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Dear Editor,

We are pleased to submit the manuscript "Estimating age at a specified length from the von Bertalanffy Growth Function" as a Technical Note to *Fisheries Research*. This manuscript describes a parameterization of the von Bertalanffy growth function (VBGF) that, with a simple choice made by the analyst, can be used to estimate the usual  $L_{\infty}$  and K parameters along with a parameter that is either the mean length at a chosen age ( $L_r$ ) or the age at chosen mean length ( $t_r$ ). Thus, this parameterization can be used to provide a direct (rather than derived) estimate of age at an important mean length (e.g., length at recruitment, legal harvest, or maturity). Direct estimation of this age, rather than deriving it from other parameter estimates, allows more options for computing measures of uncertainty and comparison of estimates among populations. In the manuscript, we demonstrate these advantages with two datasets. Additionally, by setting one of two values to zero, this parameterization can represent the traditional VBGF of Beverton and Holt (1957) or the original VBGF of von Bertalanffy (1938). Thus, this parameterization also demonstrates a conceptual link between the two most common parameterizations of the VBGF. Therefore, we feel that this parameterization of the VBGF will be of applied (directly estimate a more interesting parameter) and theoretical (demonstrate similarity among common parameterizations of the VBGF) value to fisheries scientists.

The code underlying the analyses in this manuscript is available at

https://raw.githubusercontent.com/droglenc/ModifiedVonB/master/code/OgleIsermann.R

Thus, reviewers can run this code to recreate the figures, tables, and analyses of this manuscript.

We do not have any conflicts of interest or financial or material benefit interests related to the publication of this manuscript. We have followed the Guide to Authors and Submission Checklist found on your website.

Thank you for your consideration. We look forward to your response about the suitability of this note for publication in *Fisheries Research*. Please feel free to contact me if you have any questions or concerns related to this manuscript.

Respectfully,

Dr. Derek H. Ogle

DU HOK

Professor of Mathematical Sciences and Natural Resources

## \*Highlights (for review)

- A parameterization of the von Bertalanffy growth model that includes a parameter that represents age at a mean length is derived.
- The typical Beverton and Holt (1957) and von Bertalanffy (1938) growth models are special cases of the new parameterization.
- A comparison of the estimate age at a mean length between sexes is demonstrated.

### **Estimating Age at a Specified Length from the von Bertalanffy Growth Function**

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- 8 This draft manuscript is distributed solely for purposes of scientific peer review. Its content is
- 9 deliberative and predecisional, so it must not be disclosed or released by reviewers. Because the
- 10 manuscript has not yet been approved for publication by the U.S. Geological Survey (USGS), it
- 11 does not represent any official USGS finding or policy.
- 13 Abstract
- 14 Estimating the time required (i.e., age) for fish in a population to reach a specific length (e.g.,
- 15 legal harvest length) is useful for understanding population dynamics and simulating the
- potential effects of length-based harvest regulations. The age at which a population reaches a
- 17 specific mean length is typically estimated by fitting a von Bertalanffy growth function to length-
- at-age data and then rearranging the best-fit equation to solve for age at the specified length. This
- 19 process precludes use of some statistical methods for computing confidence intervals and
- 20 comparing estimates of age at the specified length among populations. We provide a
- 21 parameterization of the von Bertalanffy growth function that allows age at a specified length to
- be directly estimated so that standard methods to construct confidence intervals and make

- 23 among-group comparisons for this parameter can be used. We demonstrate use of the new
- 24 parameterization with two datasets.

- 26 Keywords: Nonlinear modeling, Lake Whitefish, Walleye, von Bertalanffy
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#### 1. Introduction

