Levenberg Marquardt - Point to Line Iterative Closest Point (LM-PLICP)

Gradient-Descent

$$x_{i+1} = x_i + \lambda \nabla_{f(x)}$$

Newton-Rapshon

$$x_{i+1} = x_i + (\nabla^2_{f(x_i)})^{-1} \nabla_{f(x)}$$

Gauss-Newton

$$x_{i+1} = x_i + (J^T_{f(x)} J_{f(x)})^{-1} \nabla_{f(x)}$$

Levenberg-Marquardt

$$x_{i+1} = x_i + (\nabla^2_{f(x_i)} + \lambda \ \mathrm{diag}(\nabla^2_{f(x_i)}))^{-1} \nabla_{f(x)}$$

Point to Line Iterative Closest Point

$$E = \underset{R,t}{\mathrm{argmin}} \sum_{i=1}^{n} ||(y_i - Rx_i + t) n_{y,i}||^2$$

$n_{y,i}$ vektor normal pada point cloud reference dihitung dengan $\frac{-dy}{dx}$

Approximation Hessian $H = J^T J$

Fungsi loss E merupakan fungsi non-linear *differentiable* dan smooth dikarenakan terdapat matriks rotasi R, sehingga Taylor-Expansion valid untuk menyelesaikan fungsi non-linear E

$$\text{Taylor-Expansion :}$$
$$F(x + \Delta x) = F(x) + \frac{1}{1!} F'(x) \Delta x + \frac{1}{2!} F''(x) \Delta x^2 + \ldots$$

Sehingga,

$$y(x) = f(x; s)$$
$$r_i = y_i - f(x_i; s)$$

$f(x; s)$ merupakan fungsi non-linear, r_i merupakan residual, y_i merupakan target, dan s merupakan state yaitu (t_x, t_y, θ) yang akan dioptimisasi, dengan **First Order Taylor-Expansion** untuk menghitung perubahan pada state.

$$f(x_i; s') = \underbrace{f(x_i; s)}_{y_i - r_i} + \frac{\partial f}{\partial s}(s' - s) = y_i$$

Agar $f(x_1; s') = y_i$ maka $\frac{\partial f}{\partial s}(s' - s) = r_i$, ubah s menjadi (t_x, t_y, θ),

$$\begin{gathered}
\frac{\partial f}{\partial t_x}\Delta t_x + \frac{\partial f}{\partial t_y}\Delta t_y + \frac{\partial f}{\partial \theta}\Delta\theta = r_1 \\
\frac{\partial f}{\partial t_x}\Delta t_x + \frac{\partial f}{\partial t_y}\Delta t_y + \frac{\partial f}{\partial \theta}\Delta\theta = r_2 \\
\vdots \quad + \quad \vdots \quad + \quad \vdots \quad = \vdots \\
\frac{\partial f}{\partial t_x}\Delta t_x + \frac{\partial f}{\partial t_y}\Delta t_y + \frac{\partial f}{\partial \theta}\Delta\theta = r_n
\end{gathered}$$

Sehingga dapat disederhanakan menjadi,

$$\begin{array}{c} x = x_1 \\ \vdots \\ x = x_n \end{array} \begin{bmatrix} \frac{\partial f}{\partial t_x} & \frac{\partial f}{\partial t_y} & \frac{\partial f}{\partial \theta} \\ \vdots & \vdots & \vdots \\ \frac{\partial f}{\partial t_x} & \frac{\partial f}{\partial t_y} & \frac{\partial f}{\partial \theta} \end{bmatrix} \begin{bmatrix} \Delta t_x \\ \Delta t_y \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix}$$

Sehingga,

$$J\Delta s = r$$

Dikarenakan matriks J bukan matriks persegi, sehingga matriks J ***non-invertible*** matriks. Agar dapat menyelesaikan persamaan tersebut, kedua sisi dikalikan dengan J^T, menjadi

$$\begin{gathered}
(J^T J)\Delta s = J^T r \\
\Delta s = (J^T J)^{-1} J^T r
\end{gathered}$$

References :

- METHODS FOR NON-LINEAR LEAST SQUARES PROBLEMS
- Nonlinear Least Squares