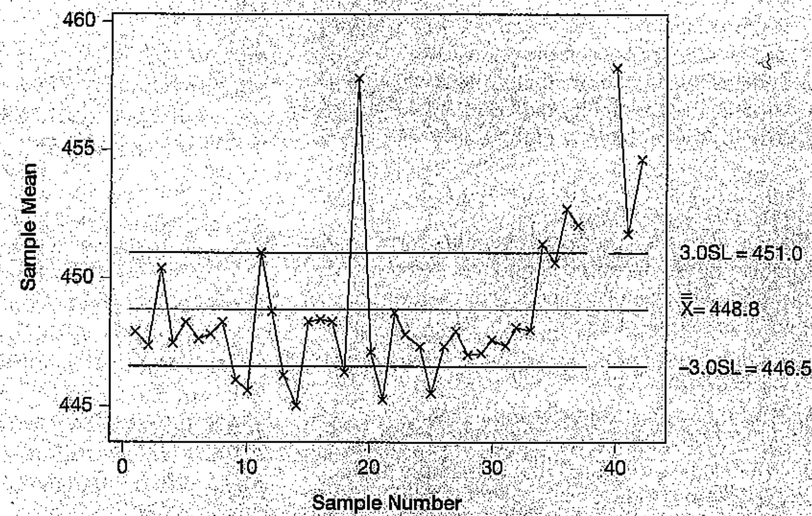
Figure 7.12 X-Bar Chart for Cereal Package Weight



monitoring limits on product quality—and numerous highly successful firms do—the central limit theorem provides the rationale for using the normal distribution to establish limits for the small sample means. Thus, a fundamentally important statistical theory drives a key management process.

INTERPRETATION

In this chart SL is the standard deviation for the sample mean. The upper and lower limits are set at $\pm 3\sigma_{\bar{X}}$ instead of $\pm 1.96\sigma_{\bar{X}}$, or 95%, the acceptance interval used in the previous example. The interval $\bar{\bar{X}} \pm 3\sigma_{\bar{X}}$ (Minitab labels the mean for the entire population as $\bar{\bar{X}}$) includes almost all of the sample means under the normal distribution, given a stable mean and variance. Thus, a sample mean outside of the control limits indicates that something has changed and corrections should be made. Given the number of points outside the acceptance interval, we recommend that the process be stopped and adjusted.

Exercises

Basic Exercises

7.5 Given a population with mean $\mu = 100$ and variance $\sigma^2 = 81$, the central limit applies when the sample size $n \geq 25$. A random sample of size $n = 25$ is obtained.

a. What are the mean and variance of the sampling distribution for the sample means?
b. What is the probability that $\bar{x} > 102$?
c. What is the probability that $98 \leq \bar{x} \leq 101$?
d. What is the probability that $\bar{x} \leq 101.5$?

7.6 Given a population with mean $\mu = 100$ and variance $\sigma^2 = 900$, the central limit applies when the sample size $n \geq 25$. A random sample of size $n = 30$ is obtained.

a. What are the mean and variance of the sampling distribution for the sample means?
b. What is the probability that $\bar{x} > 109$?
c. What is the probability that $96 \leq \bar{x} \leq 110$?
d. What is the probability that $\bar{x} \leq 107$?

7.7 Given a population with mean $\mu = 200$ and variance $\sigma^2 = 625$, the central limit applies when the sample size $n \geq 25$. A random sample of size $n = 25$ is obtained.

a. What are the mean and variance of the sampling distribution for the sample mean?
b. What is the probability that $\bar{x} > 209$?
c. What is the probability that $198 \leq \bar{x} \leq 211$?
d. What is the probability that $\bar{x} \leq 202$?

