7.5 $\quad \mu = 100 \quad \sigma^2 = 81 \quad n = 25$

a. $E(\bar{X}) = \mu = 100$

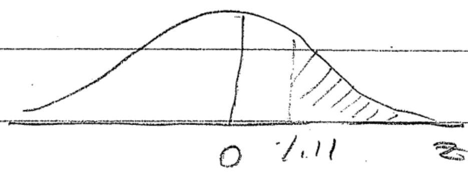
$V(\bar{X}) = \frac{\sigma^2}{n} = \frac{81}{25} = 3.24$

$S(\bar{X}) = \sqrt{3.24} = 1.8$

b. $P(\bar{X} > 102) = P\left(z > \frac{102-100}{1.8}\right)$

$= P(z > .62) = 1 - P(z < 1.11)$

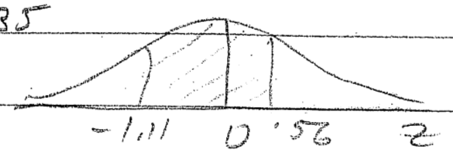
$= 1 - .8665 = .1335$



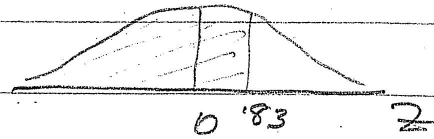
c. $P(98 < \bar{X} < 101) = P\left(\frac{98-100}{1.8} < z < \frac{101-100}{1.8}\right) = P(-1.11 < z < .56)$

$= P(z < .56) - P(z < -1.11) = .7123 - .1335$

$= .5788$



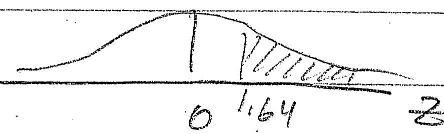
d. $P(\bar{X} < 101.5) = P\left(Z < \frac{101.5-100}{1.8}\right)$
$= P(Z < .83) = .7967$



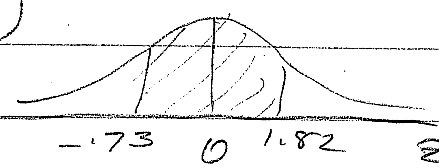
(7.6) $\mu = 100$ $\sigma^2 = 900$ $\sigma = 30$ $n = 30$

a. $E(\bar{X}) = \mu = 100$
$V(\bar{X}) = \frac{\sigma^2}{n} = \frac{900}{30} = 30$

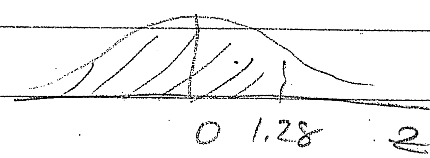
b. $P(\bar{X} > 109) = P\left(Z > \frac{109-100}{5.48}\right)$
$= P(Z > 1.64)$
$= 1 - P(Z < 1.64) = 1 - .9495 = .05$



c. $P(96 < \bar{X} < 110) = P\left(\frac{96-100}{5.48} < Z < \frac{110-100}{5.48}\right)$
$= P(-.73 < Z < 1.82)$
$= P(Z < 1.82) - P(Z < -.73)$
$= .9656 - .2327 = .7329$



d. $P(\bar{X} < 107) = P\left(Z < \frac{107-100}{5.48}\right)$
$= P(Z < 1.28) = .8997$



7.7 $\mu = 200 \quad \sigma^2 = 625 \quad n = 25$

a. $E(\bar{X}) = \mu = 200$

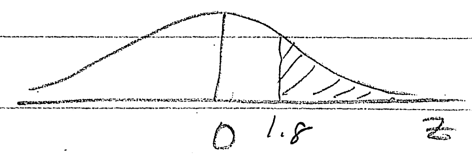
$V(\bar{X}) = \frac{\sigma^2}{n} = \frac{625}{25} = 25$

$\sigma(\bar{X}) = 5$

b. $P(\bar{X} > 209) = P\left(Z > \frac{209-200}{5}\right)$

$= P(Z > 1.8) = 1 - P(Z < 1.8)$

$= 1 - .9641 = .0359$

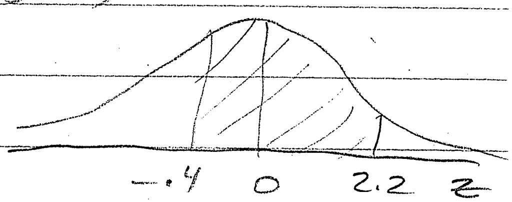


c. $P(198 < \bar{X} < 211) = P\left(\frac{198-200}{5} < Z < \frac{211-200}{5}\right)$

$= P(-.4 < Z < 2.2)$

$= P(Z < 2.2) - P(Z < -.4)$

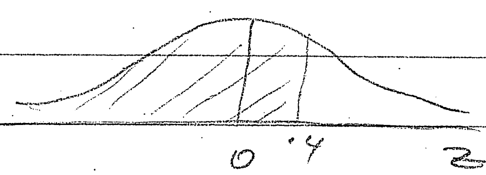
$= .9861 - .3446 = .6415$



d. $P(\bar{X} < 202) = P\left(Z < \frac{202-200}{5}\right)$

$= P(Z < .4)$

$= .6554$

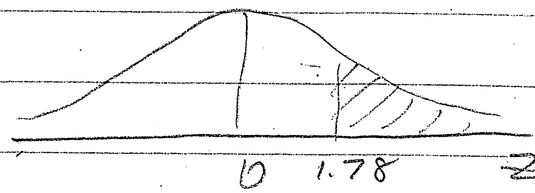


(7.8) $\mu = 400 \quad \sigma^2 = 1600 \quad n = 35$

a. $E(\bar{X}) = \mu = 400$

$V(\bar{X}) = \frac{\sigma^2}{n} = \frac{1600}{35} = 45.71$

$S(\bar{X}) = 6.76$



b. $P(\bar{X} > 412) = P\left(z > \frac{412-400}{6.76}\right)$

$= P(z > 1.78)$

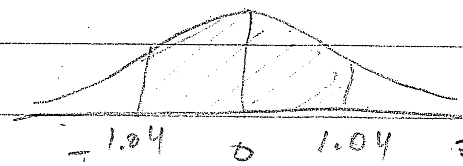
$= 1 - P(z < 1.78) = 1 - .9625 = .0375$

c. $P(393 < \bar{X} < 407) = P\left(\frac{393-400}{6.76} < z < \frac{407-400}{6.76}\right)$

