Modified von Bertalanffy Growth Function to Directly Estimate the Age at a Critical Length

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1 Critical Time Concept

2 t_r Often Estimated by Inverting the VBGF

3 Reparamereterize the VBGF to Estimate t_r

4 An Example with Lake Michigan Lake Whitefish

5 Summary

Definitions

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  - e.g., age at maturity, recruitment, a length defined by a regulation.

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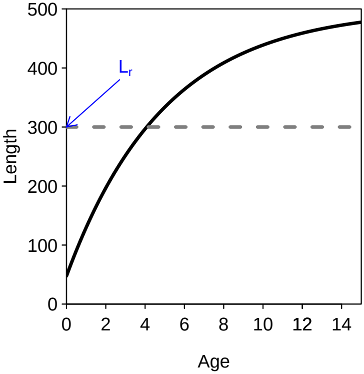

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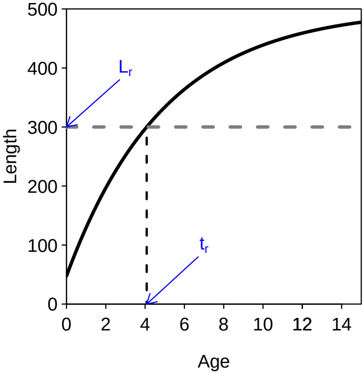
  - e.g., age at maturity, recruitment, a length defined by a regulation.

- L_r
  - Length at critical point.
  - Defined by scientist.
- t_r
  - Age (time) at the critical point.
  - To be estimated.



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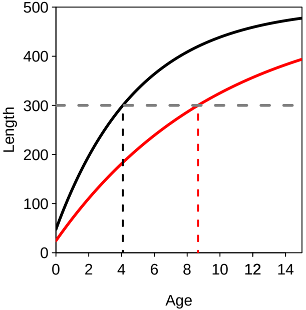

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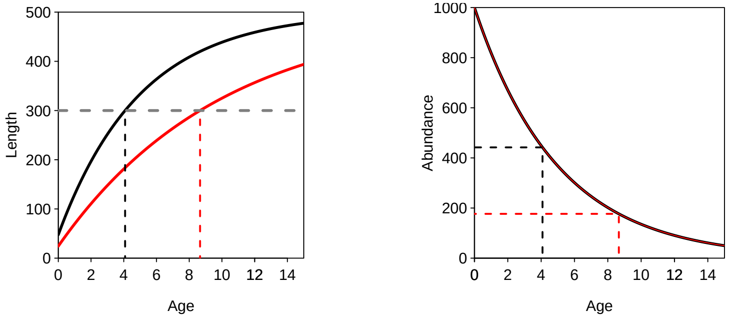

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North American Journal of Fisheries Management 27:918–931, 2007 [Article]

Yellow Perch in South Dakota: Population Variability and Predicted Effects of Creel Limit Reductions and Minimum Length Limits

DANIEL A. ISERMANN*[1] AND DAVID W. WILLIS
Department of Wildlife and Fisheries Sciences, South Dakota State University, Box 2140B, Brookings, South Dakota 57007, USA

Length limit modeling.—We conducted length limit modeling for four of the six lakes where yellow perch population characteristics were described. We excluded East 81 Slough and Enemy Swim Lake from length limit modeling because fishing mortality may not be a significant source of mortality in these populations (Blackwell 2005a; Isermann et al. 2005) and because growth of yellow perch in these systems is slow. Dynamic-pool models utilizing Jones' (1957) modification to the equilibrium yield equation of Beverton and Holt (1957), as available in Fishery Analyses and Simulation Tools (FAST version 2.0; Slipke and intercept (t_0) estimates due to the lack of mean length data for ages 0 and 1. Consequently, t_0 was fixed at zero when deriving k and L_{inf} from the von Bertalanffy model and in all length limit simulations. Our length limit modeling assumed that growth remained constant at the designated level across year-classes and that density-dependent growth responses did not occur. To further describe growth, we used von Bertalanffy models to estimate the time (years) required to reach lengths of 178 (t_{178}), 229 (t_{229}), 254 (t_{254}), and 279 mm (t_{279}).

Using the customized recruitment option available in

Importance

- Foundational value in *yield-per-recruit* and *dynamic pool* models.
  - Commonly used to examine impact of length regulations.

