

### Example 6.2 Home Heating Costs (Mean and Standard Deviation)

A homeowner estimates that within the range of likely temperatures her January heating bill,  $Y$ , in dollars, will be

$$Y = 290 - 5T$$

where  $T$  is the average temperature for the month, in degrees Fahrenheit. If the average January temperature can be represented by a random variable with mean 24 and standard deviation 4, find the mean and standard deviation of this homeowner's January heating bill.

**Solution** The random variable  $T$  has mean  $\mu_T = 24$  and standard deviation  $\sigma_T = 4$ . Therefore, the expected heating bill is

$$\begin{aligned}\mu_Y &= 290 - 5\mu_T \\ &= 290 - (5)(24) = \$170\end{aligned}$$

The standard deviation is

$$\sigma_Y = |-5|\sigma_T = (5)(4) = \$20$$

## EXERCISES

### Basic Exercises

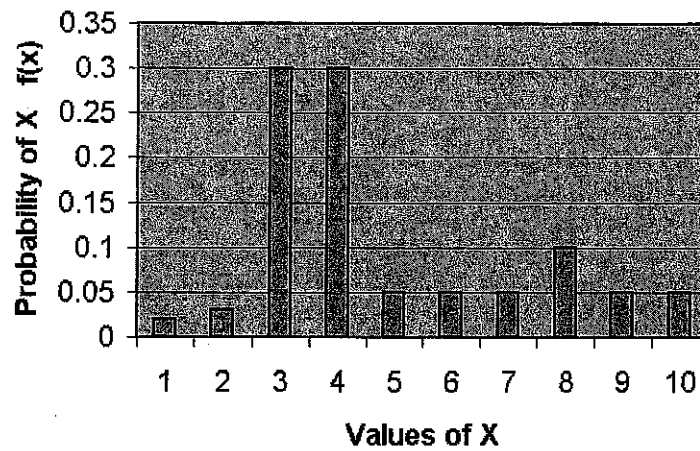
- 6.9 The total cost for a production process is equal to \$1,000 plus 2 times the number of units produced. The mean and variance for the number of units produced are 500 and 900, respectively. Find the mean and variance of the total cost.
- 6.10 The profit for a production process is equal to \$1,000 minus 2 times the number of units produced. The mean and variance for the number of units produced are 50 and 90, respectively. Find the mean and variance of the profit.
- 6.11 The profit for a production process is equal to \$2,000 minus 2 times the number of units produced. The mean and variance for the number of units produced are 500 and 900, respectively. Find the mean and variance of the profit.
- 6.12 The profit for a production process is equal to \$6,000 minus 3 times the number of units produced. The mean and variance for the number of units produced are 1,000 and 900, respectively. Find the mean and variance of the profit.

