MATH 160

EXAM 3

30 March 2012

Your name:

Pledge:

There are 9 problems, and the point values of each problem are shown. A perfect score is 100 points. Calculator use is not permitted. I'll be in my office (Arter 104B) during the test if you have questions.

Good luck!!

1. (6 points) Give an example of a continuous function which illustrates that the converse of Fermat's Theorem is false. In other words, give an example of a continuous function with a critical point that does not correspond to a local maximum or a local minimum. (No explanation is required.)

$$f(x) = x^3$$

2. (12 points) Find the absolute maximum and absolute minimum values for the function $f(x) = x^3 - 6x^2 + 9x$ on the interval $[-1, 2]$.

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3).$$

CP: $x = 1$, $\cancel{x = 3}$ NOT IN $[-1,2]$.

$f(-1) = -16$ absolute min

$f(1) = 4$ absolute max

$f(2) = 2$

3. (10 points) Give the precise statement of the Mean Value Theorem.

Let f be continuous on $[a,b]$ and differentiable on (a,b).

Then there exists a number c in (a,b) for which

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

4. (12 points) Use the Mean Value Theorem to show that the equation

$$5x + \cos x = 0$$

has at most one real root.

Let $f(x) = 5x + \cos x$. Assume f has two roots, a and b.

By MVT, there exists c with $f'(c) = \frac{f(b)-f(a)}{b-a}$. Then $f'(c) = 0$.

But $f'(x) = 5 - \sin x$. So $0 = 5 - \sin(c)$, so $\sin(c) = 5$.

Since this is impossible, f cannot have two roots.

5. (12 points) Find the intervals where the function $f(x) = 9x^2 - 2x^3 + 3$ is increasing and where it is decreasing.

$$f'(x) = 18x - 6x^2 = 6x(3-x) \qquad CP: x = 0, x = 3$$

f': $-$ | $+$ | $-$

f: $\searrow$ 0 $\nearrow$ 3 $\searrow$

f decreases on $(-\infty, 0)$ and $(3, \infty)$

f increases on $(0, 3)$

6. (12 points) Find the intervals where the function $f(x) = x^{4/3} + 4x^{1/3}$ is concave up and where it is concave down.

$$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3}$$

$$f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3} = \frac{4}{9} \cdot \frac{1}{x^{5/3}}(x-2)$$

f'': $+$ | $-$ | $+$

f: $\cup$ 0 $\cap$ 2 $\cup$

f concave up on $(-\infty, 0)$ and $(2, \infty)$

f concave down on $(0, 2)$

7. (12 points) Draw the graph of the function

$$f(x) = \frac{x+2}{x-2}$$

and label all intercepts, asymptotes, relative extrema, and inflection points. To save time, I've computed the first and second derivatives for you:

$$f'(x) = \frac{-4}{(x-2)^2} \qquad f''(x) = \frac{8}{(x-2)^3}.$$

I. Intercepts:

$(0,-1)$ y-int $\qquad$ $(-2,0)$ x-int

II. Asymptotes:

$\lim\limits_{x \to \pm\infty} \frac{x+2}{x-2} = 1$ $\qquad$ $y = 1$ HA

$\lim\limits_{x \to 2^+} \frac{x+2}{x-2} = +\infty$ $\qquad$ $x = 2$ VA

$\lim\limits_{x \to 2^-} \frac{x+2}{x-2} = -\infty$

III. Rel. Ext.:

f': $-$ | $-$ (at 2)

