A Tangram Puzzle Solver in Common Lisp

Michael Wessel

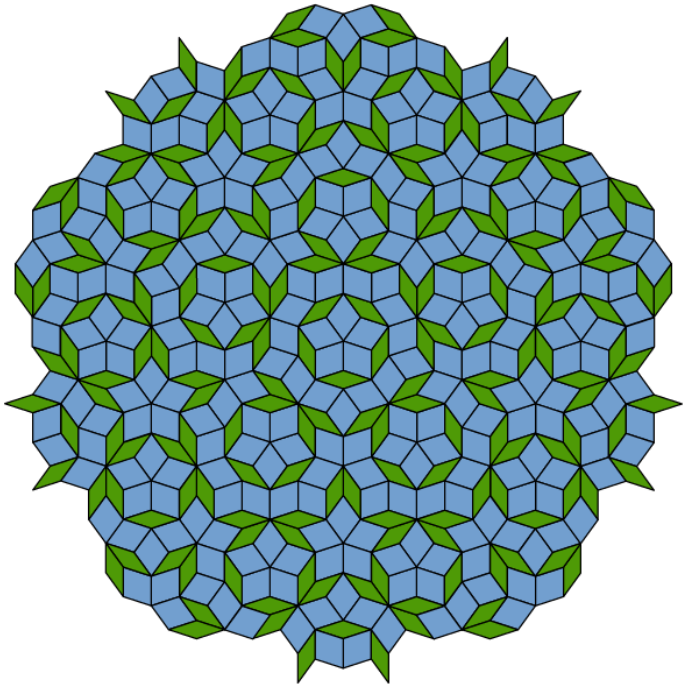
May 3rd 2021

ELS‘21

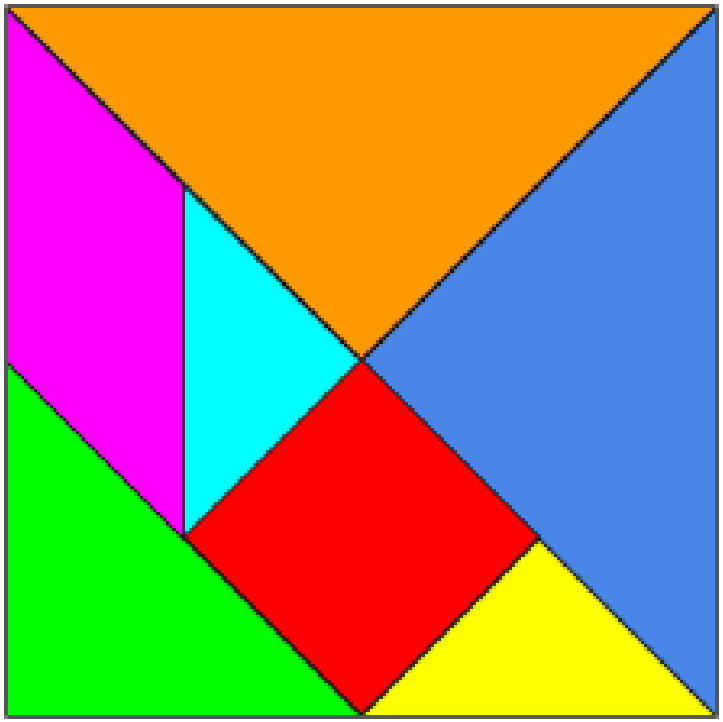
Talk Outline

- **Disclaimer**: this is a demo talk
  - Video of program in action & some Lisp code
  - No deep theoretical insights, but some practical insights
  - This is a fun recreational spare time project
  - Not associated with my employer in any way
- The program was written in 2003, reactivated now
  - To get back to Lisp – **Q: Would I still be able to maintain and extent my Lisp code from almost 20 years ago?**
  - To **finish what was started** – the original program had some deficiencies & bugs
  - To **catch up with the Lisp community**
    (paper motivation – last time I contributed was 15 years ago)
  - Because of a **recent book that really fascinated me** (see at the end)

Tiling Problems are Fascinating!
Penrose Tiling & Tangram Set



Aperiodic Penrose Tiling
(Extends Infinitely)
Source : Wikipedia CC

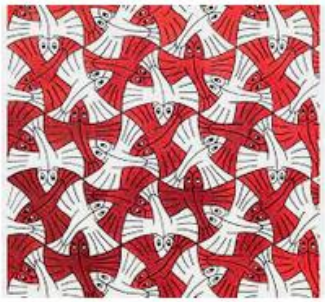


Tangram Tiling
(Finite)
Source: Wikipedia CC

Google tiling escher

Dog Pattern | Schnauzer | M. C. E...
pinterest.com

How to make an Escher tiling
static1.squarespace.com



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Tessellation Patterns - From ...
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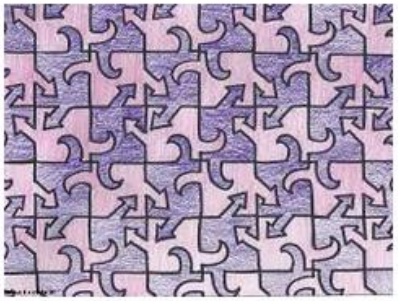

