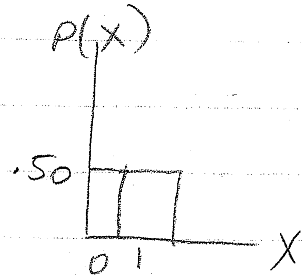
ECO 202
CHAPTER 5

(5.9)

# of Heads (X)	P(X)
0	.50
1	.50



(5.10)

# of Heads (X)	P(x)
0	.50
1	.50

(5.11)

# of Heads in 3 (x)	P(x)
0	1/8
1	3/8
2	3/8
3	1/8

SS: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

(5.12) You can pick your own values, both for X & for P(X).

X (# of absences)	P(X)	F(X)
0	.85	.85
1	.10	.95
2	.03	.98
3	.02	1.00

5.13

X	$P(X)$
0	.05
1	.10
2	.20
3	.20
4	.20
5	.15
6	.10

a. $P(3 \leq X < 6) = P(X=3) + P(X=4) + P(X=5)$
$= .20 + .20 + .15$
$= .55$

b. $P(X > 3) = P(X=4) + P(X=5) + P(X=6)$
$= .20 + .15 + .10$
$= .45$

c. $P(X \leq 4) = P(X=4) + P(X=3) + P(X=2)$
$+ P(X=1) + P(X=0)$
$= .20 + .20 + .20 + .10 + .05$
$= .75$

d. $P(2 < X \leq 5)$
$= P(X=3) + P(X=4) + P(X=5)$
$= .20 + .20 + .15$
$= .55$

5.14

X	$P(X)$	a) $F(X)$
0	.10	.10
1	.08	.18
2	.07	.25
3	.15	.40
4	.12	.52
5	.08	.60
6	.10	.70
7	.12	.82
8	.08	.90
9	.10	1.00

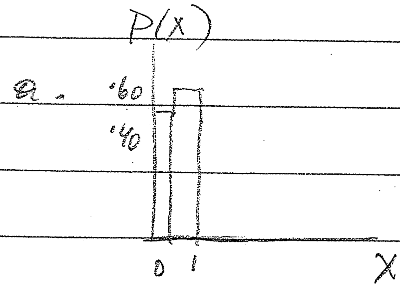
b) $P(X \geq 5) = 1 - F(X=4)$
$= 1 - .52$
$= .48$

c) $P(3 \leq X \leq 7) = F(X=7) - F(X=2)$
$= .82 - .25$
$= .57$

5.15

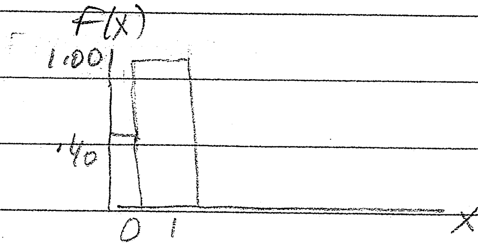
X	$P(x)$
0	.40
1	.60

a.



b.

X	$F(x)$
0	.40
1	1.00



c. $E(X) = \sum x P(X) = (0)(.40) + (1)(.60) = .60$

d. $V(x) = \sum x^2 P(X) - [E(X)]^2 = (0)^2(.40) + (1)^2(.60) - (.60)^2$

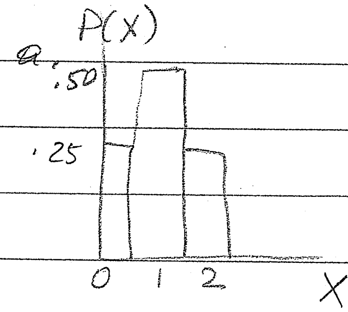
$= .60 - .36$

$= .24$

5.16

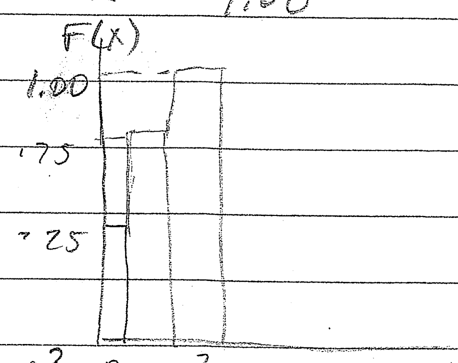
X	$P(X)$
0	.25
1	.50
2	.25

a.



b.

X	$F(X)$
0	.25
1	.75
2	1.00



c. $E(X) = \sum x P(X) = (0)(.25) + (1)(.50) + (2)(.25)$

$= .50 + .50 = 1.00$

d. $V(x) = \sum x^2 P(x) - [E(X)]^2$

$= (0)^2(.25) + (1)^2(.50) + (2)^2(.25) - (1.00)^2$

$= .50 + 1.00 - 1.00 = .50$

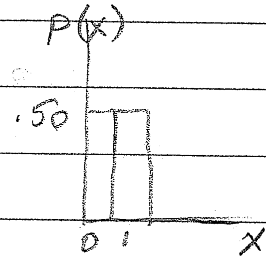
OR $(0-1)^2(.25) + (1-1)^2(.50) + (2-1)^2(.25)$

$= .25 + .25 = .50$

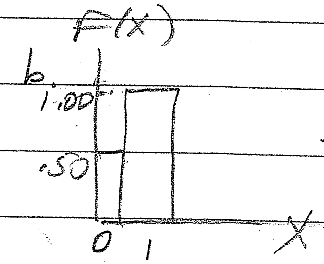
5.17

X	$P(X)$	$F(X)$
0	.50	.50
1	.50	1.00

a.



b.



c. $E(X) = \sum X\,P(X) = (0)(.50) + (1)(.50) = .50$

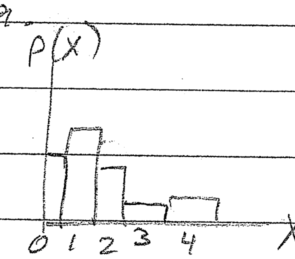
d. $V(X) = \sum X^2 P(X) - [E(X)]^2 = (0)^2(.50) + (1)^2(.50) - (.50)^2$

$= .50 - .25 = .25$

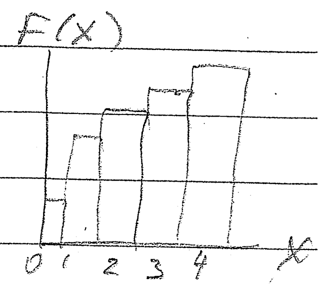
5.18

# of Returns X	Prop (Probab.)	Cum Prob
0	.28	.28
1	.36	.64
2	.23	.87
3	.09	.96
4	.04	1.00

a.



b.



c. $E(X) = \sum X\,P(X) = (0)(.28) + (1)(.36) + (2)(.23) + (3)(.09) + (4)(.04)$

$= 0 + .36 + .46 + .27 + .16$

$= 1.25$

d. $s^2 = \sum x^2 P(X) - [E(X)]^2 = (0)^2(.28) + (1)^2(.36) + (2)^2(.23) + (3)^2(.09)$