$= P(-1.04 < z < 1.04)$

$= P(z < 1.04) - P(z < -1.04)$

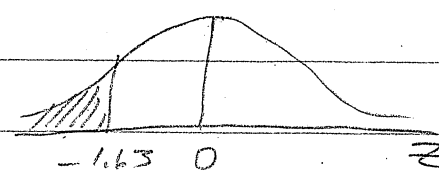
$= .8508 - .1492 = .7016$



d. $P(\bar{X} < 389) = P\left(z < \frac{389-400}{6.76}\right)$

$= P(z < -1.63)$

$= .0516$

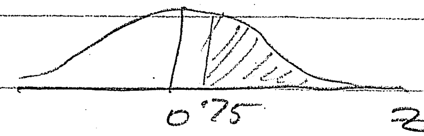


7.9 $\mu = 92.0 \quad \sigma = 3.6 \quad n = 4$

a. $E(\bar{X}) = \mu = 92.0$

b. $V(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{3.6}{2} = 1.8$

c. $s(\bar{X}) = 1.34$



d. $P(\bar{X} > 93) = P\left(z > \frac{93-92}{1.34}\right)$

$= P(z > .75) = 1 - P(z < .75) = 1 - .7734 = .2537$

7-10 $\mu = 1200 \quad \sigma = 400 \quad n = 9$

a. $E(\bar{X}) = \mu = 1200$

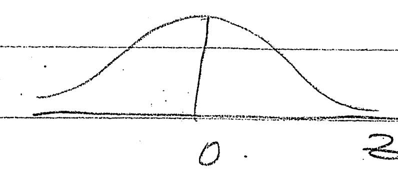
b. $V(\bar{X}) = \frac{\sigma^2}{n} = \frac{160000}{9} = 17,778$

c. $s(\bar{X}) = \sqrt{17,778} = 133.33$

d. $P(\bar{X} < 1050) = P\left(z < \frac{1050-1200}{133.33}\right)$

$= P(z < -1.125)$

$= .1292$

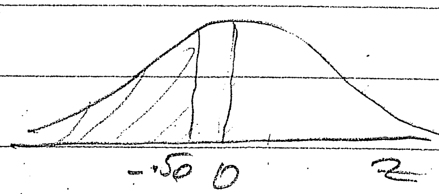


7.11 $\mu = 25 \quad \sigma = 2$

a. (i) $P(\bar{X} < 24) \qquad n = 1 \qquad s_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{1} = 2$

$P\left(z < \frac{24-25}{2}\right)$

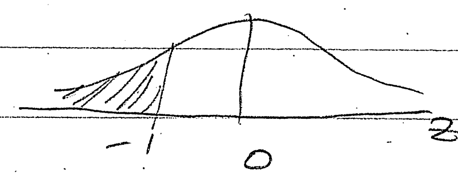
$= P(z < -.50) = .3085$



(ii) $P(\bar{X} < 24) \qquad n = 4 \qquad s_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{2} = 1$

$= P\left(z < \frac{24-25}{1}\right)$

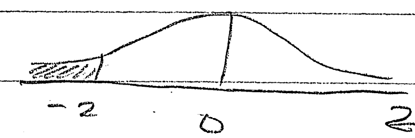
$= P(z < -1) = .1587$



(iii) $P(\bar{X} < 24) \qquad n = 16 \qquad s_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{4} = .5$

$= P\left(z < \frac{24-25}{.5}\right)$

$= P(z < -2) = .0228$



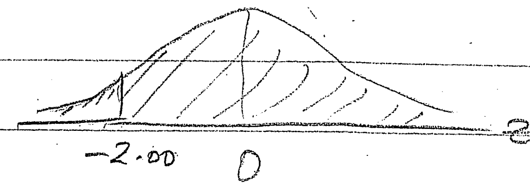
7.12 $\mu = 115000 \quad \sigma = 25{,}000 \quad n = 100 \quad S(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{25000}{\sqrt{100}} = 2500$

a. $P(\bar{X} > 110{,}000) = P\left(Z > \frac{110000 - 115000}{2500}\right)$

$= P(Z > -2.0)$

$= 1 - P(Z < -2.0) = 1 - .0228$

$= .9772$



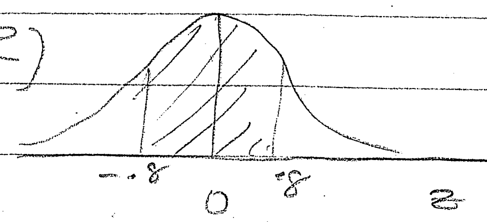
b. $P(113{,}000 < \bar{X} < 117{,}000)$

$= P\left(\frac{113{,}000 - 115{,}000}{2500} < Z < \frac{117{,}000 - 115{,}000}{2500}\right)$

$= P(-.8 < Z < .8)$

$= P(Z < .8) - P(Z < -.8)$

$= .7881 - .2119 = .5762$



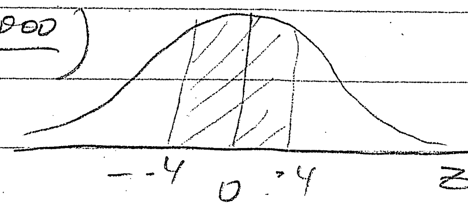
c. $P(114{,}000 < \bar{X} < 116{,}000)$

$= P\left(\frac{114{,}000 - 115{,}000}{2500} < Z < \frac{116000 - 115000}{2500}\right)$

$= P(-.4 < Z < .4)$

$= P(Z < .4) - P(Z < -.4)$

$= .6554 - .3446 = .3108$



d. 114,000 – 116,000 because it is centered around the mean of 115,000 with most probability (area under the curve.)

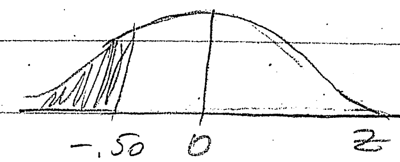
e. $\bar{X}$ would still be normally distributed by CLT.

7.13 $\mu = 280$ $\sigma = 60$ $n = 9$ a. $s(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{60}{3} = 20$

b. $P(\bar{X} < 270) = P(Z < \frac{270-280}{20})$

$= P(Z < -.50)$

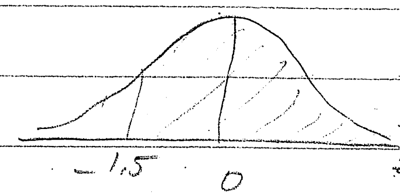
$= .3085$



c. $P(\bar{X} > 250) = P(Z > \frac{250-280}{20})$

$= P(Z > -1.5)$

$= 1 - P(Z < -1.5) = 1 - .0668$

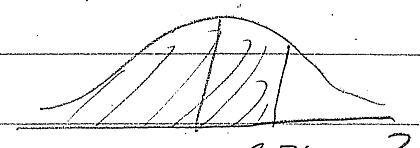


d. $\sigma = 40$ With smaller σ, $s(\bar{x})$ would also be smaller. Then (b) would be smaller

(c) would be larger

7.14 $n=16 \quad \mu=87 \quad \sigma=22$

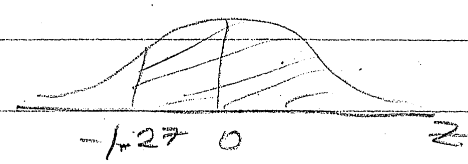
a. $s(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{22}{\sqrt{16}} = \frac{22}{4} = 5.5$



b. $P(\bar{X}<100) = P(Z < \frac{100-87}{5.5})$

$= P(Z<2.36)$

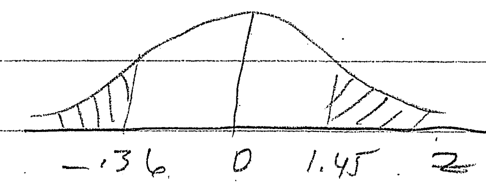
$= .9909$



c. $P(\bar{X}>80) = P(Z> \frac{80-87}{5.5})$

$= P(Z>-1.27) = 1-P(Z<-1.27)$

$= 1-.102 = .8980$



d. $P(85>X>95)$

$= P(\frac{85-87}{5.5} > Z > \frac{95-87}{5.5})$

$= P(-.36 > Z > 1.45)$

$= P(Z<-.36) + P(Z>1.45) = .3594 + \underbrace{1-P(Z<1.45)}_{.0735}$

$= .4329$

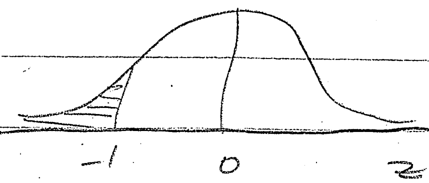
e. With n larger, $s(\bar{X})$ would become smaller. Then (b) & (c) would result in larger probabilities. (d) would be smaller.

