Back-Calculation of Previous Length

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Northland College

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Objectives

1. Concept

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2. Motivation

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3. Foundational Models

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Definition of Back-Calculation

Francis (1990) defined *back-calculation* as,

> *"... the dimensions of one or more marks in some hard part of the fish, together with its current body length, are used to estimate its length at the time of formation of each of the marks. ..."*

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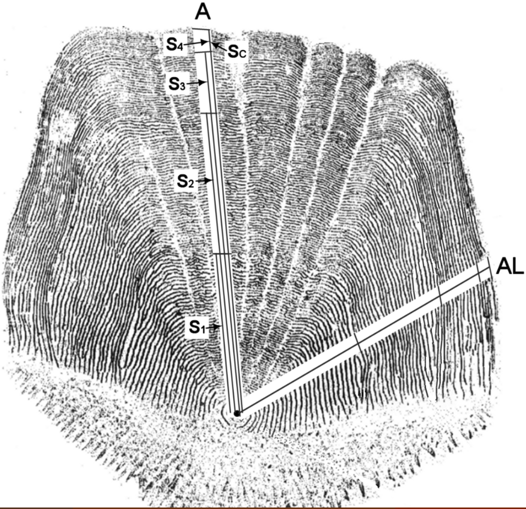

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- For example, if structure size at i is 40% of the structure size at capture than fish size at i is 40% of fish size at capture.

- Algebraically re-arrange to get simplest back-calculation model.

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- **Interest in variability in individual growth trajectories.**
  - ✓ Each fish provides individual longitudinal growth information.

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Foundational Regression Models

1. Regression of L_C on S_C.

$$E(L|S) = a + bS$$

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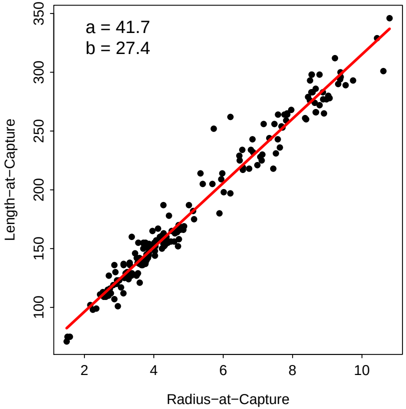

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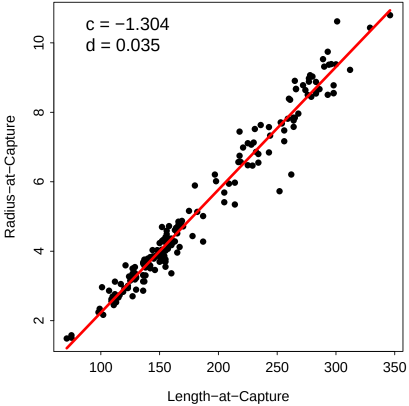

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Most Common Back-Calculation Methods

- Dahl-Lea (Direct Proportion)
- Fraser-Lee
- Body Proportional Hypothesis (BPH)
- Scale Proportional Hypothesis (SPH)
- Regression

Dahl-Lea (Direct Proportion) Method

- Derived from "structure grows in direct proportion to fish length."

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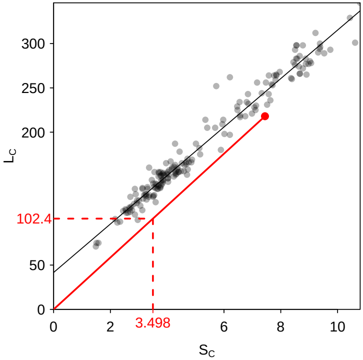

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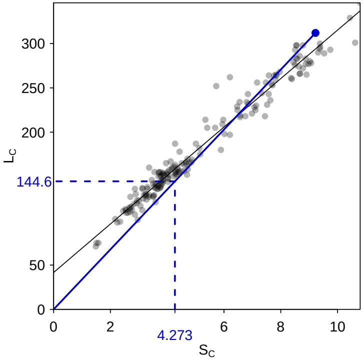

Fraser-Lee (Corrected Direct Proportion) Method

- Derived from "structure grows in direct proportion to the fish length after an initial adjustment for L when $S = 0$."

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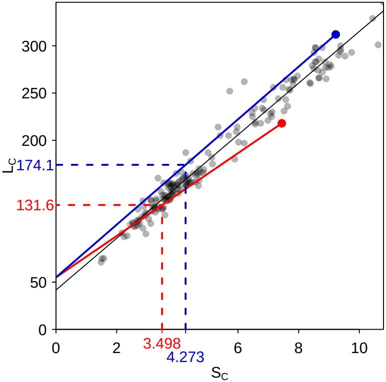
- k from
    - Known L when structure forms.
    - Published values (e.g., Carlander (1982)).
    - Intercept of L on S regression (i.e., a).

Fraser-Lee (Corrected Direct Proportion) Method

- Geometrically, L_i comes from a line between (S_C, L_C) and $(0, k)$.
  - In this example for Walleye, $k = 55$ as from Carlander (1982).



Body Proportional Hypothesis (BPH) Method

- Derived from "If L_C is 10% larger than average for a fish with S_C, then L_i was 10% larger than average for a fish with S_i."

