Nonlinear Models & Von Bertalanffy Growth

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von Bertalanffy Model – Typical

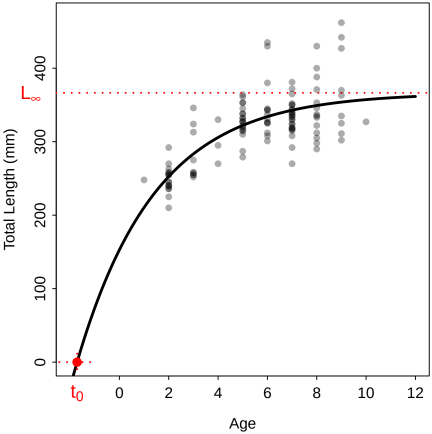
$$E[L|t] = L_\infty \left(1 - e^{-K(t-t_0)}\right)$$

where

- $E[L|t]$ is the expected (i.e., average) length at time (or age) t,

von Bertalanffy Model – Typical

- L_∞ is the asymptotic average length.
- t_0 is a modeling artifact.



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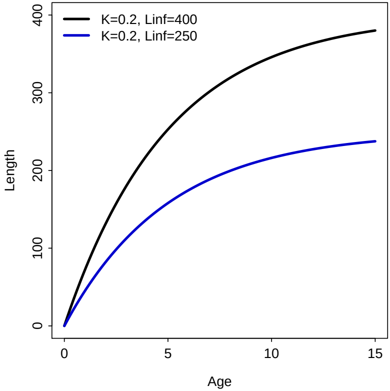

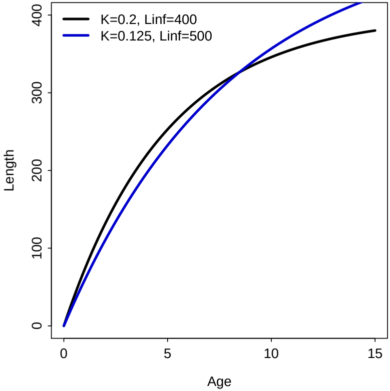

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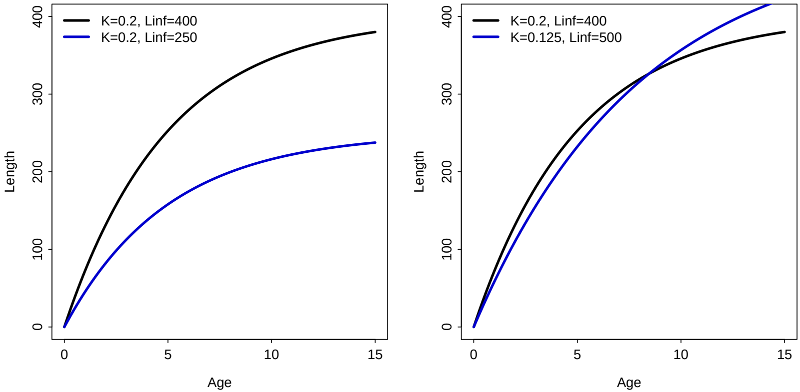

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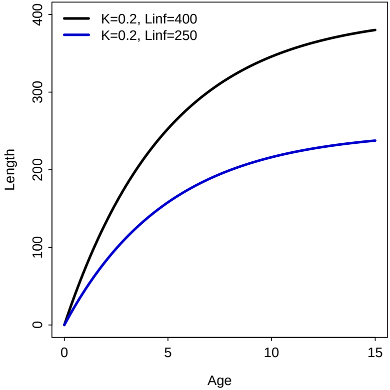

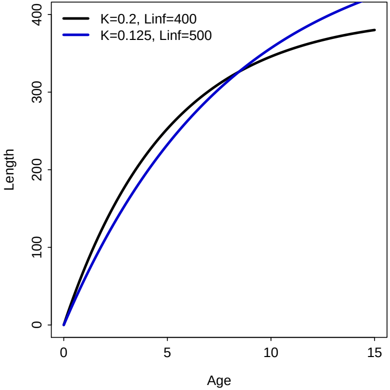

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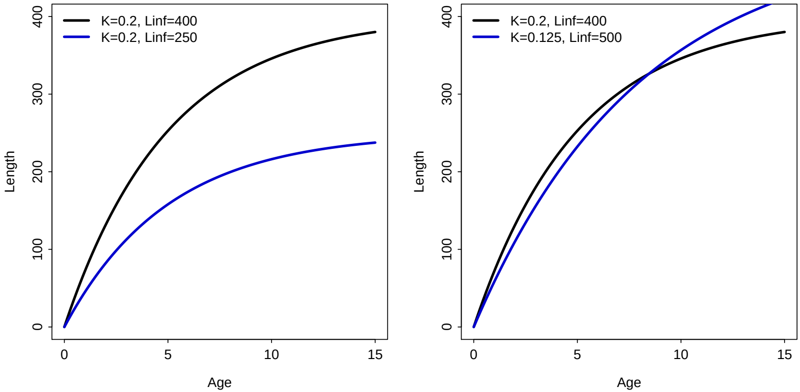

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  - $\frac{log(2)}{K}$ is "half-life" (time to reach $\frac{L_\infty}{2}$).