7.8 Given a population with mean $\mu = 400$ and variance $\sigma^2 = 1{,}600$, the central limit applies when the sample size $n \geq 25$. A random sample of size $n = 35$ is obtained.

a. What are the mean and variance of the sampling distribution for the sample means?
b. What is the probability that $\bar{x} > 412$?
c. What is the probability that $393 \leq \bar{x} \leq 407$?
d. What is the probability that $\bar{x} \leq 389$?

Application Exercises

7.9 When a production process is operating correctly, the number of units produced per hour has a normal distribution with mean 92.0 and standard deviation 3.6. A random sample of four different hours was taken.

a. Find the mean of the sampling distribution of the sample means.
b. Find the variance of the sample mean.
c. Find the standard error of the sample mean.
d. What is the probability that the sample mean exceeds 93.0 units?

7.10 The lifetimes of light bulbs produced by a particular manufacturer have a mean of 1,200 hours and a standard deviation of 400 hours. The population distribution is normal. Suppose that you purchase nine bulbs, which can be regarded as a random sample from the manufacturer's output.

a. What is the mean of the sample mean lifetime?
b. What is the variance of the sample mean?
c. What is the standard error of the sample mean?
d. What is the probability that, on average, those nine lightbulbs have lives of less than 1,050 hours?

7.11 The fuel consumption, in miles per gallon, of all cars of a particular model has mean 25 and standard deviation 2. The population distribution can be assumed to be normal. A random sample of these cars is taken.

a. Find the probability that sample mean fuel consumption will be less than 24 miles per gallon if
   i. A sample of 1 observation is taken.
   ii. A sample of 4 observations is taken.
   iii. A sample of 16 observations is taken.
b. Explain why the three answers in part (a) differ in the way they do. Draw a graph to illustrate your reasoning.

7.12 The mean selling price of new homes in a city over a year was \$115,000. The population standard deviation was \$25,000. A random sample of 100 new home sales from this city was taken.

a. What is the probability that the sample mean selling price was more than \$110,000?
b. What is the probability that the sample mean selling price was between \$113,000 and \$117,000?
c. What is the probability that the sample mean selling price was between \$114,000 and \$116,000?
d. Without doing the calculations, state in which of the following ranges the sample mean selling price is most likely to lie:

\$113,000 to \$115,000	\$114,000 to \$116,000
\$115,000 to \$117,000	\$116,000 to \$118,000

e. Suppose that, after you had done these calculations, a friend asserted that the population distribution of selling prices of new homes in this city was almost certainly not normal. How would you respond?

7.13 Candidates for employment at a city fire department are required to take a written aptitude test. Scores on this test are normally distributed with mean 280 and standard deviation 60. A random sample of nine test scores was taken.

a. What is the standard error of the sample mean score?
b. What is the probability that the sample mean score is less than 270?
c. What is the probability that the sample mean score is more than 250?
d. Suppose that the population standard deviation is, in fact, 40, rather than 60. Without doing the calculations, state how this would change your answers to parts (a), (b), and (c). Illustrate your conclusions with appropriate graphs.

7.14 A random sample of 16 junior managers in the offices of corporations in a large city center was taken to estimate average daily commuting time for all such managers. Suppose that the population times have a normal distribution with mean 87 minutes and standard deviation 22 minutes.

a. What is the standard error of the sample mean commuting time?
b. What is the probability that the sample mean is less than 100 minutes?
c. What is the probability that the sample mean is more than 80 minutes?
d. What is the probability that the sample mean is outside the range 85 to 95 minutes?
e. Suppose that a second (independent) random sample of 50 junior managers is taken. Without doing the calculations, state whether the probabilities in parts (b), (c), and (d) would be higher, lower, or the same for the second sample. Sketch graphs to illustrate your answers.

7.15 A company produces breakfast cereal. The true mean weight of the contents of its cereal boxes is 20 ounces, and the standard deviation is 0.6 ounce. The population distribution of weights is normal.

