

For exercises 4.1 - 4.4 use the sample space  $S$  defined as follows:

$$S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10}\}$$

4.1 Given  $A = [E_1, E_3, E_6, E_9]$  define  $\bar{A}$ .

4.2 Given  $A$  is  $[E_1, E_3, E_7, E_9]$  and  $B$  is  $[E_2, E_3, E_8, E_9]$

- What is  $A \cap B$ ?
- What is the union of  $A$  and  $B$ ?
- Is the union of  $A$  and  $B$  collectively exhaustive?

4.3 Given  $\bar{A} = [E_1, E_3, E_7, E_9]$  and  $\bar{B} = [E_2, E_3, E_8, E_9]$

- What is  $A \cap B$ ?
- What is the union of  $A$  and  $B$ ?
- Is the union of  $A$  and  $B$  collectively exhaustive?

4.4 Given  $A = [E_3, E_5, E_6, E_{10}]$  and  $B = [E_3, E_4, E_6, E_9]$

- What is  $A \cap B$ ?
- What is the union of  $A$  and  $B$ ?
- Is the union of  $A$  and  $B$  collectively exhaustive?

4.5 A corporation takes delivery of some new machinery that must be installed and checked before it becomes available to use. The corporation is sure that it will take no more than 7 days for this installation and check to take place. Let  $A$  be the event "It will be more than 4 days before the machinery becomes available" and  $B$  the event "It will be less than 6 days before the machinery becomes available."

- Describe the event that is the complement of event  $A$ .
- Describe the event that is the intersection of events  $A$  and  $B$ .
- Describe the event that is the union of events  $A$  and  $B$ .
- Are events  $A$  and  $B$  mutually exclusive?
- Are events  $A$  and  $B$  collectively exhaustive?
- Show that  $(A \cap B) \cup (\bar{A} \cap B) = B$
- Show that  $A \cup (\bar{A} \cap B) = A \cup B$

$$\begin{array}{ccc} A & B & \bar{A} \\ >4 & <6 & \leq 4 \end{array}$$

$$5 \quad 1, 2, 3, 4$$

$$1, 2, 3, 4, 5 \quad < 6$$

$$5 \cup (1, 2, 3, 4) = \pi$$

4.6 Consider example 4.4, with the following four basic outcomes for the Dow-Jones Industrial Average (DJIA) over 2 consecutive days:

- DJIA rises on both days.
- DJIA rises on the first day but does not rise on the second day.
- DJIA does not rise on the first day but rises on the second day.
- DJIA does not rise on either day.

Let events A and B be the following:

A : DJIA rises on the first day.

B: DJIA rises on the second day.

- a. Show that  $(A \cap B) \cup (\bar{A} \cap B) = B$
- b. Show that  $A \cup (\bar{A} \cap B) = A \cup B$

4.7 Florin Frenti operates a small used car lot that has three Mercedes ( $M_1, M_2, M_3$ ) and two Toyotas ( $T_1, T_2$ ). Two customers, Cezara and Anda, come to his lot, and each selects a car. The customers do not know each other, and there is no communication between them. Let the events A and B be defined as follows:

A: The customers select at least one Toyota.

B: The customers select two cars of the same model.

- a. Identify all possible pairs of cars (i.e. the sample space.)
- b. Describe event A (by listing outcomes included therein.)
- c. Describe event B.
- d. Describe the complement of A.
- e. Show that  $(A \cap B) \cup (\bar{A} \cap B) = B$
- f. Show that  $A \cup (\bar{A} \cap B) = A \cup B$

4.8 The sample space contains 5 A's and 7 B's. What is the probability that a randomly selected set of 2 will include 1 A and 1 B?

4.9 The sample space contains 6 A's and 4 B's. What is the probability that a randomly selected set of 3 will include 1 A and 2 B's?

4.10 The sample space contains 10 A's and 6 B's. What is the probability that a randomly selected set of 4 will include 2 A's and 2 B's?

4.11 In a city of 120,000 people, there are 20,000 Norwegians. What is the probability that a randomly selected person from the city will be Norwegian?