(7.15) $\mu = 20 \quad \sigma = .6 \quad n = 4$

a. $S(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{.6}{\sqrt{4}} = .3$

b. $P(\bar{X} < 19.7) = P\left(z < \frac{19.7 - 20}{.3}\right)$

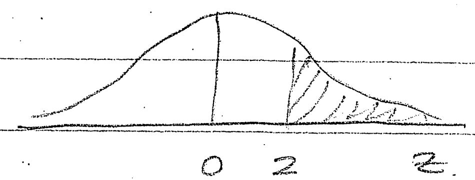
$= P(z < -1) = .1587$



c. $P(\bar{X} > 20.6) = P\left(z > \frac{20.6 - 20}{.3}\right)$

$= P(z > 2) = 1 - P(z < 2)$

$= 1 - .9772 = .0228$

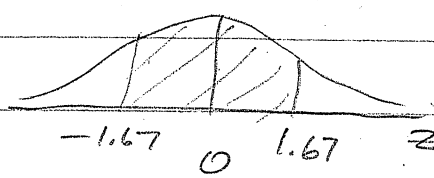


d. $P(19.5 < \bar{X} < 20.5)$

$= P\left(\frac{19.5 - 20}{.3} < z < \frac{20.5 - 20}{.3}\right)$

$= P(-1.67 < z < 1.67) = P(z < 1.67) - P(z < -1.67)$

$= .9525 - .0475 = .905$



e. $n = 2 \quad S(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{.6}{\sqrt{2}} = \frac{.6}{1.41} = .42$

$P\left(\frac{19.5 - 20}{.42} < z < \frac{20.5 - 20}{.42}\right) = P(-1.19 < z < 1.19)$

$= P(z < 1.19) - P(z < -1.19) = .8830 - .117 = .766$

7.16 $\sigma = 40 \quad n = 100$

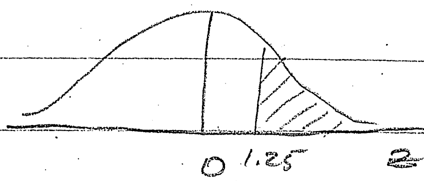
a. $V(\bar{x}) = \dfrac{\sigma^2}{n} = \dfrac{(40)^2}{100} = \dfrac{1600}{100} = 16$

$s(\bar{X}) = \sqrt{16} = 4$

b. $P(\bar{X} - \mu > 5) = P\left(\dfrac{\bar{X}-\mu}{\sigma} > \dfrac{5}{\sigma}\right)$

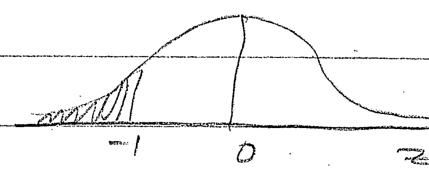
$= P\left(z > \dfrac{5}{4}\right) = P(z > 1.25)$

$= 1 - P(z < 1.25) = 1 - .8944 = .1056$



c. $P(\bar{X} - \mu < -4) = P\left(z < \dfrac{-4}{4}\right)$

$= P(z < -1) = .1587$



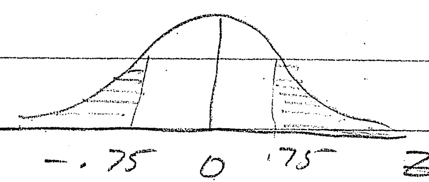
d. $P(3 < \bar{X} - \mu < -3)$

$= P\left(\dfrac{3}{4} < z < -\dfrac{3}{4}\right)$

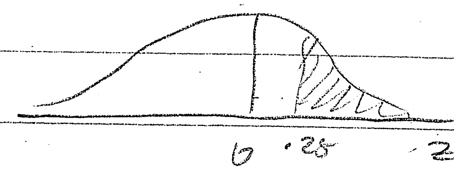
$= P(.75 < z < -.75)$

$= P(z < -.75) + [1 - P(z < .75)]$

$= .2266 + 1 - .7734 = .4532$



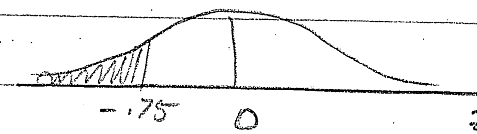
7.17 $\sigma = 8$ $n = 4$ $S(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{8}{2} = 4$



a. $P(\bar{X} - \mu > 2) = P(z > \frac{2}{4})$

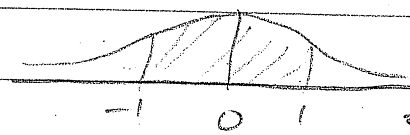
$= P(z > .50) = 1 - P(z < .50)$

$= 1 - .6915 = .3085$



b. $P(\bar{X} - \mu < -3) = P(z < \frac{-3}{4})$

$= P(z < -.75) = .2266$



c. $P(4 < X - \mu < -4)$

$= P(1 < z < -1)$

$= P(z < 1) - P(z < -1) = .8413 - .1587 = .6826$

d. $n = 10$ => smaller $S(\bar{X})$ => z-scores higher

Then, in (a) smaller prob.

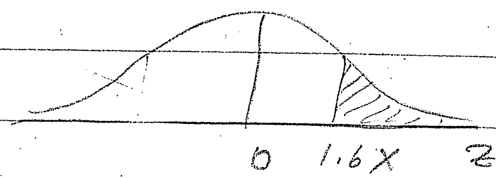
(b) smaller prob.

(c) larger prob.

7.18 $\sigma = 1.6 \quad n = 100 \quad s(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{100}} = .16$

a. $P(\bar{X} - \mu > Y) = .05$

$P\left(\frac{\bar{X}-\mu}{\sigma} > \frac{Y}{\sigma}\right) = .05$



$P\left(z > \frac{Y}{.16}\right) = .05$

$\downarrow$

$1.645 = \frac{Y}{.16} \Rightarrow Y = (1.645)(.16) = .2632$

b. $P(\bar{X} - \mu < -Y) = .10$

$P\left(z < -\frac{Y}{.16}\right) = .10$

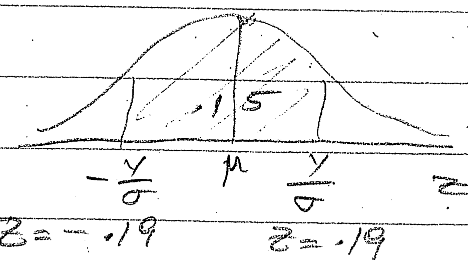
$\downarrow$

$1.28 = -\frac{Y}{.16} \Rightarrow Y = (-.16)(1.28) = -.2048$

c. $P(-Y < \bar{X} - \mu < Y) = .15$

$P\left(-\frac{Y}{\sigma} < z < \frac{Y}{\sigma}\right) = .15$

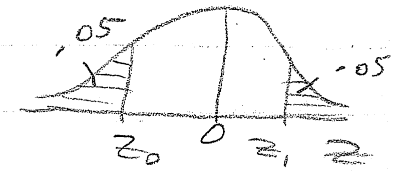
$P\left(z < \frac{Y}{.16}\right) = .5750$



$\downarrow$

$z = .19 = \frac{Y}{.16} \Rightarrow Y = (.16)(.19) = .0304$ in both directions from μ.