### Application Exercises

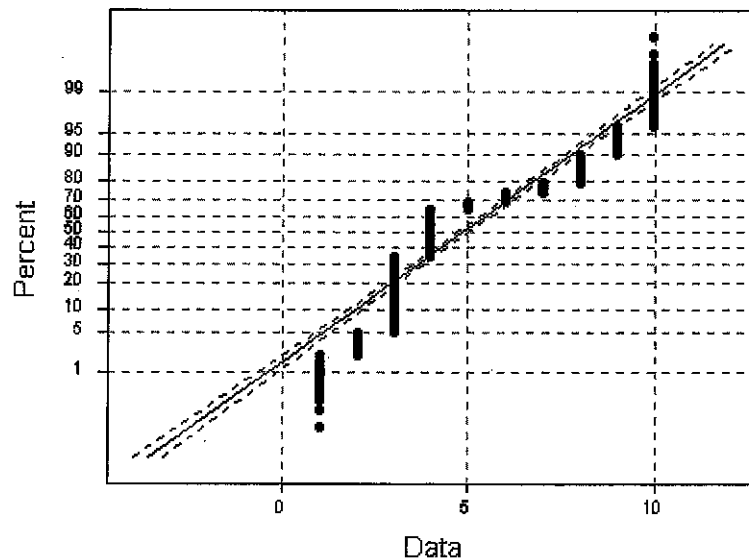
- 6.13 An author receives from a publisher a contract, according to which she is to be paid a fixed sum of \$10,000 plus \$1.50 for each copy of her book sold. Her uncertainty about total sales of the book can be represented by a random variable with mean 30,000 and standard deviation 8,000. Find the mean and standard deviation of the total payments she will receive.
- 6.14 A contractor submits a bid on a project, for which more research and development work needs to be done. It is estimated that the total cost of satisfying the project specifications will be \$20 million plus the cost of the further research and development work. The contractor views the cost of this work as a random variable with mean \$4 million and standard deviation \$1 million. The contractor wishes to submit a bid such that his expected profit will be 10% of his expected costs. What should be the bid? If this bid is accepted, what will be the standard deviation of the profit made by the project?
- 6.15 A charitable organization solicits donations by telephone. Employees are paid \$60 plus 20% of the money their calls generate each week. The amount of money generated in a week can be viewed as a random variable with mean \$700 and standard deviation \$130. Find the mean and standard deviation of an employee's total pay in a week.
- 6.16 A salesman receives an annual salary of \$6,000 plus 8% of the value of the orders he takes. The annual value of these orders can be represented by a random variable with mean \$600,000 and standard deviation \$180,000. Find the mean and standard deviation of the salesman's annual income.

Figure  
Probabi  
Density  
for a Nc  
Distribu

**Figure 6.21**  
Skewed Discrete  
Probability  
Distribution  
Function



**Figure 6.22**  
Normal  
Probability Plot  
for a Highly  
Skewed  
Distribution  
(Minitab  
Output)



you are safe in assuming that the normal model is a good approximation. Note, however, that if your plot deviates from a straight line, as do those in Figures 6.20 and 6.22, then the normal probability distribution should not be used.

## EXERCISES

### Basic Exercises

6.17 Let the random variable  $Z$  follow a standard normal distribution.

- Find  $P(Z < 1.20)$ .
- Find  $P(Z > 1.33)$ .
- Find  $P(Z < -1.70)$ .
- Find  $P(Z > -1.00)$ .
- Find  $P(1.20 < Z < 1.33)$ .

f. Find  $P(-1.70 < Z < 1.20)$ .

g. Find  $P(-1.70 < Z < -1.00)$ .

6.18 Let the random variable  $Z$  follow a standard normal distribution.

- The probability is 0.70 that  $Z$  is less than what number?
- The probability is 0.25 that  $Z$  is less than what number?

