Growth Comparison with VBGM

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Wisconsin Age & Growth Workshop
Stevens Point, WI
14&15 January 2014

Objectives

1. Motivation

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2. Model Comparisons

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3. Presenting Results

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4. An Extension I
5. An Extension II

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Motivation

- Compare "growth" (i.e., compare VBGM parameters) among groups.

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- Examples
    1. Compare between sexes.
    2. Compare between "locations" (e.g., water bodies, habitats).
    3. Compare between years.
    4. Compare between management periods.

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Example Background

- Atlantic Croaker (*Micropogonias undulatus*)

- 318 observations of length, age, and sex.

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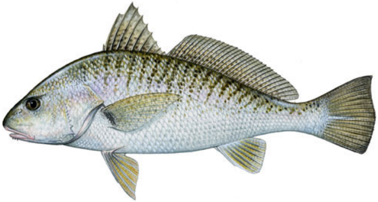
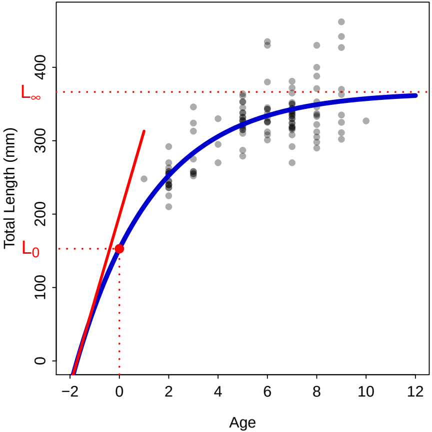


- 318 observations of length, age, and sex.

- Objectives
  1. Generally, does growth differ between male and female Atlantic Croakers?
  2. Specifically, does "early" or "mature" growth differ between sexes (i.e., does ω or L_∞ differ)?

Review Mooij Parameterization

- ω is representative of the instantaneous growth rate near t_0.
- L_∞ is the asymptotic average length.
- L_0 is the mean length at time zero (i.e., birth).



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which can be written in shorthand as

$$E[L|t] = L_\infty[sex] - (L_\infty[sex] - L_0[sex])\, e^{-\frac{\omega[sex]}{L_\infty[sex]}t}$$

where $sex = 1$ if female and $sex = 2$ if male.

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- ω in common.

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General	541854	-	3282.6
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- L_∞, but not L_0 or ω, differs between the sexes.
- Thus, the asymptotic average length differs between the sexes, but the growth rate very early in life does not.

Objectives

Coefficients Table

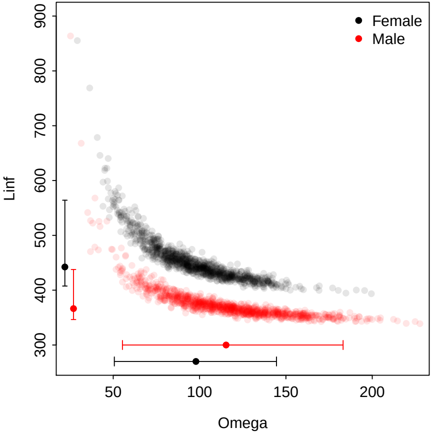
Females

	Estimate	95% LCI	95% UCI
Linf	442.3	407.5	564.1
L0	176.8	105.3	229.8
omega	97.9	50.6	144.5

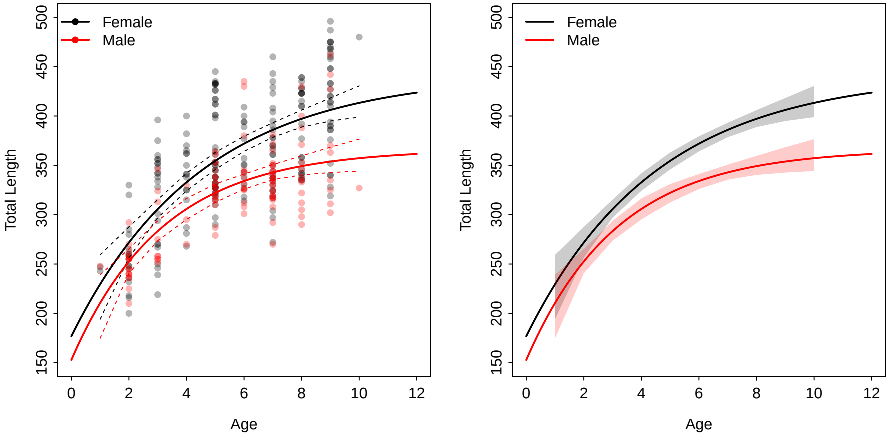
Males

	Estimate	95% LCI	95% UCI
Linf	366.4	346.4	437.7
L0	152.8	55.5	208.4
omega	115.3	55.3	183.1

Coefficients Plot



Fitted Models Plot



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1. Compare all one parameter in common models to general model.