7.19 $\sigma = 3.8$ a. $P(\bar{X} - \mu > 1.0) = .\frac{10}{2}$



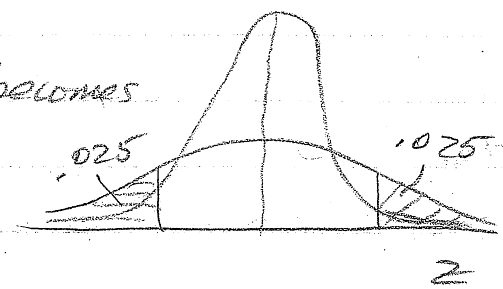
$$\Rightarrow P\left(\frac{\bar{X} - \mu}{3.8/\sqrt{n}} > \frac{1}{3.8/\sqrt{n}} \right) = .05$$

$$= P\Big(\underbrace{z > \frac{\sqrt{n}}{3.8}}_{z_1 = 1.645 = \frac{\sqrt{n}}{3.8}} \Big) = .05$$

$$\sqrt{n} = (1.645)(3.8) = 6.251$$

$$n = 39.075$$

b. With a larger n, as the distribution becomes tighter, the tail areas (i.e. prob that the sample mean sits further away from μ than 1.0) get smaller and will guarantee that these areas will be less than 5% (both tails combined)

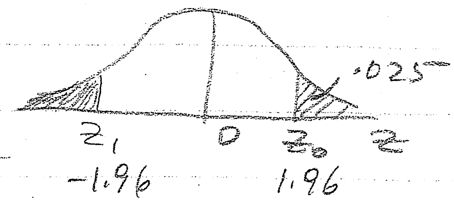


c. A larger sample (Same reasoning as above since we want to be surer that tail areas as <u>smaller</u> than .05.)

7.20 $\sigma = 8.4$

$S_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{8.4}{\sqrt{n}}$

a. $P(\bar{X} - \mu > 2) = \frac{.05}{2}$



$\Rightarrow P\left(Z > \frac{2}{8.4/\sqrt{n}}\right) = .025$

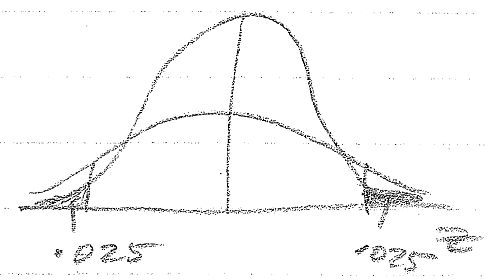
$= P\left(Z > \underbrace{\frac{2\sqrt{n}}{8.4}}\right) = .025$

$Z_0 = 1.96 = \frac{2\sqrt{n}}{8.4}$

$\sqrt{n} = \frac{(1.96)(8.4)}{2} = 8.23$

$n = (8.23)^2 = 67.77 \sim 68$

b. Smaller sample will be sufficient to make sure the tail areas add up to 10% or less



d. Larger – again to make sure that tail areas add up to less than 5% as sample means move closer to the mean in both directions (from 2 to 1.5 hrs.)

7.23

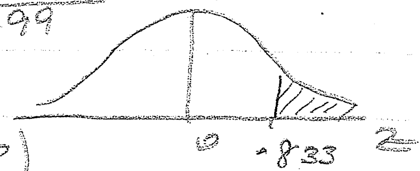
$\mu = 800,000$

$\sigma = 300,000$

$n = 100$

a. $S_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{300,000}{\sqrt{100}} \overbrace{\sqrt{\frac{500-100}{499}}}^{\text{finite pop}^n \text{ correction factor}} = 26,859.69$

b. $P(\bar{x} > 825,000)$

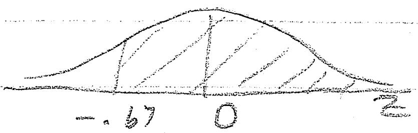


$= P\left(z > \frac{825,000-800,000}{26,859.69}\right)$

$= P(z > .93) = .50 - P(0 < z < .93)$

$= .50 - .3238 = .1762$

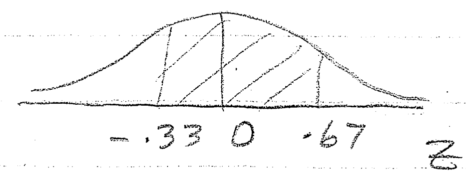
c. $P(\bar{x} > 780,000)$



$= P\left(z > \frac{780-800}{26.859}\right) = P(z > -.74)$

$= .50 + P(0 < z < .74) = .50 + .2704 = .7704$

d. $P(790 < \bar{x} < 820)$



$= P\left(\frac{790-800}{26.859} < z < \frac{820-800}{26.859}\right)$

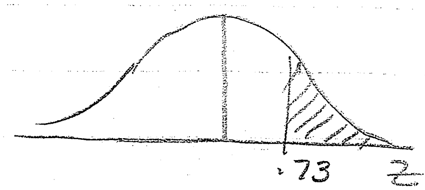
$= P(-.37 < z < .74) = P(0 < z < .37) + P(0 < z < .74)$

$= .1443 + .2704 = .4147$

7.24

$n = 50$ a. $P(\bar{X} - \mu > 2.5)$

$\sigma = 30$ $P\left(z > \frac{2.5}{3.80}\right)$



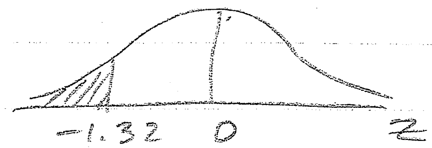
$S_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{30}{\sqrt{50}} \cdot \sqrt{\frac{N-n}{N-1}}$ $= P(z > .66)$

