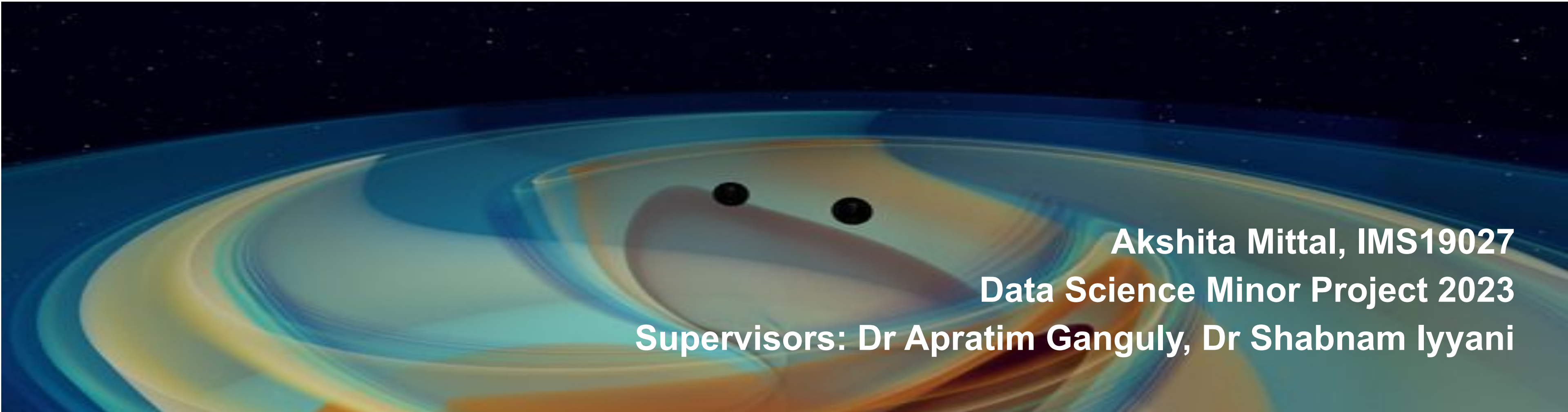
Exploring the Scope of Multiband Detection with Next-Generation Gravitational Wave Detectors



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Data Science Minor Project 2023
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1. Introduction

Understanding Gravitational Waves

- Ripples in space-time caused by accelerating massive objects
- Have two polarizations--- "plus" and "cross"
- Primary sources:
  - Binary systems
  - Supernovae
  - Continuous gravitational waves
  - Stochastic gravitational wave background

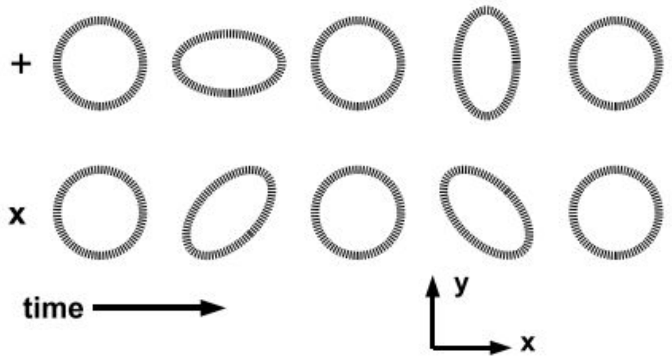


Fig. 1.1: Effect of a passing gravitational wave on test particles.