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The length of time  $(t_r)$  required for fish in a population to reach a specified mean length  $(L_r)$ is useful for understanding the dynamics of fish populations. The  $t_r$  value usually represents the age when fish become vulnerable to fishing mortality as (originally denoted as  $t_{
ho'}$ ) in Beverton-Holt equilibrium yield models (Beverton and Holt, 1957). These models have long been used to simulate fishery responses to changes in fishing mortality (Beverton and Holt, 1957; Ricker, 1975; Quinn and Deriso, 1999). Release of the Fisheries Analysis and Simulation Tools (FAST; Slipke and Maceina, 2001) and Fisheries Analysis and Modeling Simulator (FAMS, Slipke and Maceina, 2014) software packages facilitated using these Beverton-Holt models to simulate the effects of length-based harvest regulations on freshwater fisheries (e.g., Isermann et al., 2002; Brenden et al., 2007; Colvin et al., 2013). The  $t_r$  value may also be valuable outside of this modeling framework because it provides a measure of cumulative growth up to age  $t_r$  that likely responds (or is related) to abiotic and biotic factors that affect growth of fish (Brett, 1979; Lorenzen, 2016). For example, at a fixed  $L_r$ , a population with a larger  $t_r$  grows more slowly than a population with a lower  $t_r$ . Thus,  $t_r$  may be useful for comparing growth among populations. Typically,  $t_r$  has been estimated by fitting a von Bertalanffy growth function (VBGF) to length and age data and then algebraically rearranging the best-fit equation to solve for age given the specified length  $L_r$  (Beverton and Holt, 1957; Gulland, 1973; Clark, 1983; Allen and Miranda, 1995; Slipke and Maceina, 2001). The delta method (Seber and Wild, 2003; Ritz and Streibig, 2008) or bootstrapping (Hilborn and Mangel, 1997; Ritz and Streibig, 2008) may be used to approximate standard errors and confidence intervals for  $t_r$  derived in this manner. However, likelihood profiles (Hilborn and Mangel, 1997; Ritz and Streibig, 2008) cannot be

used to construct confidence intervals for the derived  $t_r$  and usual methods (extra sum-of-squares tests (Ritz and Streibig, 2008), likelihood ratio tests (Kimura, 1980), or information criterion (Burnham and Anderson, 2002) approaches) for comparing models cannot be used to determine if  $t_r$  differs among populations. These statistical shortcomings could be overcome if  $t_r$  was directly estimated as a function parameter rather than being derived from other function parameters.

Additionally, some parameters in the usual VBGF may be illogical and poorly estimated (i.e., imprecise) because they represent values outside the domain of observed ages. In some instances, these parameters have been fixed at constant values (Isermann et al., 2007; Weber et al., 2011), which may negatively affect estimates of other parameters and values derived from these parameters, such as  $t_r$ . In contrast,  $t_r$  is unlikely to be outside the domain of observed ages and, thus, is likely to be logically and precisely estimated if it is a parameter in a VBGF.

Therefore, the objectives of this note are to (1) describe a VBGF that has  $t_r$  as a directly estimated parameter and (2) demonstrate how this VBGF can be used to directly estimate  $t_r$  and identify differences in  $t_r$  between populations.

#### 2. Theory

The most commonly used parameterization of the VBGF from Beverton and Holt (1957) is

$$L_t = L_{\infty} (1 - e^{-K(t - t_0)}) \tag{1}$$

where  $L_t$  is the expected or mean length at time (hereafter, age) t,  $L_{\infty}$  is the asymptotic mean length, K is a measure of the exponential rate at which  $L_t$  approaches  $L_{\infty}$  (Schnute and Fournier, 1980), and  $t_0$  is the theoretical age at which  $L_t$  would be zero (i.e., the x-intercept; Figure 1). For use further below, Eq. (1) can be expressed as

$$L_t = 0 + (L_{\infty} - 0)(1 - e^{-K(t - t_0)})$$
 (1a)

75 The original parameterization of the VBGF from von Bertalanffy (1938) is

$$L_t = L_{\infty} + (L_{\infty} - L_0)e^{-Kt}$$
 (2)

where  $L_0$  is  $L_t$  when t = 0 (i.e., y-intercept; Figure 1). Eq (2) can be algebraically shown to equal

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$$L_t = L_0 + (L_\infty - L_0)(1 - e^{-Kt})$$

which, for use further below, can also be expressed as

$$L_t = L_0 + (L_\infty - L_0) (1 - e^{-K(t-0)})$$
 (2a)

The similarities of Eqs. (1a) and (2a) suggest that the VBGF may be expressed as

$$L_t = L_r + (L_{\infty} - L_r) (1 - e^{-K(t - t_r)})$$
(3)

- 83 where  $L_t = L_r$  when  $t = t_r$ . Thus, when  $L_r = 0$ ,  $t_r$  is the theoretical age at a mean length of zero
- 84 (i.e., the x-intercept) and Eq. (3) reduces to Eq. (1a) with  $t_r$  replaced by  $t_0$ . Similarly, when
- 85  $t_r = 0$ ,  $L_r$  is the mean length at age zero (i.e., the y-intercept) and Eq. (3) reduces to Eq. (2a)
- with  $L_r$  replaced by  $L_0$ . Thus, Eqs. (1) and (2) are special cases of Eq. (3) and only differ in
- whether they are parameterized to estimate the x- or y-intercept (Figure 1).
- Of more interest is that Eq. (3) may be used to estimate  $L_r$  or  $t_r$  for any point on the VBGF
- 89 curve (Figure 1). For example,  $t_r$  may be set to a specific age of biological interest such that the
- mean length at that age  $(L_r)$  is a parameter estimated from fitting Eq. (3) to data. Conversely, and
- 91 the focus of this note,  $L_r$  may be set to a specific length of biological interest such that the age
- 92  $(t_r)$  for fish of that mean length is a parameter estimated from fitting Eq. (3) to data. Thus,
- 93 because  $t_r$  is a parameter directly estimated from fitting Eq. (3) to data, all methods for

computing confidence intervals for function parameters may be used and common statistical methods may be used to identify differences in  $t_r$  among populations.

