MATH 205 **Foundations of Mathematics** **Spring 2013**

Problem Set 1
Due Wednesday January 23

1. Let P and Q be statements. Show that the following statements are logically equivalent.
   (a) $\neg(P \vee Q)$ and $(\neg P) \wedge (\neg Q)$
   (b) $P \Rightarrow Q$ and $(\neg P) \vee Q$.

2. Let P, Q, and R be statements. Determine whether or not the statements
$$(P \wedge Q) \Rightarrow R \text{ and } (P \Rightarrow R) \wedge (Q \Rightarrow R)$$
are logically equivalent.

3. Determine whether or not each of the following statements is true. Explain your answers.
   (a) For all real numbers x, $x^2 - 2x - 3 = 0$ only if $x = 3$.
   (b) For all real numbers x, $x^2 - 2x - 3 = 0$ if $x = 3$.

4. Find a useful denial of each of the following statements. Use mathematically precise, natural English, writing all conditional statements in the form "if ..., then ...."
   (a) I will do my homework and I will pass this class.
   (b) If $x \neq 0$, then there exists a real number y such that $xy = 1$.
   (c) The stars are green or the white horse is shining only if the world is eleven feet wide.
   (d) There are integers m and n such that for each rational number x either $m < nx$ or $n < mx$.
   (e) For every $\epsilon > 0$, there exists $\delta > 0$ such that for every $x \in \mathbb{R}$, if $|x - a| < \delta$, then $|x^2 - a^2| < \epsilon$.

5. Find the converse and contrapositive of each of the following statements. Use mathematically precise, natural English, writing all conditional statements in the form "if ..., then ...."
   (a) If I ski, I will fall.
   (b) If $y > x$ and $y > 0$, then $y > z$.
   (c) If $x \neq 0$, then there exists a real number y such that $xy = 1$.
   (d) $n - 3 \leq 6$ only if $n > 4$ or $n > 10$.
   (e) If there exist integers m and n such that $12m + 15n = 1$, then m and n are both positive.