Model	RSS	p	AIC
General	541854	-	3282.6
common L1	542744	0.47461	3281.1
common L2	592740	0.00000	3309.1
common L3	564900	0.00032	3293.8

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2. Compare two parameter in common models to L_1 in common model.

Model	RSS	p	AIC
common L1	542744	-	3281.1
common L1 and L2	612541	0.00000	3317.6
common L1 and L3	567658	0.00015	3293.4

An Extension I – Fit Francis Model

- Set parameter ages at 1 and 10 (and, thus, 5.5).
- Parameters are mean lengths at these ages.

3. Draw conclusions.
    - Mean length-at-age 1 does not differ between sexes.
    - Mean lengths-at-ages 5.5 and 10 do differ between sexes.

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An Extension II – Growth Rate at Age

- Mooij *et al.* (1999) showed that the derivative of the VBGM is a measure of the instantaneous growth rate at each age.

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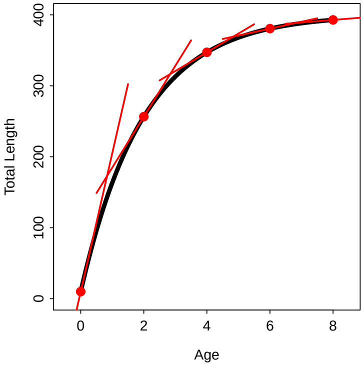

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- Predict growth rates and length at each age and group from each bootstrapped sample.
- Plot predicted growth rates vs. length for each age and group.

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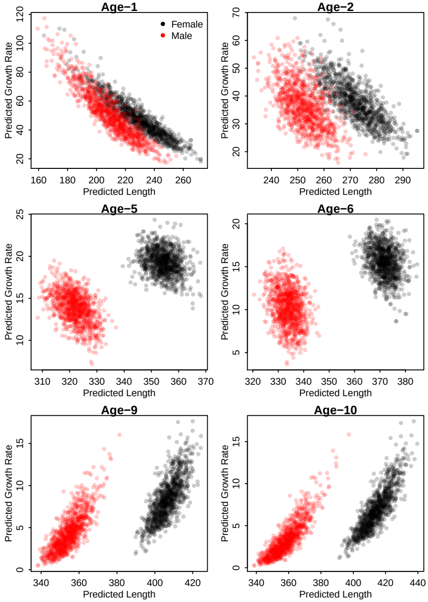

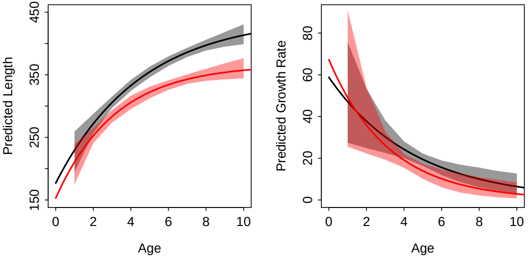

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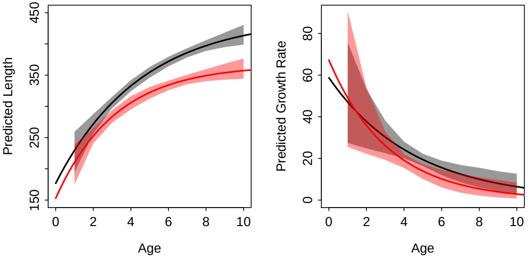

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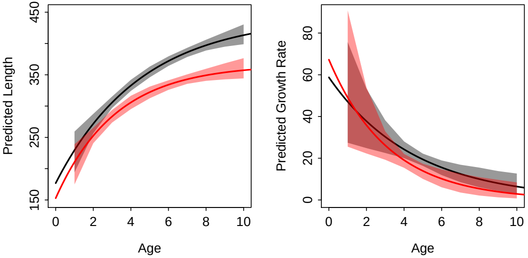

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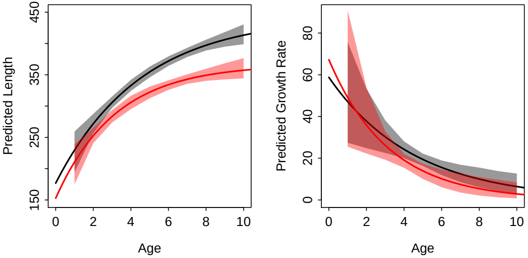

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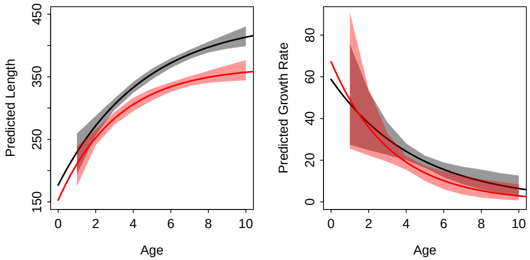

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✓ GR is similar after age-6 such that the difference in mean L is maintained.