Economics 202

Practice Problems III

Basic Exercises

5.9 What is the probability distribution function of the number of heads when a fair coin is tossed?

5.10 Show the probability distribution function of the number of heads in one toss of a fair coin.

5.11 Show the probability distribution function of the number of heads when three fair coins are tossed independently.

5.12 Let the random variable represent the number of times that you will miss class this semester. Prepare a table that shows the probability function and the cumulative probability function.

Application Exercises

5.13 The number of computers sold per day at Dan's Computer Works is defined by the following probability distribution:

X	0	1	2	3	4	5	6
P(x)	0.05	0.10	0.20	0.20	0.20	0.15	0.10

a. $P(3 \le x < 6) = ?$
b. $P(x > 3) = ?$
c. $P(x \le 4) = ?$
d. $P(2 < x \le 5) = ?$

5.14 American Travel Air has asked you to study flight delays during the week before Christmas at Midway Airport. The random variable X is the number of flights delayed per hour.

X	0	1	2	3	4	5	6	7	8	9
P(x)	0.10	0.08	0.07	0.15	0.12	0.08	0.10	0.12	0.08	0.10

a. What is the cumulative probability distribution?
b. What is the probability of five or more delayed flights?
c. What is the probability of three through seven (inclusive) delayed flights?

Basic Exercises

5.15 Consider the probability distribution function

X	0	1
Probability	0.40	0.60

a. Draw the probability distribution function.
b. Calculate and draw the cumulative probability function.
c. Find the mean of the random variable X.
d. Find the variance of X.

5.16 Given the probability distribution function

X	0	1	2
Probability	0.25	0.50	0.25

a. Draw the probability distribution function.
b. Calculate and draw the cumulative probability function.
c. Find the mean of the random variable X.
d. Find the variance of X.

5.17 Consider the probability distribution function

X	0	1
Probability	0.50	0.50

a. Draw the probability distribution function.
b. Calculate and draw the cumulative probability function.
c. Find the mean of the random variable X.
d. Find the variance of X.

5.18 An automobile dealer calculates the proportion of new cars sold that have been returned various numbers of times for the correction of defects during the warranty period. The results are shown in the table.

Number of returns	0	1	2	3	4
Proportion	0.28	0.36	0.23	0.09	0.04

a. Draw the probability distribution function.
b. Calculate and draw the cumulative probability function.
c. Find the mean of the number of returns of an automobile for corrections for defects during the warranty period.
d. Find the variance of the number of returns of an automobile for corrections for defects during the warranty period.

5.19 A company specializes in installing and servicing central heating furnaces. In the pre-winter period, service calls may result in an order for a new furnace. The table shows estimated probabilities for the numbers of new furnace orders generated in this way in the last 2 weeks of September.

Number of orders	0	1	2	3	4	5
Probability	0.10	0.14	0.26	0.28	0.15	0.07

a. Draw the probability distribution function.
b. Calculate and draw the cumulative probability function.
c. Find the probability that at least three orders will be generated in this period.
d. Find the mean of the number of orders for new furnaces in this 2-week period.
e. Find the standard deviation of the number of orders for new furnaces in this 2-week period.

Application Exercises

5.20 A corporation produces packages of paper clips. The number of clips per package varies, as indicated in the accompanying table.

Number of clips	47	48	49	50	51	52	53
Proportion of packages	0.04	0.13	0.21	0.29	0.20	0.10	0.03

a. Draw the probability function.
b. Calculate and draw the cumulative probability function.
c. What is the probability that a randomly chosen package will contain between 49 and 51 clips (inclusive)?
d. Two packages are chosen at random. What is the probability that at least one of them contains at least 50 clips?
e. Use Microsoft Excel to find the mean and standard deviation of the number of paper clips per package.
f. The cost (in cents) of producing a package of clips is 16 + 2X, where X is the number of clips in the package. The revenue from selling the package, however many clips it contains, is $1.50. If profit is defined as the difference between revenue and cost, find the mean and standard deviation of profit per package.

5.21 A municipal bus company has started operations in a new subdivision. Records were kept on the numbers of riders from this subdivision during the early-morning service. The accompanying table shows proportions over all weekdays.

