1 propositional logic

a *proposition* is a precice statement that is either true ($\top$) or false ($\bot$), but not both. for example:
- $2 + 2 = 4$ (true)
- all dogs have 3 legs (false)
- $x^2 < 0$ (false)

however, not all statements are propositions. for example:
- eliana is cool
  - *cool* is a subjective term.
- $x^3 < 0$
  - true if $x < 0$, false otherwise.
- springfield is the capital
  - true in illinois, false in massachusetts.

1.1 logical connectives

1.1.1 ¬ negation

the *negation* (or *not*) of a proposition is true *iff* the proposition is false.
- $\neg\bot = \top$
- $\neg\top = \bot$

1.1.2 ∧ conjunction

the *conjunction* (or *and*) of two propositions is true *iff* **both** propositions are true.
- $\top \land \top = \top$
- $\top \land \bot = \bot$
- $\bot \land \top = \bot$
- $\bot \land \bot = \bot$

1.1.3 ∨ disjunction

the *disjunction* (or *or*) of two propositions is true *iff* **at least one** proposition is true.
- $\top \lor \top = \top$
- $\top \lor \bot = \top$
- $\bot \lor \top = \top$
- $\bot \lor \bot = \bot$

1.1.4 $\oplus$ exclusive disjunction

the *exclusive disjunction* (or *xor*) of two propositions is true *iff* **exactly one** proposition is true.

- $\top \oplus \top = \bot$
- $\top \oplus \bot = \top$
- $\bot \oplus \top = \top$
- $\bot \oplus \bot = \bot$

1.1.5 $\rightarrow$ implication

the *implication* $p \rightarrow q$ is false if p is true, and q is false $p \rightarrow q$ is true otherwise. hint: p is called the *hypothesis*, and q is called the *conclusion*.

- $\top \rightarrow \top = \top$
- $\top \rightarrow \bot = \bot$
- $\bot \rightarrow \top = \top$
- $\bot \rightarrow \bot = \top$

1.1.6 $\leftrightarrow$ biconditional

the *biconditional* $p \leftrightarrow q$ is true *iff* p and q **assume the same** truth value.

- $\top \leftrightarrow \top = \top$
- $\top \leftrightarrow \bot = \bot$
- $\bot \leftrightarrow \top = \bot$
- $\bot \leftrightarrow \bot = \top$

1.2 logic operators order

1. $\neg$ negation
   - $\neg q \vee r$ means $(\neg q) \vee r$, not $\neg(q \vee r)$
2. $\wedge$ conjunction
3. $\vee$ disjunction
   - $q \wedge r \vee s$ means $(q \wedge r) \vee s$, not $q \wedge (r \vee s)$
4. $\rightarrow$ implication
   - $q \wedge r \rightarrow s$ means $(q \wedge r) \rightarrow s$, not $q \wedge (r \rightarrow s)$
5. $\leftrightarrow$ biconditional

1.3 other conditional statements

given an implication $p \rightarrow q$:

- $\neg p \rightarrow \neg q$ is its *inverse*
- $q \rightarrow p$ is its *converse*
- $\neg q \rightarrow \neg p$ is its *contrapositive*

1.4 activities

1.4.1 two propositions

show that an implication $p \to q$ and its *contrapositive* $\neg q \to \neg p$ always have the same value.

p, q	$\neg q, \neg p$	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to p)$
$\top$ $\top$	$\bot$ $\bot$	$\top$	$\top$	$\top$
$\top$ $\bot$	$\top$ $\bot$	$\bot$	$\bot$	$\top$
$\bot$ $\top$	$\bot$ $\top$	$\top$	$\top$	$\top$
$\bot$ $\bot$	$\top$ $\top$	$\top$	$\top$	$\top$

1.4.2 complex example 1

construct a truth table for $(p \wedge q) \to \neg r$.

p, q, r	$p \wedge q$	$\neg r$	$(p \wedge q) \to \neg r$
$\top$ $\top$ $\top$	$\top$	$\bot$	$\bot$
$\top$ $\top$ $\bot$	$\top$	$\top$	$\top$
$\top$ $\bot$ $\top$	$\bot$	$\bot$	$\top$
$\top$ $\bot$ $\bot$	$\bot$	$\top$	$\top$
$\bot$ $\top$ $\top$	$\bot$	$\bot$	$\top$
$\bot$ $\top$ $\bot$	$\bot$	$\top$	$\top$
$\bot$ $\bot$ $\top$	$\bot$	$\bot$	$\top$
$\bot$ $\bot$ $\bot$	$\bot$	$\top$	$\top$

1.5 complex example 2

construct a truth table for $p \wedge (\neg q \vee r) \to r$.

p, q, r	$\neg q$	$\neg q \vee r$	$p \wedge (\neg q \vee r)$	$p \wedge (\neg q \vee r) \to r$
$\top$ $\top$ $\top$	$\bot$	$\top$	$\top$	$\top$
$\top$ $\top$ $\bot$	$\bot$	$\bot$	$\bot$	$\top$
$\top$ $\bot$ $\top$	$\top$	$\top$	$\top$	$\top$
$\top$ $\bot$ $\bot$	$\top$	$\top$	$\top$	$\bot$
$\bot$ $\top$ $\top$	$\bot$	$\top$	$\bot$	$\top$

p, q, r	$\neg q$	$\neg q \vee r$	$p \wedge (\neg q \vee r)$	$p \wedge (\neg q \vee r) \to r$
$\bot$ $\top$ $\bot$	$\bot$	$\bot$	$\bot$	$\top$
$\bot$ $\bot$ $\top$	$\top$	$\top$	$\bot$	$\top$
$\bot$ $\bot$ $\bot$	$\top$	$\top$	$\bot$	$\top$