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# Estimating the Hubble Parameter

PHY 5132

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# Motivation and Background

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# Why is $H_0$ important?

- Hubble's Law:  $v = H_0 d$
- Establishes a relation between:
  - Recessional velocity
  - Luminosity distance
- Confirms expansion of universe
- Best evidence for Big Bang model

Hubble's Data (1929)

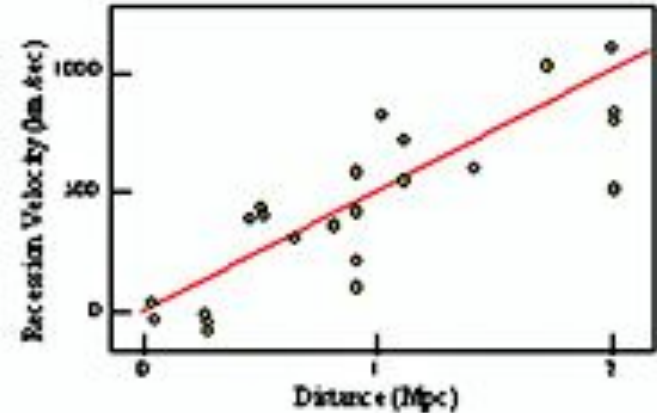


Fig. 1

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# Gravitational Waves + Hubble Parameter

- Estimation of  $H_0$  from GW signals from BNSs (and BBHs)<sup>[1]</sup>
- Done by:
  - Redshift of merger ( $z$ )
  - Luminosity distance of merger ( $D_L$ )
- Parameter estimation to determine  $D_L$
- Considered BNS event GW170817 for this study

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[1] Schutz, B. Determining the Hubble constant from gravitational wave observations. Nature 323, 310–311 (1986). <https://doi.org/10.1038/323310a0>

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# Why BNSs?

- Duration of signal longer than BBHs (40-50s)
  - Electromagnetic counterpart present
    - Easier to measure redshift
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# Why GW170817?

- Closest BNS event till date
  - Highest signal to noise ratio among BNS merger events
  - Other parameters have been well constrained by previous studies
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# Approach

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# Pipeline

- Two inputs needed–
  - Luminosity distance ( $D_L$ )
  - Recessional velocity ( $v$ )
- Luminosity distance obtained from GW Data
  - Other parameters obtained from GWOSC
  - Employing parameter estimation and Bayesian inference
  - Data from H1, L1 obtained from bilby package
- $v$  obtained from redshift of merger

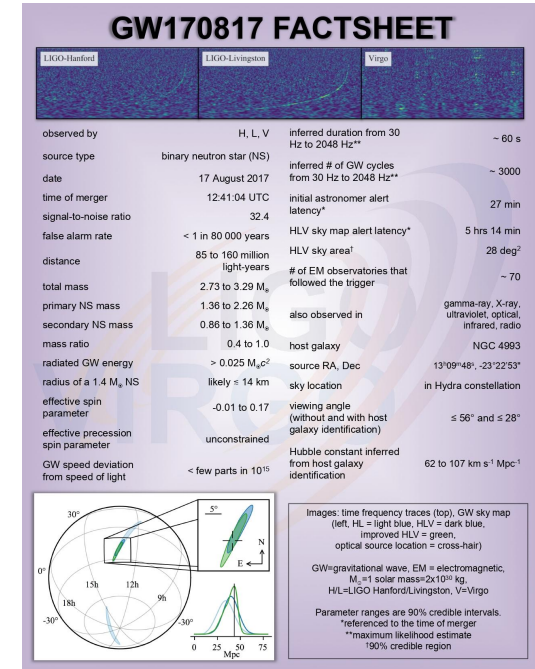


Fig. 1



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# Methods

- Injection studies run with priors on:
    - Phase
    - Psi
    - Luminosity distance
  - Waveforms used:
    - IMRPhenomPv2
    - IMRPhenomPv2\_NRTidal
    - SpinTaylorT4
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# Assumptions

