MATH 160

EXAM 2

2 March 2012

Your name:

Pledge:

There are 10 problems, and the point values of each problem are shown. A perfect score is 100 points. Calculator use is not permitted. I'll be in my office (Arter 104B) during the test if you have questions.

Good luck!!

Problem 1 (6 points) A particle moves along a straight line with equation of motion

$$s = \frac{t^2 + 6t}{\sqrt{t}} = t^{3/2} + 6t^{1/2}$$

where s is measured in meters and t in seconds. Find the velocity when $t = 4$ and give the correct units.

$$s'(t) = \frac{3}{2}t^{\frac{1}{2}} + 3t^{-1/2}$$

$$s'(4) = \frac{3}{2}\sqrt{4} + \frac{3}{\sqrt{4}} = 3 + \frac{3}{2} = \boxed{\frac{9}{2} \text{ met/sec}}$$

Problem 2 (6 points) Find the critical points for the function

$$f(x) = x^{1/5}(x-4)^2.$$

$$f'(x) = \frac{1}{5}x^{-4/5}(x-4)^2 + 2x^{1/5}(x-4)$$

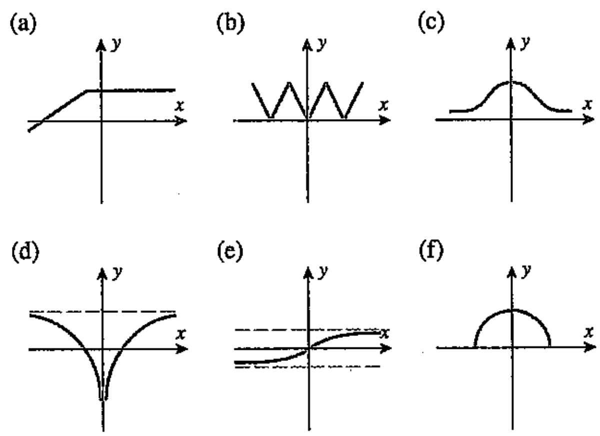
$$= \frac{(x-4)}{5x^{4/5}}\left((x-4) + 10x\right)$$

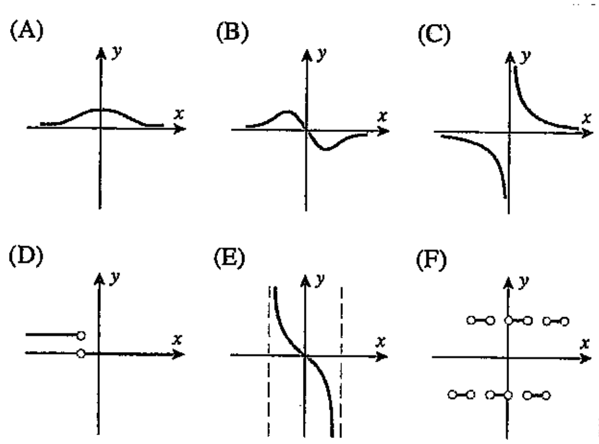
$$= \frac{(x-4)(11x-4)}{5x^{4/5}}$$

$$\text{CP: } \boxed{x = 0, 4, 4/11}$$

Problem 3 (10 points) Match the graph of each function in (a)-(f) on the top with the graph of its derivative in (A)-(F) on the bottom. You can do this by completing the following six sentences:

- The derivative of the function shown in (a) is the function shown in letter $\boxed{D}$.
- The derivative of the function shown in (b) is the function shown in letter $\boxed{F}$.
- The derivative of the function shown in (c) is the function shown in letter $\boxed{B}$.
- The derivative of the function shown in (d) is the function shown in letter $\boxed{C}$.
- The derivative of the function shown in (e) is the function shown in letter $\boxed{A}$.
- The derivative of the function shown in (f) is the function shown in letter $\boxed{E}$.





Problem 4 (8 points) State and prove the Quotient Rule.