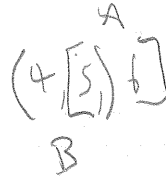
4.12 In a city of 180,000 people, there are 20,000 Norwegians. What is the probability that a random sample of 2 people from the city will contain 2 Norwegians?

4.13 Recall the corporation in exercise 4.5. Its new machinery must be installed and checked before it becomes operational. The accompanying table shows a manager's probability assessment for the number of days required before the machinery becomes operational:

<u>Number of days</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
<u>Probability</u>	<u>.08</u>	<u>.24</u>	<u>.41</u>	<u>.20</u>	<u>.07</u>

Let A be the event "It will be more than 4 days before the machinery becomes operational" and let B be the event "It will be less than 6 days before the machinery becomes available."

- Find the probability of event A.
- Find the probability of event B.
- Find the probability of complement event of A.
- Find the probability of the intersection of events A and B.
- Find the probability of the union of events A and B.



4.14 A fund manager is considering investing the stock of a healthcare provider. The manager's assessment of probabilities for rates of return on this stock over the next year is summarized in the accompanying table. Let A be the event "Rate of return will be more than 10%" and B be the event "Rate of return will be negative."

Rate of return	< -10%	-10% - 0%	0% to 10%	10% to 20%	>20%
Probability	.04	.14	.28	.33	.21

- Find the probability of event A.
- Find the probability of event B.
- Describe the event that is the complement of A.
- Find the probability of complement of A.
- Describe the event that is the intersection of A and B.
- Find the probability of intersection of A and B.
- Describe the event that is the union of A and B.
- Find the probability of the union of A and B.
- Are A and B mutually exclusive?
- Are A and B collectively exhaustive?

4.18 A corporation receives a particular part in shipments of 100. Research indicated the probabilities shown in the accompanying table for numbers of defective parts in a shipment.

Number defective	0	1	2	3	More than 3
Probability	.29	.36	.22	.10	.03

- What is the probability that there will be less than 3 defective parts in a shipment?
- What is the probability that there will be more than 1 defective part in a shipment?
- The five probabilities in the table sum up to 1. Why must this be so?

4.19 The probability of A is .60 and the probability of B is .45 and the probability of either is .80. What is the probability of both A and B?

4.20 The probability of A is .40 and the probability of B is .45 and the probability of either is .85. What is the probability of both A and B?

4.21 The probability of A is .60 and the probability of B is .40 and the probability of either is .76. What is the probability of both A and B?

4.22 The probability of A is .60 and the probability of B is .45 and the probability of both is .30. What is the probability of either A or B?

4.23 The probability of A is .60 and the probability of B is .45 and the probability of both is .30. What is the conditional probability of A, given B? Are A and B statistically independent?

4.24 The probability of A is .80 and the probability of B is .10 and the probability of both is .08. What is the conditional probability of A, given B? Are A and B statistically independent?

4.25 The probability of A is .30 and the probability of B is .40 and the probability of both is .30. What is the probability of A, given B? Are A and B statistically independent?

4.26 The probability of A is .70 and the probability of B is .80 and the probability of both is .50. What is the conditional probability of A, given B? Are A and B statistically independent?

4.27 A company knows that a rival is about to bring out a competing product. It believes that this rival has three possible packaging plans (superior, normal, cheap) in mind and that all are equally likely. Also, there are three equally likely possible marketing strategies (intense media advertising, price discounts, and use of a coupon to reduce the price of future purchases.) What is the probability that the rival will employ superior packaging in conjunction with an intense media advertising campaign? Assume that packaging plans and marketing strategies are determined independently.

4.38. It was estimated that about 30% of all seniors on a campus were seriously concerned about employment prospects, 25% were seriously concerned about grades, and 20% were seriously concerned about both. What is the probability that a randomly chosen senior from this campus is seriously concerned about at least one of these two things?

4.39 A music store owner finds that 30% of the customers entering the store ask an assistant for help and that 20% of the customers make a purchase before leaving. It is also found that 15% of all customers both ask for assistance and make a purchase. What is the probability that a customer does at least one of these things?

