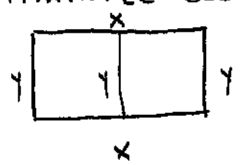
#13 1. Minimize Cost

2. 

Let y = length (in ft) of side down middle and two parallel sides.

Let x = length (in ft) of two perpendicular sides.

3. Let L = length of fencing. (To minimize cost, we minimize L.)

$L = 2x + 3y$

4. Know $xy = 1500{,}000$, and so $y = \dfrac{1500{,}000}{x}$.

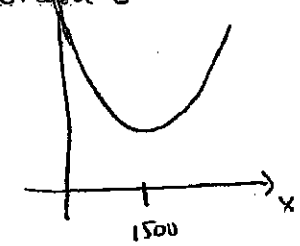
Then $L = 2x + \dfrac{3(1500{,}000)}{x}$. Domain: $0 < x < \infty$.

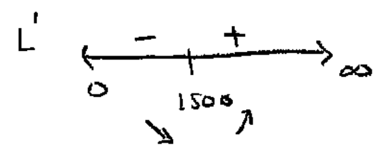
5. $L' = 2 - \dfrac{3(1500{,}000)}{x^2} = \dfrac{2x^2 - 3(1500{,}000)}{x^2}$.

$L' = 0$ when $x^2 = \dfrac{3(1500{,}000)}{2} = 3(750{,}000) = 3^2 \cdot 5^2 \cdot 100^2$.

$x = 3 \cdot 5 \cdot 100 = 1500$ (C.P.). ($x = -1500$ not in domain)

6. Graph L:

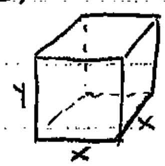




7. L is smallest when $x = 1500$ ft

and $y = 1000$ ft.

#15 1. Maximize Volume

2. 

Let x = length (in cm) of sides on bottom
Let y = height (in cm).

(no top!)

3. Let V = volume. Then $V = x^2 y$.

4. Know $1200 = x^2 + 4xy$ (amount of material)

Then $y = \dfrac{1200 - x^2}{4x}$. Then $V = x^2\left(\dfrac{1200 - x^2}{4x}\right) = \dfrac{1}{4}x\,(1200 - x^2)$.

Then $V = 300x - \frac{1}{4}x^3$. Domain: $0 < x < \sqrt{1200}$.

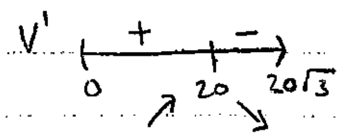
Note: $\sqrt{1200} = 20\sqrt{3}$.

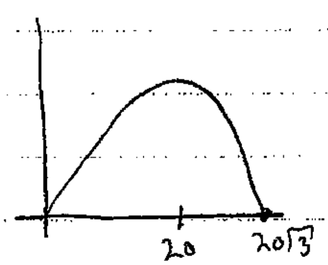
Domain: $0 < x < 20\sqrt{3}$.

5. $V' = 300 - \frac{3}{4}x^2 = \dfrac{300(4) - 3x^2}{4} = \dfrac{3}{4}(400 - x^2) = \dfrac{3}{4}(20-x)(20+x)$.

CP: $x = 20$ ($x = -20$ is not in domain).

6. Graph of V:





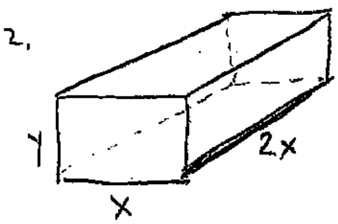
7. To get the largest possible volume, take $x = 20$ cm,

$y = \dfrac{1200 - (20)^2}{4(20)} = 10$ cm.

The volume of such a box is $V = (20)(20)(10) = 4000\ \text{cm}^3$.

#17 1. Minimize Cost

2.



Let x = width (in m)
Then $2x$ = length.
Let y = height (in m)

3. Let C = cost of box.

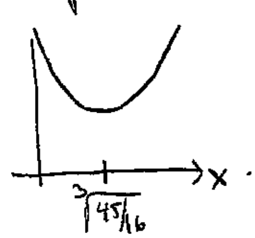
cost of base = $(\$10/m^2)\cdot(\text{area}) = 10\cdot 2x^2 = 20x^2$
cost of top = $(\$6/m^2)\cdot(\text{area}) = 6\cdot 2x^2 = 12x^2$
cost of front = ... $= 6xy$
cost of back = ... $= 6xy$
cost of right = ... $= 6(2xy) = 12xy$
cost of left = ... $= 6(2xy) = 12xy$.

