Performance Modeling and Design of Computer Systems- Ch 6 Little's Law and Other Operational Laws

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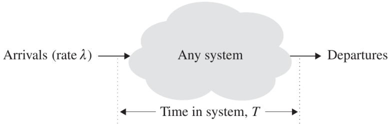
Overview

- **Little's Law :** average number of jobs in the system is equal to the product of the average arrival rate into the system and the average time a job spends in the system.
- **Little's Law for Open Systems**: For any ergodic open system

$$E[N] = \lambda E[T]$$

$E[N]$ = expected number of jobs in the system
λ= average arrival rate into the system
$E[T]$ = mean time jobs spend in the system
=Exp. time each job sepend to complete * Exp. number of jobs in system
$= \approx \frac{1}{\lambda} \cdot E[N]$ (Because λ



- **Little's Law for Closed Systems**: For any ergodic closed system

$$N = X \cdot E[T]$$

Here,
N = multiprogramming level
X= throughput (the rate of job completions for the system)
$E[T]$ = mean time jobs spend in the system

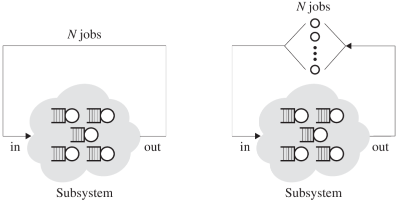


Figure: Little's Law closed systems

Restating Little's Law Using Time Average

Little's Law Proof's are in Book

- From Chapter 5
  - $$\lambda = \lim_{t\to\infty}\frac{A(t)}{t}\ ,\ X = \lim_{t\to\infty}\frac{C(t)}{t}$$
  - $A(t)$ = number of arrivals by time t
  - $C(t)$ is the number of system completions (departures) by time t
- **Little's Law for Open Systems Restated** : Given any system where $\overline{N}^{TimeAvg}$, $\overline{T}^{TimeAvg}$, λ, X exist and where $\lambda = X$,
$$\overline{N}^{TimeAvg} = \lambda\overline{T}^{TimeAvg}$$
- **Little's Law for Closed Systems Restated** : For closed system (interactive or batch) with multiprogramming level N & given that $\overline{T}^{TimeAvg}$, X exists & $\lambda = X$,
$$N = X \cdot \overline{T}^{TimeAvg}$$

The Forced Flow Law

- The Forced Flow Law relates **system throughput** to the **throughput of an individual device** as follows:

$$X_i = E[V_i] \cdot X$$

  Where, X = system throughput, X_i = throughput at device i, V_i = number of visits to device i per job = Visit ratio for device i.

- Example : page 108

Bottleneck Law

- Let, $D_i =$ total service demand on device i for all visits of a single job (i.e., a single interaction). That is,

$$D_i = \sum_{j=1}^{V_i} S_i^{(j)}$$

$S_i^{(j)} =$ service time required by the j th visit of the job to server i

- assuming that the number of visits a job makes to device i is not affected by its service demand at the device.

$$E[D_i] = E[V_i] \cdot E[S_i]$$

- **Bottleneck Law**

$$\rho_i = X \cdot E[D_i]$$

- **Explanation**: X jobs/sec arriving into system. Each arrivals into the system contributes $E[D_i]$ seconds of work for device i. So device i is busy for $X \cdot E[D_i]$ seconds out of every second (e.g., device i might be busy for half a