$+ (4)^2(.04) - (1.25)^2$

$= .36 + .92 + .81 + .64 - 1.5625$

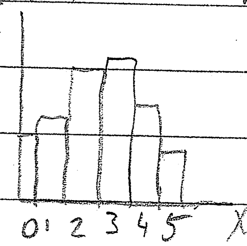
$= 1.1675$

5.19

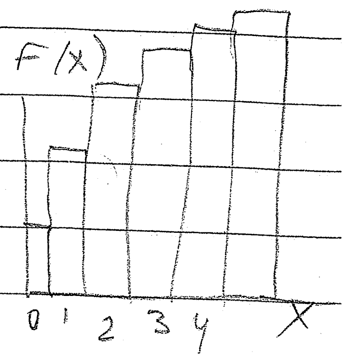
# of orders

X	P(X)	F(X)
0	.10	.10
1	.14	.24
2	.26	.50
3	.28	.78
4	.15	.93
5	.07	1.00

a. P(X)



b. F(X)



c. $P(x \geq 3) = P(X=3) + P(X=4) + P(X=5) = .28 + .15 + .07$

$= .50$

d. $E(X) = \sum x \cdot P(x) = (0)(.10) + (1)(.14) + (2)(.26) + (3)(.28)$

$+ (4)(.15) + (5)(.07)$

$= .14 + .52 + .84 + .60 + .35$

$= 2.45$

e. $s^2 = \sum x^2 P(x) - (E(X))^2 = (0)^2(.10) + (1)^2(.14) + (2)^2(.26)$

$+ (3)^2(.28) + (4)^2(.15) + (5)^2(.07)$

$- (2.45)^2$

$= .14 + 1.04 + 2.52 + 2.4 + 1.75 - 6.0025$

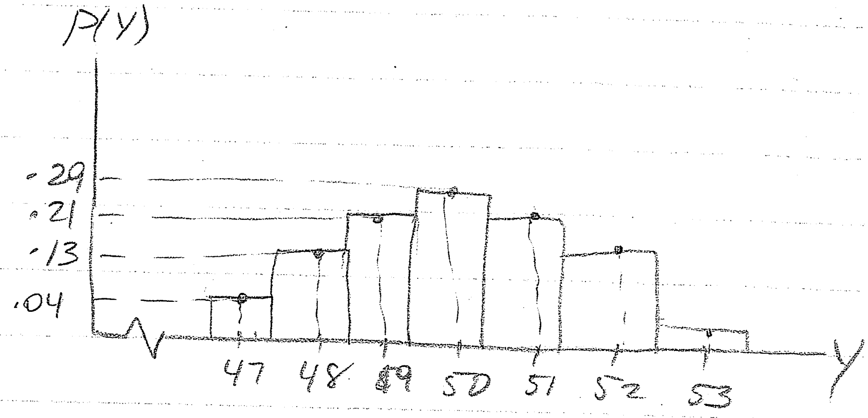
$= 1.8475$

$s = \sqrt{1.8475} = .5413$

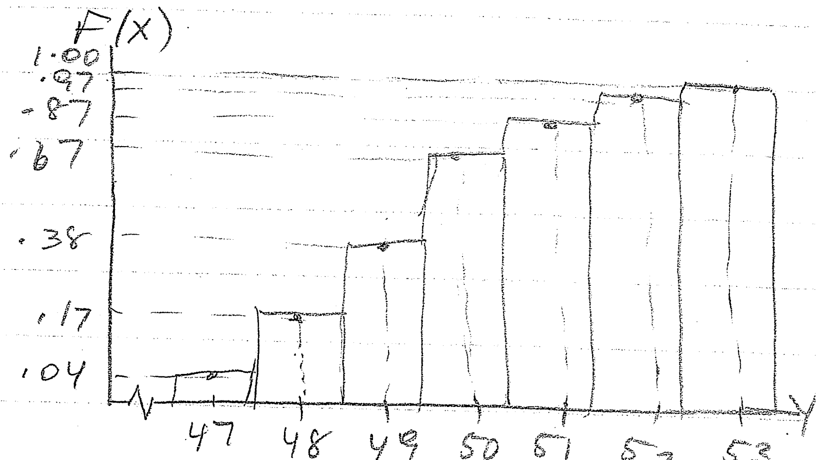
(5.20)

Y	P(Y)	F(Y)
47	.04	.04
48	.13	.17
49	.21	.38
50	.29	.67
51	.20	.87
52	.10	.97
53	.03	1.00

a)



b) function F(Y) above.



c) $P(49 \le Y \le 51)$
$= F(Y=51) - F(Y=48)$
$= .87 - .17$
$= .70$

d) P(one package contains $Y \ge 50$) $= 1 - F(Y=49) = 1 - .38 = .62$

Two packages:

P(one $Y \ge 50 \cap$ one $Y \ge 50$) $= (.62)(.62) = .3844$

P(one $Y \ge 50 \cap$ one $Y < 50$) $= 2(.62)(.38) = .4712$

$+$

$.8556$

e) $E(Y) = (47)(.04) + (48)(.13) + (49)(.21) + (50)(.29) + (51)(.20)$
$+ (52)(.10) + (53)(.03) = 1.88 + 6.24 + 10.29 + 14.5 + 10.20$
$+ 5.20 + 1.59 = 49.9$

$$V(Y) = (47)^2(.04) + (48)^2(.13) + (49)^2(.21) + (50)^2(.29) + (51)^2(.20)$$
$$+ (52)^2(.10) + (53)^2(.03) - (49.9)^2$$
$$= 88.36 + 299.52 + 504.21 + 725 + 520.2 + 270.4$$
$$+ 84.27 - 2490.01$$
$$= 1.95$$

$$s(Y) = \sqrt{1.95} = 1.3964$$

f) $C = 16 + 2X$

$\pi = 150 - (16 + 2X)$

$\pi = 134 - 2X$

Remember: C is in ¢, so revenue per package is 1.50 = 150 ¢.

$$E(\pi) = 134 - 2E(X)$$
$$= 134 - 2(49.9)$$
$$= 134 - 99.8$$
$$= 34.2 ¢$$
$$= \$.34$$

$$V(\pi) = 0 - (2)^2 V(X)$$
$$= 0 + 4(1.95)$$
$$= 7.98$$

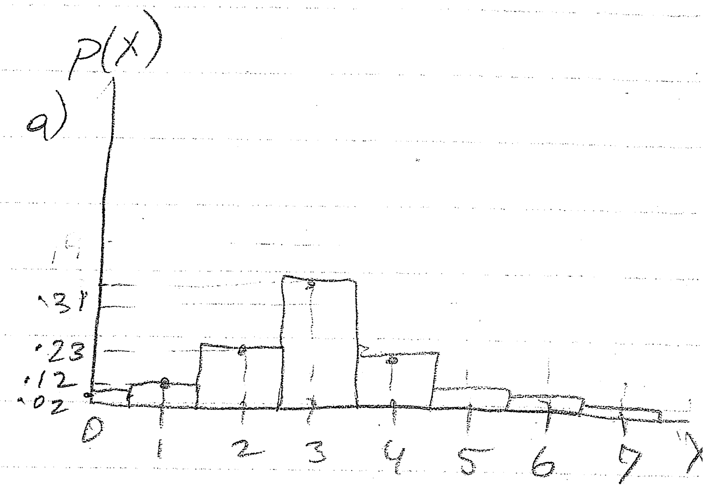
watch the sign!!

$$s(\pi) = \sqrt{7.98} = 2.793 ¢$$
$$= \$.279$$

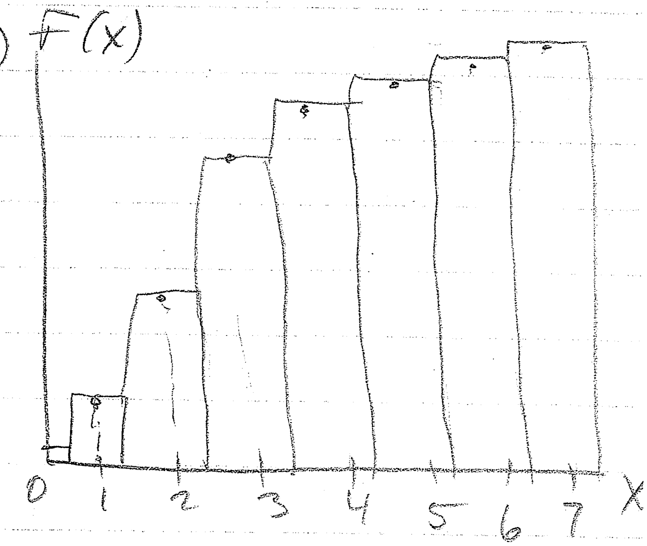
(5.21) Riders

X	$P(X)$	$F(x)$
0	.02	.02
1	.12	.14
2	.23	.37
3	.31	.68
4	.19	.87
5	.08	.95
6	.03	.98
7	.02	1.00

a)



b)



c. $P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - F(X=3)$
$= 1 - .68$
$= .32$

d. $P(X<3) = P(X \leq 2) = F(X=2) = .37$ prob. of fewer than 3 on any day

In two days:
$P(X<3 \text{ both days}) = (.37)(.37) = .1369$

e. $E(X) = (0)(.02) + (1)(.12) + (2)(.23) + (3)(.31) + (4)(.19)$
$+ (5)(.08) + (6)(.03) + (7)(.02)$
$= .12 + .46 + .93 + .76 + .4 + .18 + .14 = 2.99$

$V(X) = (0)^2(.02) + (1)^2(.12) + (2)^2(.23) + (3)^2(.31) + (4)^2(.19)$
$+ (5)^2(.08) + (6)^2(.03) + (7)^2(.02) - (2.99)^2$
$= .12 + .92 + 2.79 + 3.04 + 2 + 1.08 + .98 - 8.94$
$= 1.99$

$s(x) = \sqrt{1.99} = 1.41$

f) Total Pay $P = \$.50X$

$E(P) = (.50)E(X) = (.50)(2.99) = \$1.495$

$V(P) = (.50)^2 V(X) = (.25)(1.99) = \$.4975$

$s(P) = \sqrt{.4975} = \$.705$

5.22 $X = \#$ of defectives.