North American Journal of Fisheries Management 22:1349–1357, 2002

Predictive Evaluation of Size Restrictions as Management Strategies for Tennessee Reservoir Crappie Fisheries

DANIEL A. ISERMANN,*[1] STEVE M. SAMMONS,[2] AND PHILLIP W. BETTOLI
TIMOTHY N. CHURCHILL

North American Journal of Fisheries Management 22:1306–1313, 2002

Effect and Acceptance of Bluegill Length Limits in Nebraska Natural Lakes

CRAIG P. PAUKERT*[1] AND DAVID W. WILLIS
DONALD W. GABELHOUSE, JR.

North American Journal of Fisheries Management 31:269–279, 2011

Simulated Population Responses of Common Carp to Commercial Exploitation

Michael J. Weber,* Matthew J. Hennen,[1] and Michael L. Brown

Environ Biol Fish (2007) 79:11–25

The New River, Virginia, muskellunge fishery: population dynamics, harvest regulation modeling, and angler attitudes

Travis O. Brenden · Eric M. Hallerman · Brian R. Murphy · John R. Copeland ·

North American Journal of Fisheries Management 22:1340–1348, 2002

Rescinding a 254-mm Minimum Length Limit on White Crappies at Ft. Supply Reservoir, Oklahoma: The Influence of Variable Recruitment, Compensatory Mortality, and Angler Dissatisfaction

JEFF BOXRUCKER*

North American Journal of Fisheries Management 29:1183–1194, 2009

Fishery and Population Characteristics of Blue Catfish and Channel Catfish and Potential Impacts of Minimum Length Limits on the Fishery in Lake Wilson, Alabama

MICHAEL P. HOLLEY,[1] MATTHEW D. MARSHALL, AND MICHAEL J. MACEINA*

Transactions of the American Fisheries Society 134:1285–1298, 2005

Population Characteristics and Assessment of Overfishing for an Exploited Paddlefish Population in the Lower Tennessee River

GEORGE D. SCHOLTEN*[1] PHILLIP W. BETTOLI

North American Journal of Fisheries Management 15:766–772, 1995

An Evaluation of the Value of Harvest Restrictions in Managing Crappie Fisheries

M. S. ALLEN AND L. E. MIRANDA

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von Bertalanffy Growth Function Review

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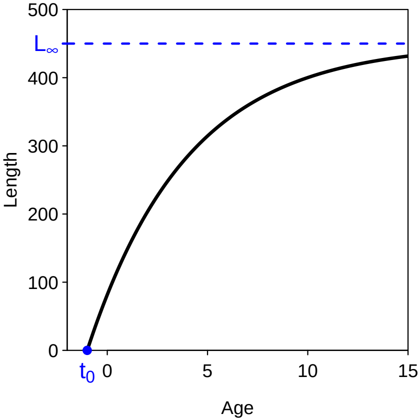

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$$t = \frac{log_e\left(1-\frac{L_t}{L_\infty}\right)}{-K} + t_0$$

- Then set L_t to the chosen L_r so that t will be t_r.

Computing t_r from Inverse VBGF

$$t = \frac{log_e\left(1 - \frac{L_t}{L_\infty}\right)}{-K} + t_0$$

- Suppose that L_∞=450, K=0.2, and t_0=-1.
- Suppose that the critical length of interest is L_r=300.

Computing t_r from Inverse VBGF

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$$t = \frac{log_e\left(1 - \frac{300}{450}\right)}{-0.2} + -1 = 4.5$$

- Thus, the estimated mean time to reach 300 mm is 4.5 years.

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  - Could *try* to fit inverse function and predict t_r at L_r.

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- BUT … how do you compute confidence intervals for t_r?
  - Could use bootstrap, delta method, or error propagation.
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- BUT … how do you compare t_r between groups?

We need a better method!

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Revisit (Typical) VBGF

- The typical VBGF ...

$$L_t = L_\infty \left(1 - e^{-K(t-t_0)}\right)$$

- ... may be rewritten as ...

$$L_t = 0 + (L_\infty - 0)\left(1 - e^{-K(t-t_0)}\right)$$

Revisit (Original) VBGF

- The original VBGF (from von Bertalanffy) ...

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- These VBGFs simply define different points on the same line.