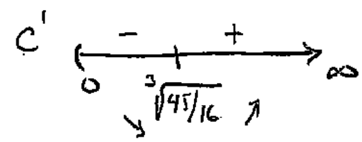
$$C = 32x^2 + 36xy.$$

4. Know $2x^2y = 10$ (volume), and so $x^2y = 5$, so $y = 5/x^2$.
Then $C = 32x^2 + 36x(5/x^2) = 32x^2 + \frac{5(36)}{x}$.
Domain: $0 < x < \infty$.

5. $C' = 64x - \frac{5(36)}{x^2} = \frac{64x^3 - 5(36)}{x^2} = \frac{4(16x^3-45)}{x^2}$.
CP: $x = \sqrt[3]{45/16}$.

6. Graph of C:

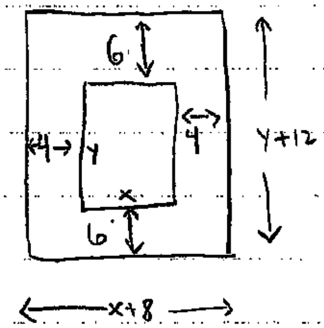




7. To minimize cost, take $x = \sqrt[3]{45/16}$ m. The minimum cost is $C(\sqrt[3]{45/16}) = 32\left(\sqrt[3]{45/16}\right)^2 + \frac{5(36)}{\sqrt[3]{45/16}} \approx \$191.28$.

#33 1. Minimize area of poster

2.



Let x = width (in cm) of printed material

Let y = height (in cm) of printed material.

3. Let A = area of entire poster.

Then $A = (x+8)(y+12)$

4. Know $xy = 384$, so $y = \frac{384}{x}$.

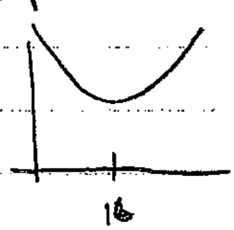
Then $A = (x+8)\left(\frac{384}{x} + 12\right) = 384 + 12x + \frac{8(384)}{x} + 8(12)$.

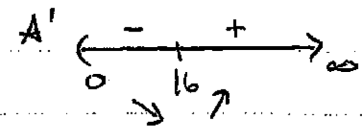
Domain: $0 < x < \infty$.

5. $A' = 12 - \frac{8(384)}{x^2} = \frac{12x^2 - 8(384)}{x^2} = \frac{12(x^2-256)}{x^2} = \frac{12(x-16)(x+16)}{x^2}$.

$x = 16$ only CP in $(0, \infty)$.

6. Graph A:



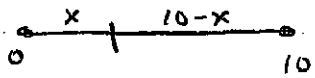


7. To minimize area, we take $x = 16$, $y = \frac{384}{16} = 24$.

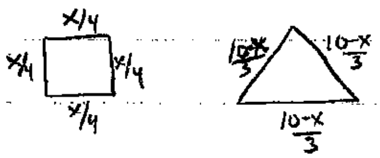
The dimensions of such a poster are

24 cm × 36 cm.

#35 1. Maximize/Minimize total area enclosed.

2. 

Let x = length (in m) used for square.
Then $10-x$ = length for triangle.



3. Let A = total area enclosed.

Area of square $= x^2/16$.

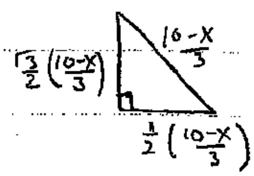
Area of triangle $= \frac{1}{2}(\text{base})\cdot(\text{height})$

$= \frac{1}{2}\left(\frac{10-x}{3}\right)\left(\frac{\sqrt{3}}{2}\cdot\frac{10-x}{3}\right)$

$= \frac{\sqrt{3}}{36}(10-x)^2$.

Then $A = x^2/16 + \frac{\sqrt{3}}{36}(10-x)^2$.

Here is the triangle, cut in half:

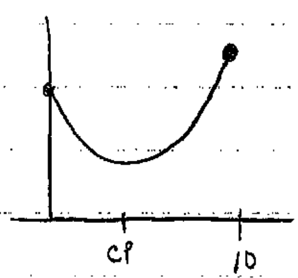


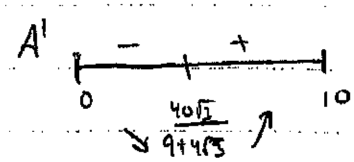
4. Domain: $0 \le x \le 10$.