Source Parameters

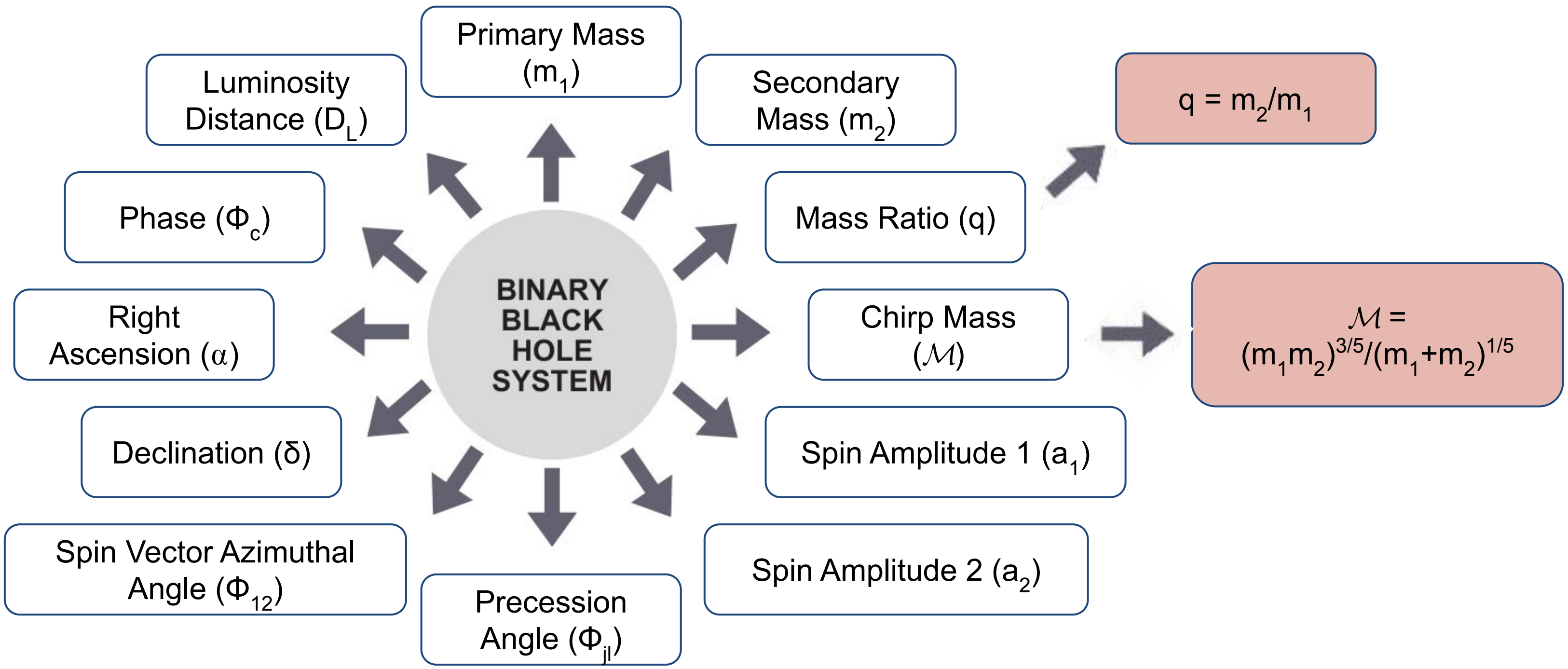


Fig. 1.2: Illustration of source parameters and definitions

Detecting Gravitational Waves

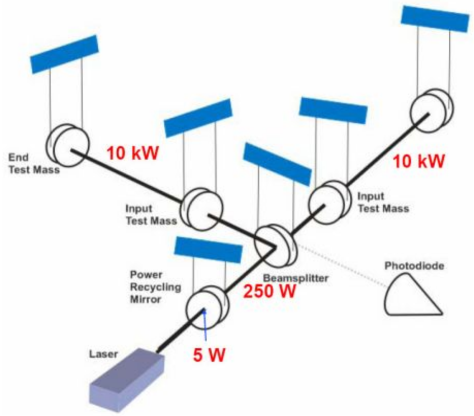


Fig. 1.3: Simple design of a LIGO interferometer.

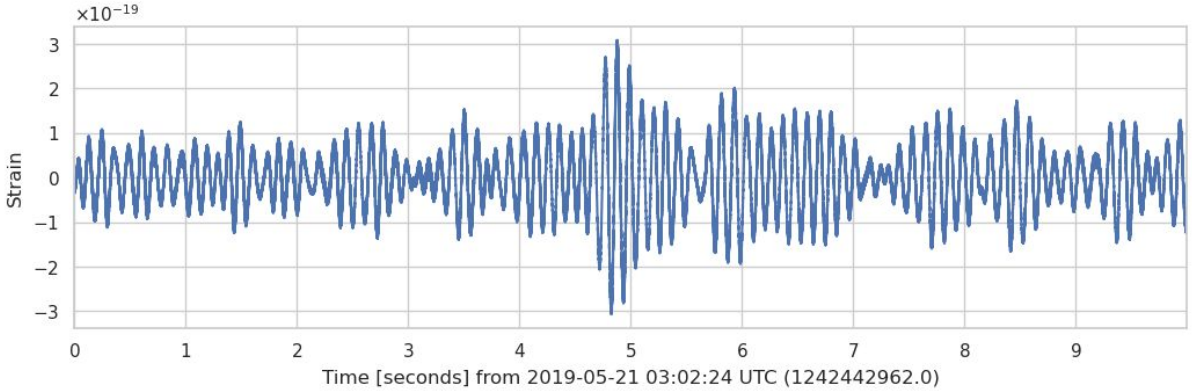


Fig. 1.4: Time series data for LIGO Hanford around GW190521

Post-Merger Data Analysis

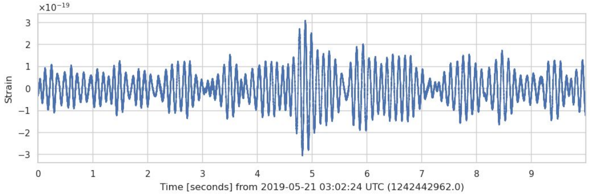


Fig. 1.4: Time series data for LIGO Hanford around GW190521

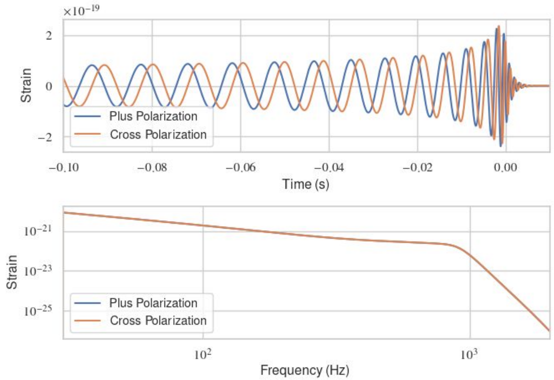


Fig. 1.5: Time and frequency domain signals for IMRPhenomD

Signal

Template

$$(s|h)(t) = 4\mathrm{Re}\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} e^{2\pi i f t}\, \mathrm{d}f. \tag{1.1}$$

Power spectral density (PSD)

Matched filtering is our end goal!