$= \frac{30}{7.07} \cdot \sqrt{\frac{200}{249}}$ $= .50 - .2454 = .2546$

$= (4.24)(.896)$

$= 3.80$

b. $P(\bar{X} - \mu < -5)$



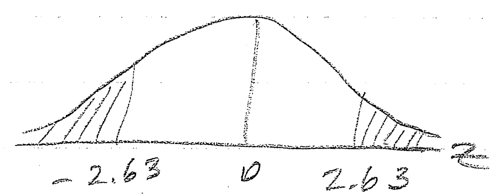
$= P(z < -5/3.8)$

$= P(z < -1.32) = .50 - P(0 < z < 1.32)$

$= .50 - .4066 = .0934$

c. $P(-10 > \bar{X} - \mu > 10)$

$= P\left(\frac{-10}{3.8} > z > \frac{10}{3.8}\right)$



$= P(-2.63 > z > 2.63) = 1 - 2P(0 < z < 2.63)$

$= 1 - 2(.4957) = 1 - .9914 = .0086$

7.25

$\mu = 32$

$\sigma = 10$

$n = 150$

$N = 600$

$$S_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$= \frac{10}{\sqrt{150}} \cdot \sqrt{\frac{600-150}{599}}$$

$$= (.82)(.87)$$

$$= .71$$

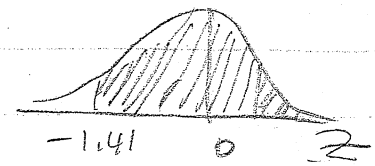
a. $P(\bar{X} > 31)$

$= P\left(z > \frac{31-32}{.71}\right)$

$= P(z > -1.41)$

$= .50 + P(0 < z < 1.41)$

$= .50 + .4207 = .9207$



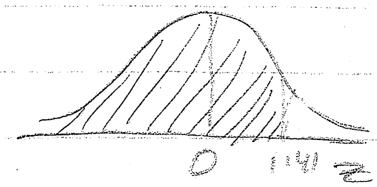
b. $P(\bar{x} < 33)$

$= P\left(z < \frac{33-32}{.71}\right)$

$= P(z < 1.41)$

$= .50 + P(0 < z < 1.41)$

$= .50 + .4207 = .9207$



c. Graphs shown above

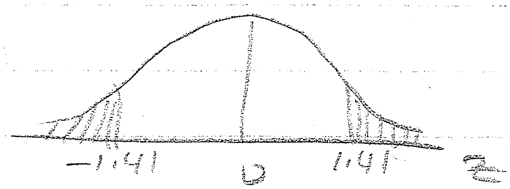
d. $P(31 > \bar{X} > 33)$

$= 1 - P(31 < \bar{X} < 33)$

$= 1 - P\left(\frac{31-32}{.71} < z < \frac{33-32}{.71}\right) = 1 - P(-1.41 < z < 1.41)$

$= 1 - 2P(0 < z < 1.41) = 1 - 2(.4207) = 1 - .8414$

$= .1586$



7.26

$P = .40$

$n = 100$

$S_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$= \sqrt{\frac{.40(.60)}{100}}$

$= \sqrt{.0024}$

$= .049$

a. $P(\hat{p} > .45)$

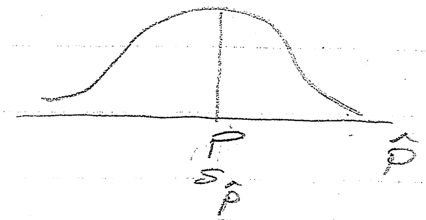
$= P\left(Z > \frac{.45 - .40}{.049}\right)$

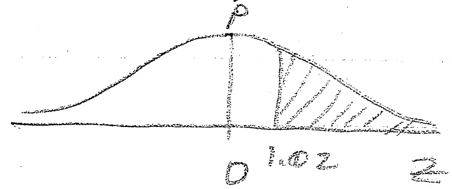
$= P(Z > 1.02)$

$= .50 - P(0 < Z < 1.02)$

$= .50 - .3461$

$= .1539$



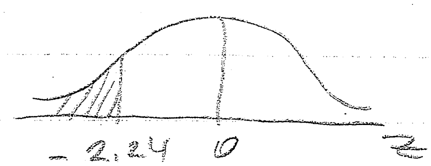


b. $P(\hat{p} < .29) = P\left(Z < \frac{.29 - .40}{.049}\right)$

$= P(Z < -2.24)$

$= .50 - P(0 < Z < 2.24)$

$= .50 - .4875 = .0125$

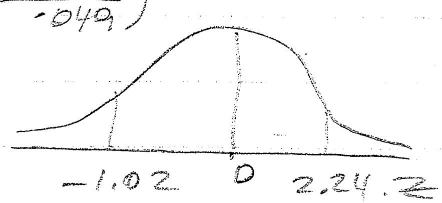


c. $P(.35 < \hat{p} < .51) = P\left(\frac{.35 - .40}{.049} < \hat{p} < \frac{.51 - .40}{.049}\right)$

$= P(-1.02 < Z < 2.24)$

$= P(0 < Z < 1.02) + P(0 < Z < 2.24)$

$= .3461 + .4875 = .8336$



7.27 $P = .25$

$n = 200$

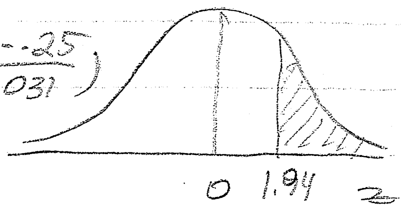
$$S_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{(.25)(.75)}{200}} = .031$$

a. $P(\hat{p} > .31) = P\left(z > \frac{.31 - .25}{.031}\right)$

$= P(z > 1.94)$

$= .50 - P(0 < z < 1.94)$

$= .50 - .4738 = .0262$

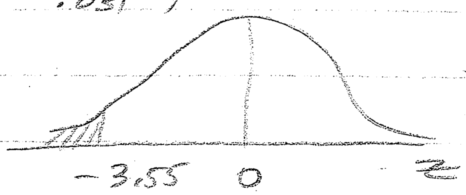


b. $P(\hat{P} < .14) = P\left(z < \frac{.14 - .25}{.031}\right)$

$= P(z < -3.55)$

$= .50 - P(0 < z < -3.55)$

$= .50 - .998$



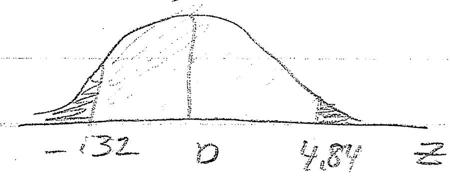
c. $P(.24 < \hat{P} < .40) = P\left(\frac{.24 - .25}{.031} < z < \frac{.40 - .25}{.031}\right)$

$= P(-.32 < z < 4.84)$

$= 1 - [P(0 < z < .32) + P(0 < z < 4.84)]$

$= 1 - (.1255 + .50)$

$= .6255$



7.28

$p = .60$

$n = 100$

$$s_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$= \sqrt{\frac{(.60)(.40)}{100}}$$

$$= \sqrt{.0024}$$

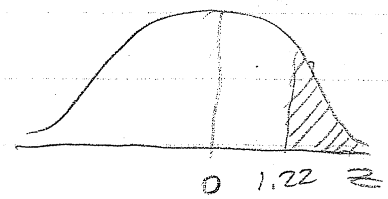
$$= .049$$

a. $P(\hat{p} > .66)$

$$= P\left(z > \frac{.66 - .60}{.049}\right)$$

$$= P(z > 1.22) = .50 - P(0 < z < 1.22)$$

$$= .50 - .3888 = .1112$$



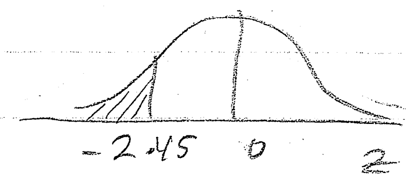
b. $P(\hat{p} < .48) = P\left(z < \frac{.48 - .60}{.049}\right)$

