Bachelor Thesis Project

Quaternion/Tensor Fields-based Implicit Surface Modeling.

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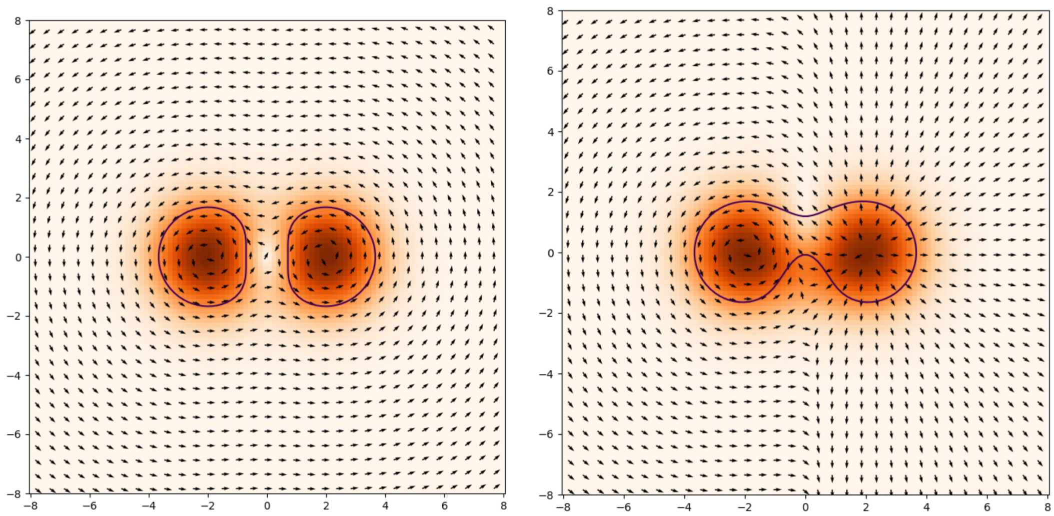

Figure 1: Examples of two anisotropic blending behaviors using complex fields in 2D. The iso-curve is shown by the red contours and is obtained in summing up two sub-fields with skeleton centered in the darker spots. Each sub-field associated to their respective point-skeleton p_i are defined as $\hat{f}_i(p) = e^{-|z|^2}\, e^{i\,\mathrm{arg}(z)+\varphi_i}$, where $z = p - p_i$. The left and right images have a different value for φ_1.

1- Context

Surfaces can be defined along two main paradigms: Either using explicit/boundary representations – and in particular with the mesh-based representation – in describing explicitly all the positions of the surface; or using an implicit representation, where a surface S is defined as an isovalue of a 3D scalar field f, with $S = \{p \in \mathbb{R}^3 | f(p) = iso\}$. Explicit and implicit representations are complementary to each other. While explicit representations allow direct model manipulation and benefits from GPU-based rendering, they are ill suited to handle topological modification such as blending two shapes together. Conversely, implicit representations enable such topological modification between shapes in using simple operations applied on their respective fields (see Fig. 2).

A successful implicit representation used for shape modeling is the so-called *skeleton-based* representation. In this representation, a skeleton i – typically a point of a segment – is associated to a scalar field f_i. The field associated to point-skeleton is often defined as a decreasing function of the distance between the skeleton position p_i and any other point in space $f_i(p) = f_i(\|p_i - p\|)$ – therefore modeling a density-like value. Among famous models are the blobs ($f_i(p) = \exp(-\|p_i - p\|^2/\sigma^2)$)[1], metaballs [7], and soft objects [4]. Such functions can be generalized to skeleton made of segments through the use of convolution surfaces [2]. Given a constant isovalue, each skeleton can be associated to a surface primitive – typically a sphere for point-skeleton, and cylinder/capsule-like shape

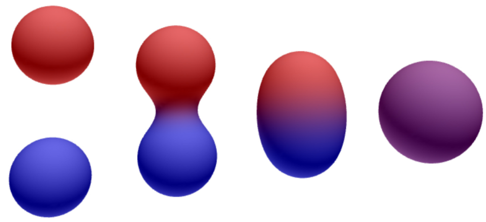

Figure 2: Implicit surfaces are used in 3D modeling for their ability to seamlessly blend shapes together.

for a segment. A complex shape can then be generated in assembling all the contributions of all the skeletons into a single field $f(p) = \bigoplus_i f_i(p)$, where $\bigoplus_i$ are the blending operators. The final surface S resulting from the blending of these primitive is therefore defined as

$$S = \left\{ p \in \mathbb{R}^3 \mid \bigoplus_i f_i(\|p - p_i\|) = iso \right\} \tag{1}$$

These operators $\bigoplus_i$ can range from a simple summation modeling smooth blending, to non smooth $\max$ operator depicting a sharp union. More recently, advanced operators were proposed, for instance in taking into account the gradient of the fields to control finely where the blending should occur depending on the relative orientation of the two surfaces [5] (see Fig. 3), or to N-ary operators that models surface contact instead of merging [3].