- Estimates on tidal deformability parameters  $\Lambda_1, \Lambda_2$  not well constrained
  - Some models do not support these parameters
  - Effect of these parameters on the wave data is low
  - Also employed calculated  $\psi$ , phase
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# Results

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**With calculated psi and phase**

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## With IMRPhenomPv2 (no prior on psi, phase)

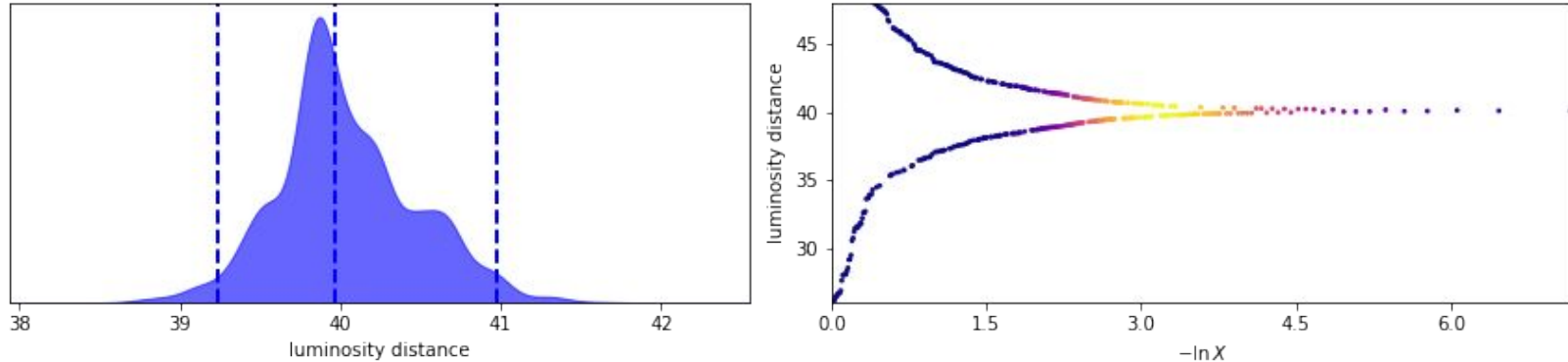


Fig. 2

- Estimate: 39.95892
  - 90% Confidence Interval: (39.37567, 40.77348 )
  - log Evidence: -372660.1356293629
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## With IMRPhenomPv2\_NRTidal (no prior on psi, phase)

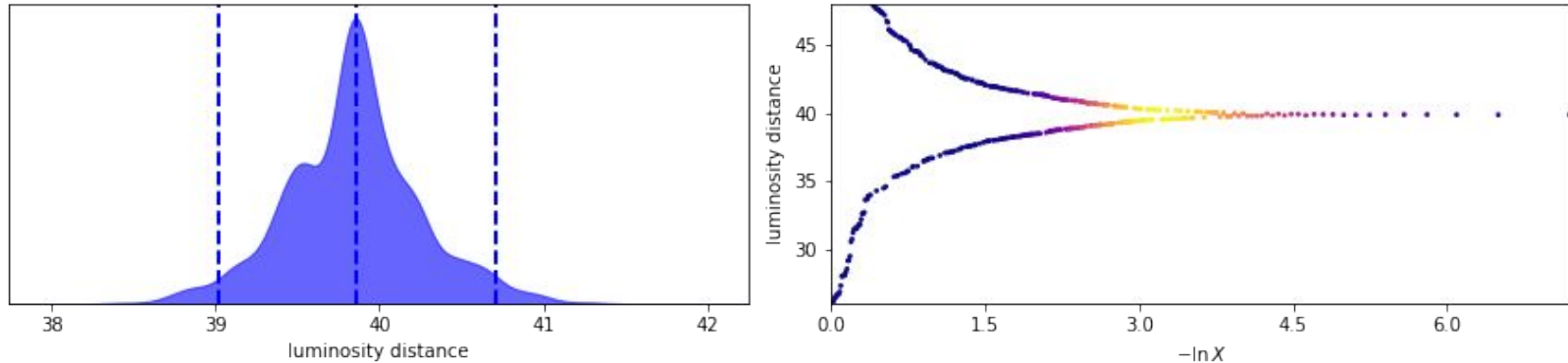


Fig. 3

- Estimate: 39.85408
  - 90% Confidence Interval: (39.12711, 40.58449)
  - log Evidence: -372658.938054884
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## With SpinTaylorT4 (no prior on psi, phase)