$$= P(z < -2.45)$$

$$= .50 - P(0 < z < -2.45)$$

$$= .50 - .4929$$

$$= .0071$$

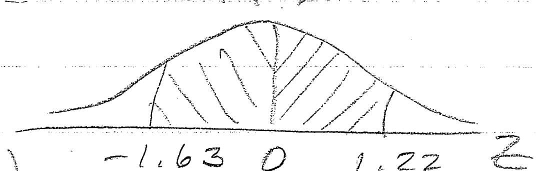


c. $P(.52 < \hat{p} < .66) = P\left(\frac{.52 - .60}{.049} < z < \frac{.66 - .60}{.049}\right)$

$$= P(-1.63 < z < 1.22)$$

$$= P(0 < z < 1.63) + P(0 < z < 1.22)$$

$$= .4484 + .3888 = .8372$$



7.29

$P = .50$

$n = 900$

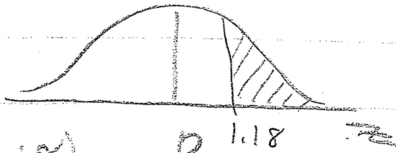
$$S_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$$

$$= \sqrt{\frac{(.50)(.50)}{900}}$$

$$= \sqrt{\frac{.25}{900}} = \sqrt{.000278}$$

$$= .017$$

a. $P(\hat{p} > .52) = P\left(Z > \frac{.52 - .50}{.017}\right)$



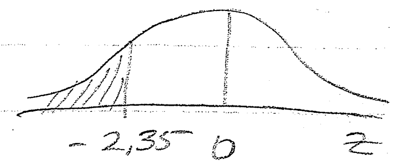
$= P(Z > 1.18)$

$= .50 - P(0 < Z < 1.18)$

$= .50 - .3810$

$= .119$

b. $P(\hat{p} < .46)$



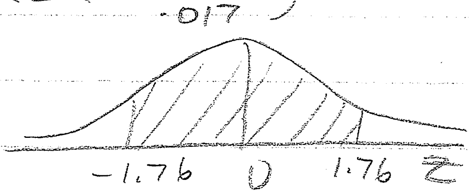
$= P\left(Z < \frac{.46 - .50}{.017}\right)$

$= P(Z < -2.35)$

$= .50 - P(0 < Z < 2.35)$

$= .50 - .4906 = .0094$

c. $P(.47 < \hat{p} < .53) = P\left(\frac{.47 - .50}{.017} < Z < \frac{.53 - .50}{.017}\right)$



$= P(-1.76 < Z < 1.76)$

$= 2(0 < Z < 1.76) = 2(.4608)$

$= .9216$

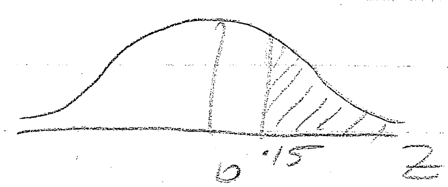
7.30 $P = .424$ a. $E(\hat{p}) = P = .424$

$n = 100$ b. $s^2_{\hat{p}} = \dfrac{P(1-P)}{n} = \dfrac{(.424)(.576)}{100} = \dfrac{.244}{100}$

$= .00244$

c. $s_{\hat{p}} = \sqrt{.00244} = .494$

d. $P(\hat{p} > .5) = P\left(z > \dfrac{.5 - .424}{.494}\right)$



$= P\left(z > \dfrac{.076}{.494}\right) = P(z > .15) = .50 - P(0 < z < .15)$

$= .50 - .0596 = .4404$

7.31 $P = .75$ a. $E(\hat{p}) = P = .75$

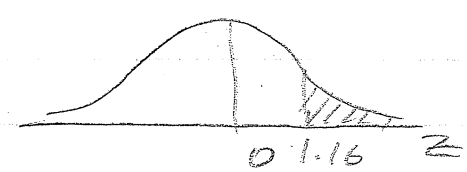
$n = 100$ b. $s^2_{\hat{p}} = \dfrac{P(1-P)}{n} = \dfrac{.75(.25)}{100} = .001875$

c. $s_{\hat{p}} = \sqrt{.001875} = .043$

d. $P(\hat{p} > .80) = P\left(z > \dfrac{.80 - .75}{.043}\right) = P(z > 1.16)$

$= .50 - P(0 < z < 1.16)$

$= .50 - .3770 = .123$



7.32 $\quad P = .20 \quad n = 180$

a. $E(\hat{p}) = P = .20$

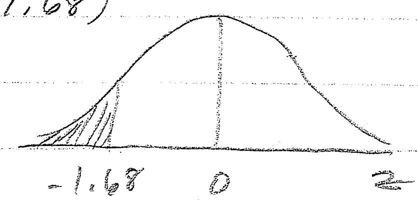
b. $s^2_{\hat{p}} = \dfrac{P(1-P)}{n} = \dfrac{(.20)(.80)}{180} = \dfrac{.16}{180} = .00089$

c. $s_{\hat{p}} = \sqrt{.00089} = .0298$

d. $P(\hat{P} < .15) = P\left(z < \dfrac{.15 - .20}{.0298}\right) = P(z < -1.68)$

$= .50 - P(0 < z < 1.68)$

$= .50 - .4535 = .0465$

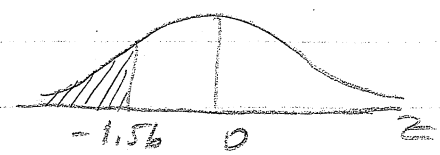


7.33 $\quad P = .30 \quad n = 200$

a. $s_{\hat{p}} = \sqrt{\dfrac{P(1-P)}{n}} = \sqrt{\dfrac{(.30)(.70)}{200}} = .032$

b. $P(\hat{P} < .25) = P\left(z < \dfrac{.25 - .30}{.032}\right)$

$= P(z < -1.56)$

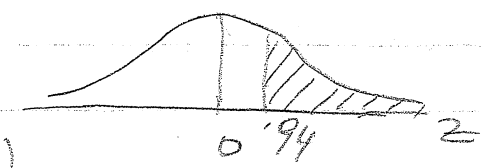


$= .50 - P(0 < z < 1.56)$

$= .50 - .4406 = .0594$

c. $P(\hat{P} > .33)$

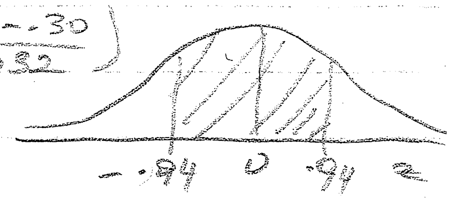
$= P\left(z > \dfrac{.33 - .30}{.032}\right)$



$= P(z > .94) = .50 - P(0 < z < .94)$

$= .50 - .3264 = .1736$

d. $P(.27 < \hat{P} < .33) = P\left(\dfrac{.27 - .30}{.032} < z < \dfrac{.33 - .30}{.032}\right)$



$= P(-.94 < z < .94) = 2P(0 < z < .94)$

$= 2(.3264) = .6528$

7.34

$n = 120$

$P = .40$

$s_{\hat{p}} = \sqrt{\dfrac{P(1-P)}{n}}$

$= \sqrt{\dfrac{(.40)(.60)}{120}}$

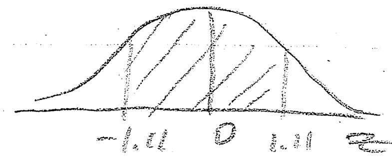
$= \sqrt{\dfrac{.24}{120}} = .045$

$P(.35 < \hat{p} < .45) = P\left(\dfrac{.35-.40}{.045} < z < \dfrac{.45-.40}{.045}\right)$