4.40 Refer to the information in Exercise 4.39, and consider the two events "Customer asks for assistance" and "Customer makes purchase". In answering the following questions, provide reasons expressed in terms of probabilities of relevant events.

- Are two events mutually exclusive?
- Are two events collectively <sup>exhaustive</sup> exclusive?
- Are two events statistically independent?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\boxed{P(A \cup B) + P(A \cap B) = P(A) + P(B)}$$

4.42 A mail-order firm considers three possible events in filling an order:

A: The wrong item is sent

B: The item is lost in transit

C: The item is damaged in transit

Assume A is independent of both B and C and that B and C are mutually exclusive. The individual event probabilities are  $P(A) = .02$ ,  $P(B) = .01$  and  $P(C) = .04$ . Find the probability that at least one of these foul-ups occurs for a randomly chosen order.

4.43 A Coach recruits for a college team a star player who is currently a high school senior. In order to play next year the senior must both complete high school with adequate grades and pass a standardized test. The coach estimates that the probability the athlete will fail to obtain adequate high school grades is .02, that the probability the athlete will not pass the standardized test is .15, and that these are independent events. According to these estimates, what is the probability that this recruit will be eligible to play in college next year?

4.44 Market research in a particular city indicated that during a week 18% of all adults watch a television program oriented to business and financial issues, 12% read a publication oriented to these issues and 10% do both.

a. What is the probability that an adult in this city who watches a television program oriented to business and financial issues reads a publication oriented to these issues?

b. What is the probability that an adult in this city who reads a publication oriented to business and financial issues watches a television program oriented to these issues?

4.45 An inspector examines items coming from an assembly line. A review of her record reveals that she accepts only 8% of all defective items. It was also found that 1% of all items from the assembly line are both defective and accepted by the inspector. What is the probability that a randomly chosen item from this assembly line is defective?

4.47 A bank classifies borrowers as high-risk or low-risk. Only 15% of its loans are made to those in the high-risk category. Of all its loans, 5% are in default, and 40% of those in default were made to high-risk borrowers. What is the probability that a high-risk borrower will default?

4.48 A conference began at noon with two parallel sessions. The session on portfolio management was attended by 40% of the delegates, while the session on chartism was attended by 50%. The evening session consisted of a talk titled "Is the Random Walk Dead?" This was attended by 80% of all delegates.

a. If attendance at the portfolio management session and attendance at the chartism session are mutually exclusive, what is the probability that a randomly chosen delegate attended at least one of these sessions?

b. If attendance at the portfolio management session and attendance at the evening session are statistically independent, what is the probability that a randomly chosen delegate attended at least one of these sessions?

c. Of those attending the chartism session, 75% also attended the evening session. What is the probability that a randomly chosen delegate attended at least one of these two sessions?

4.50 A quality control manager found that 30% of worker-related problems occurred on Mondays and that 20% occurred in the last hour of a day's shift. It was also found that 4% of worker-related problems occurred in the last hour of Monday's shift.

- a. What is the probability that a worker-related problem that occurs on a Monday does not occur in the last hour of the day's shift?
- b. Are the events "Problem occurs on Monday" and "Problem occurs in the last hour of the day's shift" statistically independent?

4.51 A corporation was concerned about the basic educational skills of its workers and decided to offer a selected group of them separate classes in reading and practical mathematics. Of these workers 40% signed up for the reading classes and 50% for the practical mathematics classes. Of those signing up for the reading classes 30% signed up for the mathematics classes.

- a. What is the probability that a randomly selected worker signed up for both classes?
- b. What is the probability that a randomly selected worker who signed up for the mathematics classes also signed up for the reading classes?
- c. What is the probability that a randomly selected worker signed up for at least one of these two classes?
- d. Are the events "Signed up for reading classes" and "Signed up for mathematics classes" statistically independent?

4.52 A lawn-care service makes telephone solicitations, seeking customers for the coming season. A review of the records indicates that 15% of these solicitations produce new customers and that, of these new customers, 80% had used some rival service in the previous year. It is also estimated that, of all solicitation calls made, 60% are to people who had used a rival service the previous year. What is the probability that a call to a person who had used a rival service the previous year will produce a new customer for the lawn-care service?