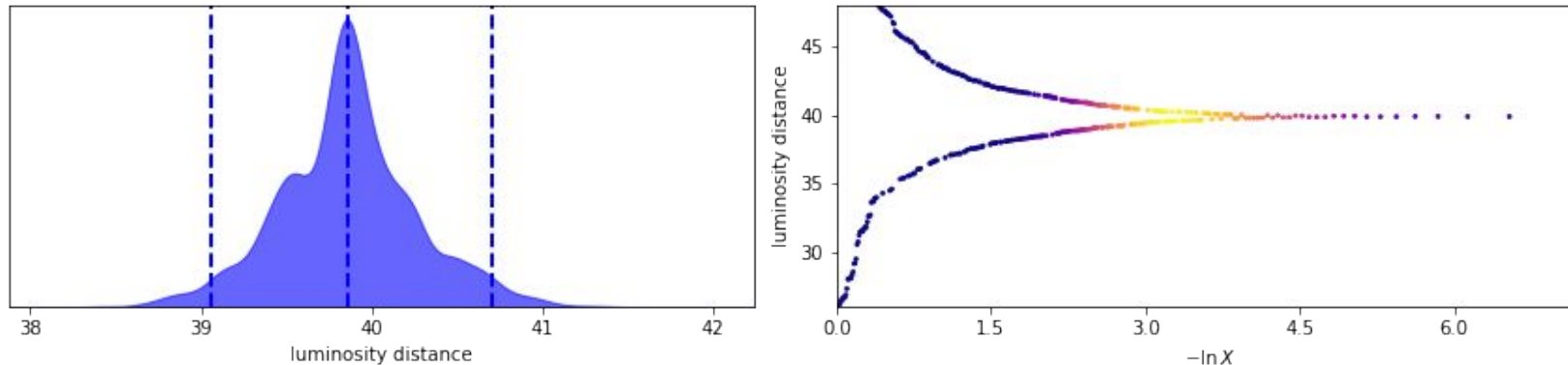


Fig. 4

- Estimate: 39.85905
  - 90% Confidence Interval: (39.19163, 40.59)
  - log Evidence: -372659.5405876796
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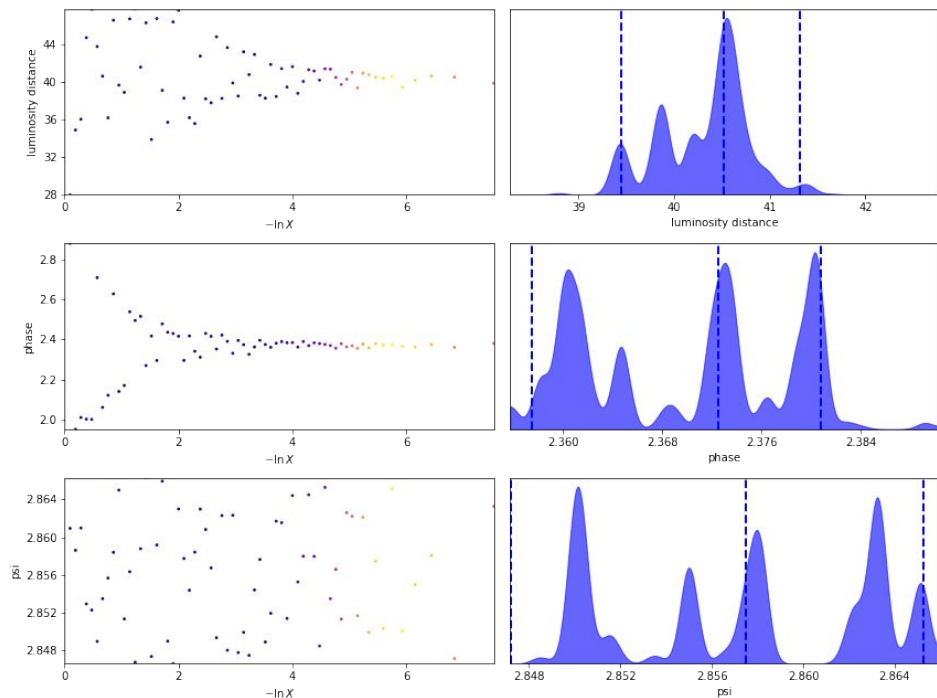
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**With prior on  $\psi$  and phase**