$P(X) = .10 \Rightarrow P(OK) = .90$ choose 2

Def OK $\Rightarrow X=1$

Def Def $\Rightarrow X=2$

OK Def $\Rightarrow X=1$

OK OK $\Rightarrow X=0$

a)

X	$P(X)$
0	$(.90)(.90) = .81$
1	$(.90)(.10)+(.10)(.90) = .18$
2	$(.10)(.10) = .01$

b) 2 out of 20 defective $\Rightarrow$ $P(def) = \frac{2}{20} = .10$

\# of defectives

Y	$P(Y)$
0	$18/20 \cdot 17/19 = .8053$
1	$2/20 \cdot 18/19 + 18/20 \cdot 2/19 = .1895$
2	$2/20 \cdot 1/19 = .005263$

In part a, selecting from a large population does not affect the likelihood of the outcome of the second selection (without replacement) enough for us to account for this influence. In part b, as the population is smaller, first selection does affect likelihood of the outcome of the second selection (again, w/o replacement).

order switched

d. $E(X) = (0)(.8053) + (1)(.1895) + (2)(.005263)$

$= .1895 + .010526 = .20$

$V(X) = (0)^2(.8053) + (1)^2(.1895) + (2)^2(.005263) - (.20)^2$

$= .1895 + .021052 - .04$

$= .210552 - .04 = .170552$

$s(X) = \sqrt{.170552} = .413$

c. $E(X) = (0)(.81) + (1)(.18) + (2)(.01)$

$= .18 + .02$

$= .20$

$V(X) = (0)^2(.81) + (1)^2(.18) + (2)^2(.01) - (.2)^2$

$= .18 + .04 - .04$

$= .18$

$s(X) = \sqrt{.18} = .424$

5.23

a)

X	$P(X)$
1	$.40$
2	$(.60)(.40) = .24$
3	$(.60)(.60)(.40) = .144$
4	$(.60)(.60)(.60)(.40) = .0864$
5	$(.60)^4(.40) = .05184$
6	$(.60)^5(.40) = .031104$
7	$(.60)^6(.40) = .01866$
8	$(.6)^7(.40) = .011197$

Generalizing from the pattern emerging in thes columns, we can write $P(X) = (.6)^{X-1}(.4)$ as the probability function for X

b)

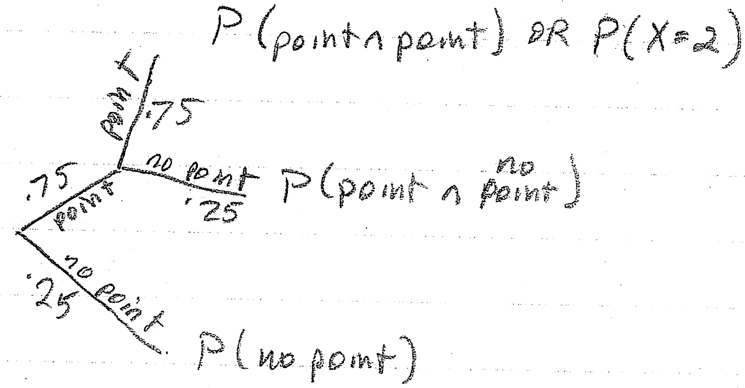
X	$F(X)$
1	$.40$
2	$.64$
3	$.784$
4	$.8704$
5	$.92224$
6	$.953344$
7	$.972004$
8	$.9832$

$$F(X) = \sum_{X=1} (.6)^{X-1}(.4)$$

Again, thru generalizing from the steps taken to compute these values, we arrive at this function

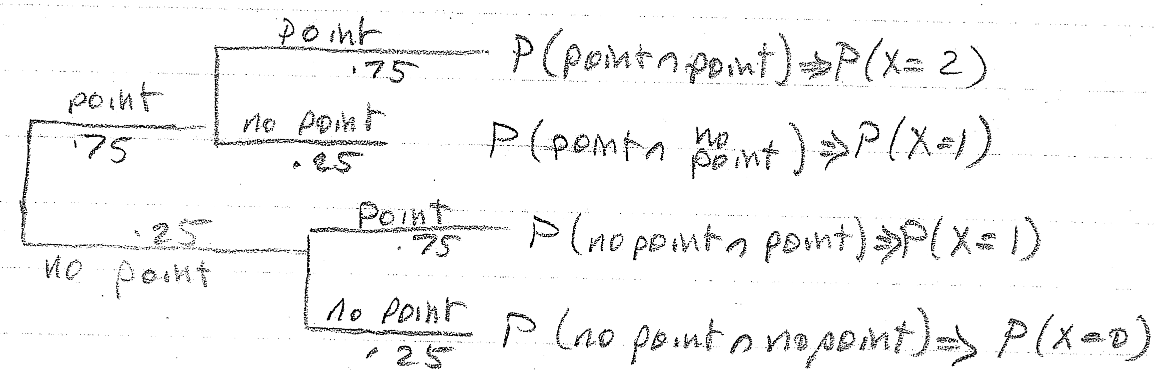
c) $P(X \geq 3) = 1 - P(X \leq 2) = F(X=2) = .64$
(from above)

5.24



X	P(x)
0	.25
1	(.75)(.25) = .1875
2	(.75)(.75) = .5625

$E(X) = (0)(.25) + (1)(.1875) + (2)(.5625) = \underline{1.3125}$



X	P(x)
0	(.25)(.25) = .0625
1	2(.75)(.25) = .375
2	(.75)(.75) = .5625

$E(X) = (0)(.0625) + (1)(.375) + (2)(.5625) = \underline{1.5}$

5.25

# of calls X	$P(X)$
0	.10
1	.15
2	.19
3	.26
4	.19
5	.11

$E(X) = (0)(.10) + (1)(.15) + (2)(.19) + (3)(.26) + (4)(.19) + 5(.11)$
$= .15 + .38 + .78 + .76 + .55$
$= 2.62$

$V(X) = (0)^2(.10) + (1)^2(.15) + (2)^2(.19) + (3)^2(.26) + (4)^2(.19) + (5)^2(.11) - (2.62)^2$
$= .10 + .15 + .76 + 2.34 + 3.04 + 2.75 - 6.86$
$= 2.28$

$S(X) = \sqrt{2.28} = 1.51$

5.26

X	$P(X)$
1	.07
2	.19
3	.28
4	.30
5	.16

$E(X) = (1)(.07) + (2)(.19) + (3)(.28) + 4(.30) + (5)(.16)$
$= .07 + .38 + .84 + 1.2 + .80$
$= 3.29$

$V(X) = (1)^2(.07) + (2)^2(.19) + (3)^2(.28) + (4)^2(.30) + (5)^2(.16) - (3.29)^2$
$= .07 + .76 + 2.52 + 4.80 + 4 - 10.8241$
$= 1.3259$