4.53 An editor may use all, some, or none of the possible strategies to enhance the sales of a book:

- a. An expensive prepublication promotion.
- b. An expensive cover design.
- c. A bonus for sales representatives who meet pre-determined sales levels.

In the past, these three strategies have been applied simultaneously to only 2% of the company's books. Twenty percent of the books have had expensive cover designs, and, of these 80% have had expensive prepublication promotion. A rival editor learns that a new book is to have both an expensive prepublication promotion and an expensive cover design and now wants to know how likely it is that a bonus scheme for sales representatives will be introduced. Compute the probability of interest to the rival editor.

Table 4.8 Probabilities for Television viewing and Income:

Viewing Frequency	High Income	Middle Income	Low Income	Totals
Regular	.10	.15	.05	.30
Occasional	.10	.20	.10	.40
Never	.05	.05	.20	.30
Totals	.25	.40	.35	1.00

For exercises 4.54 - 4.60 refer to table 4.8 above.

4.54 What is the joint probability of "High income" and "Never"?

4.55 What is the joint probability of "Low income" and "Regular"?

4.56 What is the joint probability of "Middle income" and "Never"?

4.57 What is the joint probability of "Middle income" and "Occasional"?

4.58 What is the conditional probability of "High income," given "Never"?

4.59 What is the conditional probability of "Low income," given "Occasional"?

4.60 What is the conditional probability of "Regular," given "High income"?

4.61 The probability of a sale is .80. What are the odds in favor of a sale?

4.62 The probability of a sale is .50. What are the odds in favor of a sale?

4.66 A survey carried out for a supermarket classified customers according to whether their visits to the store are frequent or infrequent and whether they often, sometimes, or never purchase generic products. The accompanying table gives the proportions of people surveyed in each of the six joint classifications.

Frequency of Visit	Purchase of Generic Products		
	Often	Sometimes	Never
Frequent	.12	.48	.19
Infrequent	.07	.06	.08

a. What is the probability that a customer both is a frequent shopper and often purchases generic products?

b. What is the probability that a customer who never buys generic products visits the store frequently?

c. Are the events "Never buys generic products" and "Visits the store frequently" independent?

- d. What is the probability that a customer who infrequently visits the store often buys generic products?
- e. Are the events "Often buys generic products" and "Visits the store infrequently" independent?
- f. What is the probability that a customer frequently visits the store?
- g. What is the probability that a customer never buys generic products?
- h. What is the probability that a customer either frequently visits the store or never buys generic products or both?

4.67 A consulting organization predicts whether corporations' earnings for the coming year will be unusually low, unusually high, or normal. Before deciding whether to continue purchasing these forecasts, a stockbroker compares past predictions with actual outcomes. The accompanying table shows proportions in the nine joint classifications.

Outcome	Prediction		
	Unusually High	Normal	Unusually Low
Unusually High	.23	.12	.03
Normal	.06	.22	.08
Unusually Low	.01	.06	.19

- a. What proportion of predictions have been for unusually high earnings?
- b. What proportion of outcomes have been for unusually high earnings?
- c. If a firm were to have unusually high earnings, what is the probability that the consulting organization would correctly predict this event?
- d. If the organization predicted unusually high earnings for a corporation, what is the probability that these would materialize?
- e. What is the probability that a corporation for which unusually high earnings had been predicted will have unusually low earnings?

4.68 Subscribers to a local newspaper were asked whether they regularly, occasionally or never read the business section and also whether they had traded common stocks (or shares in a mutual fund) over the last year. The table given here shows proportions of subscribers in six joint classifications.

Traded Stocks	Read Business Section		
	Regularly	Occasionally	Never
Yes	.18	.10	.04
No	.16	.31	.21

- a. What is the probability that a randomly chosen subscriber never reads the business section?
- b. What is the probability that a randomly chosen subscriber had traded stocks over the last year?



- c. What is the probability that a subscriber who never reads the business section has traded stocks over the last year?
- d. What is the probability that a subscriber who traded stocks over the last year never reads the business section?