QR: $$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Proof: $$\frac{d}{dx}\frac{f(x)}{g(x)} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)h}$$

$$= \lim_{h\to 0}\frac{1}{g(x+h)g(x)}\cdot\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h}$$

$$= \lim_{h\to 0}\frac{1}{g(x+h)g(x)}\cdot\left(\frac{f(x+h)-f(x)}{h}\cdot g(x) - f(x)\frac{g(x+h)-g(x)}{h}\right)$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

Problem 5 (6 points) For what values of x does the graph of $f(x) = 2x^3 + 3x^2 - 12x + 9$ have a horizontal tangent line?

$$f'(x) = 6x^2 + 6x - 12$$
$$= 6(x^2 + x - 2)$$
$$= 6(x+2)(x-1)$$

$f'(x) = 0$ when $\boxed{x = -2,\ x = 1}$

Problem 6 (6 points) Determine the value of the following limit (and show your work):

$$\lim_{x\to 0} \frac{\sin 5x}{\tan 2x} = \lim_{x \to 0} \frac{5}{2} \cdot \frac{\sin 5x}{5x} \cdot \frac{2x}{\sin 2x} \cdot \cos 2x$$

$$= \frac{5}{2} \cdot 1 \cdot 1 \cdot 1 = \boxed{\frac{5}{2}}$$

Problem 7 (6 points) Use the addition of angles formula $\cos(u+v) = \cos u \cos v - \sin u \sin v$ to prove the derivative formula

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx}\cos x = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h\to 0}\left(-\sin x \cdot \frac{\sin h}{h} + \cos x \cdot \frac{\cos h - 1}{h}\right)$$

$$= (-\sin x)\cdot 1 + (\cos x)\cdot 0$$

$$= -\sin x$$

Problem 8 (6 points) Find $\frac{dy}{dx}$ for the equation

$$y \sin x = 3 + (x+y)^4.$$

$$y' \sin x + y \cos x = 4(x+y)^3(1+y') = 4(x+y)^3 + 4(x+y)^3 y'$$

$$y' \sin x - 4(x+y)^3 y' = 4(x+y)^3 - y\cos x$$

$$\boxed{y' = \frac{4(x+y)^3 - y\cos x}{\sin x - 4(x+y)^3}}$$

Problem 9 (6 points each) Find the derivative of the six functions below. (Don't simplify.)

a. $f(x) = \dfrac{x^5 + 2x + 3}{\sqrt[3]{x} + x}$

$$f'(x) = \frac{(5x^4+2)(\sqrt[3]{x}+x) - (x^5+2x+3)(\frac{1}{3}x^{-2/3}+1)}{(\sqrt[3]{x}+x)^2}$$

b. $f(x) = \left(\dfrac{1}{x^2} + \dfrac{1}{x}\right)\left(\tan x + \cot x\right)$

$$f'(x) = (-2x^{-3} - x^{-2})(\tan x + \cot x) + \left(\frac{1}{x^2} + \frac{1}{x}\right)(\sec^2 x - \csc^2 x)$$

c. $f(x) = (x^4 + 5x)^6$

$$f'(x) = 6(x^4+5x)^5(4x^3+5)$$

d. $f(x) = \sqrt[3]{2x + \csc x}$

$f'(x) = \frac{1}{3}(2x + \csc x)^{-2/3} \cdot (2 - \csc x \cot x)$

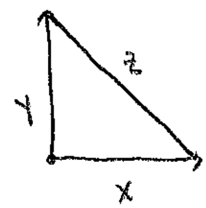
e. $f(x) = \sin(x \cos x)$

$f'(x) = \cos(x\cos x) \cdot [\cos x - x \sin x]$

f. $f(x) = \sec^3(\tan x)$

$f'(x) = 3\sec^2(\tan x)\sec(\tan x)\tan(\tan x)\sec^2 x$

Problem 10 (10 points) Two people start cycling from the same point. One cycles east at 10 mi/hr and the other cycles north at 24 mi/hr. How fast is the distance between the two people changing after 30 minutes?



$\frac{dx}{dt} = 10 \text{ m/hr}$

$\frac{dy}{dt} = 24 \text{ m/hr}$

want: $\frac{dz}{dt}$ when $t = \frac{1}{2}$ hr

$x^2 + y^2 = z^2$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

when $t = \frac{1}{2}$,

$x = 10(\frac{1}{2}) = 5$

$y = 24(\frac{1}{2}) = 12$

$z = \sqrt{5^2 + 12^2} = 13.$

When $t = \frac{1}{2}$, $\frac{dz}{dt} = \frac{5(10) + 12(24)}{13} = \frac{2(5^2 + 12^2)}{13} = \frac{2(13^2)}{13} = 2(13)$

$= \boxed{26 \text{ miles/hour}}$