$S(X) = \sqrt{1.3259} = 1.1515$

(5.27)

x	$P(x)$
0	.12
1	.16
2	.18
3	.32
4	.14
5	.08

Price = .90 cost = .70

Profit =

$\pi = .90X - .70S - (X-S).05$

$E(X) = (0)(.12) + (1)(.16)$
$+ (2)(.18) + (3)(.32)$
$+ (4)(.14) + (5)(.08)$
$= .16 + .36 + .96$
$+ .56 + .40$
$= 2.44$

5.28

X	$P(X)$
0	.10
1	.26
2	.42
3	.16
4	.06

a) $E(X) = (0)(.10)+(1)(.26)+(2)(.42)+(3)(.16) + (4)(.06) = .26+.84+.48+.24$

$= 1.82$

$V(X) = (0)^2(.10)+(1)^2(.26)+(2)^2(.42)+(3)^2(.16) + (4)^2(.06) - (1.82)^2$

$= .26+1.68+1.44+.96 - 3.3124$

$= 1.0276$

$S(X) = \sqrt{1.0276} = 1.0137$

b) Loss: $L = 1500X$

$E(L) = 1500\,E(X)$

$= 1500(1.82) = 2730$

$V(L) = (1500)^2\,V(X)$

$= 2250000\,(1.0276)$

$= 2,312,100$

$S(L) = \sqrt{2,312,100} = 1520.559$

5.29 Strategy I: $E(\pi) = 10{,}000(.15) + (-1000)(.85) = 1500 - 850 = \$650$

Strategy II: $E(\pi) = (1000)(.50) + (500)(.30) + (-500)(.20)$
$= 500 + 150 - 100$
$= \$550$

Strategy III: $E(\pi) = \$400$

Highest expected π is with Strategy I. Because the \$650 is not a sure return, I would not advise that the investor choose this strategy if s/he happens to be very risk averse. For such investors, a lower but guaranteed profit of \$400 might be much more appropriate.

5.30 $\quad P = .50 \qquad E(X) = P = .50$

$V(X) = P(1-P) = (.50)(.50) = .25$

5.31 $\quad P = .50$, $n = 12$

$$P(X=7) = \frac{12!}{7!\,5!}(.50)^7(.50)^5$$

$$= \frac{8 \cdot 9 \cdot 10 \cdot 11 \cdot 12}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}(.0078125)(.03125)$$

$$= (792)(.000244141)$$

$$= .19335$$

$$P(X<6) = P(X=0) + P(X=1) + \cdots + P(X=5)$$

From Table
$$= .0002 + .0029 + .0161 + .0537 + .1208 + .1934$$
$$= .3871$$

5.32 $\quad P = .30$, $n = 14$

$$P(X=7) = \frac{14!}{7!\,7!}(.30)^7(.70)^7$$

$$= \frac{8 \cdot 9 \cdot 10 \cdot 11 \cdot 12 \cdot 13 \cdot 14}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}(.21)^7$$

$$= (3432)(.000018011)$$

$$= .0618$$

$$P(X<6) = P(X=0) + P(X=1) + \cdots + P(X=5)$$

$$= .0068 + .0407 + .1134 + .1943 + .2290 + .1963$$

$$= .7805$$

5.33 $\quad P = .40 \quad n = 20$

$$P(X=9) = \frac{20!}{9!\,11!}(.40)^9(.60)^{11}$$

$$= \frac{\cancel{12}\cdot 13\cdot \overset{2}{\cancel{14}}\cdot \overset{3}{\cancel{15}}\cdot \overset{2}{\cancel{16}}\cdot 17\cdot \overset{2}{\cancel{18}}\cdot 19\cdot \overset{5}{\cancel{20}}}{\cancel{9}\cdot \cancel{8}\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}}(.00026)(.0036$$

$$= 167{,}960\,(.000000951)$$

$$= .1597$$

$$P(X<7) = P(x=0)+P(x=1)+\text{-----}+P(x=6)$$

$$= .000 + .0005 + .0031 + .0123 + .0350 + .0746 + .1244$$

$$= .2499$$

5.34 $\quad P = .70 \quad n = 18$

$$P(X=12) = \frac{18!}{12!\,6!}(.70)^{12}(.30)^6$$

$$= \frac{13\cdot \overset{7}{\cancel{14}}\cdot \overset{3}{\cancel{15}}\cdot \overset{4}{\cancel{16}}\cdot 17\cdot \overset{3}{\cancel{18}}}{\cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1}(.01384)(.000729)$$

$$= 18{,}564\,(.00001\,0089)$$

$$= .187$$

$$P(X<6) = P(x=0)+P(x=1)+\text{----}+P(x=5)$$

$$= .0002$$

5.35 $P = .05$

$n = 6$

a) $P(X=0) = \frac{6!}{0!6!}(.05)^0(.95)^6 = (1)(1)(.735) = .735$

b) $P(X=1) = \frac{6!}{1!5!}(.05)^1(.95)^5 = 6(.05)(.7738) = .232$

c) $P(X \geq 2) = 1 - P(X=1) - P(X=0)$

$= 1 - \left[\frac{6!}{1!5!}(.05)^1(.95)^5\right] - \left[\frac{6!}{0!6!}(.05)^0(.95)^6\right]$

$= 1 - .232 - .735$

$= .0329$

5.36 $P = .25$

$n = 5$

a) $P(X \geq 1) = 1 - P(X=0)$ where $P(X=0) = \frac{5!}{0!5!}(.25)^0(.75)^5$

$= (1)(1)(.2373)$

$= .2373$

$= 1 - .2373 = .7627$

b) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= \frac{5!}{3!2!}(.25)^3(.75)^2 + \frac{5!}{4!1!}(.25)^4(.75)^1 + \frac{5!}{5!0!}(.25)^5(.75)^0$

$= \frac{\cancel{24}\cdot 5}{\cancel{2}}(.01563)(.5625) + 5(.00391)(.75) + (1)(.0009766)(1$

$= 10(.00879) + .0146625 + .0009766$

$= .1035$

5.37 $p = .70$

$n = 6$

a) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

where $P(X=0) = \frac{6!}{0!6!}(.70)^0(.30)^6$

$= (1)(1)(.000729)$

$= .000729$

$P(X=1) = \frac{6!}{1!5!}(.70)^1(.30)^5$

$= 6(.70)(.00243)$

$= .010206$

$P(X \geq 2) = 1 - (.000729) - (.010206)$

$= .989065$

b) Redefine "success" as not giving up job with $P = .30$

Then

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$

where $P(X=0) = \frac{6!}{6!0!}(.30)^0(.70)^6$

$= (1)(1)(.11765)$

$= .11765$

$P(X=1) = \frac{6!}{1!5!}(.30)^1(.70)^5$

$= (6)(.30)(.16807)$

$= .3025$

$P(X \geq 2) = 1 - .11765 - .3025 = .5798$

5.38 $P = .50$ majority of weeks of 7 $\Rightarrow$ 4 or more weeks.

$n = 7$

$P(X=4) = \frac{7!}{4!\,3!}(.50)^4(.50)^3 = \frac{\cancel{5}\cdot\cancel{6}\cdot 7}{\cancel{3}\cdot\cancel{2}\cdot 1}(.50)^7$

$= 35(.0078125) = .2734$

$P(X=5) = \frac{7!}{5!\,2!}(.50)^5(.50)^2 = \frac{{}^{3}\cancel{6}\cdot 7}{\cancel{2}}(.50)^7$

$= 21(.0078125)$

$= .1641$

$P(X=6) = \frac{7!}{6!\,1!}(.50)^6(.50)^1 = (7)(.50)^7$

$= 7(.0078125)$

$= .05469$

$P(X=7) = \frac{7!}{7!\,0!}(.50)^7(.50)^0 = (1)(.0078125)$

$= .0078125$

Adding all Up: $P(X \geq 4) = .2734 + .1641 + .05469$

$+ .0078125$

$= .50$

5.39 $P = .15$

$n = 6$

a) $P(X=6) = \frac{6!}{6!0!} (.15)^6 (.85)^0 = (1)(.000011391)(1)$

$= .000011391$

b) $P(X=0) = \frac{6!}{0!6!} (.15)^0 (.85)^6 = (1)(1)(.3771495)$

$= .3771495$

c) $P(X>1) = 1 - P(X=0) - P(X=1)$

$= 1 - \left[\frac{6!}{6!0!} (.15)^0 (.85)^6\right] - \left[\frac{6!}{1!5!} (.15)^1 (.85)^6\right]$

$= 1 - .3771495 - .3394$

$= .2834$

5.40 $P = .40$

$n = 5$

a) $P(X=5) = \frac{5!}{5!0!} (.40)^5 (.60)^0 = (1)(.01024)(1) = .01024$

b) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= \frac{5!}{3!2!} (.40)^3 (.60)^2 + \frac{5!}{4!1!} (.40)^4 (.60)^1 + \frac{5!}{5!0!} (.40)^5 (.60)^0$

