MODULE 17

1-SAMPLE T-TEST

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PRIOR TO THIS MODULE, hypothesis testing methods required knowing σ , which is a parameter that is seldom known. When σ is replaced by its estimator, s, the test statistic follows a Student's t rather than a standard normal (Z) distribution. In this module, the t-distribution is described and a 1-Sample t-Test for testing that the mean from one population equals a specific value is discussed.

17.1 t-distribution

A t-distribution is similar to a standard normal distribution (i.e., N(0,1)) in that it is centered on 0 and is bell shaped (Figure 17.1). The t-distribution differs from the standard normal distribution in that it is heavier in the tails, flatter near the center, and its exact dispersion is dictated by a quantity called the degrees-of-freedom (df). The t-distribution is "flatter and fatter" because of the uncertainty surrounding the use of s rather than σ in the standard error calculation. The degrees-of-freedom are related to n and generally come from the denominator in the standard deviation calculation. As the degrees-of-freedom increase, the t-distribution becomes narrower, taller, and approaches the standard normal distribution (Figure 17.1).

¹Recall that the sample standard deviation is a statistic and is thus subject to sampling variability.

Figure 17.1. Standard normal (black) and t-distributions (red) with varying degrees-of-freedom.

Proportional areas on a t-distribution are computed using distrib() similar to what was described for a normal distribution in Modules 8 and 13. The major exceptions for using distrib() with a t-distribution is that distrib="t" must be used and the degrees-of-freedom must be given in df= (how to find df is discussed in subsequent sections). For example, the area right of t = -1.456 on a t-distribution with 9 df is 0.9103 (Figure 17.2).

```
> ( distrib(-1.456,distrib="t",df=9,lower.tail=FALSE) )
[1] 0.9103137
```

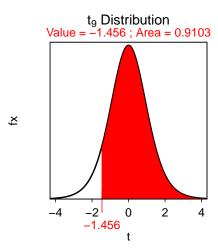


Figure 17.2. Depiction of the area to the right of t = -1.456 on a t-distribution with 9 df.

Similarly, the t with an upper-tail area of 0.95 on a t-distribution with 19 df is -1.729 (Figure 17.3).²

```
> ( distrib(0.95,distrib="t",type="q",df=19,lower.tail=FALSE) )
[1] -1.729133
```

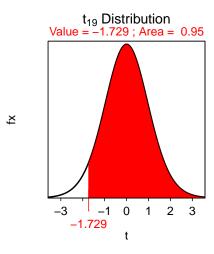


Figure 17.3. Depiction of the value of t with an area to the right of 0.95 on a t-distribution with 19 df.

17.2 1-Sample t-Test Specifics

A 1-Sample t-Test is similar to a 1-Sample Z-test in that both test the same H_0 . The difference, as discussed above, is that when σ is replaced by s, the test statistic becomes t and the scaling factor for confidence regions becomes a t^* . Other aspects are similar between the two tests as shown in Table 17.1.³

Table 17.1. Characteristics of a 1-Sample t-Test.

- Hypothesis: $H_0: \mu = \mu_0$
- Statistic: \bar{x}
- Test Statistic: $t = \frac{\bar{x} \mu_0}{\frac{s}{\sqrt{n}}}$
- Confidence Region: $\bar{x} + t^* \frac{s}{\sqrt{n}}$
- **df**: *n* − 1
- Assumptions:
 - 1. σ is UNknown
 - 2. $n \ge 40$, $n \ge 15$ and the **sample** (i.e., histogram) is not strongly skewed, OR the **sample** is normally distributed.
- When to Use: Quantitative response, one population, σ is UNknown.

²This "reverse" calculation would be t^* for a 95% lower confidence bound.

³Compare Table 17.1 to Table 16.1.

Example - Purchase Catch of Salmon? 17.2.1

Below are the 11-steps (Section 16.1) for completing a full hypothesis test for the following situation:

A prospective buyer will buy a catch of several thousand salmon if the mean weight of all salmon in the catch is at least 19.9 lbs. A random selection of 50 salmon had a mean of 20.1 and a standard deviation of 0.76 lbs. Should the buyer accept the catch at the 5% level?