Number of riders	0	1	2	3	4	5	6	7
Proportion	0.02	0.12	0.23	0.31	0.19	0.08	0.03	0.02

a. Draw the probability function.
b. Calculate and draw the cumulative probability function.
c. What is the probability that on a randomly chosen weekday there will be at least four riders from the subdivision on this service?
d. Two weekdays are chosen at random. What is the probability that on both of these days there will be fewer than three riders from the subdivision on this service?
e. Find the mean and standard deviation of the number of riders from this subdivision on this service on a weekday.
f. If the cost of a ride is 50 cents, find the mean and standard deviation of the total payments of riders from this subdivision on this service on a weekday.

5.22

a. A very large shipment of parts contains 10% defectives. Two parts are chosen at random from the shipment and checked. Let the random variable X denote the number of defectives found. Find the probability function of this random variable.
b. A shipment of 20 parts contains 2 defectives. Two parts are chosen at random from the shipment and checked. Let the random variable Y denote the number of defectives found. Find the probability function of this random variable. Explain why your answer is different from that for part (a).
c. Find the mean and variance of the random variable X in part (a).
d. Find the mean and variance of the random variable Y in part (b).

5.24 A college basketball player who sinks 75% of his free throws comes to the line to shoot a "one and one" (if the first shot is successful, he is allowed a second shot, but no second shot is taken if the first is missed; one point is scored for each successful shot). Assume that the outcome of the second shot, if any, is independent of that of the first. Find the expected number of points resulting from the "one and one." Compare this with the expected number of points from a "two-shot foul," where a second shot is allowed, irrespective of the outcome of the first.

5.25 A professor teaches a large class and has scheduled an examination for 7:00 P.M. in a different classroom. She estimates the probabilities in the table for the number of students who will call her at home, in the hour before the examination, asking in which classroom it will be held.

Number of calls	0	1	2	3	4	5
Probability	0.10	0.15	0.19	0.26	0.19	0.11

Find the mean and standard deviation of the number of calls.

5.26 Students in a large accounting class were asked to rate the course by assigning a score of 1, 2, 3, 4, or 5 to the course. A higher score indicates that the students received greater value from the course. The accompanying table shows proportions of students rating the course in each category.

Rating	1	2	3	4	5
Proportion	0.07	0.19	0.28	0.30	0.16

Find the mean and standard deviation of the ratings.

5.27 A store owner stocks an out-of-town newspaper, which is sometimes requested by a small number of customers. Each copy of this newspaper costs him 70 cents, and he sells them for 90 cents each. Any copies left over at the end of the day have no value and are destroyed. Any requests for copies that cannot be met because stocks have been exhausted are considered by the store owner as a loss of 5 cents in goodwill. The probability distribution of the number of requests for the newspaper in a day is shown in the accompanying table. If the store owner defines total daily profit as total revenue from newspaper sales, less total cost of newspapers ordered, less goodwill loss from unsatisfied demand, how many copies per day should he order to maximize expected profit?

Number of requests	0	1	2	3	4	5
Probability	0.12	0.16	0.18	0.32	0.14	0.08

5.28 A factory manager is considering whether to replace a temperamental machine. A review of past records indicates the following probability distribution for the number of breakdowns of this machine in a week.

Number of breakdowns	0	1	2	3	4
Probability	0.10	0.26	0.42	0.16	0.06

a. Find the mean and standard deviation of the number of weekly breakdowns.
b. It is estimated that each breakdown costs the company $1,500 in lost output. Find the mean and standard deviation of the weekly cost to the company from breakdowns of this machine.

5.29 An investor is considering three strategies for a $1,000 investment. The probable returns are estimated as follows:

• Strategy 1: A profit of $10,000 with probability 0.15 and a loss of $1,000 with probability 0.85

• Strategy 2: A profit of $1,000 with probability 0.50, a profit of $500 with probability 0.30, and a loss of $500 with probability 0.20

• Strategy 3: A certain profit of $400

Which strategy has the highest expected profit? Would you necessarily advise the investor to adopt this strategy?

