# Compare VBGFs

### **Preliminaries**

#### Load Necessary Packages

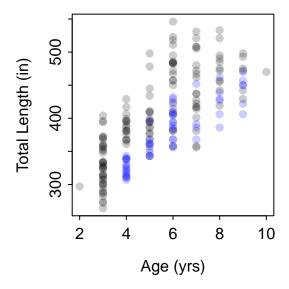
```
> library(FSA)  # for filterD(), headtail(), col2rgbt(), vbFuns(), vbStart()
> library(dplyr)  # for mutate(), select()
> library(nlstools)  # for nlsBoot()
```

#### Load Data and Make Some Preparations

#### **Quick Summaries**

> clr2 <- col2rgbt(clr1,1/5)

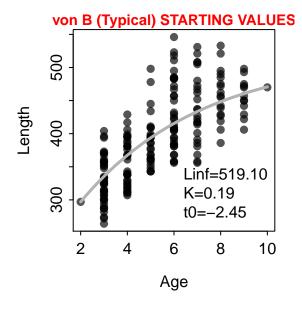
```
> plot(len~age,data=wae,pch=19,col=clr2[waterbody],xlab=xlbl,ylab=ylbl)
```



```
> Summarize(len~age+waterbody,data=wae,digits=1)
Warning: First RHS variable was converted to a factor.
   age
           waterbody n nvalid mean
                                       sd min
                                                 Q1 median
                                                              Q3 max percZero
                                       NA 297 297.0
                                                     297.0 297.0 297
    2 LAKE CHIPPEWA 1
                            1 297.0
2
    3 LAKE CHIPPEWA 40
                            40 328.6 35.7 264 301.5
                                                     326.5 356.5 404
                                                                             0
3
    4 LAKE CHIPPEWA 17
                           17 387.9 18.4 361 376.0
                                                     386.0 396.0 429
                                                                            0
    5 LAKE CHIPPEWA 18
                           18 403.4 39.5 343 380.0
                                                     395.0 410.5 498
    6 LAKE CHIPPEWA 21
                            21 460.9 50.4 358 424.0
                                                     472.0 485.0 546
                                                                            0
    7 LAKE CHIPPEWA 20
                            20 446.1 52.5 356 416.2
                                                     437.0 486.2 531
7
    8 LAKE CHIPPEWA 11
                           11 476.7 33.4 419 456.0
                                                     475.0 492.5 533
                                                                            0
    9 LAKE CHIPPEWA 6
                           6 468.2 27.5 429 449.5
                                                     473.5 488.5 498
8
    10 LAKE CHIPPEWA 1
9
                            1 470.0
                                      NA 470 470.0
                                                     470.0 470.0 470
                                                                            0
10
    4
           SAND LAKE 16
                           16 324.8 12.0 307 314.2
                                                     324.0 338.0 343
                                                                            0
    5
          SAND LAKE 9
                            9 359.6 16.4 343 348.0
                                                     361.0 363.0 396
                                                                            0
11
                           16 394.1 23.8 356 380.0
12
    6
          SAND LAKE 16
                                                     394.0 409.0 432
    7
13
           SAND LAKE
                            4 399.2 36.5 368 381.5
                                                     388.5 406.2 452
                                                                            0
14
    8
           SAND LAKE 6
                             6 424.8 26.3 386 411.2
                                                     428.0 436.5 462
                                                                            0
15
          SAND LAKE
                             6 442.5 23.8 406 429.0
                                                     450.0 455.2 470
```

### Fitting Most Complex Model and Checking Assumptions

```
> ( sv0m <- vbStarts(len~age,data=wae,plot=TRUE) )</pre>
```



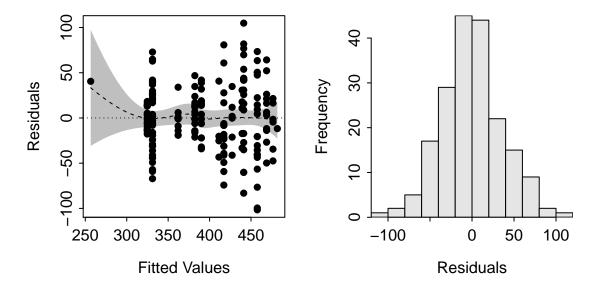
```
$Linf
[1] 519.1043

$K
[1] 0.1905709

$t0
[1] -2.454813
```

```
> ( svLKt \leftarrow Map(rep,svOm,c(2,2,2)) )
$Linf
[1] 519.1043 519.1043
$K
[1] 0.1905709 0.1905709
$t0
[1] -2.454813 -2.454813
> svLKt <- list(Linf=c(550,510),K=c(0.2,0.2),t0=c(-2.5,-2.5))
```

```
> vbLKt <- len~Linf[waterbody]*(1-exp(-K[waterbody]*(age-t0[waterbody])))</pre>
> fitLKt <- nls(vbLKt,data=wae,start=svLKt)</pre>
> residPlot(fitLKt)
```