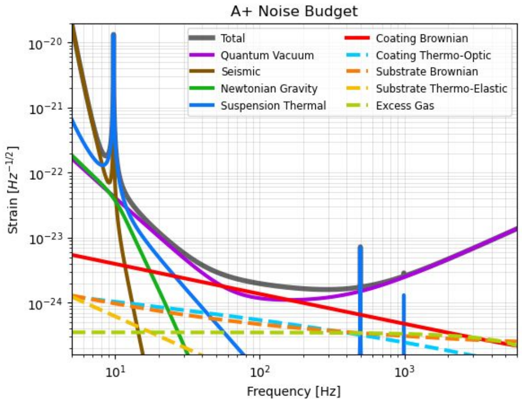


Fig. 1.6: Noise budget evaluated using GWINC

Present Status

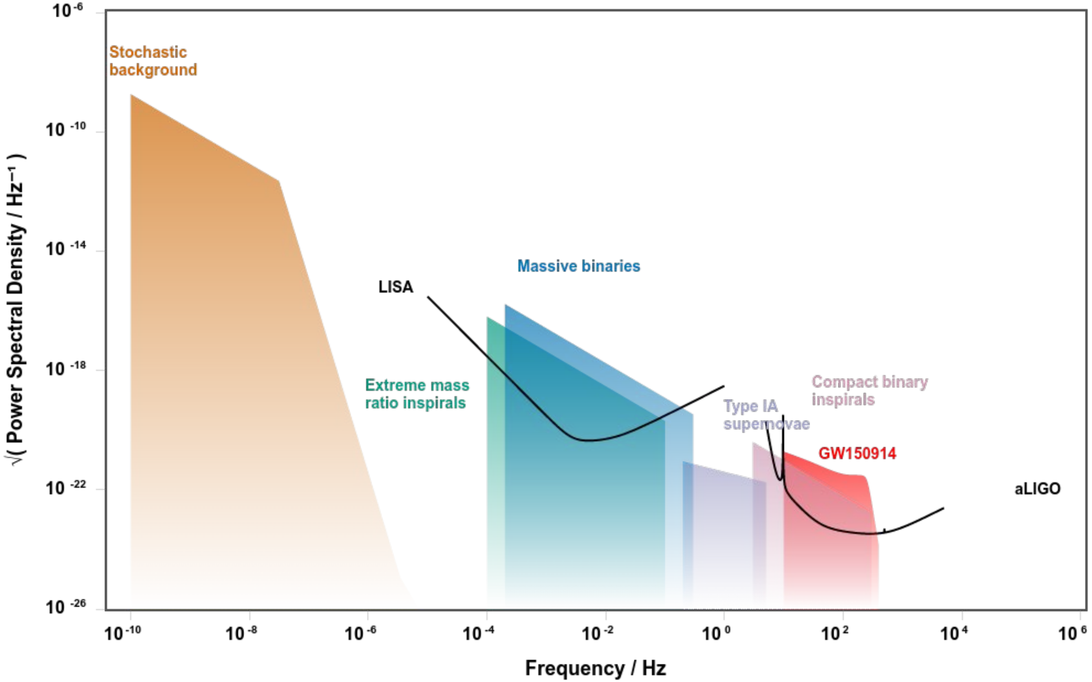


Fig. 1.7: To illustrate the frequency bands of active and future detectors. Made using GWPlotter

Objective

- To understand the detection capability of a proposed deci-hertz detector
- Multiband detection of heavier-mass black holes

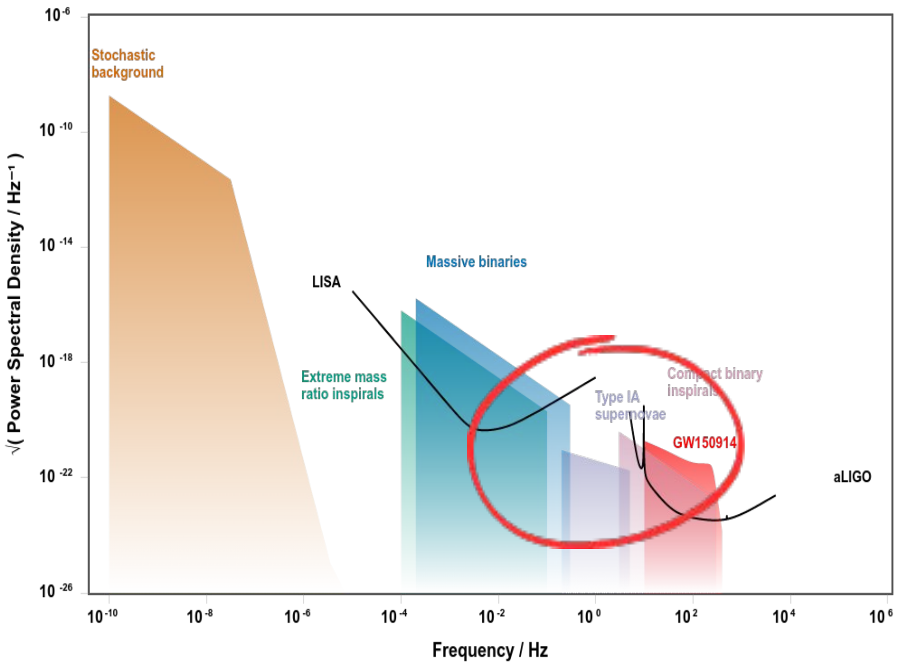


Fig. 1.8: To illustrate the placement of a deci-hertz detector

2. Framework

Salpeter Mass Distribution

- Power law function with index (α) = -2.35
- Gives relation between number of stars with mass m in mass range dm