Note that Eq. (3) appears to have four parameters, but either  $L_r$  is set to a constant value and  $t_r$  is estimated or  $t_r$  is set to a constant value and  $L_r$  is estimated. Thus, Eq. (3) has three estimable parameters, as do Eqs. (1) and (2).

## 3. Methods

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We demonstrate use of Eq. (3) for estimating  $t_r$  with two examples. First, length-at-age data for Lake Michigan lake whitefish (Coregonus clupeaformis) are used to demonstrate that the fit of Eq. (3) is equivalent to the fits of Eqs. (1) and (2), and that direct estimates of  $t_r$  from Eq. (3) equal derived estimates of  $t_r$  from Eqs. (1) and (2). Second, length-at-age data for Lake Winnibigoshish (Minnesota) walleye (Sander vitreus) are used to show how model comparison methods can be used to assess differences in  $t_r$  (and other function parameters) between groups (i.e., sexes). Lake whitefish were captured by commercial trap-netters from locations in and around Green Bay, Lake Michigan in October 2012 and 2013 and were genetically assigned to the Big Bay de Noc stock. Total length (TL) was measured to the nearest mm and integer ages were estimated from thin-sectioned otoliths. Full collection details for these data are in Belnap (2014). As in Belnap (2014), we estimate the at which a mean TL of 480 mm was reached (i.e.,  $t_{480}$ ), which is the TL at which lake whitefish are fully vulnerable to commercial and tribal harvest in Lake Michigan (Ebener et al., 2008). Eqs. (1)-(3) were fit to these data using the default Gauss-Newton algorithm of the nls() function in the R environment (R Development Core Team, 2017). Starting values were obtained by visually fitting each equation to the observed data (Ritz and

Streibig, 2008; Ogle, 2016). Alternative starting values were used to confirm that a global rather

than a local minimum was obtained (McCullough, 2008). Results from fitting Eqs. (1) and (2) were algebraically rearranged to estimate  $t_{480}$ . For each equation, 999 non-parametric bootstrap samples of mean-centered residuals were computed with the nlsBoot() function from the nlstools package v1.0-2 (Baty et al., 2015). A  $t_{480}$  was derived for each bootstrap sample for Eqs. (1) and (2). To further compare the equivalency of Eqs. (1)-(3), predicted mean lengths at ages 8 and 20 were computed from each bootstrap sample for all three equations. Approximate 90% confidence intervals (CI) for each function parameter, derived  $t_{480}$  estimate, and predicted mean length-atage were the 5th and 95th percentile values of the 999 bootstrap estimates. The 90% confidence intervals were used to eliminate the tail portion of the bootstrapped distributions to better compare the equivalency of estimated parameters and derived values across equations.

Gillnets were used to capture walleye from two locations in Lake Winnibigoshish in September 2012. Total length was measured to the nearest mm, integer ages were estimated from cracked otoliths viewed with a fiber optic light, and sex was determined by visually examining gonads. We estimated  $t_{432}$  because 432 mm was the lower end of a protective slot limit for Lake Winnibigoshish walleye in 2012. We used extra sum-of-squares tests in a sequential step-down process (as described in Ogle, 2016) to identify which of eight possible models best fit these data. The eight models were modifications of Eq. (3) where all, two, one, or no parameters differed between the two sexes. All models were fit with the default Gauss-Newton algorithm in nls() of R. The confint() function from the MASS package (Venables and Ripley, 2002) was used to construct 95% profile likelihood CI for all function parameters in the final model. The profile likelihood method, rather than bootstrapping, was used for these CI to illustrate that the likelihood profile method can be used to estimate CI for  $t_{432}$  from Eq. (3).

#### 4. Results

Point estimates for all parameters and derived values, including  $t_{480}$ , shared between Eqs. (1)-(3) were equivalent (Table 1). Confidence intervals for all parameters and derived values shared between Eqs. (1)-(3) were similar, but not exactly equal due to the inherent stochasticity of the bootstrap method (Table 1). Lake whitefish from the Big Bay de Noc genetic stock reached a total length of 480 mm at approximately 8 years of age.