## Are There Any Differences?

1vA 189 -990.004 186 -957.712 3 -32.292 64.584 6.157e-14

```
> vb0m <- len~Linf*(1-exp(-K*(age-t0)))
> fit0m <- nls(vb0m,data=wae,start=sv0m)</pre>
> extraSS(fitOm,com=fitLKt,sim.name="{Omega}",com.name="{Linf,K,t0}")
Model 1: {Omega}
Model A: {Linf,K,t0}
    DfO
          RSSO DfA
                     RSSA Df
                                 SS
                                              Pr(>F)
1vA 189 338461 186 241781 3 96679 24.791 1.535e-13
> lrt(fit0m,com=fitLKt,sim.name="{Omega}",com.name="{Linf,K,t0}")
Model 1: {Omega}
Model A: {Linf,K,t0}
    DfO logLikO DfA logLikA Df
                                 logLik Chisq Pr(>Chisq)
```

### Is the Most Complex Model Warranted?

```
> vbLK <- len~Linf[waterbody]*(1-exp(-K[waterbody]*(age-t0)))
> (svLK \leftarrow Map(rep,svOm,c(2,2,1)))
$Linf
[1] 519.1043 519.1043
[1] 0.1905709 0.1905709
$t0
[1] -2.454813
> fitLK <- nls(vbLK,data=wae,start=svLK)</pre>
> vbLt <- len~Linf[waterbody]*(1-exp(-K*(age-t0[waterbody])))</pre>
> svLt <- Map(rep,svOm,c(2,1,2))
> fitLt <- nls(vbLt,data=wae,start=svLt)</pre>
> vbKt <- len~Linf*(1-exp(-K[waterbody]*(age-t0[waterbody])))</pre>
> svKt \leftarrow Map(rep,svOm,c(1,2,2))
> fitKt <- nls(vbKt,data=wae,start=svKt)</pre>
> extraSS(fitLK,fitLt,fitKt,com=fitLKt,com.name="{Linf,K,t0}",
          sim.names=c("{Linf,K}","{Linf,t0}","{K,t0}"))
Model 1: {Linf,K}
Model 2: {Linf,t0}
Model 3: {K,t0}
Model A: {Linf,K,t0}
    Df0
               RSSO DfA
                              RSSA Df
                                                        F Pr(>F)
1vA 187 241786.383 186 241781.303 1
                                            5.081 0.0039 0.9502
2vA 187 242080.068 186 241781.303 1
                                          298.765 0.2298 0.6322
3vA 187 241871.187 186 241781.303 1
                                        89.884 0.0691 0.7929
```

# Can the Model be Reduced to Only One Parameter that Differs?

```
> vbL <- len-Linf[waterbody]*(1-exp(-K*(age-t0)))
> ( svL <- Map(rep,svOm,c(2,1,1)) )
$Linf
[1] 519.1043 519.1043

$K
[1] 0.1905709

$t0
[1] -2.454813

> fitL <- nls(vbL,data=wae,start=svL)
> vbK <- len-Linf*(1-exp(-K[waterbody]*(age-t0)))
> svK <- Map(rep,svOm,c(1,2,1))
> fitK <- nls(vbK,data=wae,start=svK)
Error in nls(vbK, data = wae, start = svK): step factor 0.000488281 reduced below 'minFactor' of 0.000976562

> fitK <- nls(vbK,data=wae,start=svK,control=list(minFactor=1e-15,maxiter=500))
> extraSS(fitL,fitK,com=fitLK,com.name="{Linf,K}",sim.names=c("{Linf}","{K}"))
```

```
Model 1: {Linf}
Model 2: {K}
Model A: {Linf,K}

DfO RSSO DfA RSSA Df SS F Pr(>F)

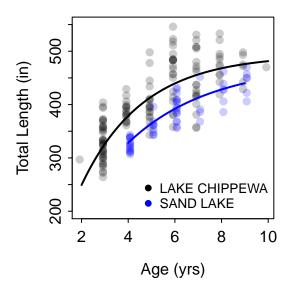
1vA 188 246207.2 187 241786.4 1 4420.8 3.4191 0.06602

2vA 188 242717.7 187 241786.4 1 931.3 0.7203 0.39713
```