$= \frac{\cancel{4}^2 \cdot 5}{\cancel{2}} (.064)(.36) + (5)(.0256)(.60) + (1)(.01024)(1)$

$= .2304 + .0768 + .01024 = .31744$

c) $P(X \geq 2) = \frac{4!}{2!2!} (.40)^2 (.60)^2 + \frac{4!}{3!1!} (.40)^3 (.60)^1 + \frac{4!}{4!0!} (.40)^4 (.60)$

out of 4 remaining games

$= \frac{3 \cdot \cancel{4}^2}{\cancel{2}} (.0576) + 4(.0384) + (1)(.0256)(1)$

$= .3456 + .1536 + .0256 = .5248$

d. $E(X) = nP = 5(.40) = 2.0$ games.

e. $E(X \text{ in } 4 \text{ games}) = 4(.40) = 1.6$

+ 1 game already won

= 2.6 games

(5.42) $P = .40$ a) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$n = 4$

$$= 1 - \left[\frac{4!}{0!4!}(.40)^0(.60)^4\right] - \left[\frac{4!}{1!3!}(.40)^1(.60)^3\right]$$

$$= 1 - (.1296) - .3456$$

$$= .5248$$

b) $E(X) = nP = 4(.4) = 1.6$

$V(X) = nP(1-P) = 4(.4)(.6) = .96$

$S(X) = \sqrt{.96} = .9798$

(5.43) $P = .15$ a) $E(X) = nP = 50(.15) = 7.5$

$n = 50$ $V(X) = nP(1-P) = 50(.15)(.85) = 6.375$

$s(X) = \sqrt{6.375} = 2.5249$

b) Cost: $C = 250X$

$E(C) = 250\,E(X) = 250(7.5) = \$1875$

$V(C) = (250)^2 V(X) = 62500(6.375) = 398437.5$

$s(C) = \sqrt{398437.5}$

$= \$631.22$

5.44

$P = .032$ a) $E(X) = nP = 2000(.032) = 64$

$n = 2000$ $V(X) = nP(1-P) = 2000(.032)(.968)$

$= 61.95$

$s(X) = \sqrt{61.95} = 7.87$

b) Cost : $C = 10X$

$E(C) = 10\,E(X) = 10(64) = \$640$

$V(C) = (10)^2 V(X) = 100(61.95) = 6195$

$s(C) = \sqrt{6195} = \$78.71$

5.46

$P = .78$ a) $E(X) = nP = 620(.78) = \$483.6$

$n = 620$ $V(X) = nP(1-P) = 620(.78)(.22) = 106.39$

$s(X) = \sqrt{106.39} = \$10.3146$

b) Fine : $F = (\$2)X$

$E(F) = 2\,E(X) = 2(483.6) = \$967.20$

$V(F) = (2)^2 V(X) = 4(106.39) = 425.56$

$s(F) = \sqrt{425.56} = \$20.6291$

5.47 $n=16$

a) $P=.05$ $\quad P(X<2) = P(X=0)+P(X=1)$

$$= \frac{16!}{0!\,16!}(.05)^0(.95)^{16} + \frac{16!}{1!\,15!}(.05)^1(.95)^{15}$$

$$= (1)(1)(.44012)+16(.05)$$

$$= .440127 + .46329$$

$$= .90342$$

b) $P=.15$ $\quad P(X<2) = P(X=0)+P(X=1)$

$$= \frac{16!}{0!\,16!}(.15)^0(.85)^{16} + \frac{16!}{1!\,15!}(.15)^1(.85)^{15}$$

$$= (1)(1)(.07425) + 16(.15)(.087354)$$

$$= .07425 + .20965$$

$$= .2839$$

c) $P=.25$ $\quad P(X<2) = P(X=0)+P(X=1)$

$$= \frac{16!}{0!\,16!}(.25)^0(.75)^{16} + \frac{16!}{1!\,15!}(.25)^1(.75)^{15}$$

$$= (1)(1)(.010023)+16(.25)(.013363)$$

$$= .010023 + .05345$$

$$= .06348$$

5.48 $P = .20$

$n = 10$

To Accept: $P(X=0) = \frac{10!}{0!10!}(.20)^0(.80)^{10}$

$= (1)(1)(.107374)$

$= .107374$

$n = 20$

$P(X=0) + P(X=1)$

$= \frac{20!}{0!20!}(.20)^0(.80)^{20} + \frac{20!}{1!19!}(.2)^1(.8)^{19}$

$= (1)(1)(.0115292) + (20)(.2)(.0144)$

$= .0115292 + .0576$

$= .06913$

This acceptance rule allows accepting a shipment with smaller probability.

5.49 A: .70 of supply w/ .10 defectives $P(X \mid A) = .10$ $P(A) = .70$

B: .30 of supply w/ .20 defectives $P(X \mid B) = .20$ $P(B) = .30$

$n = 20$ $X = 1$

$P(A \mid X=1) = \frac{P(A \cap X=1)}{P(X=1)} = \frac{P(X=1 \mid A)P(A)}{P(X=1 \mid A)P(A) + P(X=1 \mid B)P(B)}$

Where $P(X=1 \mid A) = \frac{20!}{1!19!}(.10)^1(.90)^{19}$

$= (20)(.10)(.13509)$

$= .2702$

$P(X=1 \mid B) = \frac{20!}{1!19!}(.20)^1(.80)^{19}$

$= (20)(.20)(.01441)$

$= .057646$

$= \frac{(.2702)(.70)}{(.2702)(.70) + (.057646)(.30)}$

$= \frac{.18914}{.18914 + .017293} = \frac{.18914}{.206433}$

$= .9162$

(5.50) $n=12$, $N=50$, $S=25$

$$P(X=5)=\frac{\left(\frac{25!}{5!\,20!}\right)\left(\frac{25!}{7!\,18!}\right)}{\frac{50!}{12!\,38!}}$$

$$\frac{\left(\frac{21\cdot 22\cdot 23\cdot 24\cdot 25}{5\cdot 4\cdot 3\cdot 2\cdot 1}\right)\left(\frac{19\cdot 20\cdot 21\cdot 22\cdot 23\cdot 24\cdot 25}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\right)}{\frac{39\cdot 40\cdot 41\cdot 42\cdot 43\cdot 44\cdot 45\cdot 46\cdot 47\cdot 48\cdot 49\cdot 50}{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

$$=\frac{(53130)(480{,}700)}{121399651100}=.210376$$

(5.51) $n=14$, $N=60$, $S=25$

$$P(X=7)=\frac{\left(\frac{25!}{7!\,18!}\right)\left(\frac{35!}{7!\,28!}\right)}{\frac{60!}{14!\,46!}}$$

$$=\frac{\left(\frac{19\cdot 20\cdot 21\cdot 22\cdot 23\cdot 24\cdot 25}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}\right)\left(\frac{29\cdot 30\cdot 31\cdot 32\cdot 33\cdot 34\cdot 35}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\right)}{\frac{59\cdot 60\cdot 47\cdot 48\cdot 49\cdot 50\cdot 51\cdot 52\cdot 53\cdot 54\cdot 55\cdot 56\cdot 57\cdot 58}{14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}}$$

$$=\frac{(480700)(33{,}622{,}600)}{17345898650000}=.93177$$

(5.52) $n=20$, $N=80$, $S=42$

$$P(X=9)=\frac{\left(\frac{42!}{9!\,33!}\right)\left(\frac{38!}{11!\,27!}\right)}{\frac{80!}{20!\,60!}}=.151769$$

5.53

$n=5$

$N=40$

$S=25$

$$P(X=3) = \frac{\left(\frac{25!}{3!\,22!}\right)\left(\frac{15!}{2!\,13!}\right)}{\left(\frac{40!}{5!\,35!}\right)}$$

$$= \frac{\left(\frac{23\cdot 24\cdot 25}{3\cdot 2}\right)\left(\frac{14\cdot 15}{2}\right)}{\frac{36\cdot 37\cdot 38\cdot 39\cdot 40}{5\cdot 4\cdot 3\cdot 2\cdot 1}} = \frac{(2300)(105)}{658008}$$

$$= .367$$

5.54

$n=15$

$N=400$

$S=200$

$$P(X=8) = \frac{\left(\frac{200!}{8!\,192!}\right)\left(\frac{200!}{7!\,193!}\right)}{\frac{400!}{15!\,385!}} = .1999$$