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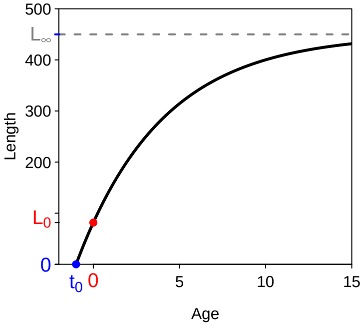

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- Can a more useful point on the line be defined?
    - A more general VBGF is

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    - Original VBGF sets $t_r = 0$ and estimates $L_r = L_0$.
  - **However, we can also set L_r to a critical length and estimate t_r.**

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Example with Lake Michigan Lake Whitefish

- Data from Belnap (2014).[1]
    - Fish from commercial trapnets in six management zones.
    - Measured total length (TL; mm).
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    - Interested in t_{480} (480 mm is length at full vulnerability to harvest).

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  - Compared parameter estimates and predicted mean lengths-at-age.

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- Fit typical and modified VBGF to fish from Big Bay de Noc.
    - Compared parameter estimates and predicted mean lengths-at-age.
- Fit modified VBGF to fish from Big Bay de Noc and Green Bay to illustrate t_r comparison (following methods in Ogle (2016)[2]).

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Comparison of VBGF Results

Typical VBGF

	Estimate	LCI	UCI
L_∞	549.51	541.56	560.99
K	0.29	0.19	0.39
t_0	2.70	-0.32	4.32
t_{480}	9.88	-	-

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t_{480}	9.88	9.43	10.28

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Age	Pred Len
8.00	429.97
15.00	533.56
25.00	548.61
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- Fit all (8) models where all, two, one, or no parameters differ between the two locations (Big Bay de Noc and Green Bay).

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Model	Params	AICc	ΔAICc	Weight
L_∞, t_{480}	6	3203.3	0.00	0.38
K, t_{480}	6	3203.5	0.19	0.34
L_∞, K, t_{480}	7	3204.7	1.40	0.19
t_{480}	5	3206.2	2.90	0.09
L_∞	5	3221.2	17.91	0.00
K	5	3222.5	19.14	0.00
L_∞, K	6	3223.3	19.97	0.00
None	4	3230.0	26.68	0.00

Comparing t_r Between Groups

Big Bay de Noc

	Estimate	LCI	UCI
L_∞	549.51	541.79	560.50
K	0.29	0.20	0.38
t_{480}	9.88	9.44	10.27

Green Bay

	Estimate	LCI	UCI
L_∞	555.91	546.84	571.16
K	0.39	0.18	1.23
t_{480}	7.56	6.20	8.48

Comparing t_r Between Groups

Big Bay de Noc

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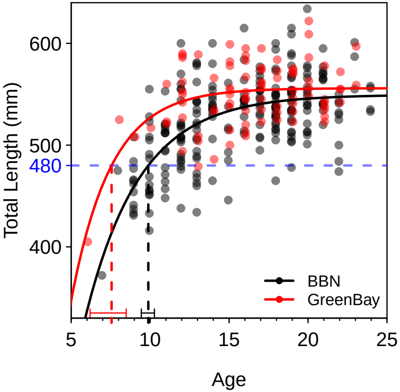
t_{480}	9.88	9.44	10.27

Green Bay

	Estimate	LCI	UCI
t_{480}	7.56	6.20	8.48



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Summary

$$L_t = L_r + (L_\infty - L_r)\left(1 - e^{-K(t-t_r)}\right)$$

- A simple modification of the VBGF allows direct estimation of a parameter of interest, t_r.
  - Same estimates of L_∞ and K as with the typical VBGF.
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  - Easy interval estimates of t_r.
  - Compare t_r between groups with standard (ANCOVA-like) methods.

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- Benefits
  - Easy interval estimates of t_r.
  - Compare t_r between groups with standard (ANCOVA-like) methods.
- Costs
  - No direct estimate of t_0 (in the typical VBGF).

Recommendation

$$L_t = L_r + (L_\infty - L_r)\left(1 - e^{-K(t-t_r)}\right)$$

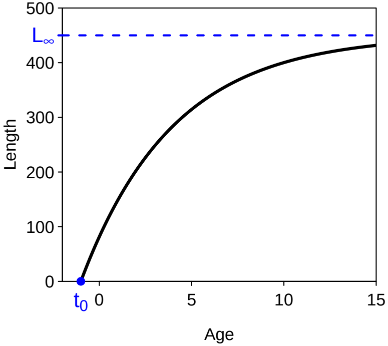
Use this modified VBGF in place of the typical VBGF.

