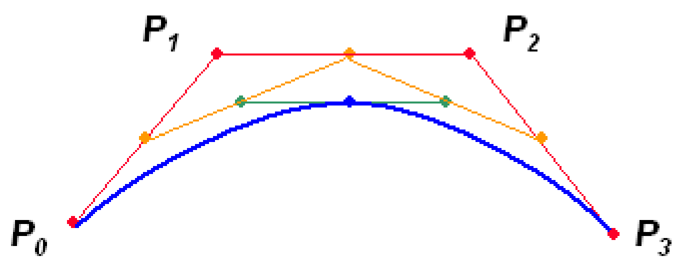
We would approximates the Quarter circle (of unit radius) with Bezier Curve then by symmetry complete circle can be easily approximated using four bezier curves.

Set control points so that the midpoint of the Bezier curve locates on the mid point of quarter circle

<u>Mid Point of Quarter Circle</u>

Mid Point of Quarter Circle= $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ =(0.7071,0.7071)

<u>Mid Point of Cubic Bezier Curve that approximates quarter circle:</u>



P_0, P_1, P_2 and P_3 are the four control points of the Bezier curve. To approximate quarter unit circle P_0 and P_3 are at (1,0) and (0,1) respectively (for first quadrant). The middle two control points P_1 and P_2 must be chosen so that the curve is tangent to the quarter circle. P_1 must have the same x coordinate as P_0 and P_2 must have the same y coordinate as P_3. Since quarter circle is symmetric, the best approximating Bezier curve should also be symmetric. Now our four control point have following values.

P_0=(1,0)
P_1=(1,k)
P_2=(k,1)
P_3=(0,1)

Using the midpoint subdivision rule:

$A = \frac{(2,k)}{2}$ (Mid Point of P_0 and P_1)

$B = \frac{(k+1, k+1)}{2}$ (Mid Point of P_1 and P_2)

$C = \frac{(k,2)}{2}$ (Mid Point of P_2 and P_3)

Similarly

$D = \frac{(k+3, 2k+1)}{4}$ (Mid Point of A and B)

$E = \frac{(2k+1, k+3)}{4}$ (Mid Point of B and C)

Finally

$\mathrm{F} = \frac{(3k+4, 3k+4)}{8}$ (Mid Point of D and E)

Since F should be on midpoint of quarter circle

Therefore

$$F = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{(3k+4, 3k+4)}{8}$$

$$\frac{(3k+4)}{8} = \frac{1}{\sqrt{2}}$$

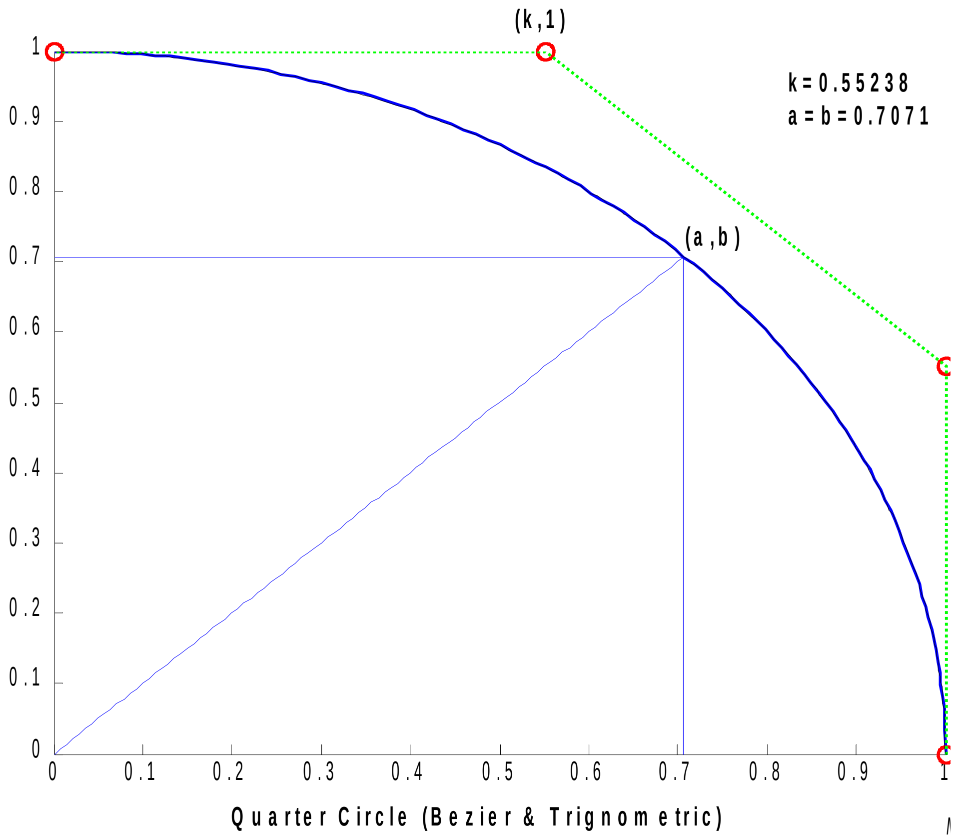
Solving for k

k*=*0.55228474983079339840225163227960

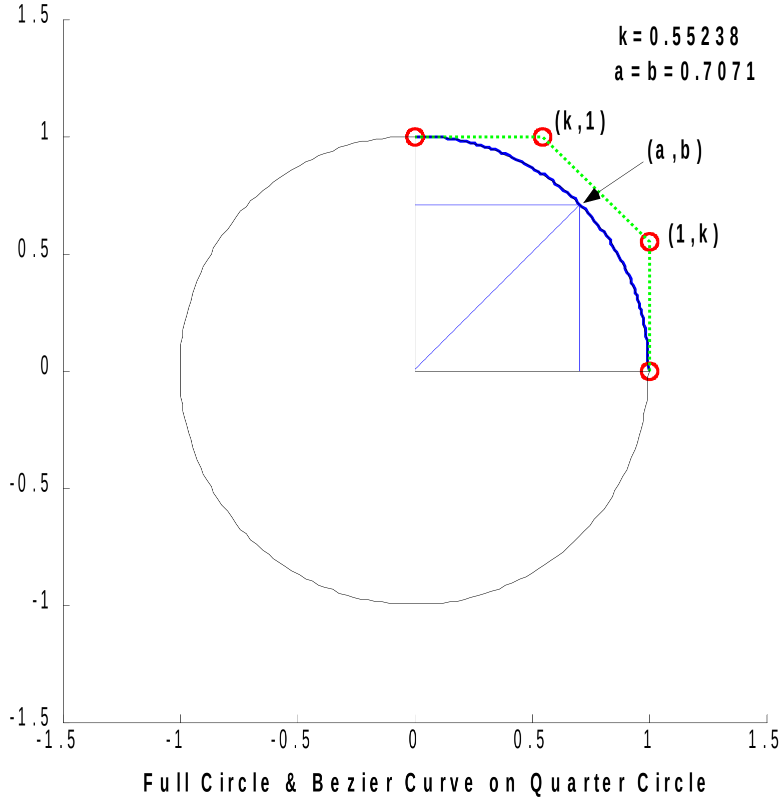
Hence

P_1=(1, *0.5522847*)

P_2=(*0.5522847*,1)



(k,1)
k=0.55238
a=b=0.7071
(a,b)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Quarter Circle (Bezier & Trignometric)


k = 0.55238
a = b = 0.7071
(k,1)
(a,b)
(1,k)
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
Full Circle & Bezier Curve on Quarter Circle