5.56

$N=16$

$M=8$

$W=8$

$n=8$

$$P(X=4) = \frac{\left(\frac{8!}{4!\,4!}\right)\left(\frac{8!}{4!\,4!}\right)}{\frac{16!}{8!\,8!}} = \frac{\left(\frac{5\cdot 6\cdot 7\cdot 8}{4\cdot 3\cdot 2}\right)^2}{\frac{9\cdot 10\cdot 11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

$$= \frac{(70)^2}{12870} = \frac{4900}{12870} = .3807$$

5.57

$N=12$

$n=3$

$S=4$

$$P(X\geq 2) = P(X=2)+P(X=3)$$

$$= \frac{\left(\frac{4!}{2!\,2!}\right)\left(\frac{8!}{1!\,7!}\right)}{\left(\frac{12!}{3!\,9!}\right)} + \frac{\left(\frac{4!}{3!\,1!}\right)\left(\frac{8!}{0!\,8!}\right)}{\left(\frac{12!}{3!\,9!}\right)}$$

$$= \frac{\left(\frac{3\cdot 4}{2}\right)(8) + (4)(1)}{\frac{10\cdot 11\cdot 12}{3\cdot 2}} = \frac{48+4}{220} = \frac{52}{220} = .2364$$

5.58

$N=10$
$S=5$
$n=6$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$$P(X=0) = \frac{\left(\frac{5!}{0!5!}\right)\left(\frac{5!}{6!(-1)!}\right)}{\left(\frac{10!}{6!4!}\right)} \Leftarrow \text{can't happen} = 0$$

$$P(X=1) = \frac{\left(\frac{5!}{1!4!}\right)\left(\frac{5!}{5!0!}\right)}{\frac{10!}{6!4!}} = \frac{(5)(1)}{\frac{\cancel{7}\cdot\cancel{8}\cdot\cancel{9}\cdot\cancel{10}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}} = \frac{5}{210} = .0238$$

$$P(X=2) = \frac{\left(\frac{5!}{2!3!}\right)\left(\frac{5!}{4!1!}\right)}{210} = \frac{\left(\frac{\cancel{4}\cdot 5}{\cancel{2}}\right)(5)}{210} = \frac{50}{210} = .238$$

Adding all Up $\Rightarrow$ $P(X<3) = .0238 + .238$
$= .2618$

5.73 a.

X	P(X)
1	.50
2	.50

marginal prob fcn for X

Y	P(Y)
0	.50
1	.50

marginal prob. function for Y

b. $Cov(X,Y) = \sum X \cdot Y\, P(X,Y) - E(X)E(Y)$.

$X \cdot P(X)$	$Y \cdot P(Y)$
$(1)(.50) = .50$	$(0)(.50) = 0$
$(2)(.50) = 1.00$	$(1)(.50) = .50$
$E(X) = \sum X P(X) = 1.50$	$E(Y) = \sum Y P(Y) = .50$

$X^2 P(X)$	$Y^2 P(Y)$
$(1)^2(.50) = .50$	$(0)^2(.50) = 0$
$(2)^2(.50) = 2.00$	$(1)^2(.50) = .50$
$\sum X^2 P(X) = 2.50$	$\sum Y^2 P(Y) = .50$

$V(X) = \sum X^2 P(X) - [E(X)]^2 = 2.50 - (1.50)^2 = 2.50 - 2.25 = .25$

$S(X) = \sqrt{.25} = .5$

$V(Y) = \sum Y^2 P(Y) - [E(Y)]^2 = .50 - (.25) = .25$

$S(Y) = \sqrt{.25} = .5$

$Cov(X,Y) = (1)\cancel{(0)(.25)}^{0} + (1)(1)(.25) + (2)\cancel{(0)(.25)}^{0} + (2)(1)(.25)$

$- (1.50)(.50)$

$= .25 + .50 - .75 = 0$

$$CORR(X,Y) = \frac{COV(X,Y)}{S(X)\cdot S(Y)} = \frac{0}{(.5)(.5)} = \frac{0}{.25} = 0$$

5.74

a.

X	$P(x)$	$X\cdot P(x)$	$X^2\cdot P(x)$
1	.50	$(1)(.50)=.50$	$(1)^2(.50)=.50$
2	.50	$(2)(.50)=1.00$	$(2)^2(.50)=2.00$

Marg. Prob. Distr.

Y	$P(Y)$	$Y\cdot P(Y)$	$Y^2\cdot P(Y)$
0	.45	$(0)(.45)=0$	$(0)^2(.45)=0$
1	.55	$(1)(.55)=.55$	$(1)^2(.55)=.55$

Marg. Prob. Distr.

b. $E(X)=\sum X\cdot P(x) = .50+1.00 = 1.50$

$E(Y)=\sum Y\cdot P(Y) = 0+.55 = .55$

$V(X)=\sum X^2P(X)-(E(X))^2 = 2.50-(1.50)^2 = 2.50-2.25 = .25$

$S(X)=\sqrt{.25} = .5$

$V(Y)=\sum Y^2P(Y)-(E(Y))^2 = .55-(.55)^2 = .55-.3025$

$= .2475$

$S(Y)=\sqrt{.2475} = .4975$

$COV(X,Y)=\sum X\cdot Y P(X,Y) - E(X)E(Y)$

$= \cancel{(1)(0)(.20)}+(1)(1)(.30)+\cancel{(2)(0)(.25)}+(2)(1)(.25)-(1.50)(.55)$

$= .30+.50-.825 = -.025$

$$CORR(X,Y) = \frac{COV(X,Y)}{S(X)S(Y)} = \frac{-.025}{(.5)(.4975)} = -.1005$$

(5.75) a.

X	$P(X)$
1	.50
2	.50

b.

$X \cdot P(X)$	$X^2 \cdot P(X)$
$(1)(.50) = .50$	$(1)^2(.50) = .50$
$(2)(.50) = 1.00$	$(2)^2(.50) = 2.00$
$\sum XP(X) = 1.50$	$\sum X^2P(X) = 2.50$

Y	$P(Y)$
0	.50
1	.50

$Y \cdot P(Y)$	$Y^2 P(Y)$
$(0)(.50) = 0$	$(0)^2(.50) = 0$
$(1)(.50) = .50$	$(1)^2(.50) = .50$
$\sum Y \cdot P(Y) = .50$	$\sum Y^2 P(Y) = .50$

$E(X) = \sum X \cdot P(X) = 1.50$

$E(Y) = \sum Y \cdot P(Y) = .50$

$V(X) = \sum X^2 \cdot P(X) - (E(X))^2 = 2.50 - (1.50)^2 = 2.5 - 2.25$
$= .25$

$S(X) = \sqrt{.25} = .5$

$V(Y) = \sum Y^2 \cdot P(Y) - (E(Y))^2 = .50 - (.5)^2 = .50 - .25$
$= .25$

$S(Y) = \sqrt{.25} = .5$

$cov(X,Y) = (1)(0)(.25) + (1)(1)(.25) + (2)(0)(.25) + (2)(1)(.25)$
$= .25 + .50 - (1.50)(.50)$
$= .75 - .75 = 0$

$$CORR(X,Y) = \frac{cov(X,Y)}{S(X)\,S(Y)} = \frac{0}{(.5)(.5)} = 0$$

c. $W = X + Y$ $\quad E(W) = E(X) + E(Y) = 1.50 + .50 = 2.00$

$V(W) = V(X) + V(Y) + 2\,cov(X,Y)$
$= .25 + .25 + 2(0)$
$= .50$

5.76 a.

X	$P(X)$
0	.55
1	.45

b.