$$N(m)\mathrm{d}m \propto m^{-\alpha} \tag{2.1}$$

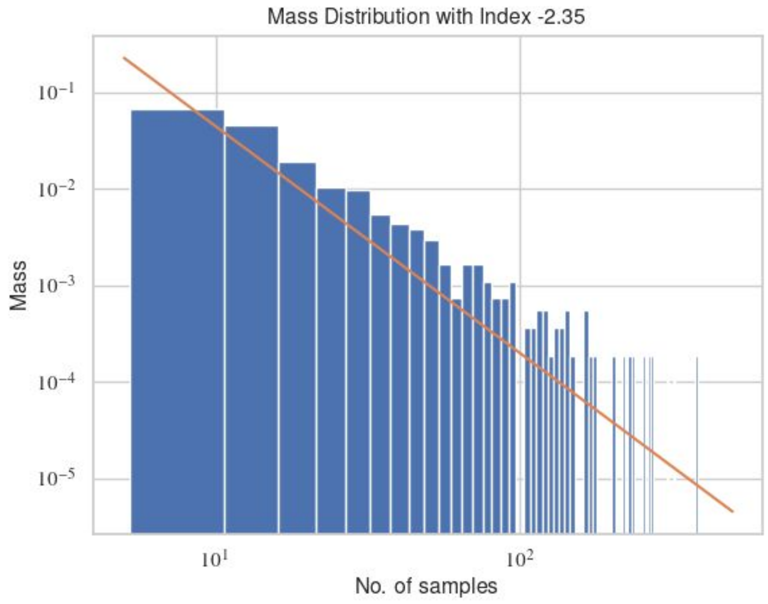


Fig. 2.1: Salpeter mass distribution. Plotted in log scale.

Optimal SNR (σ)

- σ^2 is defined as the inner product of a signal with itself
- Quantifies loudness of the noise of a detector
- We use it to quantify the loudness of the signal
- For a signal h,

$$\sigma^2 = (h|h) \implies \sigma = \sqrt{(h|h)}$$

(2.2)

$$\sigma^2 = 2\int_{-\infty}^{\infty} df \frac{\tilde{h_c}^*(f)\tilde{h_c}(f)}{S_h(|f|)}$$

(2.3)

3. Methodology

Generating a BBH Population

- Population size = 1000
- Salpeter mass distribution for primary mass
- Uniform priors placed on other parameters

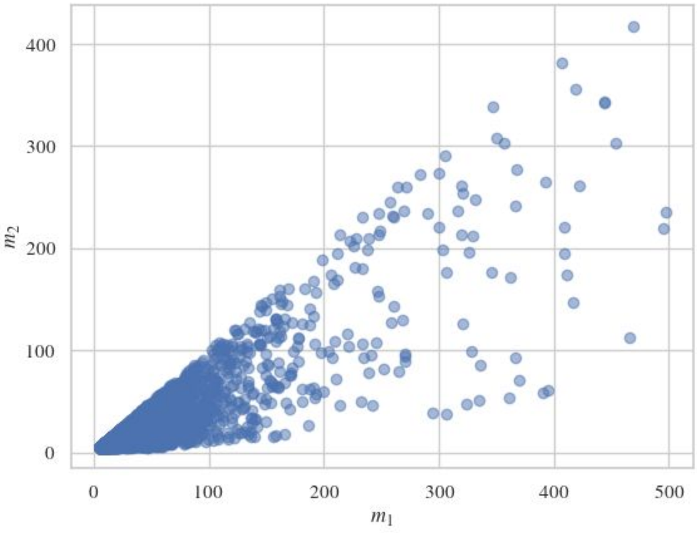


Fig. 3.1: Mass distribution for generated BBH population.

