1 propositional logic

a *proposition* is a precice statement that is either `true` ($\top$) or false ($\bot$), but not both. for example:
- $2 + 2 = 4$ (true)
- all dogs have 3 legs (false)
- $x^2 < 0$ (false)

however, not all statements are propositions. for example:
- eliana is cool
  - ‣ *cool* is a subjective term.
- $x^3 < 0$
  - ‣ true if $x < 0$, false otherwise.
- springfield is the capital
  - ‣ true in illinois, false in massachusetts.

1.1 logical connectives

1.1.1 ¬ negation

the *negation* (or *not*) of a proposition is true *iff* the proposition is false.
- $\neg\bot = \top$
- $\neg\top = \bot$

1.1.2 ∧ conjunction

the *conjunction* (or *and*) of two propositions is true *iff* **both** propositions are true.
- $\top \wedge \top = \top$
- $\top \wedge \bot = \bot$
- $\bot \wedge \top = \bot$
- $\bot \wedge \bot = \bot$

1.1.3 ∨ disjunction

the *disjunction* (or *or*) of two propositions is true *iff* **at least one** proposition is true.
- $\top \vee \top = \top$
- $\top \vee \bot = \top$
- $\bot \vee \top = \top$
- $\bot \vee \bot = \bot$

1.1.4 ⊕ exclusive disjunction

the *exclusive disjunction* (or *xor*) of two propositions is true *iff* **exactly one** proposition is true.

- $\top \oplus \top = \bot$
- $\top \oplus \bot = \top$
- $\bot \oplus \top = \top$
- $\bot \oplus \bot = \bot$

1.1.5 → implication

the *implication* $p \to q$ is false if p is true, and q is false $p \to q$ is true otherwise. hint: p is called the *hypothesis*, and q is called the *conclusion*.

- $\top \to \top = \top$
- $\top \to \bot = \bot$
- $\bot \to \top = \top$
- $\bot \to \bot = \top$

1.1.6 ↔ biconditional

the *biconditional* $p \leftrightarrow q$ is true *iff* p and q **assume the same** truth value.

- $\top \leftrightarrow \top = \top$
- $\top \leftrightarrow \bot = \bot$
- $\bot \leftrightarrow \top = \bot$
- $\bot \leftrightarrow \bot = \top$

1.2 logic operators order

1. ¬ negation
   - $\neg q \lor r$ means $(\neg q) \lor r$, not $\neg(q \lor r)$
2. ∧ conjunction
3. ∨ disjunction
   - $q \land r \lor s$ means $(q \land r) \lor s$, not $q \land (r \lor s)$
4. → implication
   - $q \land r \to s$ means $(q \land r) \to s$, not $q \land (r \to s)$
5. ↔ biconditional

1.3 other conditional statements

given an implication $p \to q$:

- $\neg p \to \neg q$ is its *inverse*
- $q \to p$ is its *converse*
- $\neg q \to \neg p$ is its *contrapositive*

1.4 activities

1.4.1 two propositions

show that an implication $p \rightarrow q$ and its *contrapositive* $\neg q \rightarrow \neg p$ always have the same value.

p, q	$\neg q, \neg p$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow p)$
$\top \top$	$\bot \bot$	$\top$	$\top$	$\top$
$\top \bot$	$\top \bot$	$\bot$	$\bot$	$\top$
$\bot \top$	$\bot \top$	$\top$	$\top$	$\top$
$\bot \bot$	$\top \top$	$\top$	$\top$	$\top$

1.4.2 complex example 1

construct a truth table for $(p \wedge q) \rightarrow \neg r$.

p, q, r	$p \wedge q$	$\neg r$	$(p \wedge q) \rightarrow \neg r$
$\top \top \top$	$\top$	$\bot$	$\bot$
$\top \top \bot$	$\top$	$\top$	$\top$
$\top \bot \top$	$\bot$	$\bot$	$\top$
$\top \bot \bot$	$\bot$	$\top$	$\top$
$\bot \top \top$	$\bot$	$\bot$	$\top$
$\bot \top \bot$	$\bot$	$\top$	$\top$
$\bot \bot \top$	$\bot$	$\bot$	$\top$
$\bot \bot \bot$	$\bot$	$\top$	$\top$

1.5 complex example 2

construct a truth table for $p \wedge (\neg q \vee r) \rightarrow r$.

p, q, r	$\neg q$	$\neg q \vee r$	$p \wedge (\neg q \vee r)$	$p \wedge (\neg q \vee r) \rightarrow r$
$\top \top \top$	$\bot$	$\top$	$\top$	$\top$
$\top \top \bot$	$\bot$	$\bot$	$\bot$	$\top$
$\top \bot \top$	$\top$	$\top$	$\top$	$\top$
$\top \bot \bot$	$\top$	$\top$	$\top$	$\bot$
$\bot \top \top$	$\bot$	$\top$	$\bot$	$\top$

p, q, r	$\neg q$	$\neg q \vee r$	$p \wedge (\neg q \vee r)$	$p \wedge (\neg q \vee r) \rightarrow r$
$\bot \top \bot$	$\bot$	$\bot$	$\bot$	$\top$
$\bot \bot \top$	$\top$	$\top$	$\bot$	$\top$
$\bot \bot \bot$	$\top$	$\top$	$\bot$	$\top$