- c. The probability is 0.2 that  $Z$  is greater than what number?
- d. The probability is 0.6 that  $Z$  is greater than what number?
- 6.19 Let the random variable  $X$  follow a normal distribution with  $\mu = 50$  and  $\sigma^2 = 64$ .
- Find the probability that  $X$  is greater than 60.
  - Find the probability that  $X$  is greater than 35 and less than 62.
  - Find the probability that  $X$  is less than 55.
  - The probability is 0.2 that  $X$  is greater than what number?
  - The probability is 0.05 that  $X$  is in the symmetric interval about the mean between which two numbers?
- 6.20 Let the random variable  $X$  follow a normal distribution with  $\mu = 80$  and  $\sigma^2 = 100$ .
- Find the probability that  $X$  is greater than 60.
  - Find the probability that  $X$  is greater than 72 and less than 82.
  - Find the probability that  $X$  is less than 55.
  - The probability is 0.1 that  $X$  is greater than what number?
  - The probability is 0.08 that  $X$  is in the symmetric interval about the mean between which two numbers?
- 6.21 Let the random variable  $X$  follow a normal distribution with  $\mu = 0.2$  and  $\sigma^2 = .0025$ .
- Find the probability that  $X$  is greater than 0.4.
  - Find the probability that  $X$  is greater than 0.15 and less than 0.28.
  - Find the probability that  $X$  is less than 0.10.
  - The probability is 0.2 that  $X$  is greater than what number?
  - The probability is 0.05 that  $X$  is in the symmetric interval about the mean between which two numbers?
- e. You want to find a range of dollar spending on textbooks in a year that includes 80% of all students on this campus. Explain why any number of such ranges could be found, and find the shortest one.
- 6.23 Anticipated consumer demand for a product next month can be represented by a normal random variable with mean 1,200 units and standard deviation 100 units.
- What is the probability that sales will exceed 1,000 units?
  - What is the probability that sales will be between 1,100 and 1,300 units?
  - The probability is 0.10 that sales will be more than how many units?
- 6.24 The tread life of a particular brand of tire has a normal distribution with mean 35,000 miles and standard deviation 4,000 miles.
- What proportion of these tires has a tread life of more than 38,000 miles?
  - What proportion of these tires has a tread life of less than 32,000 miles?
  - What proportion of these tires has a tread life of between 32,000 and 38,000 miles?
  - Draw a graph of the probability density function of tread lives, illustrating:
    - Why the answers to parts (a) and (b) are the same
    - Why the answers to parts (a), (b), and (c) sum to 1
- 6.25 An investment portfolio contains stocks of a large number of corporations. Over the last year the rates of return on these corporate stocks followed a normal distribution with mean 12.2% and standard deviation 7.2%.
- For what proportion of these corporations was the rate of return higher than 20%?
  - For what proportion of these corporations was the rate of return negative?
  - For what proportion of these corporations was the rate of return between 5% and 15%?
- 6.26 A company produces bags of a chemical, and it is concerned about impurity content. It is believed that the weights of impurities per bag are normally distributed with mean 12.2 grams and standard deviation 2.8 grams. A bag is chosen at random.
- What is the probability that it contains less than 10 grams of impurities?
  - What is the probability that it contains more than 15 grams of impurities?
  - What is the probability that it contains between 12 and 15 grams of impurities?

### Application Exercises

- 6.22 It is known that amounts of money spent on textbooks in a year by students on a particular campus follow a normal distribution with mean \$380 and standard deviation \$50.
- What is the probability that a randomly chosen student will spend less than \$400 on textbooks in a year?
  - What is the probability that a randomly chosen student will spend more than \$360 on textbooks in a year?
  - Draw a graph to illustrate why the answers to parts (a) and (b) are the same.
  - What is the probability that a randomly chosen student will spend between \$300 and \$400 on textbooks in a year?

- d. It is possible, without doing the detailed calculations, to deduce which of the answers to parts (a) and (b) will be the larger. How?

6.27 A contractor regards the cost of fulfilling a particular contract as a normally distributed random variable with mean \$500,000 and standard deviation \$50,000.

- What is the probability that the cost of fulfilling the contract will be between \$460,000 and \$540,000?
- The probability is 0.2 that the contract will cost less than how much to fulfill?
- Find the shortest range such that the probability is 0.95 that the cost of fulfilling the contract will fall in this range.

6.28 Scores on a test follow a normal distribution. What is the probability that a randomly selected student will achieve a score that exceeds the mean score by more than 1.5 standard deviations?

6.29 A new television series is to be shown. A broadcasting executive feels that his uncertainty about the rating which the show will receive in its first month can be represented by a normal distribution with mean 18.2 and standard deviation 1.6. According to this executive, the probability is 0.1 that the rating will be less than what number?

6.30 A broadcasting executive is reviewing the prospects for a new television series. According to her judgment, the probability is 0.25 that the show will achieve a rating higher than 17.8, and the probability is 0.15 that it will achieve a rating higher than 19.2. If the executive's uncertainty about the rating can be represented by a normal distribution, what are the mean and variance of that distribution?

6.31 Scores on an examination taken by a very large group of students are normally distributed with mean 700 and standard deviation 120.

- An A is awarded for a score higher than 820. What proportion of all students obtain an A?
- A B is awarded for scores between 730 and 820. An instructor has a section of 100 students who can be viewed as a random sample of all students in the large group. Find the expected number of students in this section who will obtain a B.
- It is decided to give a failing grade to 5% of students with the lowest scores. What is the minimum score needed to avoid a failing grade?

