MATH 160

EXAM 1

10 February 2012

Your name:

Pledge:

There are 12 problems, and the point values of each problem are shown. A perfect score is 100 points. Calculator use is not permitted. I'll be in my office (Arter 104B) during the test if you have questions.

Good luck!!

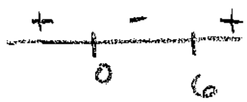
1. (5+5 points) Consider the function

$$h(x) = \frac{1}{\sqrt[4]{x^2 - 6x}}.$$

a. Determine the domain of $h(x)$.

Need $x^2 - 6x > 0$

$x(x-6) > 0$

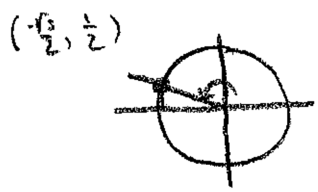


Domain: $(-\infty, 0) \cup (6, \infty)$

b. Determine functions f and g that satisfy $f(g(x)) = h(x)$.

$f(x) = \frac{1}{\sqrt[4]{x}}$, $g(x) = x^2 - 6x$

2. (6 points) Determine the value of $\cos(\frac{5\pi}{6})$.

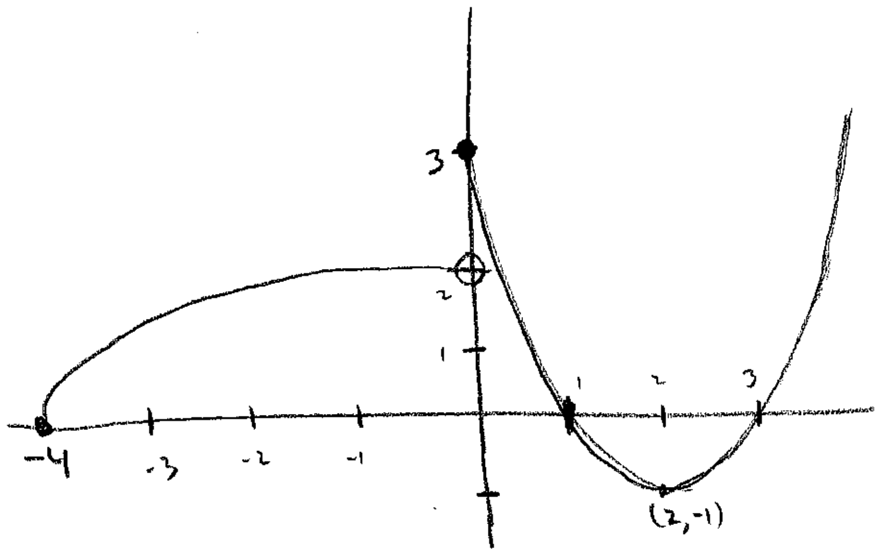


$\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$

3. (6+3+3 points) Consider the function

$$f(x) = \begin{cases} (x-2)^2 - 1 & \text{if } x \geq 0 \\ \sqrt{x+4} & \text{if } x < 0. \end{cases}$$

a. Draw the graph of $f(x)$.



b. Determine the value of the one sided limits:

$\lim_{x \to 0+} f(x) = 3$

$\lim_{x \to 0-} f(x) = 2$

4. (6+6+6+6 points) Determine the following limits.

a. $\lim_{x\to -3} \frac{x^2-9}{x+3}$ $= \lim_{x\to -3} \frac{(x+3)(x-3)}{x+3} = -6$

b. $\lim_{x\to 1} \frac{1-x}{1-\sqrt{x}} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}} = \lim_{x\to 1} \frac{(1-x)(1+\sqrt{x})}{1-x} = 2$

c. $\lim_{x\to 3^-} \frac{x+3}{x-3}$ $= -\infty$

$\frac{6}{0}$, $\frac{+}{-}$

d. $\lim_{x\to -\infty} \frac{5x+10}{\sqrt{9x^2+6x+1}}$ $= \lim_{x\to -\infty} \frac{5x}{3\sqrt{x^2}} = \lim_{x\to -\infty} \frac{5x}{-3x} = \frac{-5}{3}$

5. (3 points) Is the following statement true or false? (No explanation is required.)

If $\lim_{x \to a} f(x) = 0$, then $\lim_{x \to a} f(x)g(x) = 0$.

False

6. (6 points) Use the Squeeze Theorem to prove that

$$\lim_{x \to 0} x^4 \cos\left(\frac{7}{x^3}\right) = 0.$$

Since $-1 \le \cos\left(\frac{7}{x^3}\right) \le 1$ for all $x \neq 0$,

we get $-x^4 \le x^4 \cos\left(\frac{7}{x^3}\right) \le x^4$ for all $x \neq 0$.