Tessellation art ...
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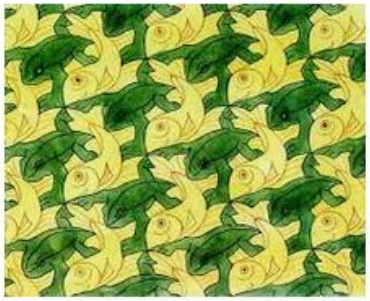

Tessellations - The Curious Question...
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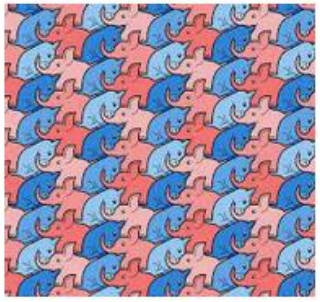

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tangram - Google Search

https://www.google.com/search?q=tangram&tbm=isch&ved=2ahUKEwj

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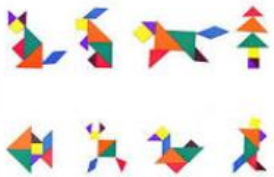

Colorful Wooden Tangram Brain

$3.88

eBay

Free shipping

Wooden Tangrams Puzzles, 28 Pieces

$9.99

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Tangram S Sticker | R

$2.50

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Tangram - Wikipedia

en.wikipedia.org

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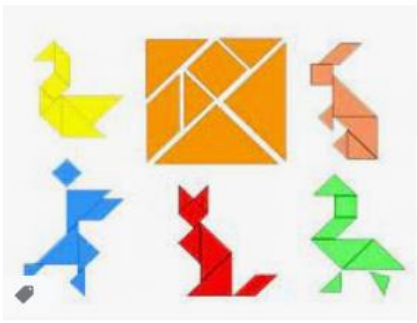

tangram puzzle in wood - Logica Puzz...

logicagiochi.com

Wooden Tangrams Puzzles, 28 Pie...

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tangram bat puzzle - Google S

https://www.google.com/search?q=tangram+bat+puzzle&client=firefox

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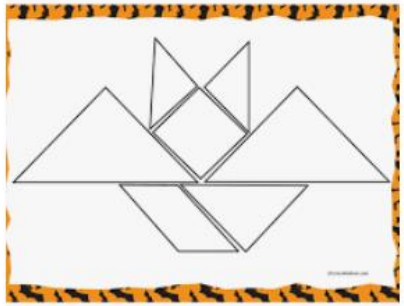