6.32 I am considering two alternative investments. In both cases I am unsure about the percentage return but believe that my uncertainty can be represented by normal distributions with the means and standard deviations shown in the accompanying table. I want to make the investment that is more likely to produce a return of at least 10%. Which should I choose?

	Mean	Standard Deviation
Investment A	10.4	1.2
Investment B	11.0	4.0

6.33 A company can purchase raw material from either of two suppliers and is concerned about the amounts of impurity the material contains. A review of the records for each supplier indicates that the percentage impurity levels in consignments of the raw material follow normal distributions with the means and standard deviations given in the following table. The company is particularly anxious that the impurity level in a consignment not exceed 5% and wants to purchase from the supplier more likely to meet that specification. Which supplier should be chosen?

	Mean	Standard Deviation
Supplier A	4.4	0.4
Supplier B	4.2	0.6

6.34 An instructor has found that the time spent by students on a particular homework assignment follows a normal distribution with mean 150 minutes and standard deviation 40 minutes.

- The probability is 0.9 that a randomly chosen student spends more than how many minutes on this assignment?
- The probability is 0.8 that a randomly chosen student spends less than how many minutes on this assignment?
- Two students are chosen at random. What is the probability that at least one of them spends at least 2 hours on this assignment?

6.35 A company services copiers. A review of its records shows that the time taken for a service call can be represented by a normal random variable with mean 75 minutes and standard deviation 20 minutes.

- What proportion of service calls takes less than 1 hour?
- What proportion of service calls takes more than 90 minutes?
- Sketch a graph to show why the answers to parts (a) and (b) are the same.
- The probability is 0.1 that a service call takes more than how many minutes?

6.36 Scores on an achievement test are known to be normally distributed with mean 420 and standard deviation 80.

- For a randomly chosen person taking this test, what is the probability of a score between 400 and 480?
- What is the minimum test score needed in order to be in the top 10% of all people taking the test?



- c. For a randomly chosen individual, state, without doing the calculations, in which of the following ranges his or her score is most likely to be: 400–439, 440–479, 480–519, or 520–559.
- d. In which of the ranges listed in part (c) is the individual's score least likely to be?
- e. Two people taking the test are chosen at random. What is the probability that at least one of them scores more than 500 points?
- 6.37 It is estimated that the time that a well-known rock band, the Living Ingrates, spends on stage at their concerts follows a normal distribution with mean 200 minutes and standard deviation 20 minutes.
- a. What proportion of concerts played by this band lasts between 180 and 200 minutes?
- b. An audience member smuggles a tape recorder with reel-to-reel tapes with a capacity of 245 minutes into a Living Ingrates concert. What is the probability that this capacity will be insufficient to record the entire concert?
- c. If the standard deviation of concert time was only 15 minutes, state, without doing the calculations, whether the probability that a concert would last more than 245 minutes would be larger than, smaller than, or the same as that found in part (b). Sketch a graph to illustrate your answer.
- d. The probability is 0.1 that a Living Ingrates concert will last less than how many minutes? (Assume, as originally, that the population standard deviation is 20 minutes.)
- 6.38 An economics test is taken by a large group of students. The test scores are normally distributed with mean 70, and the probability that a randomly chosen student receives a score less than 85 is 0.9332. Four students are chosen at random. What is the probability that at least one of them scores more than 80 points on this test?

**Figure 6.**  
Binomial  
Distribution  
with  $n = 100$   
 $P = 0.50$

## 6.4 NORMAL DISTRIBUTION APPROXIMATION FOR BINOMIAL DISTRIBUTION

In this section we show how the normal distribution can be used to approximate the discrete binomial and proportion random variables that are used extensively in business and economics. This approximation can be used to compute probabilities for larger sample sizes when tables are not readily available. The normal distribution approximation of the binomial distribution also provides a benefit for applied problem solving. We learn that procedures based on the normal distribution can also be applied in problems involving binomial and proportion random variables. Thus, you can reduce the number of different statistical procedures that you need to know to solve business problems.