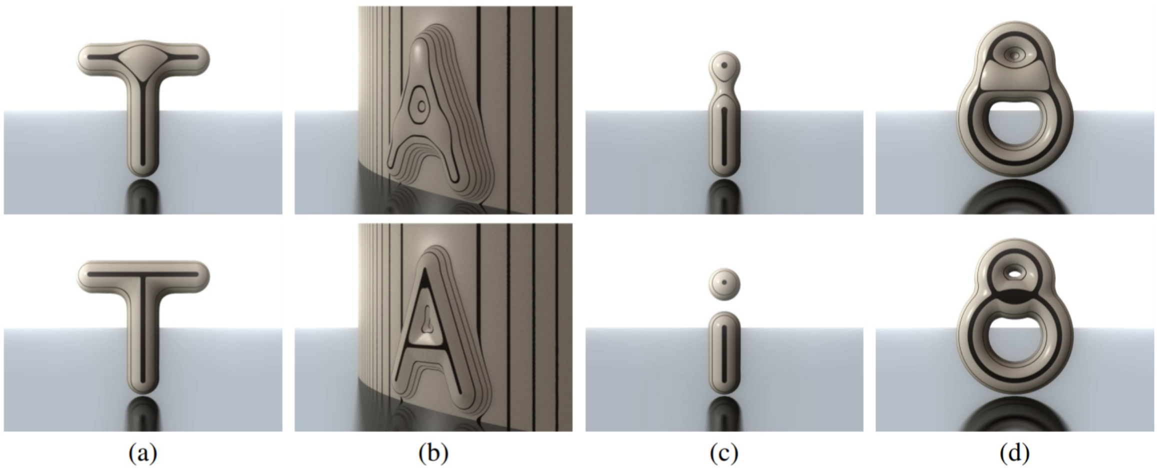

Figure 3: The use of Gradient Blending [5] improves how two primitives blend (top: standard smooth blending, bottom: use of gradient blending). However the blending remains anisotropic.

While more advanced operators representing for instance wrinkling patterns can be proposed in adding extra orientation information, designing them is rather complex as they have to be set up in the field space, possibly extended with the gradient space. In addition, the way that the shape blends is not intrinsically associated to the primitive, but is instead associated to the operator we apply between two fields. We propose in this project the use of more complex fields representation in order to model anisotropic effects such as wrinkle-like blending between surface, while preserving simple operators when merging the surfaces together.

2- Research objective

We aim at extending the definition of the field used in implicit modeling to quaternion and/or tensor fields, and explore this representation for 3D modeling purpose with a focus on anisotropic blending exhibiting oscillatory behavior. In the case of point-based skeleton, the isosurface will be defined as

$$S = \left\{ p \in \mathbb{R}^3 \mid \bigoplus_i \|\hat{f}_i(p, p_i)\| = iso \right\}, \tag{2}$$

where $\hat{f}_i$ are now Quaternions and/or Tensors. At the opposite of scalar fields, $\hat{f}_i$ have intrinsic orientation embedded in their definition. Therefore a simple operator such as $+$ may generate an anisotropic behavior without requiring the use of more involved operators (see for instance the 2D example using complex fields in Fig. 1). In addition, the anisotropic behavior of the skeleton i can then be fully embedded in its associated field $\hat{f}_i$, instead of the binary operator, thus modeling a specific type of material attached to the primitive itself rather than to its operator.

The main objective is to explore these complex fields in defining and parameterizing relevant individual functions $\hat{f}_i$ from a point and segment skeleton in order to model anisotropic blends from simple operators.

3- Methodology

This project will require to set-up a basic demonstrator in 3D

- Set-up a generic visualizer for implicit surface (ex. using Marching Cubes [6]).
- Program skeleton-based complex and/or tensor fields generation.
- Explore quaternion and/or tensor based *primitives* parameterization (magnitude/orientation for quaternions, eigenvalue/vectors for tensors) for point skeleton through blending with the operator +.
- Adapt the field parameterization (and/or operator) to model a wrinkle-like blending effect; Extend the model to segment skeleton.

References

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