

HYPOTHESIS TESTING • MTH107

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1-Sample Z-Test

$H_0: \mu = \mu_0$ (where μ_0 = specific value)

Statistic: \bar{X} Test Statistic: $Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ Conf. Region: $\bar{X} \pm Z^* \frac{\sigma}{\sqrt{n}}$

Assumptions: 1) σ is known
2) $n \geq 30$, $n \geq 15$ and population not strongly skewed, OR population is normal

R: `z.test()`

1-Sample t-Test

$H_0: \mu = \mu_0$ (where μ_0 = specific value)

Statistic: \bar{X} Test Statistic: $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ Conf. Region: $\bar{X} \pm t^* \frac{s}{\sqrt{n}}$ df: $n-1$

Assumptions: 1) σ is UNknown,
2) $n \geq 40$, $n \geq 15$ & histogram not strongly skewed, OR histogram is normal

R: `t.test()`, `hist()`

2-Sample t-Test

$H_0: \mu_1 = \mu_2$ Statistic: $\bar{X}_1 - \bar{X}_2$

Test Statistic: $t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$ where $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Conf. Region: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$ df: $n_1 + n_2 - 2$

Assumptions:
1) Individuals in groups are independent
2) $n_1 + n_2 \geq 40$, $n_1, n_2 \geq 15$ and both histograms are not strongly skewed, OR both histograms are normal
3) Group population variances are equal (use Levene's Test)

R: `t.test()`, `leveneTest()`, `hist()`

Choosing a Hypothesis Test

1. If response variable is QUANTITATIVE, GOTO 2; if CATEGORICAL, GOTO 5.

Quantitative Response

2. If 1 group/population, GOTO 3; if 2 or more groups/populations, GOTO 4.
3. If σ is KNOWN, then 1-Sample Z; If σ is UNKNOWN, then 1-Sample t.
4. If individuals are INDEPENDENT between groups, then 2-Sample t; otherwise, Paired t.

Categorical Response

5. If 1 group, then Goodness-of-Fit; if 2 or more groups, then Chi-Square.

Making a Decision about H_0

If p-value < α , then REJECT H_0 , otherwise DNR H_0 .

Chi-Square Test

H_0 : "Distribution of individuals into response levels is the same for all groups"

H_A : "Distribution of individuals into response levels is NOT the same for all groups"

Statistic: Observed frequency table

Test Statistic: $\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$ df: (rows-1)(columns-1)

Assumptions: ≥ 5 in each cell of the EXPECTED table

R: `xtabs()`, `matrix()`, `chisq.test()`, `percTable()`

Goodness-of-Fit Test

H_0 : "Distribution of individuals into response levels follows the theoretical distribution"

H_A : "Distribution of individuals into response levels does NOT follow the theoretical distribution"

Statistic: Observed frequency table

Test Statistic: $\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$ df: cells-1

Assumptions: ≥ 5 in each cell of the EXPECTED table

R: `xtabs()`, `c()`, `chisq.test()`, `percTable()`, `chiGOF()`

11 STEPS FOR ANY HYPOTHESIS TEST

- 1) State the rejection criterion (α)
- 2) State the null & alternative hypotheses and define the parameter(s)
- 3) Determine which test to perform – Explain!
- 4) Collect the data (address type of study and randomization)
- 5) Check all necessary assumption(s)
- 6) Calculate the appropriate statistic(s)
- 7) Calculate the appropriate test statistic
- 8) Calculate the p-value
- 9) State your rejection decision
- 10) Summarize your findings in terms of the problem
- 11) Compute and interpret a **100(1- α)%** *confidence region* for parameter