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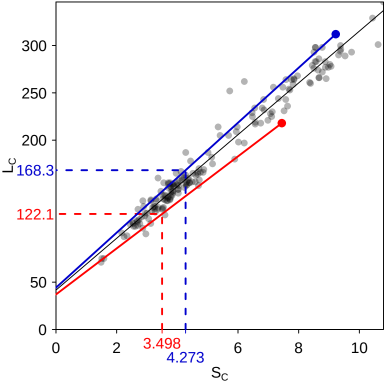

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$$L_i = \frac{S_i}{S_C}\left(L_C + \frac{c}{d}\right) - \frac{c}{d}$$

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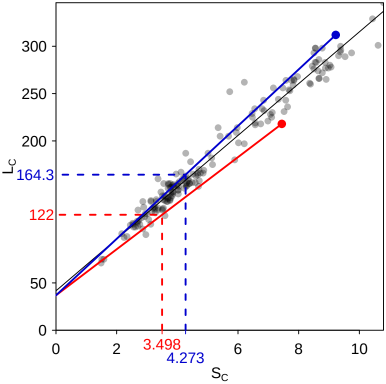

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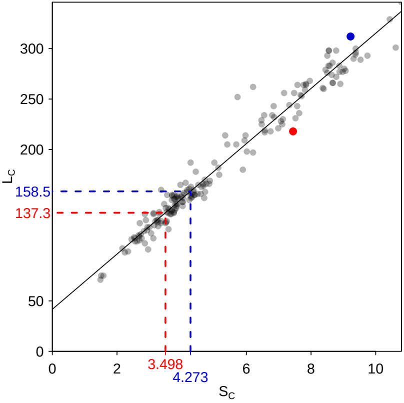

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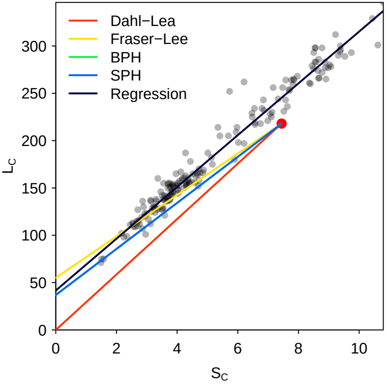

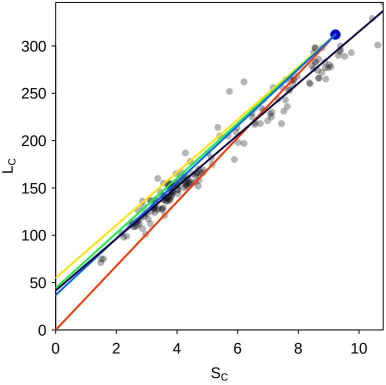

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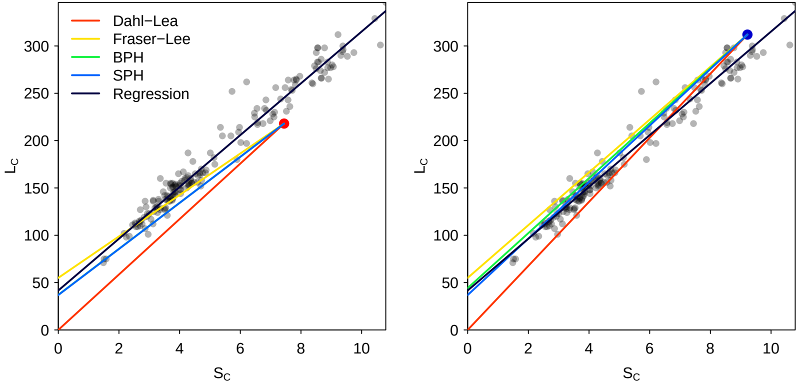

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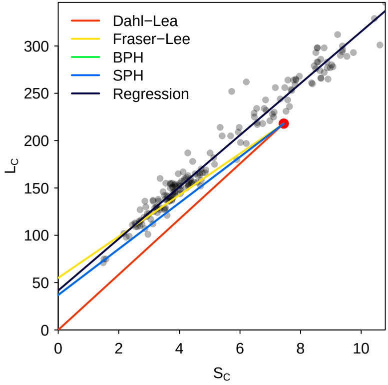

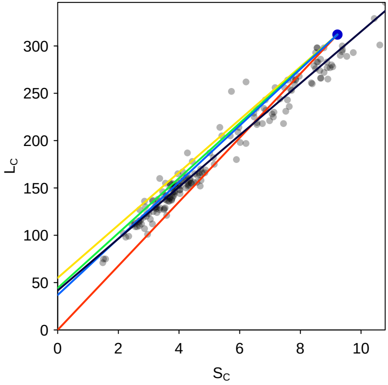

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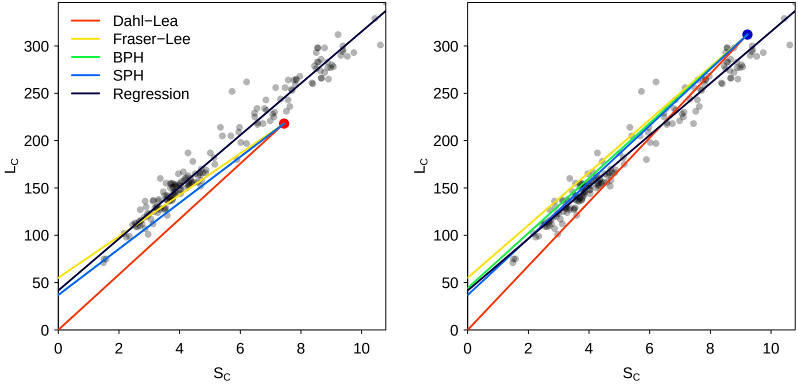