4.69 A corporation regularly takes deliveries of a particular sensitive part from three subcontractors. It found that the proportion of parts that are good or defective from the total received were as shown in the following table:

Part	Subcontractor		
	A	B	C
Good	.27	.30	.33
Defective	.02	.05	.03

- a. If a part is chosen randomly from all those received, what is the probability that it is defective?
- b. If a part is chosen randomly from all those received, what is the probability it is from subcontractor B?
- c. What is the probability that a part from subcontractor B is defective?
- d. What is the probability that a randomly chosen defective part is from subcontractor B?
- e. Is the quality of a part independent of the source of supply?
- f. In terms of quality, which of the three subcontractors is most reliable?

4.70 Students in a business statistics class were asked what grade they expected in the course and whether they worked additional problems beyond those assigned by the instructor. The following table gives proportions of students in each of eight joint classifications.

Worked Problems	Expected Grade			
	A	B	C	Below C
Yes	.12	.06	.12	.02
No	.13	.21	.26	.08

- a. Find the probability that a randomly chosen student from this class worked additional problems.
- b. Find the probability that a randomly chosen student from this class expects an A.
- c. Find the probability that a randomly chosen student who worked additional problems expects an A.
- d. Find the probability that a randomly chosen student who expects an A worked additional problems.
- e. Find the probability that a randomly chosen student who worked additional problems expects a grade below B.
- f. Are "Worked additional problems" and "Expected grade" statistically independent?

4.71 The accompanying table shows proportions of computer sales people classified according to marital status and whether they left their jobs or stayed over a period of 1 year.

<u>Marital Status</u>	<u>Stayed 1 Year</u>	<u>Left</u>
Married	.64	.13
Single	.17	.06

- What is the probability that a randomly chosen sales person is married?
- What is the probability that a randomly chosen sales person left the job within the year?
- What is the probability that a randomly chosen single sales person left the job within the year?
- What is the probability that a randomly chosen sales person who stayed in the job over the year is married?

4.72 The accompanying table shows proportions of adults in nonmetropolitan areas, categorized as to whether they are readers or nonreaders of newspapers and whether or not they voted in the last election.

<u>Voted</u>	<u>Readers</u>	<u>Non readers</u>
Yes	.63	.13
No	.14	.10

- What is the probability that a randomly chosen adult from this population voted?
- What is the probability that a randomly chosen adult from this population reads newspapers?
- What is the probability that a randomly chosen adult from this population who did not read newspapers did not vote?

4.73 A campus student club distributed material about membership to new students attending an orientation meeting. Of those receiving this material 40% were men and 60% were women. Subsequently, it was found that 7% of the men and 9% of the women who received this material joined the club.

- Find the probability that a randomly chosen new student who receives the membership material will join the club.
- Find the probability that a randomly chosen new student who joins the club after receiving the membership material is a woman.

4.74 An analyst attempting to predict a corporation's earnings next year believes that the corporation's business is quite sensitive to the level of interest rates. She believes that, if average rates in the next year are more than 1% higher than this year, the probability of significant earnings growth is .1. If average rates next year are more than 1% lower than this year, the probability of significant earnings growth is estimated to be .8. Finally, if average interest rates next year are within 1% of this year's rates, the probability for significant earnings growth is put at .5. The analyst estimates that the probability is .25 that rates next year will be more than 1% higher than this year and .15 that they will be more than 1% lower than this year.

- What is the probability that both interest rates will be 1% higher and significant earnings growth will result?

- b. What is the probability that this corporation will experience significant earnings growth?
- c. If the corporation exhibits significant earnings growth, what is the probability that interest rates will have been more than 1% lower than in the current year?

4.75 42% of a corporation's blue collar employees were in favor of a modified health care plan, and 22% of its blue collar employees favored a proposal to change the work schedule. 34% of those favoring the health plan modification favored the work schedule change.

- a. What is the probability that a randomly selected blue collar employee is in favor of both the modified health care plan and the changed work schedule?
- b. What is the probability that a randomly chosen blue collar employee is in favor of at least one of the two changes?
- c. What is the probability that a blue collar employee favoring the work schedule change also favors the modified health plan?