Generating a BBH Population

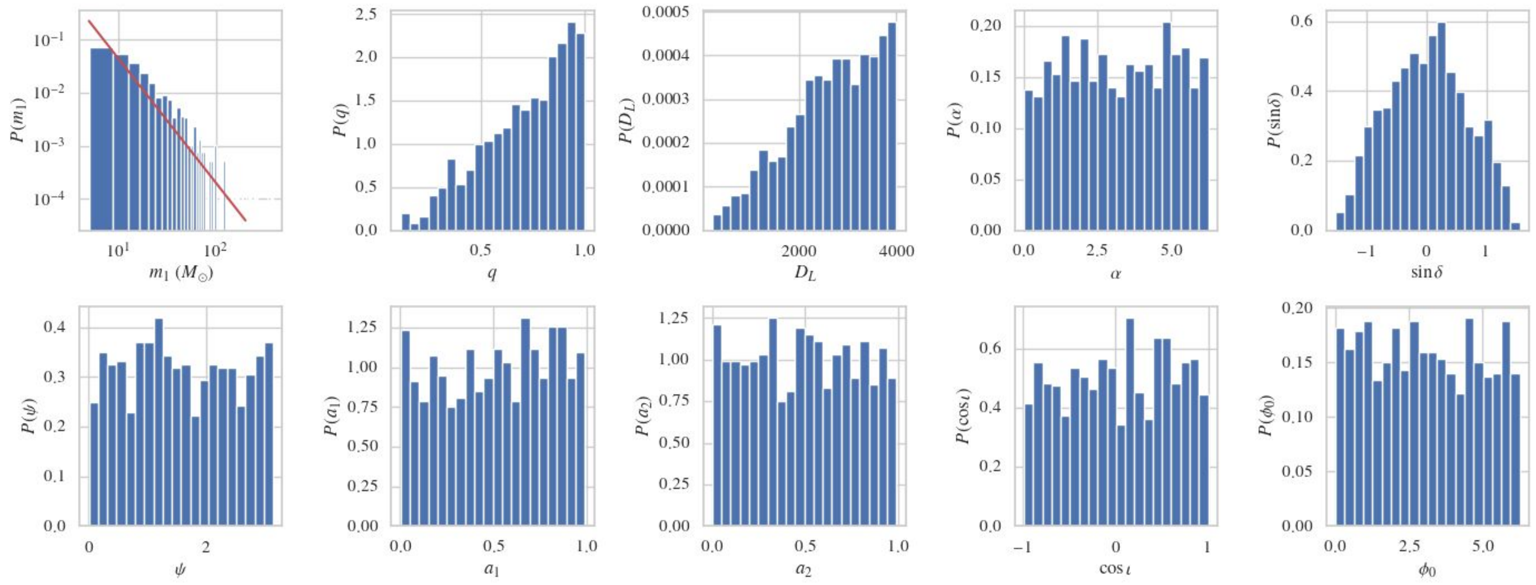


Fig. 3.2: Distribution of parameters for generated population

Generating PSDs

- PSDs of HLV obtained from LIGO DCC
- PSD for the proposed deci-hertz observatory empirically determined
- Sampling rate chosen to be 4096Hz
- Lower bounds on HLV at 10Hz, deci-hertz at 0.05Hz

$$S_n^{deci} = (10^{-25} f_{deci}^{-2} + 10^{-24} f_{deci}^{1} + 10^{-23} f_{deci}^{0})^2$$

(3.1)

Generating PSDs

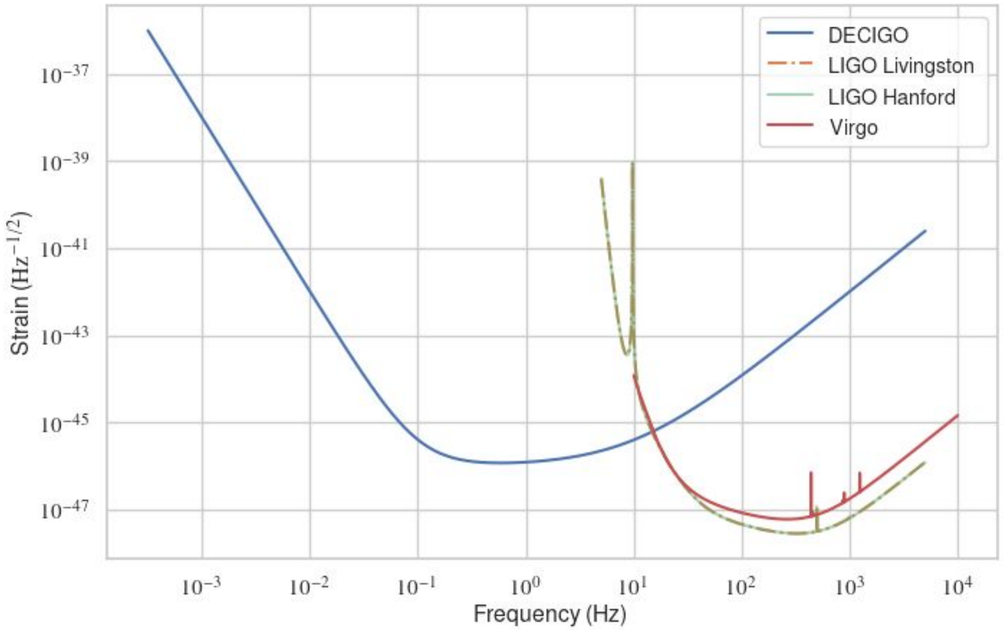


Fig. 3.3: Theoretical PSDs for proposed deci-hertz observatory and HLV

Generating Signals

- IMRPhenomD used to create frequency domain signals for 1000 systems
- Antenna pattern functions for HLV and deci-hertz observatory taken into account
- High frequency cutoff placed at f_{high}, where