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- Regression method likely differs from other three at older ages for fish well off the regression line.
- Fraser-Lee, BPH, SPH likely similar for older ages, may differ more (but variably) for younger ages.

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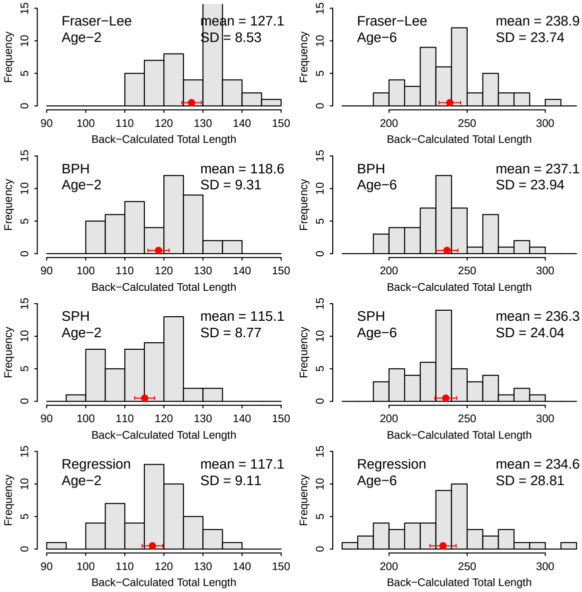

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	species	lake	gear	yearcap	fish	agecap	lencap	anu1	anu2	anu3	anu4	anu5	radcap	prvL1	prvL2	prvL3	prvL4
2	SMB	WB	T	1988	18	4	165	1.967	3.440	4.877	6.638		6.638	87.6	112.0	135.8	
3	SMB	WB	T	1988	19	4	198	2.720	3.424	4.565	5.411		6.281	116.9	133.0	158.9	
4	SMB	WB	E	1988	28	5	180	1.012	2.521	3.419	4.693	6.133	6.133	75.6	106.4	124.7	150.7
5	SMB	WB	E	1988	29	5	180	1.240	2.369	3.382	4.868	6.112	6.112	80.4	103.4	124.2	154.6

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```
> wb90r <- gReshape(wb90,in.pre="anu")
> wb90r$fl.len <- with(wb90r,(anu/radcap)*(lencap-55)+55)
```

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  - Fraser-Lee model modified to go through the fish and otolith length corresponding to the initiation of proportionality between fish and otolith growth.
  - "In many cases, the biological intercept could be determined by simple measurements of otolith and fish size in newly-hatched larvae in the laboratory" (Campana 1990).

Further Thoughts – Analysis of Back-Calculated Lengths

- Should be examined for evidence of “Lee’s Phenomenon.”
  - The tendency for back-calculated lengths at a given age in the same cohort of fish to be smaller as the fish they are computed from get older.

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	1	2	3	4	5	6	7
5	127.5	152.5	170.0	189.5	212.1	-	-
6	150.2	171.7	186.6	202.2	219.5	240.5	-
7	153.8	176.2	192.5	208.5	227.7	249.4	269.1

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- Successive lengths back-calculated from the same fish are not independent observations.
  - Must be analyzed with repeated mesures or mixed model methods.
  - See Jones (2000), Vigliola and Meekan (2009), and several others.

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- Suppose that you have fish captured in May (right when the annulus forms) and September (these contain "plus" growth).
- Can you combine the data (as is) from these samples for the growth analysis?

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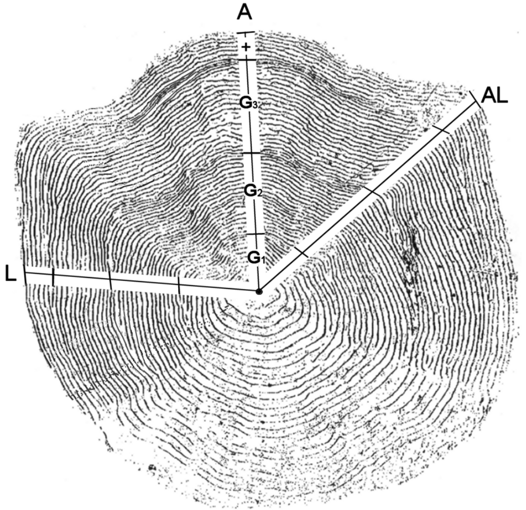
- **YES**, but need to back-calculate length of September-caught fish to annulus before “plus growth.”



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