$X \cdot P(X)$	$X^2 P(X)$
$(0)(.55) = 0$	$(0)^2(.55) = 0$
$(1)(.45) = .45$	$(1)^2(.45) = .45$

Y	$P(Y)$	$YP(Y)$	$Y^2P(Y)$
0	.50	$(0)(.50) = 0$	$(0)^2(.50) = 0$
1	.50	$(1)(.50) = .50$	$(1)^2(.50) = .50$

$$E(X) = \sum XP(X) = 0 + .45 = .45$$
$$E(Y) = \sum YP(Y) = (0) + .50 = .50$$
$$V(X) = \sum X^2P(X) - (E(X))^2 = 0 + .45 - (.45)^2$$
$$= .45 - .2025 = .2475$$
$$V(Y) = \sum Y^2P(Y) - (E(Y))^2 = 0 + .50 - (.50)^2$$
$$= .50 - .25 = .25$$
$$s(X) = \sqrt{.2475} = .4975$$
$$s(Y) = \sqrt{.25} = .5$$

$$\operatorname{cov}(X,Y) = \sum X \cdot Y P(x,y) - E(X)E(Y)$$
$$= \cancel{(0)(0)(.30) + (1)(0)(.25) + (1)(0)(.20)} + (1)(1)(.25)$$
$$- (.45)(.50)$$
$$= .25 - .225 = .025$$

$$\operatorname{CORR}(X,Y) = \frac{\operatorname{cov}(X,Y)}{s(X)\,s(Y)} = \frac{.025}{(.4975)(.50)} = .1005$$

c. $W = 2X + Y$ $\quad E(W) = 2E(X) + E(Y) = 2(.45) + (.50) = 1.40$

$$V(W) = (2)^2V(X) + V(Y) + 2(2)(1)\operatorname{cov}(X,Y)$$
$$= 4(.2475) + (.25) + 4(.025)$$
$$= 1.34$$

5.77 a.

X	$P(X)$
1	(.70)
2	(.30)

b.

$X \cdot P(X)$	$X^2 P(X)$
$(1)(.70) = .70$	$(1)^2(.70) = .70$
$(2)(.30) = .60$	$(2)^2(.30) = 1.20$

Y	$P(Y)$
0	(.70)
1	(.30)

$Y \cdot P(Y)$	$Y^2 P(Y)$
$(0)(.70) = 0$	$(0)^2(.70) = 0$
$(1)(.30) = .30$	$(1)^2(.30) = .30$

$$E(X) = \sum X P(X) = .7 + .6 = 1.30$$

$$E(Y) = \sum Y P(Y) = 0 + .30 = .30$$

$$V(X) = \sum X^2 P(X) - (E(X))^2 = .70 + 1.20 - (1.30)^2 = 1.90 - 1.69$$

$$= .21$$

$$S(X) = \sqrt{.21} = .458$$

$$V(Y) = \sum Y^2 P(Y) - (E(Y))^2 = 0 + .30 - (.3)^2 = .3 - .09$$

$$= .21$$

$$S(Y) = \sqrt{.21} = .458$$

$$\text{cov}(X,Y) = \cancel{(1)(0)(.70)} + \cancel{(2)(0)(0)} + \cancel{(1)(1)(.0)} + (1)(2)(.30)$$

$$-(1.30)(.30)$$

$$= .60 - .39 = .21$$

$$\text{CORR}(X,Y) = \frac{\text{cov}(X,Y)}{S(X)S(Y)} = \frac{.21}{(.458)(.458)} = \frac{.21}{.21} = 1$$

c. $E(W) = 3E(X) + 4E(Y) = 3(1.3) + 4(.30)$

$$= 3.9 + 1.20 = 5.10$$

$$V(W) = 3^2 V(X) + 4^2 V(Y) + 2(3)(4)\text{cov}(X,Y)$$

$$= 9(.21) + 16(.21) + 24(1)$$

$$= 29.25.$$

5.79 a.

X	$P(X)$
1	.55
2	.45

b.

$X \cdot P(X)$	$X^2 \cdot P(X)$
$(1)(.55) = .55$	$(1)^2(.55) = .55$
$(2)(.45) = .90$	$(2)^2(.45) = 1.80$

Y	$P(Y)$
0	.50
1	.50

$Y \cdot P(Y)$	$Y^2 \cdot P(Y)$
$(0)(.50) = 0$	$(0)^2(.50) = 0$
$(1)(.50) = .50$	$(1)^2(.50) = .50$

$E(X) = \sum X \cdot P(X) = .55 + .90 = 1.45$

$E(Y) = \sum Y P(Y) = 0 + .50 = .50$

$V(X) = \sum X^2 P(X) - (E(X))^2 = .55 + 1.80 - (1.45)^2$

$= 2.35 - 2.1025 = .2475$

$V(Y) = \sum Y^2 P(Y) - (E(Y))^2 = 0 + .50 - (.50)^2$

$= .50 - .25 = .25$

$S(X) = \sqrt{.2475} = .4975$

$S(Y) = \sqrt{.25} = .50$

$cov(X,Y) = \sum X \cdot Y \, P(X,Y) - E(X)E(Y)$

$= \cancel{(0)(1)(.30)} + (1)(1)(.25) + \cancel{(2)(0)(.20)} + (2)(1)(.25)$

$- (1.45)(.50)$

$= .25 + .50 - .725 = .75 - .725 = .025$

$CORR = \dfrac{cov(X,Y)}{S(X)S(Y)} = \dfrac{.025}{(.4975)(.50)} = .1005$

c. $W = 2X + Y$ $\quad E(W) = 2E(X) + E(Y) = 2(1.45) + .50$

$= 3.40$

$V(W) = (2)^2 V(X) + V(Y) + 2(2)(1)\, cov(X,Y)$

$= 4(.2475) + .25 + 4(.025)$

$= 1.34$

5.80 a.

X	$P(X)$
1	.40
2	.60

b.

$X^2P(X)$	$XP(X)$
$(1)^2(.40)=.40$	$(1)(.40)=.40$
$(2)^2(.6)=2.4$	$(2)(.60)=1.20$

Y	$P(Y)$	$Y^2P(Y)$	$YP(Y)$
0	.60	$(0)^2(.60)=0$	$(0)(.60)=0$
1	.40	$(1)^2(.40)=.40$	$(1)(.40=.40$

$E(X)=\sum X\cdot P(X)= .40+1.20=1.60$

$E(Y)=\sum Y P(Y)= 0+.40=.40$

$V(X)=\sum X^2P(X)-(E(X))^2= .40+2.4-(1.6)^2=2.80-2.56$

$=.24$

$S(X)=.4898$

$V(Y)=\sum Y^2P(Y)-(E(Y))^2= 0+.40-(.4)^2$

$=.40-.16=.24$

$S(Y)=.4898$

$\text{cov}(X,Y)=(1)\cancel{(0)}(0)+(1)(1)(.40)+(2)(0)\cancel{(.60)}+(2)(1)(0)$

$-(1.60)(.40)=.40-.64=-.24$

$\text{CORR}(X,Y)=\dfrac{\text{cov}(X,Y)}{S(X)S(Y)}=\dfrac{-.24}{(.4898)(.4898)}=\dfrac{-.24}{.2399}$

$=-1.0004$

d. $W=2X-4Y$ $\quad E(W)=2E(X)-4E(Y)$

$=2(1.60)-4(.40)=3.20-1.60$

$=1.6$

$V(W)=(2)^2V(X)+(-4)^2V(Y)+2(2)(-4)\text{cov}(X,Y)$

$=4(.24)+16(.24)+(-16)(-.24)=8.64$

5.81 a.

X	$P(X)$
1	.70
2	.30

b.