$$f_{high} = \frac{0.1}{M_T m_1 (1+q)} \tag{3.2}$$

M_T = Solar mass in seconds, m_1= Primary mass, q = Mass ratio

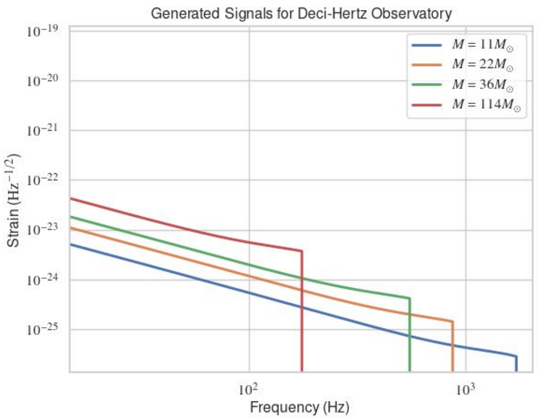


Fig. 3.4 (a): Sample of generated signals for deci-hertz

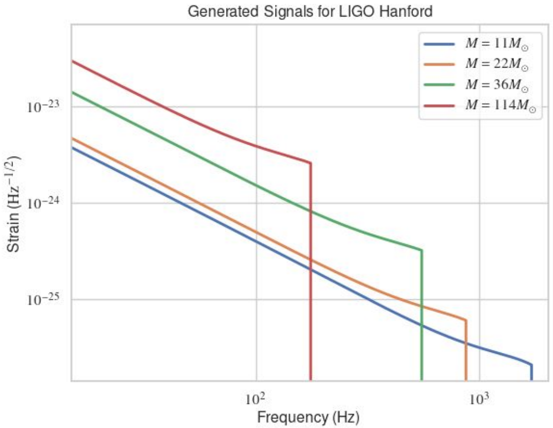


Fig. 3.4 (b): Sample of generated signals for H1

Calculation of σ

- Evaluation of σ for 1000 samples
- Signal classified as detectable for σ>8
- Example events:
  - Stellar-mass binary merger (A)
    - m_1=15$M_\odot$, m_2=12$M_\odot$, D_L=3000MPc
  - Intermediate mass binary merger (B)
    - m_1=150$M_\odot$, m_2=120$M_\odot$, D_L=2500MPc

	LIGO Hanford	Deci-hertz
σ	14	200

Table 3.1: σ of stellar-mass binary merger.

	LIGO Hanford	Deci-hertz
σ	84	1436

Table 3.2: σ of intermediate-mass binary merger.

4. Results

Detectable Events

σ >8 events	**LIGO Hanford**	**Deci-hertz**
Out of 1000 events	607	1000

Table 4.1: Number of events with σ>8 out of 1000.

Distribution of SNR (σ)

Fig. 4.1: Distribution of σ for decihertz and LIGO Hanford (H1).

SNR (σ) v/s Luminosity Distance

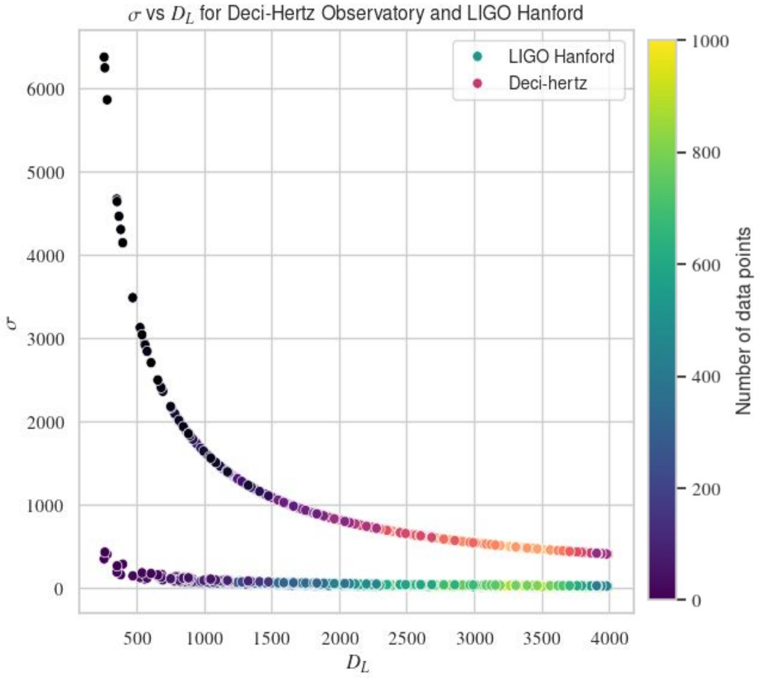


Fig. 4.2: σ vs luminosity distance for same mass systems for deci-hertz observatory and H1.