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Suppose that you purchase four boxes, which can be regarded as a random sample of all those produced.

a. What is the standard error of the sample mean weight?
b. What is the probability that, on average, the contents of these four boxes will weigh less than 19.7 ounces?
c. What is the probability that, on average, the contents of these four boxes will weigh more than 20.6 ounces?
d. What is the probability that, on average, the contents of these four boxes will weigh between 19.5 and 20.5 ounces?
e. Two of the four boxes are chosen at random. What is the probability that, the average, contents of these two boxes will weigh between 19.5 and 20.5 ounces?

7.16 Assume that the standard deviation of monthly rents paid by students in a particular town is $40. A random sample of 100 students was taken to estimate the mean monthly rent paid by the whole student population.

a. What is the standard error of the sample mean monthly rent?
b. What is the probability that the sample mean exceeds the population mean by more than $5?
c. What is the probability that the sample mean is more than $4 below the population mean?
d. What is the probability that the sample mean differs from the population mean by more than $3?

7.17 Times spent studying by students in the week before final exams follow a normal distribution with standard deviation 8 hours. A random sample of 4 students was taken in order to estimate the mean study time for the population of all students.

a. What is the probability that the sample mean exceeds the population mean by more than 2 hours?
b. What is the probability that the sample mean is more than 3 hours below the population mean?
c. What is the probability that the sample mean differs from the population mean by more than 4 hours?
d. Suppose that a second (independent) random sample of 10 students was taken. Without doing the calculations, state whether the probabilities in parts (a), (b), and (c) would be higher, lower, or the same for the second sample.

7.18 An industrial process produces batches of a chemical whose impurity levels follow a normal distribution with standard deviation 1.6 grams per 100 grams of chemical. A random sample of 100 batches is selected in order to estimate the population mean impurity level.

a. The probability is 0.05 that the sample mean impurity level exceeds the population mean by how much?
b. The probability is 0.10 that the sample mean impurity level is below the population mean by how much?
c. The probability is 0.15 that the sample mean impurity level differs from the population mean by how much?

7.19 The price-earnings ratios for all companies whose shares are traded on the New York Stock Exchange follow a normal distribution with a standard deviation 3.8. A random sample of these companies is selected in order to estimate the population mean price-earnings ratio.

a. How large a sample is necessary in order to ensure that the probability that the sample mean differs from the population mean by more than 1.0 is less than 0.10?
b. Without doing the calculations, state whether a larger or smaller sample than that in part (a) would be required to guarantee that the probability that the sample mean differs from the population mean by more than 1.0 is less than 0.05.
c. Without doing the calculations, state whether a larger or smaller sample than that in part (a) would be required to guarantee that the probability that the sample mean differs from the population mean by more than 1.5 hours is less than 0.05.

7.20 The number of hours spent studying by students on a large campus in the week before final exams follows a normal distribution with standard deviation 8.4 hours. A random sample of these students is taken to estimate the population mean number of hours studying.

a. How large a sample is needed to ensure that the probability that the sample mean differs from the population mean by more than 2.0 hours is less than 0.05?
b. Without doing the calculations, state whether a larger or smaller sample than that in part (a) would be required to guarantee that the probability that the sample mean differs from the population mean by more than 2.0 hours is less than 0.10.
c. Without doing the calculations, state whether a larger or smaller sample than that in part (a) would be required to guarantee that the probability that the sample mean differs from the population mean by more than 1.5 hours is less than 0.05.

7.21 In Table 7.1 and Example 7.1, we considered samples of $n = 2$ observations from a population of $N = 6$ values of years on the job for employees. The population mean is $\mu = 5.5$ years.

a. Confirm from the six population values that the population variance is

$$\sigma^2 = 3.92$$

b. Confirm, following the approach of Example 7.1, that the variance of the sampling distribution of the sample mean is

$$\sigma_{\bar{x}}^2 = \sum_{i=1}^{15} (\bar{x}_i - \mu)^2 P(x_i) = 1.57$$

c. Verify for this example that

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

7.22 In taking a sample of n observations from a population of N members, the variance of the sampling distribution of the sample means is

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

The quantity $\frac{(N-n)}{(N-1)}$ is called the *finite population correction factor*.

a. To get some feeling for possible magnitudes of the finite population correction factor, calculate it for samples of $n = 20$ observations from populations of $N = 20$, 40, 100, 1,000, and 10,000 members.
b. Explain why the result for $N = 20$ found in part (a), is precisely what one should expect on intuitive grounds.
c. Given the results in part (a), discuss the practical significance of using the finite population correction factor for samples of 20 observations from populations of different sizes.