$X \cdot P(X)$	$X^2 \cdot P(X)$
$(1)(.70) = .70$	$(1)^2(.70) = .70$
$(2)(.30) = .60$	$(2)^2(.30) = 1.20$

Y	$P(Y)$	$Y \cdot P(Y)$	$Y^2 \cdot P(Y)$
0	.70	$(0)(.70) = 0$	$(0)^2(.70) = 0$
1	.30	$(1)(.30) = .30$	$(1)^2(.30) = .30$

$E(X) = \sum X \cdot P(X) = .70 + .60 = 1.30$

$E(Y) = \sum Y \cdot P(Y) = 0 + .30 = .30$

$V(X) = \sum X^2 \cdot P(X) - (E(X))^2 = .70 + 1.20 - (1.30)^2 = 1.90 - 1.69$
$= .21$

$V(Y) = \sum Y^2 \cdot P(Y) - (E(Y))^2 = (0) + (.30) - (.30)^2 = .30 - .09$
$= .21$

$s(X) = \sqrt{.21} = 4.5825$

$s(Y) = \sqrt{.21} = 4.5825$

$cov(X, Y) = \sum X \cdot Y \cdot P(X, Y) - E(X)E(Y)$
$= \cancel{(1)(0)(.70)} + \cancel{(1)(1)(0)} + (1)(2)(.3) + \cancel{(2)(0)(0)}$
$- (1.3)(.3)$
$= .3 - .39 = -.09$

$$CORR(X, Y) = \frac{cov(X, Y)}{s(X) \cdot s(Y)} = \frac{-.09}{(4.5825)(4.5825)} = \frac{-.09}{21}$$
$= -.00428$

c. $W = 10X - 8Y$ $\quad E(W) = 10E(X) - 8E(Y)$
$= 10(1.30) - 8(.3) = 13 - 2.4$
$= 10.6$

$V(W) = 10^2 V(X) + (-8)^2 V(Y) + 2(10)(-8) cov(X, Y)$
$= 100(.21) + 64(.21) + (-160)(-.09) = 48.84$

5.82 a.

X	$P(X)$
0	.22
1	.26
2	.43
3	.09

$$E(X) = \sum X P(X)$$
$$= (0)(.22) + (1)(.26) + (2)(.43) + (3)(.09)$$
$$= .26 + .86 + .27 = 1.40$$

b.

Y	$P(Y)$
0	.23
1	.21
2	.30
3	.26

$$E(Y) = \sum Y \cdot P(Y)$$
$$= (0)(.23) + (1)(.21) + (2)(.30) + (3)(.26)$$
$$= .21 + .60 + .78$$
$$= 1.59$$

c.

$(Y \mid X=3)$	$P(Y \mid X=3)$
0	$.01/.09 = .11$
1	$.01/.09 = .11$
2	$.03/.09 = .33$
3	$.04/.09 = .44$

Given student is taking 3 tests on same day. The probabilities of 0, 1, 2, 3 snacks are given by this function

d. $$\text{cov}(X,Y) = (0)(0)(.07) + (0)(1)(.09) + (0)(2)(.06) + (0)(3)(.01)$$
$$+ (1)(0)(.07) + (1)(1)(.06) + (1)(2)(.07) + (1)(3)(.01)$$
$$+ (2)(0)(.06) + (2)(1)(.07) + (2)(2)(.14) + (2)(3)(.03)$$
$$+ (3)(0)(.02) + (3)(1)(.04) + (3)(2)(.16) + (3)(3)(.04)$$
$$- (1.4)(1.59)$$
$$= .06 + .14 + .03 + .14 + .56 + .18 + .12 + .96$$
$$+ .36 - 2.226$$
$$= .324$$

e. $$P(Y=2) \stackrel{?}{=} P(Y=2 \mid X=3)$$
$$.30 \neq .33 \Rightarrow \text{NOT S.I.}$$

5.84 a.

Y	P(Y)
0	.12
1	.24
2	.23
3	.23
4	.18

b.

(Y/X=3)	P(Y/X=3)
0	.01/.26
1	.03/.26
2	.06/.26
3	.08/.26
4	.08/.26

c. $P(Y=1) \stackrel{?}{=} P(Y=1 \mid X=3)$

$.24 \neq .115$ NOT SI

5.87

X: # Food Compl

Y: # Service Compl.

	0	1	2	3	
0	.0216	.0522	.0756	.0306	.18
1	.0456	.1102	.1596	.0646	.38
2	.0408	.0986	.1428	.0578	.34
3	.012	.029	.042	.017	.10
	.12	.29	.42	.17	1.00

5.88 $T = X + Y$

$E(X) = (0)(.12) + (1)(.29) + (2)(.42) + (3)(.17)$

$= .29 + .84 + .51 = 1.64$

$E(Y) = (0)(.18) + (1)(.38) + (2)(.34) + (3)(.10)$

$= .38 + .68 + .3 = 1.36$

$V(Y) = (0)^2(.18) + (1)^2(.38) + (2)^2(.34) + (3)^2(.10) - (1.36)^2$

$= .38 + 1.36 + .9 - 1.8496$

$= .7904$

$s(Y) = \sqrt{.7904} = .8890$

$V(X) = (0)^2(.12) + (1)^2(.29) + (2)^2(.42) + (3)^2(.17) - (1.64)^2 = .29 + 1.68 + 1.53 - 2.6896$

$= .8104$

$s(X) = \sqrt{.8104} = .9$

$E(T) = E(X) + E(Y) = 1.64 + 1.36 = 3.00$

$V(T) = V(X) + V(Y) + 2\,cov(X,Y)$ where $cov(X,Y) = 0$ by assumption of S.I.

$= .8104 + .7904 = 1.6008$

$s(T) = \sqrt{1.6008} = 1.265$

5.89

X	P(X)	X·P(X)	X²·P(X)
0	.08	0	0
1	.16	.16	.16
2	.28	.56	1.12
3	.32	.96	2.88
4	.10	.40	1.60
5	.06	.30	1.50
		2.38	7.26

$E(X) = \sum X \cdot P(X)$
$= 2.38$

$V(X) = \sum X^2 \cdot P(X) - (E(X))^2$
$= 7.26 - (2.38)^2$
$= 7.26 - 5.6644$
$= 1.5956$

$s(X) = \sqrt{1.5956} = 1.263$

Y	P(Y)	Y·P(Y)	Y²·P(Y)
0	.18	0	0
1	.26	.26	.26
2	.36	.72	1.44
3	.13	.39	1.17
4	.07	.28	1.96
		1.95	4.83

$E(Y) = \sum Y \cdot P(Y)$
$= 1.95$

$V(Y) = \sum Y^2 \cdot P(Y) - (E(Y))^2$
$= 4.83 - (1.95)^2$
$= 4.83 - 3.8025$
$= 1.0275$

$s(Y) = \sqrt{1.0275} = 1.0137$

$cov(X,Y) = 0$ as they are given to be S.I.

Then: $T = X + Y$

$E(T) = E(X) + E(Y) = 2.38 + 1.95 = 4.33$

$V(T) = V(X) + V(Y) + 2\,cov(X,Y) = 1.5956 + 1.0275 + 0$
$= 2.6231$

3.78 a.

X	$P(X)$
1	.50
2	.50

b.

$X \cdot P(X)$	$X^2 \cdot P(X)$
$(1)(.50) = .50$	$(1)^2(.50) = .50$
$(2)(.50) = 1.00$	$(2)^2(.50) = 2.00$

Y	$P(Y)$
0	.50
1	.50

$Y \cdot P(Y)$	$Y^2 \cdot P(Y)$
$(0)(.50) = 0$	$(0)^2(.50) = 0$
$(1)(.50) = .50$	$(1)^2(.50) = .50$

$$E(X) = \sum X \cdot P(X) = .50 + 1.00 = 1.50$$
$$E(Y) = \sum Y \cdot P(Y) = 0 + .50 = .50$$
$$V(X) = \sum X^2 \cdot P(X) - (E(X))^2 = .50 + 2.00 - (1.5)^2$$
$$= 2.50 - 2.25 = .25$$
$$V(Y) = \sum Y^2 P(Y) - (E(Y))^2 = 0 + .50 - (.50)^2$$
$$= .50 - .25 = .25$$
$$S(X) = \sqrt{.25} = .5$$
$$S(Y) = \sqrt{.25} = .5$$

$$\text{cov}(X,Y) = \cancel{(1)(0)(.25)} + (1)(1)(.25) + (2)(1)(.25) + \cancel{(2)(0)(.25)}$$
$$- (1.50)(.50)$$
$$= .25 + .50 - .75 = .75 - .75 = 0$$

$$\text{CORR}(X,Y) = \frac{\text{cov}(X,Y)}{S(X)\,S(Y)} = \frac{0}{(.5)(.5)} = 0$$

c. $W = X + Y$

$$E(W) = E(X) + E(Y) = 1.50 + .50 = 2.00$$
$$V(W) = V(X) + V(Y) + 2(1)(1)\,\text{cov}(X,Y)$$
$$= .25 + .25 + 2(0)$$
$$= .50$$