SNR (σ) v/s Chirp Mass

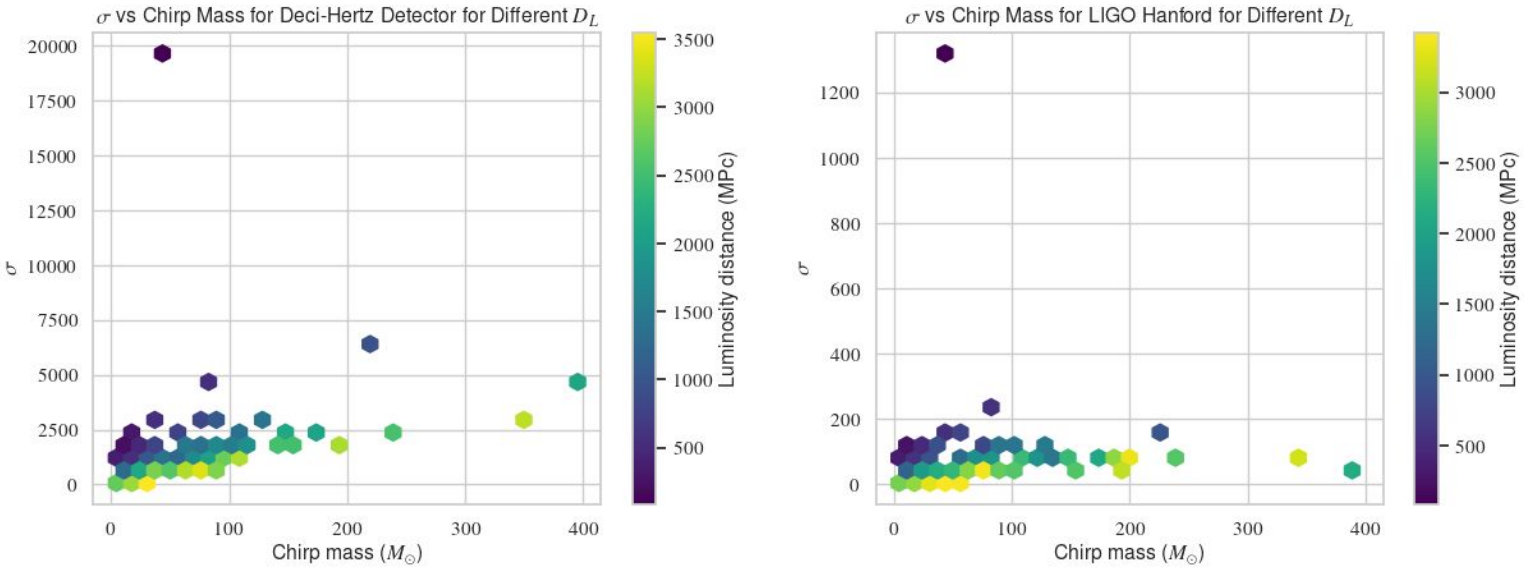
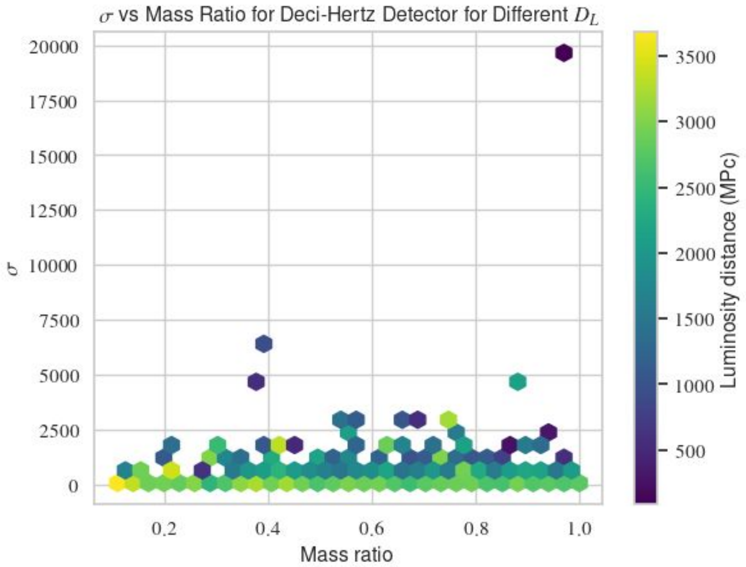


Fig. 4.3: σ vs chirp mass for different distances for deci-hertz observatory (left) and H1 (right).

SNR (σ) v/s Mass Ratio



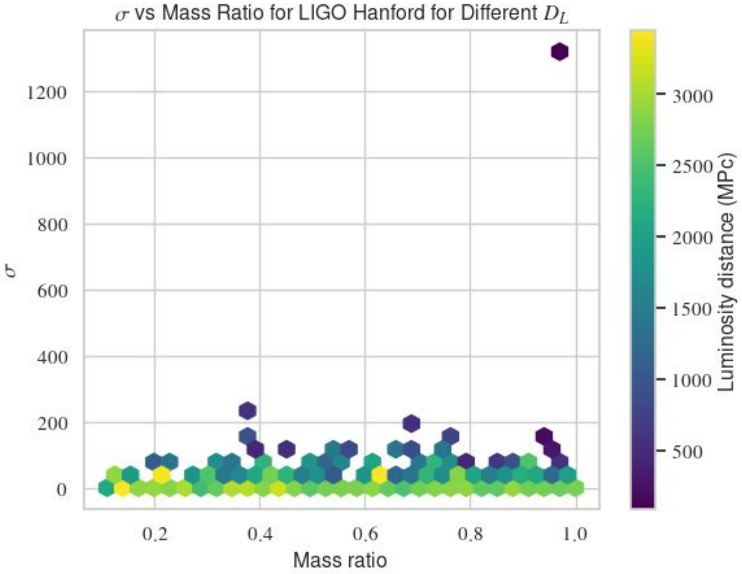


Fig. 4.4: σ vs mass ratio for different distances for deci-hertz observatory (left) and H1 (right).