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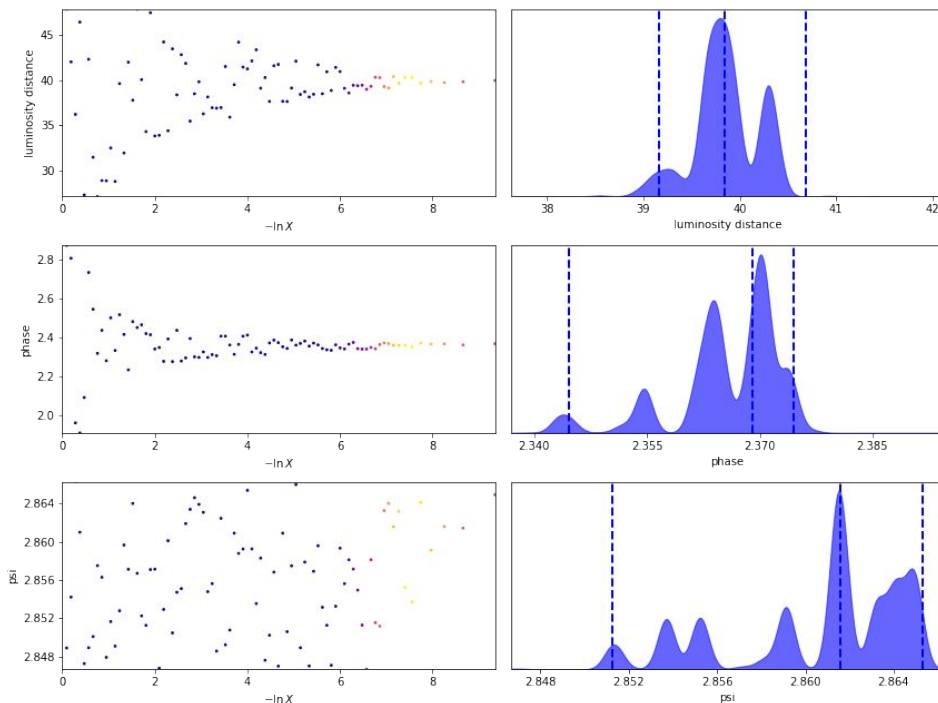
# With IMRPhenomPv2 (with prior on psi, phase)



- Estimate: 40.46503
- 90% Confidence Interval: (39.44871, 40.91322)
- log Evidence: -372663.40605726547