Basic Exercises

5.30 For a Bernoulli random variable with probability of success $P = 0.5$, compute the mean and variance.

5.31 For a binomial probability function with $P = 0.5$ and $n = 12$, find the probability that the number of successes is equal to 7 and the probability that the number of successes is less than 6.

5.32 For a binomial probability function with $P = 0.3$ and $n = 14$, find the probability that the number of successes is equal to 7 and the probability that the number of successes is less than 6.

5.33 For a binomial probability function with $P = 0.4$ and $n = 20$, find the probability that the number of successes is equal to 9 and the probability that the number of successes is less than 7.

5.34 For a binomial probability function with $P = 0.7$ and $n = 18$, find the probability that the number of successes is equal to 12 and the probability that the number of successes is less than 6.

Application Exercises

5.35 A production manager knows that 5% of components produced by a particular manufacturing process have some defect. Six of these components, whose characteristics can be assumed to be independent of each other, are examined.

a. What is the probability that none of these components has a defect?
b. What is the probability that one of these components has a defect?
c. What is the probability that at least two of these components have a defect?

5.36 A politician believes that 25% of all macroeconomists in senior positions will strongly support a proposal he wishes to advance. Suppose that this belief is correct and that five senior macroeconomists are approached at random.

a. What is the probability that at least one of the five will strongly support the proposal?
b. What is the probability that a majority of the five will strongly support the proposal?

5.37 A public interest group hires students to solicit donations by telephone. After a brief training period, students make calls to potential donors and are paid on a commission basis. Experience indicates that early on these students tend to have only modest success and that 70% of them give up their jobs in their first 2 weeks of employment. The group hires six students, which can be viewed as a random sample.

a. What is the probability that at least two of the six will give up in the first 2 weeks?
b. What is the probability that at least two of the six will not give up in the first 2 weeks?

5.38 Suppose that the probability is 0.5 that the value of the U.S. dollar will rise against the Japanese yen over any given week and that the outcome in one week is independent of that in any other week.

What is the probability that the value of the U.S. dollar will rise against the Japanese yen in a majority of weeks over a period of 7 weeks?

5.39 A company installs new central heating furnaces and has found that for 15% of all installations a return visit is needed to make some modifications. Six installations were made in a particular week. Assume independence of outcomes for these installations.

a. What is the probability that a return visit will be needed in all of these cases?
b. What is the probability that a return visit will be needed in none of these cases?
c. What is the probability that a return visit will be needed in more than one of these cases?

5.40 The Cubs are to play a series of five games in St. Louis against the Cardinals. For any one game it is estimated that the probability of a Cubs win is 0.4. The outcomes of the five games are independent of one another.

a. What is the probability that the Cubs will win all five games?
b. What is the probability that the Cubs will win a majority of the five games?
c. If the Cubs win the first game, what is the probability that they will win a majority of the five games?
d. Before the series begins, what is the expected number of Cubs wins in these five games?
e. If the Cubs win the first game, what is the expected number of Cubs wins in the five games?

5.41 A small commuter airline flies planes that can seat up to eight passengers. The airline has determined that the probability that a ticketed passenger will not show up for a flight is 0.2. For each flight the airline sells tickets to the first 10 people placing orders. The probability distribution for the number of tickets sold per flight is shown in the accompanying table. For what proportion of the airline's flights does the number of ticketed passengers showing up exceed the number of available seats? (Assume independence between the number of tickets sold and the probability that a ticketed passenger will show up.)

Number of tickets	6	7	8	9	10
Probability	0.25	0.35	0.25	0.10	0.05

5.42 Following a touchdown, a college football coach has the option to elect to attempt a "2-point conversion"; that is, 2 additional points are scored if the attempt is successful and none if it is unsuccessful. The coach believes that the probability is 0.4 that his team will be successful in any attempt and that outcomes of different attempts are independent of each other. In a particular game the team scores four touchdowns and 2-point conversion attempts were made each time.

a. What is the probability that at least two of these attempts will be successful?
b. Find the mean and standard deviation of the total number of points resulting from these four attempts.