Conclusions

- Proposed deci-hertz observatory has a 100% detection rate
- Consistently outperforms the current hecto-hertz detectors
- Negative relationship between SNR (σ) and luminosity distance verified
- SNR (σ) increases with chirp mass but trend dominated by luminosity distance

Future Directions

- Bigger population of BBHs should be used
- Conduct signal injections with actual noise and evaluate matched filter SNR
- Analysis for IMRIs limited by IMRPhenomD
- Data should not be downsampled

Thank you.

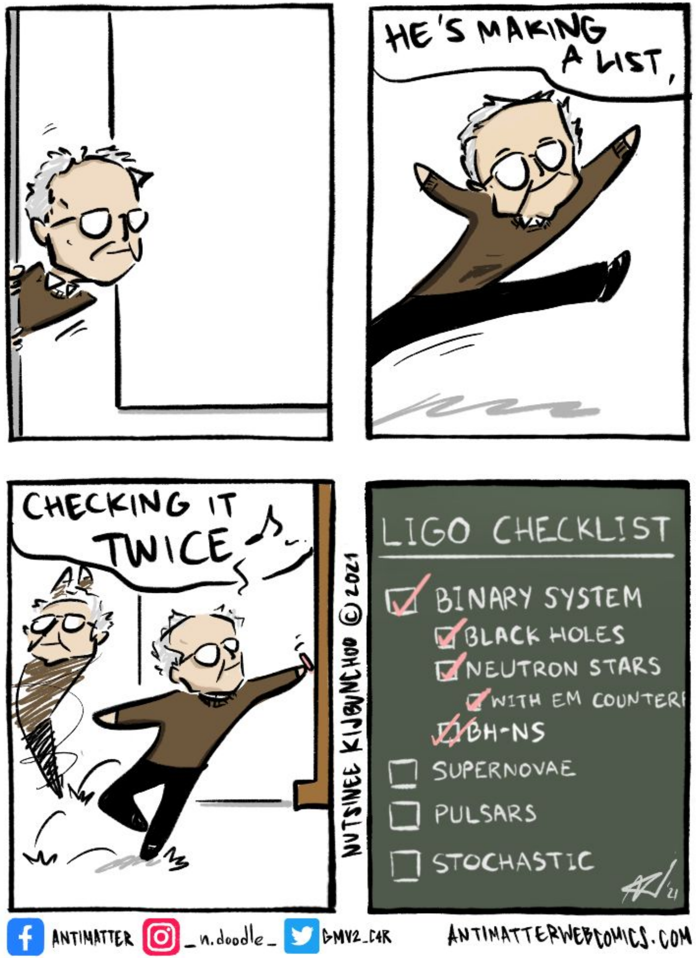


Fig.: Rainer Weiss (American physicist)

Follow-up

Discussion

More on Source Parameters

- Phase: Position of BHs in orbit around each other
- Spin vector azimuthal angle: Angle made by spins with direction perpendicular to orbital plane
- Precession angle: Angle between total angular momentum and direction of wave propagation, determines the polarization of GWs
- Tilt: Angle between the total angular momentum and the LoS from the observer to the binary system

Innermost Stable Circular Orbit (ISCO)

- Smallest possible stable orbit that two black holes can have around each other
- f_{ISCO} determined by the properties of black holes
  - More massive black holes have a lower f_{ISCO}
- Why f_{ISCO} as f_{high}?
  - Final frequency as BHs start to merge
  - More massive BH formed has weak GW signal
  - Dominated by noise above f_{ISCO}

$$f_{high} = \frac{0.1}{M_T m_1 (1+q)} \tag{3.2}$$

Empirical Form of Deci-Hertz PSD

- Based on current observations from LIGO noise curve
- 10^{-24} dominated by flat noise
- Downward slope corresponding to 10^{-25}
- 10^{-23} corresponds to shot and quantum noise

$$S_n^{deci} = (10^{-25} f_{deci}^{-2} + 10^{-24} f_{deci}^{1} + 10^{-23} f_{deci}^{0})^2$$

(3.1)

Why this sampling rate?

- Sampling rate of a digital signal evaluated by Nyquist sampling theorem
- $f_s > 2f_{max}$ where f_{max} is the highest frequency present in the signal
- 16384Hz captures content upto 8192Hz
- Why 16384 and not 16000?
    - To make it a multiple of the resonant frequencies of the interferometer
    - Resonant frequency determined by arm length
    - Resonant frequency ≠ f_s produces beat notes

Threshold at SNR>8

- Corresponds to a statistical significance of 5σ (σ=standard deviation)
- General accepted value for statistical significance for claiming a discovery
- Probability of a false positive detection due to random noise fluctuations extremely low (<1/3.5 million)
- Good compromise between sensitivity and computational cost

Why IMRPhenomD?

- Most recent model from the IMRPhenom family
    - Newer than IMRPhenomA/B/C
- Takes into account precession of orbits, and finite sizes
- Has Post-Newtonian correction terms upto 3.5 orders
    - What is PN?
        - Extends Newtonian gravity to include higher order of velocity terms
        - Includes relativistic effects (time dilation and space-time curvature)