#### Summarize Final Model

```
> summary(fitK,correlation=TRUE)
Formula: len ~ Linf * (1 - exp(-K[waterbody] * (age - t0)))
Parameters:
       Estimate Std. Error t value Pr(>|t|)
Linf 4.922e+02 1.711e+01 28.774 < 2e-16
      3.743e-01 8.318e-02
                            4.500 1.19e-05
K2
      2.638e-01 4.688e-02
                            5.627 6.57e-08
    -9.452e-04 5.156e-01 -0.002
t0
                                      0.999
Residual standard error: 35.93 on 188 degrees of freedom
Correlation of Parameter Estimates:
  Linf K1
             K2
K1 - 0.95
K2 -0.94 0.98
t0 -0.87 0.97 0.96
Number of iterations to convergence: 16
Achieved convergence tolerance: 1.431e-06
> round(cbind(coef(fitK),confint(fitK)),3)
Waiting for profiling to be done...
                2.5%
                      97.5%
Linf 492.234 467.551 539.594
K1
       0.374
             0.230 0.540
K2
       0.264
              0.175
                      0.352
  -0.001 -1.277 0.768
> vb <- vbFuns("typical")</pre>
> # Lake Chippewa
> crLC <- filterD(wae, waterbody=="LAKE CHIPPEWA")</pre>
> svLC <- list(Linf=492,K=0.37,t0=0)
> fitLC <- nls(len~vb(age,Linf,K,t0),data=crLC,start=svLC)</pre>
> # Sand Lake
> crSL <- filterD(wae, waterbody=="SAND LAKE")</pre>
> svSL <- list(Linf=492, K=0.26, t0=0)
> fitSL <- nls(len~vb(age,Linf,K,t0),data=crSL,start=svSL)</pre>
> offset <- 0.08
> # Lake Chippewa
> plot(len~I(age-offset),data=crLC,pch=19,col=clr2[1],ylim=c(200,550),xlab=xlbl,ylab=ylbl)
> curve(vb(x-offset,coef(fitLC)),from=2,to=10,col=clr1[1],lwd=2,add=TRUE)
```

```
> # Sand Lake
> points(len~I(age+offset),data=crSL,pch=19,col=clr2[2])
> curve(vb(x+offset,coef(fitSL)),from=4,to=9,col=clr1[2],lwd=2,add=TRUE)
> legend("bottomright",levels(wae$waterbody),pch=19,col=clr1,bty="n",cex=0.8)
```



### **Using Information Criterion**

Fit the Only Other Model not Fit Above

```
> vbt <- len~Linf*(1-exp(-K*(age-t0[waterbody])))
> svt <- Map(rep,sv0m,c(1,1,2))
> fitt <- nls(vbt,data=wae,start=svt)</pre>
```

#### AICc Table

```
> library(AICcmodavg)
> ms <- list(fitOm,fitL,fitK,fitt,fitLK,fitLt,fitKt,fitLKt)</pre>
> mnames <- c("{Omega}","{Linf}","{K}","{t0}","{Linf,K}","{Linf,t0}","{K,t0}","{Linf,K,t0}")
> aictab(ms,mnames)
Model selection based on AICc:
                 AICc Delta_AICc AICcWt Cum.Wt
{K}
                                           0.35 -958.08
            5 1926.49
                             0.00
                                    0.35
{Linf,K}
            6 1927.88
                             1.39
                                    0.17
                                          0.52 - 957.71
{K,t0}
            6 1927.95
                             1.46
                                    0.17
                                           0.69 - 957.75
                                           0.85 -957.83
{Linf,t0}
            6 1928.12
                             1.63
                                    0.16
{Linf}
            5 1929.23
                             2.74
                                    0.09
                                           0.94 - 959.45
{Linf,K,t0} 7 1930.03
                             3.54
                                    0.06
                                           1.00 -957.71
{t0}
            5 1935.54
                             9.05
                                    0.00
                                           1.00 -962.61
            4 1988.22
                            61.73
                                    0.00
                                           1.00 -990.00
{Omega}
```