7.23 A town has 500 real estate agents. The mean value of the properties sold in a year by these agents is \$800,000, and the standard deviation is \$300,000. A random sample of 100 agents is selected, and the value of the properties they sold in a year is recorded.

a. What is the standard error of the sample mean?
b. What is the probability that the sample mean exceeds \$825,000?
c. What is the probability that the sample mean exceeds \$780,000?
d. What is the probability that the sample mean is between \$790,000 and \$820,000?

7.24 An economics course was taken by 250 students. Each member of a random sample of 50 of these students was asked to estimate the amount of time he or she spent on the previous week's assignment. Suppose that the population standard deviation is 30 minutes.

a. What is the probability that the sample mean exceeds the population mean by more than 2.5 minutes?
b. What is the probability that the sample mean is more than 5 minutes below the population mean?
c. What is the probability that the sample mean differs from the population mean by more than 10 minutes?

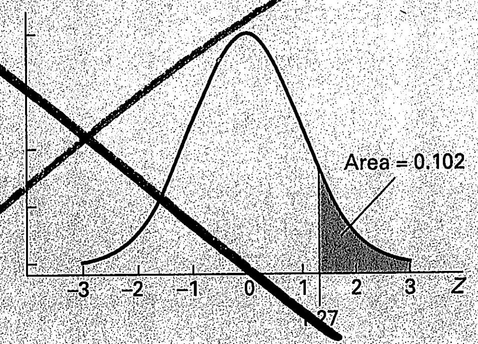
7.25 For an audience of 600 people attending a concert, the average time on the journey to the concert was 32 minutes, and the standard deviation was 10 minutes. A random sample of 150 audience members was taken.

a. What is the probability that the sample mean journey time was more than 31 minutes?
b. What is the probability that the sample mean journey time was less than 33 minutes?
c. Draw a graph to illustrate why the answers to parts (a) and (b) are the same.
d. What is the probability that the sample mean journey time was not between 31 and 33 minutes?

7.3 Sampling Distributions of Sample Proportions

In Section [illegible] we developed the binomial distribution as the sum of n independent Bernoulli random variables, each with probability of success P. To characterize the distribution, we need a value for P. Here, we indicate how we can use the sample proportion to obtain inferences about the population proportion. The proportion random variable has many applications, including percent market share, percent successful business investments, and outcomes of elections.

Figure 7.14 The Probability That a Standard Normal Random Variable Exceeds 1.27



EXERCISES

Basic Exercises

7.26 Suppose that we have a population with proportion $P = 0.40$ and a random sample of size $n = 100$ drawn from the population.

a. What is the probability that the sample proportion is greater than 0.45?
b. What is the probability that the sample proportion is less than 0.29?
c. What is the probability that the sample proportion is between 0.35 and 0.51?

7.27 Suppose that we have a population with proportion $P = 0.25$ and a random sample of size $n = 200$ drawn from the population.

a. What is the probability that the sample proportion is greater than 0.31?
b. What is the probability that the sample proportion is less than 0.14?
c. What is the probability that the sample proportion is between 0.24 and 0.40?

7.28 Suppose that we have a population with proportion $P = 0.60$ and a random sample of size $n = 100$ drawn from the population.

a. What is the probability that the sample proportion is greater than 0.66?
b. What is the probability that the sample proportion is less than 0.48?
c. What is the probability that the sample proportion is between 0.52 and 0.66?