Since $\lim_{x \to 0} -x^4 = 0$ and $\lim_{x \to 0} x^4 = 0$,

we get $\lim_{x \to 0} x^4 \cos\left(\frac{7}{x^3}\right) = 0$ by Squeeze Theorem.

7. (5 points) Give the precise statement of the Intermediate Value Theorem.

Suppose that f is continuous on $[a,b]$, that $f(a) \neq f(b)$, and that N is any number between $f(a)$ and $f(b)$. Then there exists a number c in (a,b) for which $f(c) = N$.

8. (4 points) There are two equations listed below, equation (a) and equation (b). For one of these two equations, the Intermediate Value Theorem can be used to show that the equation has a root. Which equation is it, (a) or (b)? (No explanation is required.)

(a) $\cos x + x^2 + 3 = 0$

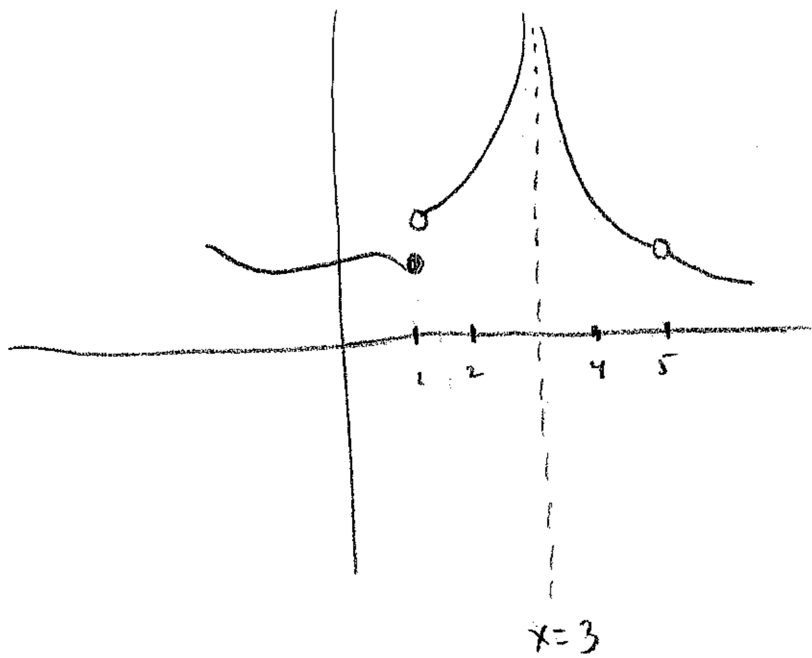
(b) $\cos x + x + 3 = 0$

(If $f(x) = \cos x + x + 3$, then $f(-10) < 0$ and $f(10) > 0$.

Also, $\cos x + x^2 + 3 \geq -1 + 0 + 3 > 0$ for all x.)

9. (3+3+3 points) Draw the graph of one function f for which all of the following properties hold:

   a. f has a jump discontinuity at $x = 1$
   b. f has an infinite discontinuity at $x = 3$
   c. f has a removable discontinuity at $x = 5$.



10. (4+3 points) Consider the function

$$f(x) = \frac{x^3 - 2x^2 - 3x}{x - 3}.$$

a. Show that the discontinuity of f at $x = 3$ is removable by computing

$\lim_{x \to 3} f(x).$ $= \lim_{x \to 3} \frac{x(x^2-2x-3)}{x-3} = \lim_{x \to 3} \frac{x(x-3)(x+1)}{x-3}$

$= \lim_{x \to 3} x(x+1) = 12$

Since $\lim_{x \to 3} f(x)$ exists, but is not equal to $f(3)$, f has a removable discontinuity at $x=3$.

b. Find a function $g(x)$ that agrees with $f(x)$ where $x \neq 3$, but is continuous at $x = 3$.

$g(x) = x(x+1)$

11. (6 points) If $f(x) = x^3 + x$, it can be shown that $f'(x) = 3x^2 + 1$. Use this information to find the equation of the tangent line to the graph of f at $x = 1$.

$f(1) = 2$ Known point : $(1,2)$

$f'(1) = 4$ slope : 4.

Tangent line : $y - 2 = 4(x-1)$

12. (8 points) Use the definition of the derivative to find the derivative of the function

$$f(x) = \sqrt{x+4}.$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+4} - \sqrt{x+4}}{h} \cdot \frac{\sqrt{x+h+4} + \sqrt{x+4}}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$= \lim_{h \to 0} \frac{x+h+4 - (x+4)}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$= \frac{1}{2\sqrt{x+4}}.$$