The  $L_{\infty}$  ( $F_{1,482}$  = 147.43, P < 0.001) and  $t_{432}$  ( $F_{1,482}$  = 128.30, P < 0.001) parameters, but not K ( $F_{1,481}$  = 3.21, P = 0.074), differed significantly between male and female Lake Winnibigoshish walleye (Figure 2). The  $L_{\infty}$  was greater for female (95% CI: 641-707 mm) than male (95% CI: 560-616 mm) walleye, whereas  $t_{432}$  was lower for female (95% CI: 3.78-3.95 years) than male (95% CI: 4.61-4.93 years) walleye. These results suggest that female walleye in Lake Winnibigoshish reached the minimum slot length limit (432 mm) before and achieved a longer maximum mean length than males.

#### 5. Conclusion

Eq. (3) is a simple parameterization of the VBGF that includes the typical and original VBGF parameterizations as special cases. However, Eq. (3) is flexible in that it may also be used to estimate mean length for any specific age or age for any specific mean length, rather than only intercept values as with the typical and original VBGFs. We expect the primary use of Eq. (3) among fisheries scientists will be to estimate age at a specific length (i.e.,  $t_r$ ). Thus, we demonstrated that point- and bootstrapped-interval estimates for  $t_r$  from Eq. (3) match those derived from parameters estimated with Eqs. (1) and (2). We also showed how Eq. (3) allows use of likelihood profile methods to estimate confidence intervals and model selection procedures to statistically determine if age at the specified mean length differs among populations.

A direct estimate of  $t_r$  (but as  $t_0$ ) may also be made by replacing  $L_t$  in Eq. (1) with  $L_t - L_r$  (i.e., subtracting  $L_r$  from each observed length). However,  $L_\infty$  from fitting this modified equation is underestimated by a constant  $L_r$ . If  $L_r$  is also subtracted from  $L_\infty$  in Eq. (1), then  $L_\infty$  will be estimated on the original scale. These two *ad hoc* modifications simply convert Eq. (1) to Eq. (3). Thus, for conceptual consistency with previous parameterizations of the VBGF and because of the flexibility afforded by Eq. (3), we suggest using Eq. (3), rather than *ad hoc* approaches, when interest lies in estimating or testing for differences among populations in  $L_\infty$ , K, and a specific point on the growth curve, such as  $t_r$ .

## Acknowledgments

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## 178 **Supplementary Information**

179 R code for all figures and analyses.

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Table 1. Estimated parameters ( $L_{\infty}$ , K,  $t_0$ ,  $L_0$ , and, for Eq. (3),  $t_{480}$ ), derived variables ( $t_{480}$  for Eqs. (1) and (2)), and predicted mean lengths-at-ages 8 ( $L_8$ ) and 20 ( $L_{20}$ ), with 90% confidence intervals in parentheses, and residual sum-of-squares (RSS) from fitting Eqs. (1)-(3) to the Big Bay de Noc genetic stock of lake whitefish.

Parameter/ Variable	Eq. (1)	Eq. (2)	Eq. (3)
$L_{\infty}$	550.83 (540.45, 572.97)	550.83 (540.99, 574.34)	550.83 (541.33, 577.59)
K	0.197 (0.108, 0.300)	0.197 (0.097, 0.297)	0.197 (0.093, 0.306)
$t_0$	-2.386 (-9.834, 1.027)		
$L_0$		206.31 (-214.67, 380.72)	
$t_{480}$	$8.04 (7.09, 8.65)^{a}$	$8.04 (7.02, 8.67)^{a}$	8.04 (7.03, 8.64)
$L_8$	479.38 (469.10, 489.68)	479.38 (468.89, 489.62)	479.38 (469.42, 489.57)
$L_{20}$	544.08 (537.65, 550.31)	544.08 (538.22, 549.73)	544.08 (538.65, 550.62)
RSS	320685.4	320685.4	320685.4

<sup>&</sup>lt;sup>a</sup>Value derived by rearranging the equation to solve for *t* with a length of 480 mm.