5.43 An automobile dealer mounts a new promotional campaign. Purchasers of new automobiles may, if dissatisfied for any reason, return them within 2 days of purchase and receive a full refund. The cost to the dealer of such a refund is $250. The dealer estimates that 15% of all purchasers will indeed return automobiles and obtain refunds. Suppose that 50 automobiles are purchased during the campaign period.

a. Find the mean and standard deviation of the number of these automobiles that will be returned for refunds.
b. Find the mean and standard deviation of the total refund costs that will accrue as a result of these 50 purchases.

5.44 A family of mutual funds maintains a service that allows clients to switch money among accounts through a telephone call. It was estimated that 3.2% of callers either get a busy signal or are kept on hold so long that they may hang up. Fund management assesses any failure of this sort as a $10 goodwill loss. Suppose that 2,000 calls are attempted over a particular period.

a. Find the mean and standard deviation of the number of callers who will either get a busy signal or may hang up after being kept on hold.
b. Find the mean and standard deviation of the total goodwill loss to the mutual fund company from these 2,000 calls.

5.46 A campus finance officer finds that, for all parking tickets issued, fines of 78% are paid. The fine is $2. In the most recent week 620 parking tickets have been issued.

a. Find the mean and standard deviation of the number of these tickets for which the fines will be paid.
b. Find the mean and standard deviation of the amount of money that will be obtained from the payment of these fines.

5.47 A company receives a very large shipment of components. A random sample of 16 of these components will be checked, and the shipment will be accepted if fewer than 2 of these components are defective. What is the probability of accepting a shipment containing

a. 5% defectives?
b. 15% defectives?
c. 25% defectives?

5.48 The following two acceptance rules are being considered for determining whether to take delivery of a large shipment of components:

• A random sample of 10 components is checked, and the shipment is accepted only if none of them is defective.

• A random sample of 20 components is checked, and the shipment is accepted only if no more than 1 of them is defective.

Which of these acceptance rules has the smaller probability of accepting a shipment containing 20% defectives?

5.49 A company receives large shipments of parts from two sources. Seventy percent of the shipments come from a supplier whose shipments typically contain 10% defectives, while the remainder are from a supplier whose shipments typically contain 20% defectives. A manager receives a shipment but does not know the source. A random sample of 20 items from this shipment is tested, and 1 of the parts is found to be defective. What is the probability that this shipment came from the more reliable supplier? [*Hint*: Use Bayes' theorem.]

Basic Exercises

5.50 Compute the probability of 5 successes in a random sample of size n = 12 obtained from a population of size N = 50 that contains 25 successes.

5.51 Compute the probability of 7 successes in a random sample of size n = 14 obtained from a population of size N = 60 that contains 25 successes.

5.52 Compute the probability of 9 successes in a random sample of size n = 20 obtained from a population of size N = 80 that contains 42 successes.

5.53 Compute the probability of 3 successes in a random sample of size n = 5 obtained from a population of size N = 40 that contains 25 successes.

5.54 Compute the probability of 8 successes in a random sample of size n = 15 obtained from a population of size N = 400 that contains 200 successes.

Application Exercises

5.56 A committee of eight members is to be formed from a group of eight men and eight women. If the choice of committee members is made randomly, what is the probability that precisely half of these members will be women?

5.57 A bond analyst was given a list of 12 corporate bonds. From that list she selected 3 whose ratings she felt were in danger of being downgraded in the next year. In actuality, a total of 4 of the 12 bonds on the list had their ratings downgraded in the next year. Suppose that the analyst had simply chosen 3 bonds randomly from this list. What is the probability that at least 2 of the chosen bonds would be among those whose ratings were to be downgraded in the next year?

5.58 A bank executive is presented with loan applications from 10 people. The profiles of the applicants are similar, except that 5 are minorities and 5 are not minorities. In the end the executive approves 6 of the applications. If these 6 approvals are chosen at random from the 10 applications, what is the probability that less than half the approvals will be of applications involving minorities?

Basic Exercises

5.73 Consider the joint probability distribution

		X	
		1	2
Y	0	0.25	0.25
	1	0.25	0.25

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.