7.29 Suppose that we have a population with proportion $P = 0.50$ and a random sample of size $n = 900$ drawn from the population.

a. What is the probability that the sample proportion is greater than 0.52?
b. What is the probability that the sample proportion is less than 0.46?
c. What is the probability that the sample proportion is between 0.47 and 0.53?

Application Exercises

7.30 In 1992, Canadians voted in a referendum on a new constitution. In the province of Quebec, 42.4% of those who voted were in favor of the new constitution. A random sample of 100 voters from the province was taken.

a. What is the mean of the distribution of the sample proportion in favor of a new constitution?
b. What is the variance of the sample proportion?
c. What is the standard error of the sample proportion?
d. What is the probability that the sample proportion is bigger than 0.5?

7.31 According to the Internal Revenue Service, 75% of all tax returns lead to a refund. A random sample of 100 tax returns is taken.

a. What is the mean of the distribution of the sample proportion of returns leading to refunds?
b. What is the variance of the sample proportion?
c. What is the standard error of the sample proportion?
d. What is the probability that the sample proportion exceeds 0.8?

7.32 A record store owner finds that 20% of customers entering her store make a purchase. One morning 180 people, who can be regarded as a random sample of all customers, enter the store.

a. What is the mean of the distribution of the sample proportion of customers making a purchase?

b. What is the variance of the sample proportion?
c. What is the standard error of the sample proportion?
d. What is the probability that the sample proportion is less than 0.15?

7.33 An administrator for a large group of hospitals believes that of all patients 30% will generate bills that become at least 2 months overdue. A random sample of 200 patients is taken.

a. What is the standard error of the sample proportion that will generate bills that become at least 2 months overdue?
b. What is the probability that the sample proportion is less than 0.25?
c. What is the probability that the sample proportion is more than 0.33?
d. What is the probability that the sample proportion is between 0.27 and 0.33?

7.34 A corporation receives 120 applications for positions from recent college graduates in business. Assuming that these applicants can be viewed as a random sample of all such graduates, what is the probability that between 35% and 45% of them are women if 40% of all recent college graduates in business are women?

7.35 A charity has found that 42% of all donors from last year will donate again this year. A random sample of 300 donors from last year was taken.

a. What is the standard error of the sample proportion who will donate again this year?
b. What is the probability that more than half of these sample members will donate again this year?
c. What is the probability that the sample proportion is between 0.40 and 0.45?
d. Without doing the calculations, state in which of the following ranges the sample proportion is more likely to lie: 0.39 to 0.41, 0.41 to 0.43, 0.43 to 0.45, 0.45 to 0.47.

7.36 A corporation is considering a new issue of convertible bonds. Management believes that the offer terms will be found attractive by 20% of all its current stockholders. Suppose that this belief is correct. A random sample of 130 current stockholders is taken.

a. What is the standard error of the sample proportion who find this offer attractive?
b. What is the probability that the sample proportion is more than 0.15?
c. What is the probability that the sample proportion is between 0.18 and 0.22?
d. Suppose that a sample of 500 current stockholders had been taken. Without doing the calculations, state whether the probabilities in parts (b) and (c) would have been higher, lower, or the same as those found.

7.37 A store has determined that 30% of all lawn mower purchasers will also purchase a service agreement. In 1 month 280 lawn mowers are sold to customers who can be regarded as a random sample of all purchasers.

a. What is the standard error of the sample proportion of those who will purchase a service agreement?
b. What is the probability that the sample proportion will be less than 0.32?
c. Without doing the calculations, state in which of the following ranges the sample proportion is most likely to be: 0.29 to 0.31, 0.30 to 0.32, 0.31 to 0.33, 0.32 to 0.34.

7.38 A random sample of 100 voters is taken to estimate the proportion of a state's electorate in favor of increasing the gasoline tax to provide additional revenue for highway repairs. What is the largest value that the standard error of the sample proportion in favor of this measure can take?