$= P(-1.11 < z < 1.11)$

$= 2P(0 < z < 1.11)$

$= 2(.3665) = .733$



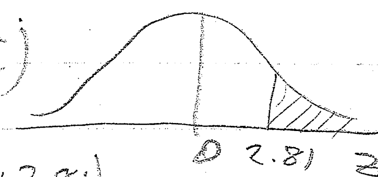
7.35

$P = .42$

$n = 300$

a. $s_{\hat{p}} = \sqrt{\dfrac{(.42)(.58)}{300}} = \sqrt{\dfrac{.2436}{300}} = \sqrt{.000812} = .0285$

b. $P(\hat{p} > .50) = P\left(z > \dfrac{.50-.42}{.0285}\right)$



$= P(z > 2.807) = .50 - P(0 < z < 2.81)$

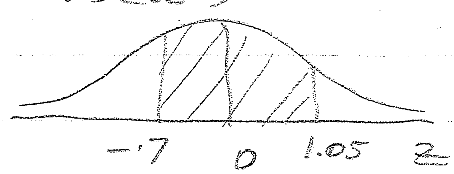
$= .50 - .4975 = .0025$

c. $P(.40 < \hat{p} < .45) = P\left(\dfrac{.40-.42}{.0285} < z < \dfrac{.45-.42}{.0285}\right)$

$= P(-.70 < z < 1.05)$

$= P(0 < z < .70) + P(0 < z < 1.05)$

$= .2580 + .3531 = .6111$



d. .41 – .43 as it is centered around the mean of .42.

7.36 $P = .20$

$n = 130$

a. $s_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{(.20)(.80)}{130}} = \sqrt{\frac{.16}{130}} = \sqrt{.00123}$

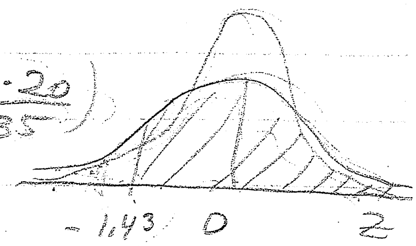
$= .035$

b. $P(\hat{p} > .15) = P\left(Z > \frac{.15 - .20}{.035}\right)$

$= P(Z > -1.43)$

$= .50 + P(0 < Z < 1.43)$

$= .50 + .4236 = .9236$

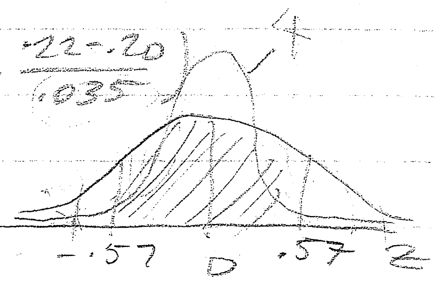


c. $P(.18 < \hat{P} < .22) = P\left(\frac{.18 - .20}{.035} < Z < \frac{.22 - .20}{.035}\right)$

$= P(-.57 < Z < .57)$

$= 2P(0 < Z < .57) = 2(.2157)$

$= .4314$



d. $n = 500$. Higher n makes $s_{\hat{p}}$ become smaller. When $\hat{p}$ is distributed more tightly, areas under the curve for each of the values above will become larger.

7.37 $P = .30$ $n = 280$

a. $s_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{(.30)(.70)}{280}} = \sqrt{\frac{.21}{280}} = \sqrt{.00075}$

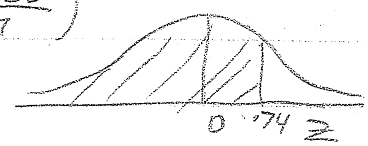
$= .027$

b. $P(\hat{p} < .32) = P\left(z < \frac{.32 - .30}{.027}\right)$

$= P(z < .74)$

$= .50 + P(0 < z < .74)$

$= .50 + .2704 = .7704$



c. .29 – .31, as this range is centered around the mean of .30.

7.38 $n = 100$ $s_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$ For $s_{\hat{p}}$ to be the highest it can be, $P = .50$ so that

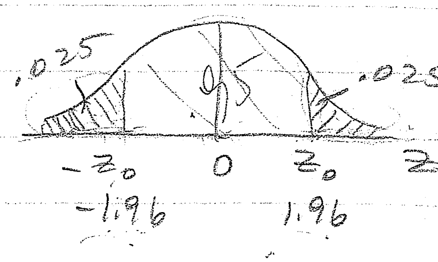
$P(1-P) = (.5)(.5) = .25$

and $n = 1$ so that

$s_{\hat{p}} = \sqrt{\frac{.25}{100}} = .05$

7.39 $P(-.03 > \hat{P} - p > .03) < .05$

$$= P\Big(\underbrace{\frac{-.03}{\sqrt{\frac{P(1-P)}{n}}}}_{-z_0} > z > \underbrace{\frac{.03}{\sqrt{\frac{P(1-P)}{n}}}}_{z_0}\Big) < .05$$



$$1.96 = \frac{.03}{\sqrt{\frac{P(1-P)}{n}}} = \frac{.03\sqrt{n}}{\sqrt{P(1-P)}}$$

$$\frac{1.96\sqrt{P(1-P)}}{.03} = \sqrt{n}$$

$$\frac{(1.96)^2 P(1-P)}{(.03)^2} = n$$

$$4268\,P(1-P) = n$$

$$4268(.1)(.9) = n$$

$$384.12 = n$$

$P = .001$

use $P = .1$ for smallest n can be while still meeting the requirement that $P(-.03 < \hat{p} - p < .03) < .05$

7.40

$n = 120$

$p = .25$

$$S_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

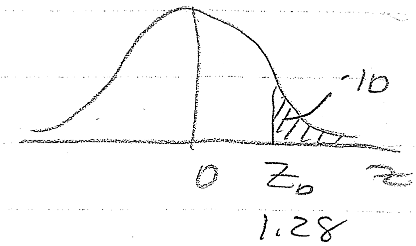
$$= \sqrt{\frac{(.25)(.75)}{120}}$$

$$= \sqrt{\frac{.1875}{120}}$$

$$= \sqrt{.0015625} = .0395$$

a. $P(\hat{p} - p > x_0) = .10$

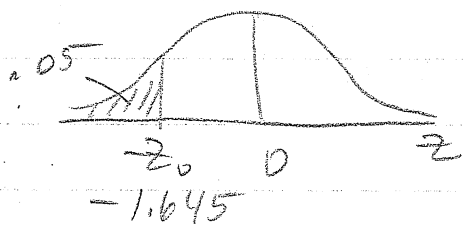
$$P\Big(z > \underbrace{\frac{x_0}{.0395}}\Big) = .10$$



$$z_0 = 1.28 = \frac{x_0}{.0395}$$

$$x_0 = (1.28)(.0395) = .0506$$

b. $P(\hat{p} - p < x_0) = .05$



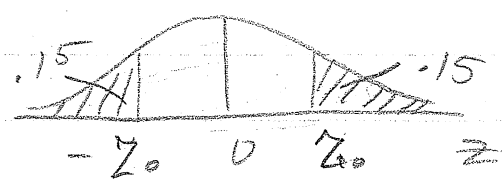
$$P\Big(z < \underbrace{\frac{x_0}{.0395}}\Big) = .05$$