5.74 Consider the joint probability distribution

		X	
		1	2
Y	0	0.20	0.25
	1	0.30	0.25

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.

5.75 Consider the joint probability distribution

		X	
		1	2
Y	0	0.20	0.25
	1	0.30	0.25

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function $W = X + Y$.

5.76 Consider the joint probability distribution

		X	
		0	1
Y	0	0.30	0.20
	1	0.25	0.25

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function $W = 2X + Y$.

5.77 Consider the joint probability distribution

		X	
		1	2
Y	0	0.70	0.0
	1	0.0	0.30

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function $W = 3X + 4Y$.

5.78 Consider the joint probability distribution

		X	
		1	2
Y	0	0.25	0.25
	1	0.25	0.25

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function $W = X + Y$.

5.79 Consider the joint probability distribution

		X	
		1	2
Y	0	0.30	0.20
	1	0.25	0.25

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function $W = 2x + Y$.

5.80 Consider the joint probability distribution

		X	
		1	2
Y	0	0.0	0.60
	1	0.40	0.0

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function *W = 2X – 4Y*.

5.81 Consider the joint probability distribution

		X	
		1	2
Y	0	0.70	0.0
	1	0.0	0.30

a. Compute the marginal probability distributions for *X* and *Y*.
b. Compute the covariance and correlation for *X* and *Y*.
c. Compute the mean and variance for the linear function *W = 10X – 8Y*.

Application Exercises

5.82 A researcher suspected that the number of between-meal snacks eaten by students in a day during final examinations might depend on the number of tests a student had to take on that day. The accompanying table shows joint probabilities, estimated from a survey.

	Number of Tests (*X*)			
Number of Snacks (*Y*)	0	1	2	3
0	0.07	0.09	0.06	0.01
1	0.07	0.06	0.07	0.01
2	0.06	0.07	0.14	0.03
3	0.02	0.04	0.16	0.04

a. Find the probability function of *X* and hence the mean number of tests taken by students on that day.
b. Find the probability function of *Y* and hence the mean number of snacks eaten by students on that day.
c. Find and interpret the conditional probability function of *Y*, given *X* = 3.
d. Find the covariance between *X* and *Y*.
e. Are number of snacks and number of tests independent of each other?

5.84 The accompanying table shows, for credit card holders with one to three cards, the joint probabilities for number of cards owned (*X*) and number of credit purchases made in a week (*Y*).

	Number of Purchases in Week (*Y*)				
Number of Cards (*X*)	0	1	2	3	4
1	0.08	0.13	0.09	0.06	0.03
2	0.03	0.08	0.08	0.09	0.07
3	0.01	0.03	0.06	0.08	0.08

a. For a randomly chosen person from this group, what is the probability function for number of purchases made in a week?
b. For a person in this group who has three cards, what is the probability function for number of purchases made in the week?
c. Are number of cards owned and number of purchases made statistically independent?

5.87 A restaurant manager receives occasional complaints about the quality of both the food and the service. The marginal probability functions for the number of weekly complaints in each category are shown in the accompanying table. If complaints about food and service are independent of each other, find the joint probability function.

Number of Food Complaints	Probability	Number of Service Complaints	Probability
0	0.12	0	0.18
1	0.29	1	0.38
2	0.42	2	0.34
3	0.17	3	0.10

5.88 Refer to the information in Exercise 5.87. Find the mean and standard deviation of the total number of complaints received in a week. Having reached this point, you are concerned that the numbers of food and service complaints may not be independent of each other. However, you have no information about the nature of their dependence. What can you now say about the mean and standard deviation of the total number of complaints received in a week?

5.89 A company has 5 representatives covering large territories and 10 representatives covering smaller territories. The probability distributions for the numbers of orders received by each of these types of representatives in a day are shown in the accompanying table. Assuming that the number of orders received by any representative is independent of the number received by any other, find the mean and standard deviation of the total number of orders received by the company in a day.

Number of Orders (Large Territory)	Probability	Number of Orders (Smaller Territory)	Probability
0	0.08	0	0.18
1	0.16	1	0.26

2	0.28	2	0.36
3	0.32	3	0.13
4	0.10	4	0.07
5	0.06		