7.39 In Exercise 7.38 above, suppose that it is decided that a sample of 100 voters is too small to provide a sufficiently reliable estimate of the population proportion. It is required instead that the probability that the sample proportion differs from the population proportion (whatever its value) by more than 0.03 should not exceed 0.05. How large a sample is needed to guarantee that this requirement is met?

7.40 A company wants to estimate the proportion of people who are likely to purchase electric shavers and who watch the nationally telecast baseball playoffs. A random sample obtained information from 120 people who were identified as likely to purchase electric shavers. Suppose that the proportion of those likely to purchase electric shavers in the population who watch the telecast is 0.25.

a. The probability is 0.10 that the sample proportion watching the telecast exceeds the population proportion by how much?
b. The probability is 0.05 that the sample proportion is lower than the population proportion by how much?
c. The probability is 0.30 that the sample proportion differs from the population proportion by how much?

7.41 Suppose that 50% of all adult Americans believe that a major overhaul of the nation's health care delivery system is essential. What is the probability that more than 56% of a random sample of 150 adult Americans would hold this belief?

7.42 Suppose that 50% of all adult Americans believe that federal budget deficits at recent levels cause long-term harm to the nation's economy. What is the probability that more than 58% of a random sample of 250 adult Americans would hold this belief.

7.43 A journalist wanted to learn the views of the chief executive officers of the 500 largest U.S. corporations on program trading of stocks. In the time available, it was only possible to contact a random sample of 81 of these chief executive officers. If 55% of all the population members believe that program trading should be banned, what is the probability that less than half the sample members hold this view?

7.44 A small college has an entering freshman class of 528 students. Of these, 211 have brought their own personal computers to campus. A random sample of 120 entering freshmen was taken.

a. What is the standard error of the sample proportion bringing their own personal computers to campus?
b. What is the probability that the sample proportion is less than 0.33?
c. What is the probability that the sample proportion is between 0.5 and 0.6?

7.45 A manufacturing plant has 438 blue-collar employees. Of this group, 239 are concerned about future health care benefits. A random sample of 80 of these employees was questioned to estimate the population proportion concerned about future health care benefits.

a. What is the standard error of the sample proportion who are concerned?
b. What is the probability that the sample proportion is less than 0.5?
c. What is the probability that the sample proportion is between 0.5 and 0.6?

7.46 The annual percentage salary increases for the chief executive officers of all midsize corporations are normally distributed with mean 12.2% and standard deviation 3.6%. A random sample of 81 of these chief executive officers was taken. What is the probability that more than half the sample members had salary increases of less than 10%?

7.4 Sampling Distributions of Sample Variances

Now that sampling distributions for sample means and proportions have been developed, we will consider sampling distributions of sample variances. As business and industry increase their emphasis on producing products that satisfy customer quality standards, there is an increased need to measure and reduce population variance. High variance for a process implies a wider range of possible values for important product characteristics. This wider range of outcomes will result in more individual products that perform below an acceptable standard. After all, a customer does not care if a product performs well "on average." She is concerned that the particular item that she purchased works. High-quality products can be obtained from a manufacturing process if the process has a low population variance, so that fewer units are below the desired quality standard. By understanding the sampling distribution of sample variances, we can make inferences about the population variance. Thus, processes that have high variance can be identified and corrected. In addition, a smaller population variance improves our ability to make inferences about population means using sample means.

We begin by considering a random sample of n observations drawn from a population with unknown mean μ and unknown variance σ^2. Denote the sample members as $x_1, x_2, \ldots, x_n$. The population variance is the expectation

$$\sigma^2 = E[(X - \mu)^2]$$

which suggests that we consider the mean of $(x_i - \bar{x})^2$ over n observations. Since μ is unknown, we will use the sample mean $\bar{x}$ to compute a sample variance.