4.76 The grades of a freshman college class, obtained after the first year of college, were analyzed. 70% of the students in the top quarter of the college class had graduated in the upper 10% of their high school class, as had 50% of the students in the middle half of the college class and 20% of the students in the bottom quarter of the college class.

- a. What is the probability that a randomly chosen freshman graduated in the upper 10% of his or her high school class?
- b. What is the probability that a randomly chosen freshman who graduated in the upper 10% of his or her high school class will be in the top quarter of the college class?
- c. What is the probability that a randomly chosen freshman who did not graduate in the upper 10% of his or her high school class will not be in the top quarter of the college class?

4.77 Before books aimed at preschool children are marketed, reactions are obtained from a panel of preschool children. These reactions are categorized as "favorable," "neutral," or "unfavorable." Subsequently, book sales are categorized as "high," "moderate," or "low," according to the norms of this market. Similar panels have evaluated 1,000 books in the past. The accompanying table shows their reactions and the resulting market performance of the books.

<u>Sales</u>	<u>Panel Reaction</u>		
	<u>Favorable</u>	<u>Neutral</u>	<u>Unfavorable</u>
High	173	101	61
Moderate	88	211	70
Low	42	113	141

- a. If the panel reaction is favorable, what is the probability that sales will be high?
- b. If the panel reaction is unfavorable, what is the probability that sales will be low?
- c. If the panel reaction is neutral or better, what is the probability that sales will be low?
- d. If sales are low, what is the probability that the panel reaction was neutral or better?

4.78 A manufacturer produces boxes of candy, each containing 10 pieces. Two machines are used for this purpose. After a large batch has been produced, it is discovered that one of the machines, which produces 40% of the total output, has a fault that has led to the introduction of an impurity into 10% of the pieces of candy it makes. From a single box of candy, one piece is selected at random and tested. If that piece contains no impurity, what is the probability that the faulty machine produced the box from which it came?

4.79 A student feels that 70% of his college courses have been enjoyable and the remainder have been boring. This student has access to student evaluations of professors and finds out that professors who had previously received strong positive evaluations from their students have taught 60% of his enjoyable courses and 25% of his boring courses. Next semester the student decides to take three courses, all from professors who have received strongly positive student valuations. Assume that this student's reactions to the three courses are independent of one another.

- What is the probability that this student will find all three courses enjoyable?
- What is the probability that this student will find at least one of the courses enjoyable?

In the following exercises, use a sample space defined by events  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$ .

4.80 Given  $P(A_1) = .40$ ,  $P(B_1|A_1) = .60$ , and  $P(B_1|A_2) = .70$  what is the probability of  $(A_1|B_1)$ ?

4.81 Given  $P(A_1) = .80$ ,  $P(B_1|A_1) = .60$  and  $P(B_1|A_2) = .20$  what is the probability of  $(A_1|B_1)$ ?

4.82 Given  $P(A_1) = .50$ ,  $P(B_1|A_1) = .40$ , and  $P(B_1|A_2) = .70$  what is the probability of  $(A_1|B_2)$ ?

4.83 Given  $P(A_1) = .40$ ,  $P(B_1|A_1) = .60$ , and  $P(B_1|A_2) = .70$  what is the probability of  $(A_2|B_2)$ ?

4.84 Given  $P(A_1) = .60$ ,  $P(B_1|A_1) = .60$ , and  $P(B_1|A_2) = .40$  what is the probability of  $(A_1|B_1)$ ?

4.85 A publisher sends advertising materials for an accounting text to 80% of all professors teaching the appropriate accounting course. 30% of the professors who received this material adopted the book, as did 10% of the professors who did not receive the material. What is the probability that a professor who adopts the book has received the advertising material?

4.86 A stock market analyst examined the prospects of the shares of a large number of corporations. When the performance of these stocks was investigated one year later, it turned out that 25% performed much better than the market average, 25% much worse, and the remaining 50% about the same as the average. 40% of the stocks that turned out to do much better than the market were rated "good buys" by the analyst, as were 20% of those that did about as well as the market and 10% of those that did much worse. What is the probability that a stock rated a "good buy" by the analyst performed much better than the average?