Acknowledgments

- Matthew Belnap for collection and initial processing of Whitefish data.
- Ben Wegleitner, Zach Kleemann, Andrew Gullickson, and Connie Isermann for help processing Whitefish otoliths.
- David Staples (Minnesota DNR) and Joshua McCormick (Oregon Department of Fisheries & Wildlife) for comments on modified VBGF.

Revisit t_0

- Recall that t_0 is the x-intercept (value of X when $Y = 0$).



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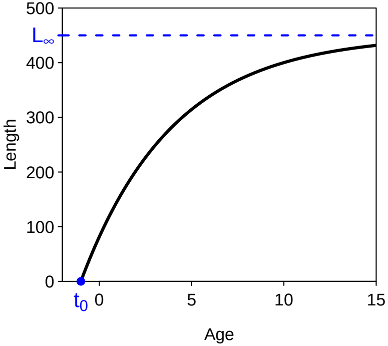

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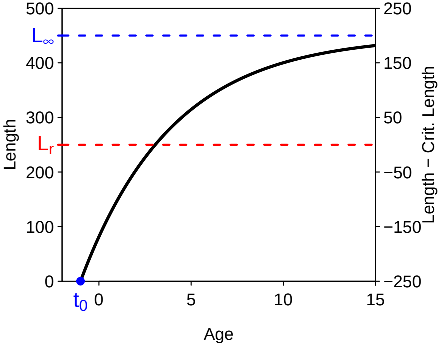

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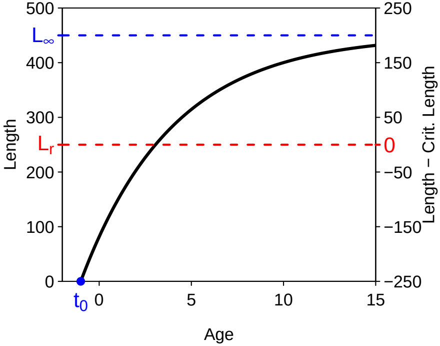

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- Thus, $Y = 0$ means that $L_t^* = 0$, $L_t - L_r = 0$, and $L_t = L_r$.

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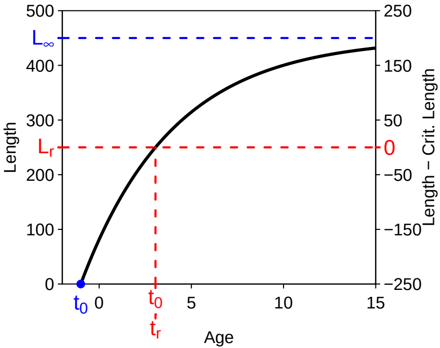

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- Thus, $Y = 0$ means that $L_t^* = 0$, $L_t - L_r = 0$, and $L_t = L_r$.
- Thus, when using L_t^*, $t_0 = t_r$.

Modified VBGF

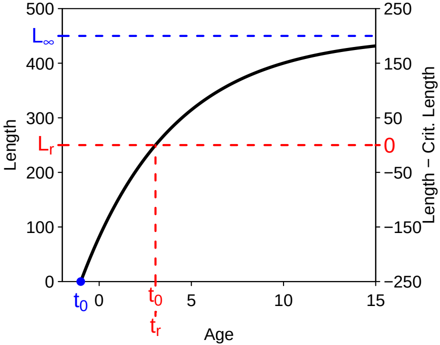
- Therefore, this simple adjustment allows direction estimation of t_r.

$$L_t - L_r = L_\infty \left(1 - e^{-K(t-t_r)}\right)$$

$$L_t = L_r + L_\infty \left(1 - e^{-K(t-t_r)}\right)$$

Modified VBGF

- However, L_∞ is now incorrect.



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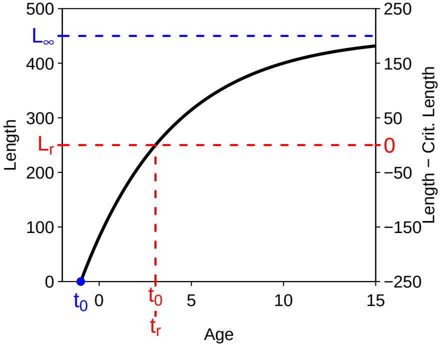

- But this is easily corrected.

$$L_t = L_r + (L_\infty - L_r)\left(1 - e^{-K(t-t_r)}\right)$$