Fig. 5

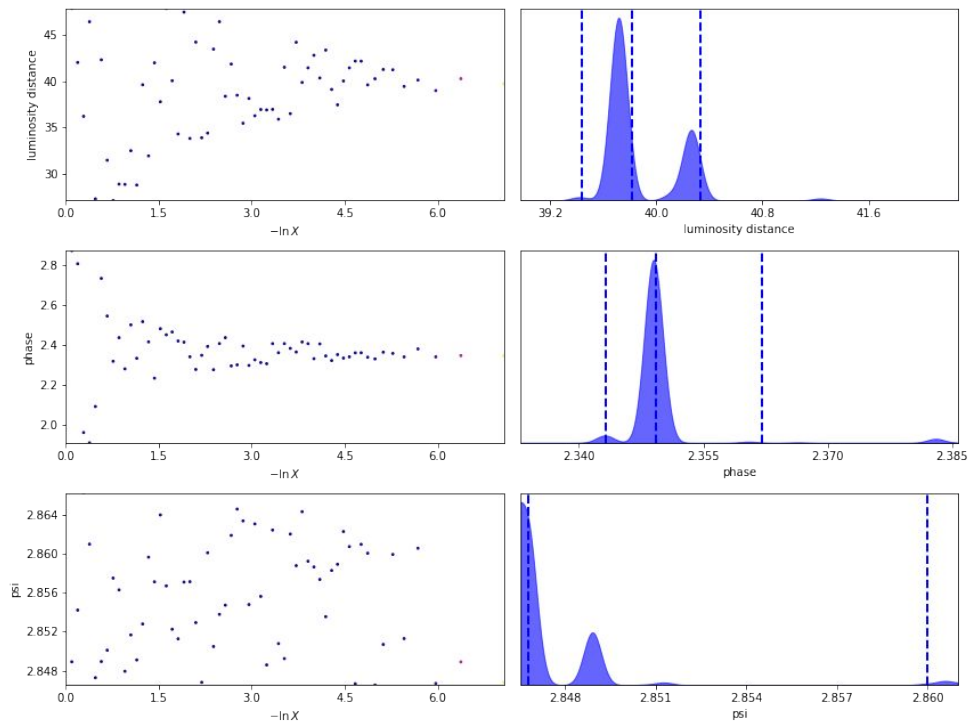
# With IMRPhenomPv2\_NRTidal (with prior on psi, phase)



- Estimate: 39.81041
- 90% Confidence Interval: (39.13407, 40.30831)
- log Evidence: -372663.5570583796

Fig. 6

# With SpinTaylorT4 (with prior on psi, phase)



- Did not converge
- Computationally expensive

Fig. 7

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# Which model performs best?

- Calculating Log Bayes Factor
    - Log Bayes Factor can be used to compare two models
    - Equals Difference of log Evidence
    - Positive/Negative logBF = Better First/Second model
  - We note that IMRPhenomPv2\_NRTidal was the best fitting model
  - IMRPhenomPv2 was the least fitting model
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# Limitations and Future Directions

- Priors on three parameters insufficient
    - Number of calculations scales exponentially with number of priors
  - Higher order modes neglected
    - Detectors not sensitive enough in 2017 to detect them
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# Limitations and Future Directions

- Only one merger studied so far
  - No well constrained parameters for other BNS mergers
  - Rate of BNS detections is improving

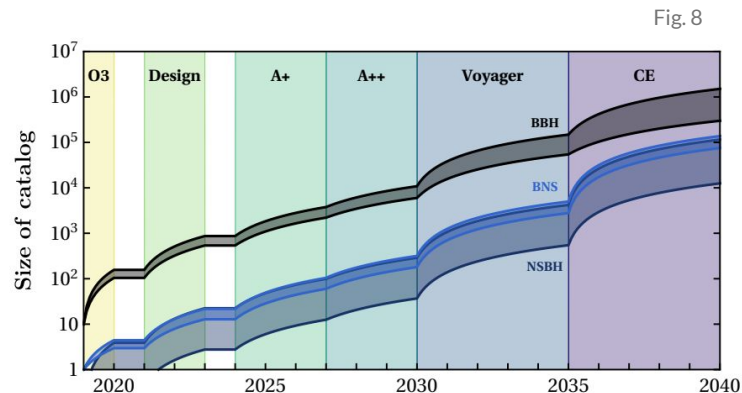


FIG. 4. Growth of catalog size as detectors improve for models in agreement with current observations. The timeline for different detectors and their upgrades is estimated following Refs. [53–55]. We assume an optimistic duty cycle of 100%, which is compatible with expectations for future observations with multiple detectors.

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# Limitations and Future Directions

- To be extended to BBHs
  - Can use  $D_L$  to measure redshift
  - Standard siren to measure cosmological distances