17.3. 1-SAMPLE T-TEST IN R

- 1. $\alpha = 0.05$.
- 2. $H_0: \mu = 19.9$ lbs vs. $H_A: \mu > 19.9$ lbs where μ is the mean weight of ALL salmon in the catch.
- 3. A 1-Sample t-Test is required because (1) a quantitative variable (weight) was measured, (ii) individuals from one population were sampled (this catch of salmon), and (iii) σ is UNknown.⁴
- 4. The data appear to be part of an observational study with random selection.
- 5. (i) n=50 > 40 and (ii) σ is unknown.
- 6. $\bar{x} = 20.1 \text{ lbs}$ (and s = 0.76 lbs).
- 7. $t = \frac{\frac{20.1 19.9}{0.76}}{\frac{0.76}{\sqrt{50}}} = \frac{0.2}{0.107} = 1.87$ with df = 50-1 = 49.
- 8. p-value = 0.0337.
- 9. H_0 is rejected because the p-value $< \alpha$.
- 10. The average weight of ALL salmon in this catch appears to be greater than 19.9 lbs; thus, the buyer should accept this catch of salmon.
- 11. I am 95% confident that the mean weight of ALL salmon in the catch is greater than 19.92 lbs (i.e., $20.1 - 1.677 \frac{0.76}{\sqrt{50}} = 20.1 - 0.18 = 19.92$).

R Appendix:

```
( pval <- distrib(1.87,distrib="t",df=49,lower.tail=FALSE) )</pre>
(zstar <- distrib(0.95.distrib="t".tvpe="q".df=49.lower.tail=FALSE))
```

17.3 1-Sample t-Test in R

If raw data exist, the calculations for a 1-Sample t-test can be efficiently computed with t.test(). The arguments to t.test() are the same as those for z.test(), with the exception that sd= is not used with t.test(). Thus, t.test() requires the vector of quantitative data as the first argument, the null hypothesized value for μ in mu=, the type of alternative hypothesis in alt= (again, can be alt="two.sided" (the default), alt="less", or alt="greater"), and the level of confidence as a proportion in conf.level= (defaults to 0.95). The use of t.test() is illustrated in the following example.

Example - Crab Body Temperature 17.3.1

Below are the 11-steps (Section 16.1) for completing a full hypothesis test for the following situation:

A marine biologist wants to determine if the body temperature of crabs exposed to ambient air temperature is different than the ambient air temperature. The biologist exposed a sample of 25 crabs to an air temperature of 24.3°C for several minutes and then measured the body temperature of each crab (shown below). Test the biologist's question at the 5% level.

```
22.9,22.9,23.3,23.5,23.9,23.9,24.0,24.3,24.5,24.6,24.6,24.8,24.8,
25.1,25.4,25.4,25.5,25.5,25.8,26.1,26.2,26.3,27.0,27.3,28.1
```

 $^{^{4}}$ If σ is given, then it will appear in the background information to the question and will be in a sentence that uses the words "population", "assume that", or "suppose that."

- 1. $\alpha = 0.05$.
- 2. $H_0: \mu = 24.3^{\circ}\text{C}$ vs. $H_A: \mu \neq 24.3^{\circ}\text{C}$, where μ is the mean body temperature of ALL crabs.
- 3. A 1-Sample t-Test is required because (1) a quantitative variable (temperature) was measured, (ii) individuals from one population were sampled (an ill-defined population of crabs), and (iii) σ is **UN**known.
- 4. The data appear to be part of an experimental study (the temperature was controlled) with no suggestion of random selection of individuals.
- 5. (i) $n = 25 \ge 15$ and the sample distribution of crab temperatures appears to be only slightly right-skewed (Figure 17.4) and (ii) σ is **UN**known.
- 6. $\bar{x} = 25.0^{\circ} \text{C}$ (Table 17.2).
- 7. t = 2.713 with 24 df (Table 17.2).
- 8. p-value = 0.0121 (Table 17.2).
- 9. H_0 is rejected because the p-value $< \alpha$.
- 10. It appears that the average body temperature of ALL crabs is greater than the ambient temperature of 24.3° C.
- 11. I am 95% confident that the mean body temperature of ALL crabs is between 24.5° C and 25.6° C (Table 17.2).

Table 17.2. Results from 1-Sample t-Test for body temperature of crabs.

```
t = 2.7128, df = 24, p-value = 0.01215
95 percent confidence interval:
  24.47413 25.58187
sample estimates:
mean of x
  25.028
```

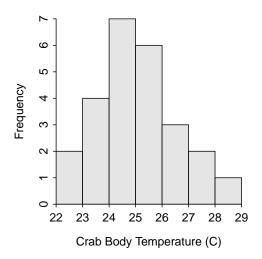


Figure 17.4. Histogram of the body temperatures of crabs exposed to an ambient temperature of 24.3° C.

R Appendix:

```
df <- read.csv("data/CrabTemps.csv")
hist(~ct,data=df,xlab="Crab Body Temp (C)")
( ct.t <- t.test(df$ct,mu=24.3,conf.level=0.95) )</pre>
```