Non-Linear Least-Squares

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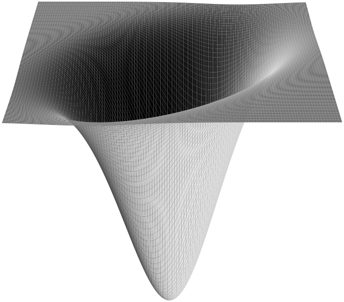
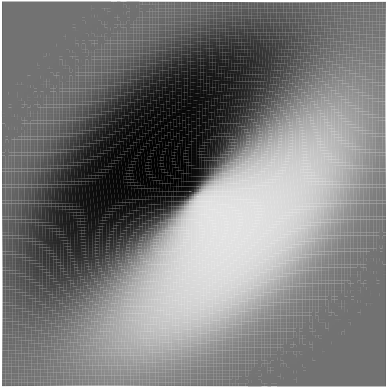
- Non-linear least-squares methods minimize RSS.

RSS Surface (side view)



Non-Linear Least-Squares

RSS Surface (top view)



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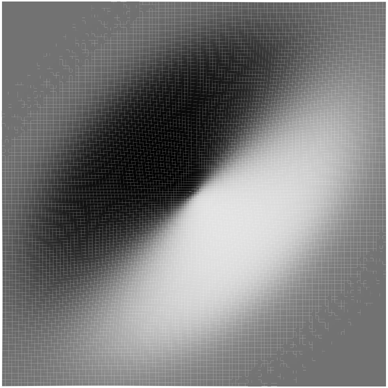

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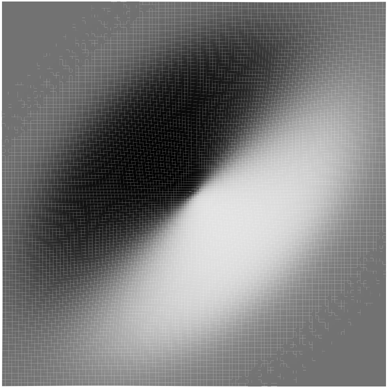

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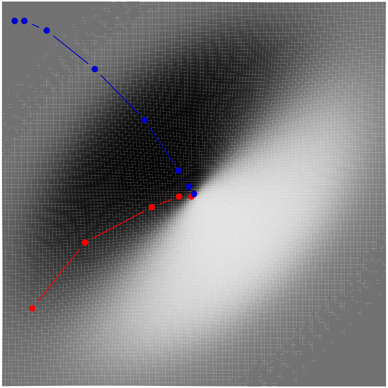

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  4. 95% CI is values of ordered parameter estimates with 2.5% of values lesser and 2.5% of values greater.

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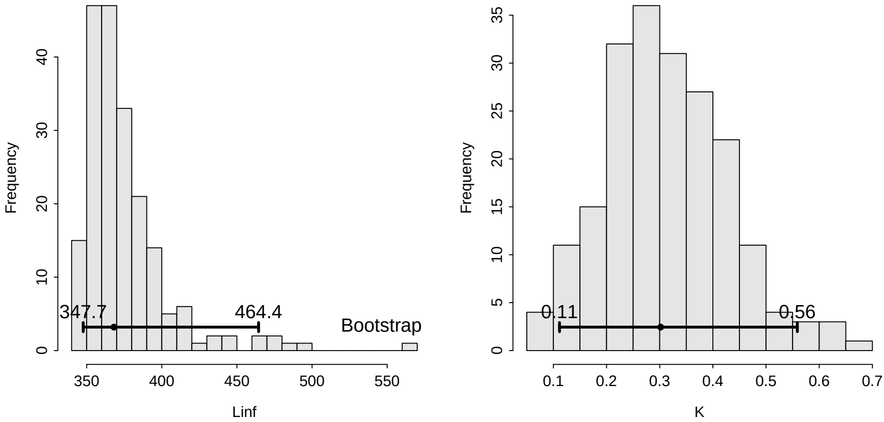

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