$$z_0 = \frac{x_0}{.0395}$$

$$-1.645 = \frac{x_0}{.0395}$$

$$x_0 = (-1.645)(.0395)$$

$$= -.06497$$



c. $P(-x_0 > \hat{p} - p > x_0) = .30$

$$\Rightarrow P(\hat{p} - p > x_0) = .15$$

$$P\Big(z > \underbrace{\frac{x_0}{.0395}}\Big) = .15$$

$$z_0 = 1.04 = \frac{x_0}{.0395} \Rightarrow x_0 = (1.04)(.0395)$$

$$= .04108$$

Then $-x_0 = -.04108$

7.41

$P = .50$

$n = 150$

$S_{\hat{p}} = \sqrt{\dfrac{P(1-P)}{n}}$

$= \sqrt{\dfrac{(.50)(.50)}{150}}$

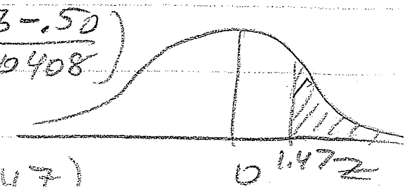
$= \sqrt{\dfrac{.25}{150}} = \sqrt{.00167} = .0408$

$P(\hat{P} > .56) = P\left(Z > \dfrac{.56 - .50}{.0408}\right)$

$= P(Z > 1.47)$

$= .50 - P(0 < Z < 1.47)$

$= .50 - .4292 = .0708$



7.42

$P = .50$

$n = 250$

$S_{\hat{p}} = \sqrt{\dfrac{P(1-P)}{n}}$

$= \sqrt{\dfrac{(.50)(.50)}{250}}$

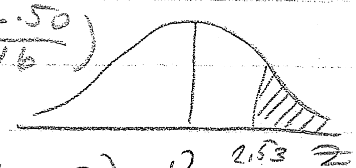
$= \sqrt{\dfrac{.25}{250}} = \sqrt{.001} = .0316$

$P(\hat{P} > .58) = P\left(Z > \dfrac{.58 - .50}{.0316}\right)$

$= P(Z > 2.53)$

$= .50 - P(0 < Z < 2.53)$

$= .50 - .4943 = .0057$



7.43

$n = 81$

$P = .55$

$S_{\hat{p}} = \sqrt{\dfrac{P(1-P)}{n}}$

$= \sqrt{\dfrac{(.55)(.45)}{81}}$

$= \sqrt{\dfrac{.2475}{81}}$

$= \sqrt{.00306} = .0553$

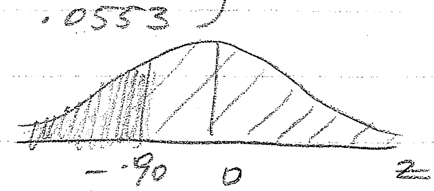
$P(\hat{P} < .50) = P\left(Z < \dfrac{.50 - .55}{.0553}\right)$

$= P(Z < -.90)$

$= .50 - P(0 < Z < .90)$

$= .50 - .3159$

$= .1841$



7.44 $n = 120$

$$P = \frac{211}{528} = .40$$

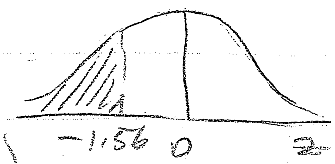
a. $$S_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$$

$$= \sqrt{\frac{(.40)(.60)}{120}}$$

$$= \sqrt{\frac{.24}{120}}$$

$$= \sqrt{.002} = .045$$

b. $P(\hat{p} < .33)$



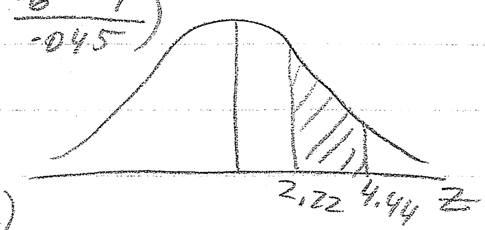
$$= P\left(z < \frac{.33 - .40}{.045}\right)$$

$$= P(z < -1.56)$$

$$= .50 - P(0 < z < 1.56)$$

$$= .50 - .4406 = .0594$$

c. $$P(.5 < \hat{p} < .6) = P\left(\frac{.5 - .4}{.045} < z < \frac{.6 - .4}{.045}\right)$$



$$= P(2.22 < z < 4.44)$$

$$= P(0 < z < 4.44) - P(0 < z < 2.22)$$

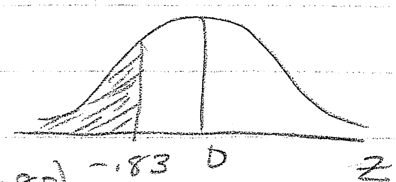
$$= .50 - .4868 = .0132$$

7.45 $P = \frac{239}{438} = .546$ a. $s_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{.546(.4521)}{80}}$

$n = 80$

$= \sqrt{.003} = .0557$

b. $P(\hat{P} < .5) = P\left(z < \frac{.5 - .546}{.0557}\right)$

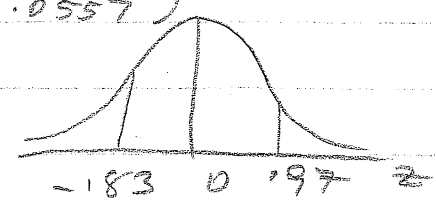


$= P(z < -.83) = .50 - P(0 < z < .83)$

$= .50 - .2967 = .2033$

c. $P(.5 < \hat{P} < .6) = P\left(\frac{.5 - .546}{.0557} < z < \frac{.6 - .546}{.0557}\right)$

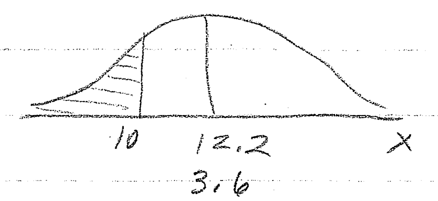
$= P(-.83 < z < .97)$



$= P(0 < z < .83) + P(0 < z < .97)$

$= .2967 + .3340 = .6307$

7.46 $\mu = 12.2\%$ $P(X < 10)$

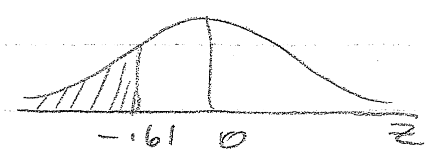


$\sigma = 3.6\%$ $= P\left(z < \frac{10 - 12.2}{3.6}\right)$

$n = 81$

$= P(z < -.61)$

$= .50 - P(0 < z < .61)$

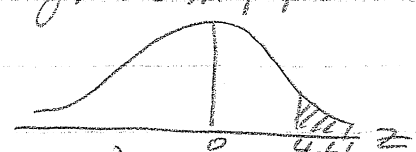


$= .50 - .2291$

$= .27$

Therefore, $P = .27$ (prob. of less than 10% pay raise in population)

Then, $s_{\hat{p}} = \sqrt{\frac{(.27)(.73)}{81}} = \sqrt{.0024} = .049$



$P(\hat{P} > .50) = P\left(z > \frac{.50 - .27}{.049}\right) = P(z > 4.69)$

$= .50 - P(0 < z < 4.69) = .50 - .50 = 0$