Let us consider a problem with  $n$  independent trials, each with probability of success  $P$ . In Section 5.4 we saw that the binomial random variable  $X$  could be written as the sum of  $n$  independent Bernoulli random variables:

$$X = X_1 + X_2 + \cdots + X_n$$

where the random variable  $X_i$  takes the value 1 if the outcome of the  $i$ th trial is "success" and 0 otherwise, with respective probabilities  $P$  and  $1 - P$ . The number  $X$  of successes that result has a binomial distribution with mean and variance

$$\begin{aligned} E(X) &= \mu = nP \\ \text{Var}(X) &= \sigma^2 = nP(1 - P) \end{aligned}$$

The plot of a binomial distribution with  $P = 0.5$  and  $n = 100$ , in Figure 6.23, shows us that the binomial has the same shape as the normal. This visual evidence that the binomial can be approximated by a normal distribution with the same mean and variance is also established in work done by mathematical statisticians. A good rule

portfolio will have a larger variance and hence a higher risk. And if the two stock prices are negatively correlated, then the resulting portfolio will have a smaller variance and hence a lower risk. The term *hedging* is often used by fund managers to describe this phenomenon. This important principle for a two-stock portfolio extends directly to a portfolio with a large number of different stocks, but in that case the algebra is more complex and is typically computed using a sophisticated computer program. Investment fund managers use this principle to select combinations of many different stocks in order to obtain the desired portfolio value and risk characteristics that are the objectives for a particular investment fund.

## EXERCISES

### Basic Exercises

- 6.59 A random variable  $X$  is normally distributed with mean 100 and variance 100, and a random variable  $Y$  is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to 0.5. Find the mean and variance of the random variable

$$W = 5X + 4Y$$

- 6.60 A random variable  $X$  is normally distributed with mean 100 and variance 100, and a random variable  $Y$  is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to  $-0.5$ . Find the mean and variance of the random variable

$$W = 5X + 4Y$$

- 6.61 A random variable  $X$  is normally distributed with mean 100 and variance 100, and a random variable  $Y$  is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to 0.5. Find the mean and variance of the random variable

$$W = 5X - 4Y$$

- 6.62 A random variable  $X$  is normally distributed with mean 500 and variance 100, and a random variable  $Y$  is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to 0.5. Find the mean and variance of the random variable

$$W = 5X - 4Y$$

- 6.63 A random variable  $X$  is normally distributed with mean 100 and variance 500, and a random variable  $Y$  is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to  $-0.5$ . Find the mean and variance of the random variable

$$W = 5X - 4Y$$

### Application Exercises

- 6.64 An investor plans to divide \$200,000 between two investments. The first yields a certain profit of 10%, while the second yields a profit with expected value 18% and standard deviation 6%. If the investor divides the money equally between these two investments, find the mean and standard deviation of the total profit.
- 6.65 A homeowner has installed a new energy-efficient furnace. It is estimated that over a year the new furnace will reduce energy costs by an amount that can be regarded as a random variable with mean \$200 and standard deviation \$60. Stating any assumptions you need to make, find the mean and standard deviation of the total energy cost reductions over a period of 5 years.
- 6.66 A consultant is beginning work on three projects. The expected profits from these projects are \$50,000, \$72,000, and \$40,000. The associated standard deviations are \$10,000, \$12,000, and \$9,000. Assuming independence of outcomes, find the mean and standard deviation of the consultant's total profit from these three projects.
- 6.67 A consultant has three sources of income—from teaching short courses, from selling computer software, and from advising on projects. His expected annual incomes from these sources are \$20,000, \$25,000, and \$15,000, and the respective standard deviations are \$2,000, \$5,000, and \$4,000. Assuming independence, find the mean and standard deviation of his total annual income.
- 6.68 Five inspectors are employed to check the quality of components produced on an assembly line. For each inspector the number of components that can be checked in a shift can be represented by a random variable with mean 120 and standard deviation 16. Let  $X$  represent the number of components checked by an inspector in a shift. Then the total number checked is  $5X$ , which has mean 600 and standard deviation 80. What is wrong with this argument? Assuming that inspectors' performances are independent of one