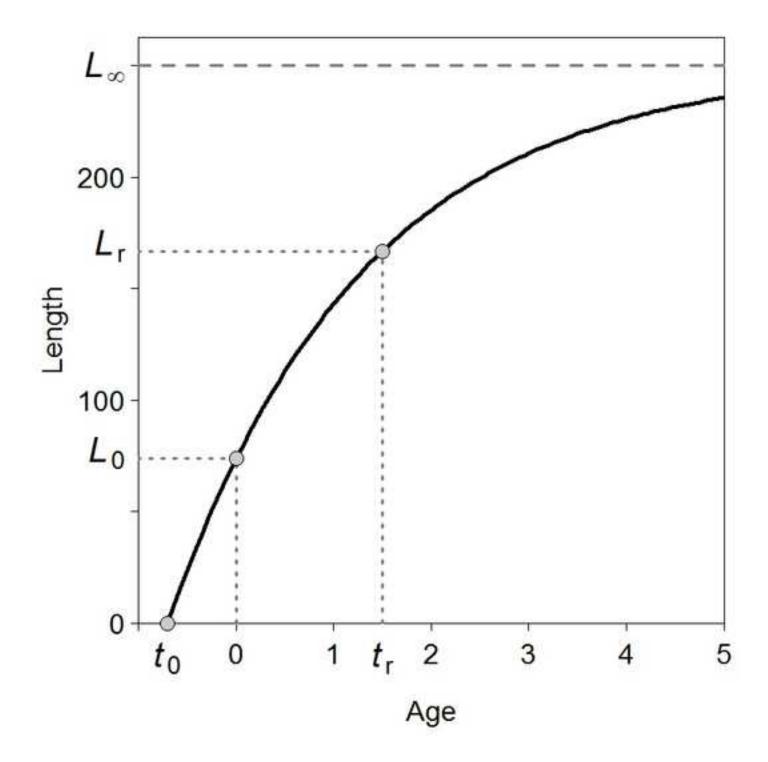
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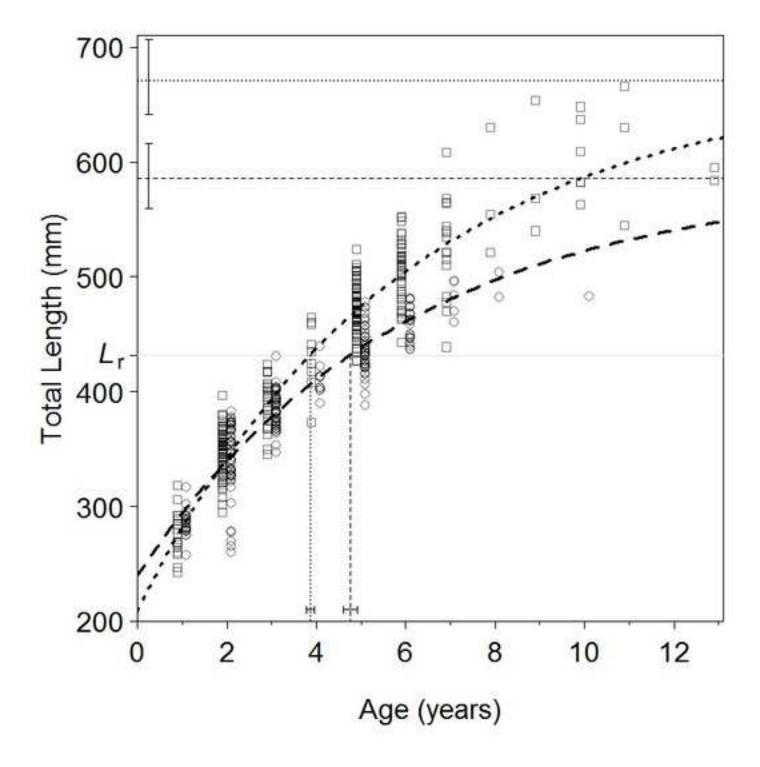
- 261 Fig. 1. Examples of Eqs. (1)-(3) with  $L_{\infty} = 250$ , K = 0.7,  $t_0 = -0.7$ , and  $L_0 = 73.8$ . Three points on
- 262 the curve are shown with gray circles --  $(t_0, 0)$  specifically defines Eq. (1),  $(0, L_0)$  specifically
- defines Eq. (2), and  $(t_r, L_r)$  generically defines Eq. (3).
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- Fig. 2. Fits of Eq. (3) to female (open squares, dotted line) and male (open circles, dashed line)
- total length-at-age data for walleye captured from Lake Winnibigoshish in September, 2012.
- Points are slightly offset from the integer ages to reduce overlap between sexes. Point estimates
- and 95% confidence intervals are shown for each sex along the y-axis for  $L_{\infty}$  and along the x-
- axis for  $t_{432}$ . The gray horizontal line is at  $L_r = 432$  mm. One 581 mm age-16 male is not shown.

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