5. $A' = \frac{x}{8} - \frac{\sqrt{3}}{18}(10-x) = \frac{9x - 4\sqrt{3}(10-x)}{72} = \frac{(9+4\sqrt{3})x - 40\sqrt{3}}{72}$.

$x = \frac{40\sqrt{3}}{9+4\sqrt{3}}$ CP. (Note $\frac{40\sqrt{3}}{9+4\sqrt{3}} < \frac{40\sqrt{3}}{4\sqrt{3}} = 10$, so the CP is in the interval $[0,10]$.)

6. Graph A:





$A(0) = \frac{100\sqrt{3}}{36} < \frac{100}{16} = A(10)$

7. To Minimize A, use $x = \frac{40\sqrt{3}}{9+4\sqrt{3}}$ m for square, rest for triangle.
To Maximize A, use all 10 m for square.

\#37 1. Minimize Surface area.

2. 

Let r = radius, h = height (both in cm).

(no top!)

3. Let S = surface area.

Then $S = \pi r^2 + 2\pi r h$.

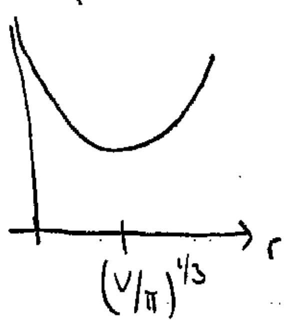
4. Know $\pi r^2 h = V$ (V is fixed). Then $h = V/\pi r^2$.

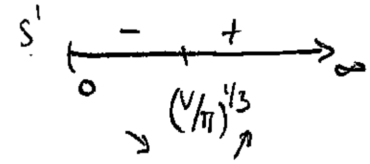
Then $S = \pi r^2 + 2\pi r\left(V/\pi r^2\right) = \pi r^2 + 2V/r$. Domain: $0 < r < \infty$.

5. $S' = 2\pi r - \frac{2V}{r^2} = \frac{2\pi r^3 - 2V}{r^2} = \frac{2(\pi r^3 - V)}{r^2}$.

Only CP in $(0,\infty)$ is $r = (V/\pi)^{1/3}$

6. Graph of S:



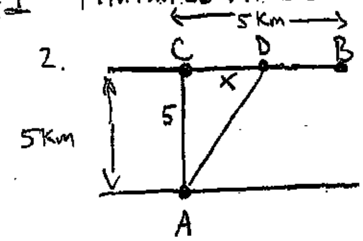


7. To minimize cost, we take $r = (V/\pi)^{1/3}$ cm

and we take $h = \frac{V}{\pi\left(\frac{V}{\pi}\right)^{2/3}} = \left(\frac{V}{\pi}\right)^{1/3}$ cm.

#47 Minimize Time.

2.



Let x = distance from C to D (in km)

3. Let T = time travelled. (in hrs)

time rowing $= \dfrac{\sqrt{x^2+25}}{6}$ hr

time running $= \dfrac{5-x}{8}$ hr.

$$T = \frac{\sqrt{x^2+25}}{6} + \frac{5-x}{8}$$

4. Domain: $0 \le x \le 5$

5. $$T' = \frac{\frac{1}{2}(x^2+25)^{-1/2}2x}{6} - \frac{1}{8} = \frac{x}{6\sqrt{x^2+25}} - \frac{1}{8}$$

$$= \frac{4x - 3\sqrt{x^2+25}}{24\sqrt{x^2+25}}.$$

If $T'=0$, then $4x = 3\sqrt{x^2+25}$. Then $16x^2 = 9(x^2+25)$.

Then $7x^2 = 9(25)$. Then $x^2 = \frac{9(25)}{7}$, $x = \pm\frac{15}{\sqrt{7}}$.

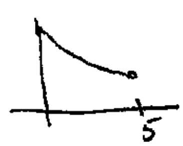
Note: $x = -15/\sqrt{7}$ is not in $[0,5]$.

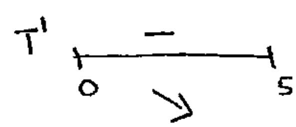
But $x = 15/\sqrt{7}$ is not in $[0,5]$ either, since

$15/\sqrt{7} > 15/\sqrt{9} = 15/3 = 5$.

So NO CP'S in $[0,5]$!!

6. Graph of T:





7. Time is minimized for $x = 5$, that is, if the person rows directly to B.