f: $\searrow$ 2 $\searrow$

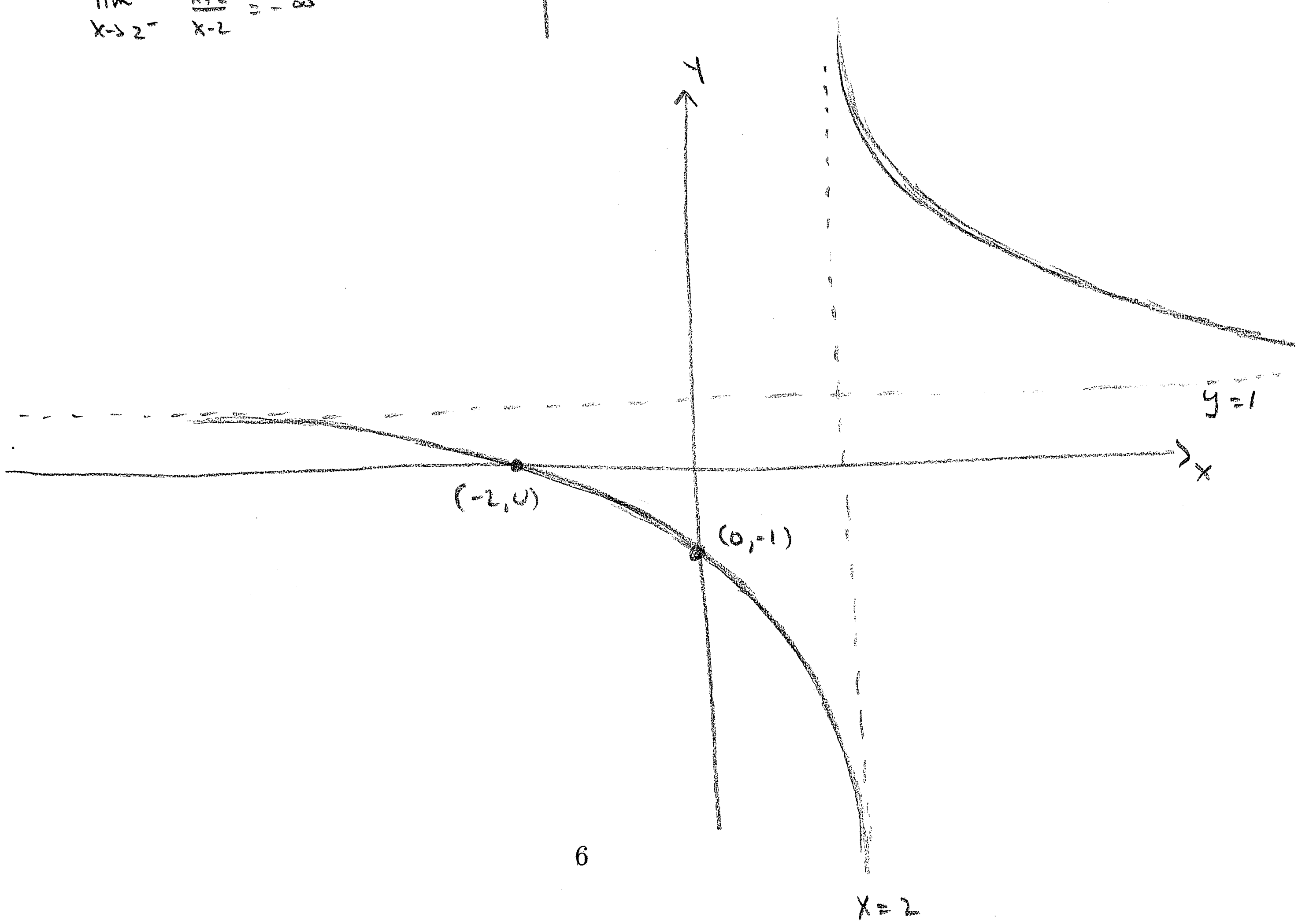
No rel. ext.

III. IP's:

f'': $-$ | $+$ (at 2)

f: $\cap$ 2 $\cup$

No IP's



8. (12 points) Draw the graph of the function

$$f(x) = x^{1/3}(x-4)$$

and label all intercepts, asymptotes, relative extrema, and inflection points. To save time, I've computed the first and second derivatives for you:

$$f'(x) = \frac{4(x-1)}{3x^{2/3}} \qquad f''(x) = \frac{4(x+2)}{9x^{5/3}}.$$

I. Intercepts

$(0,0)$ y-int $(0,0), (4,0)$ x-int

II. Asymptotes

$\lim_{x \to \pm\infty} x^{1/3}(x-4) = +\infty$ NO HA

NO VA

III. Rel. Ext.:

f': $-$ | $-$ | $+$

f: $\searrow$ 0 $\searrow$ 1 $\nearrow$

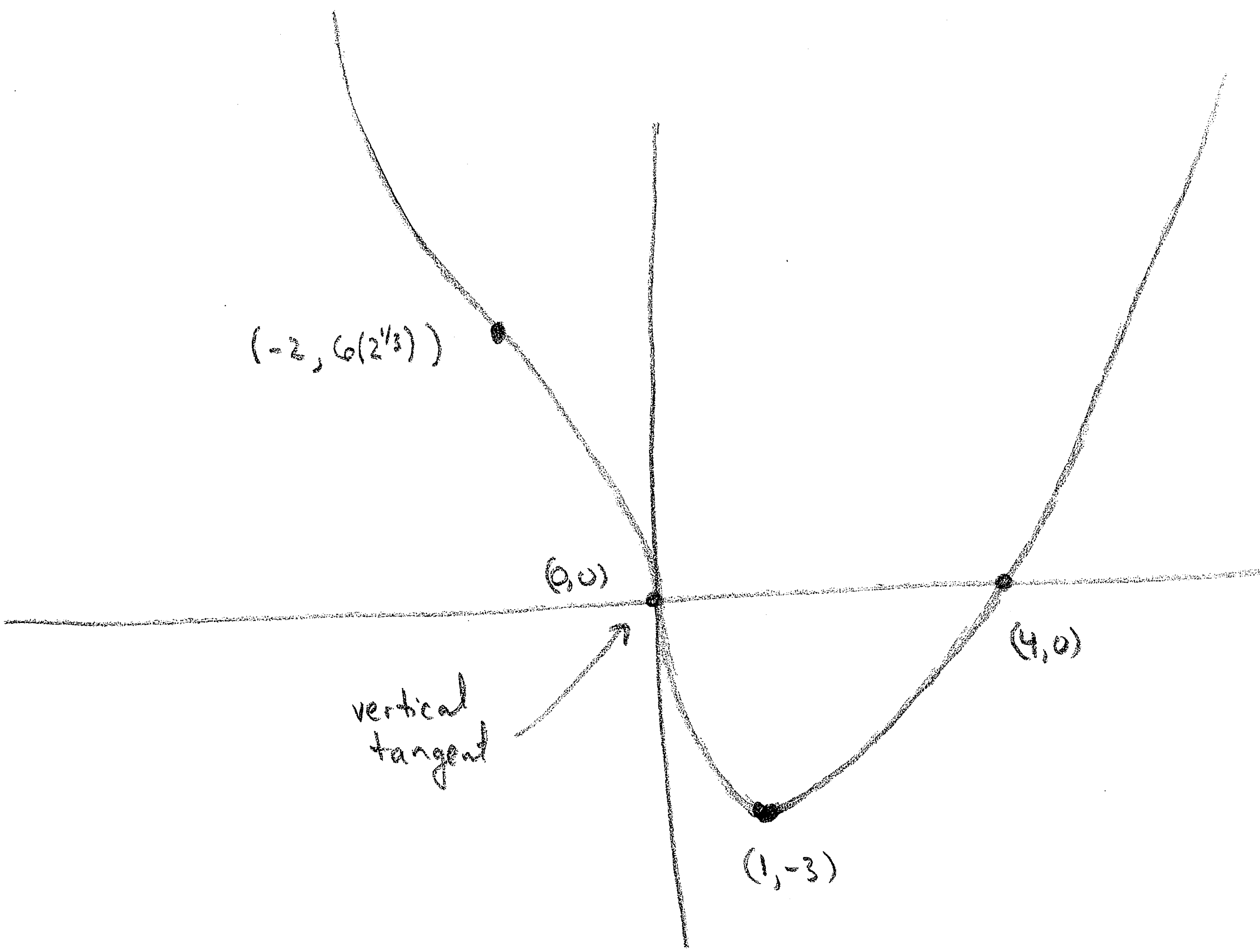
$(1,-3)$ rel. min

IV. IP's:

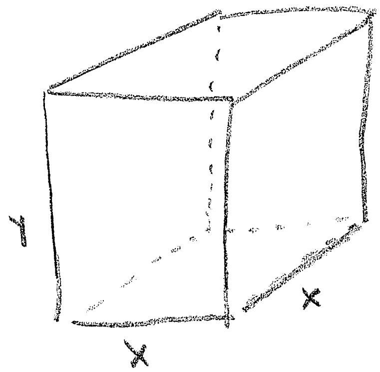
f'': $+$ | $-$ | $+$

f: $\cup$ -2 $\cap$ 0 $\cup$

$(0,0)$, $(-2, 6(2^{1/3}))$ IP.



9. (12 points) If 2400 square feet of material is available to make a closed rectangular box (which includes all four sides, the top, and the bottom) with a square base, what are the dimensions of the box with the largest possible volume?



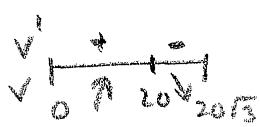
Maximize volume: $V = x^2 y$.

Given: $2x^2 + 4xy = 2400$

$x^2 + 2xy = 1200$, $y = \dfrac{1200 - x^2}{2x}$.

$V = x^2\left(\dfrac{1200 - x^2}{2x}\right) = \frac{1}{2}(1200x - x^3)$, $0 < x < 20\sqrt{3}$

$V'(x) = \frac{1}{2}(1200 - 3x^2) = \frac{3}{2}(400 - x^2)$. CP: $x = 20$.



V is maximized when $x = 20$, $y = \dfrac{1200 - 20^2}{2 \cdot 20} = 20$.

Dimensions: $20' \times 20' \times 20'$