Bat Tangram Puzzle Puzzles Handma...
alsgroup.cl

Make A Tangram Bat | Tangram, Tangram ...
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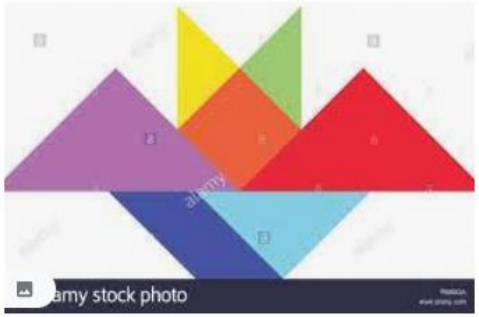
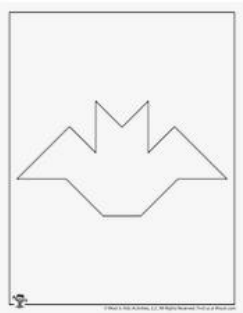
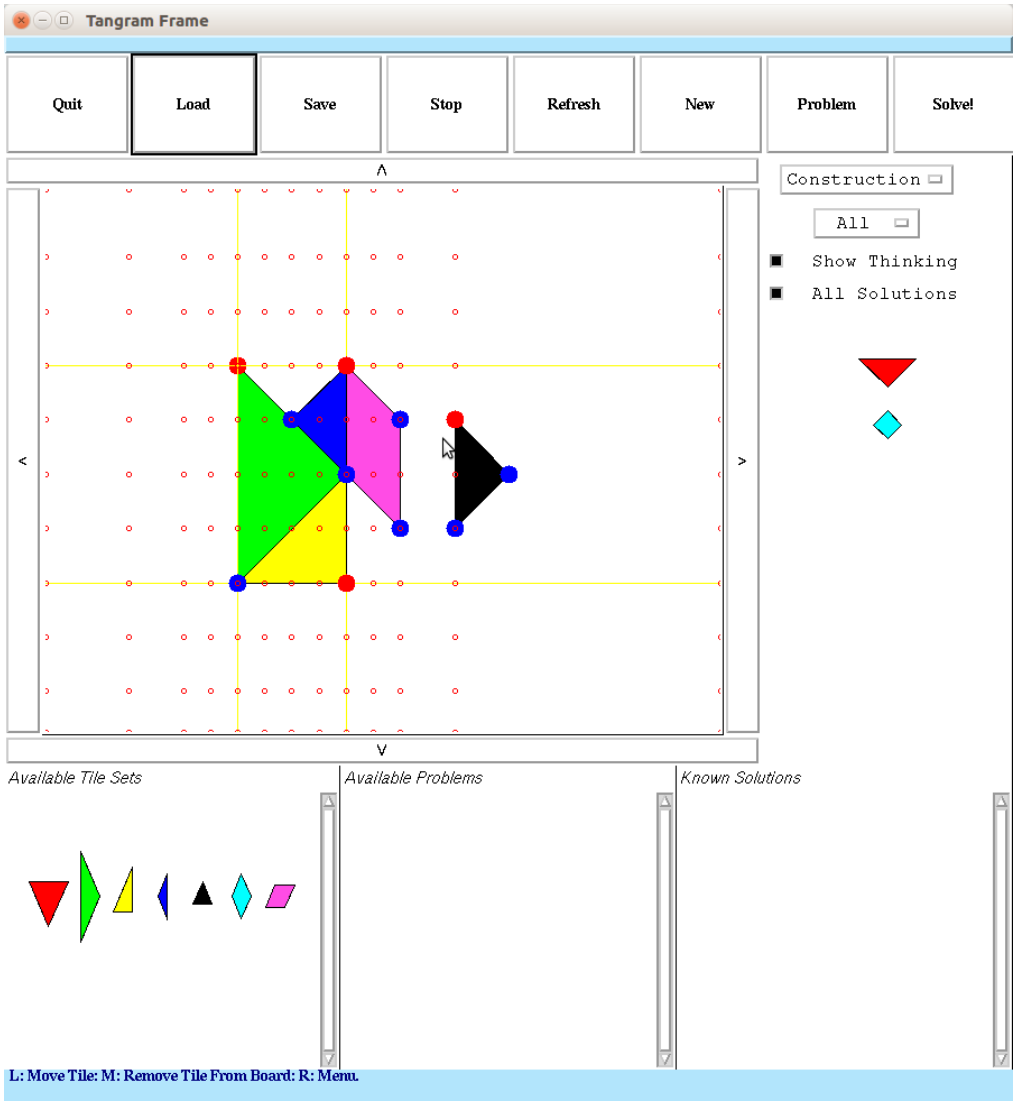


Color tangram puzzle in bat shape on ...
alamy.com



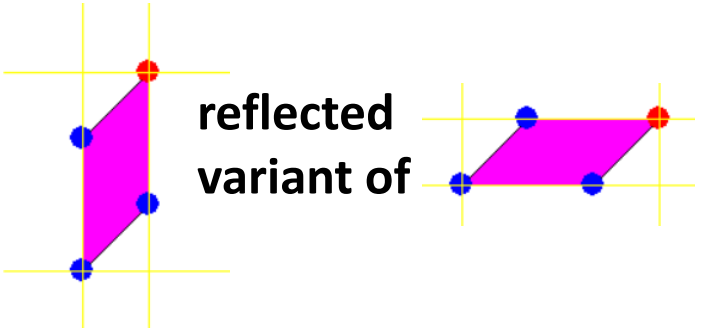
Bat Halloween Tangra...
woojr.com

Editor Demo Video

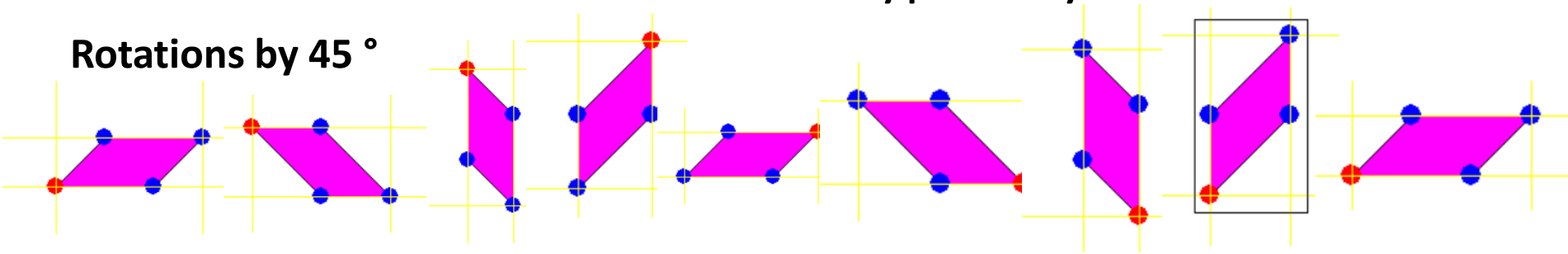


Rules of the Tangram Game

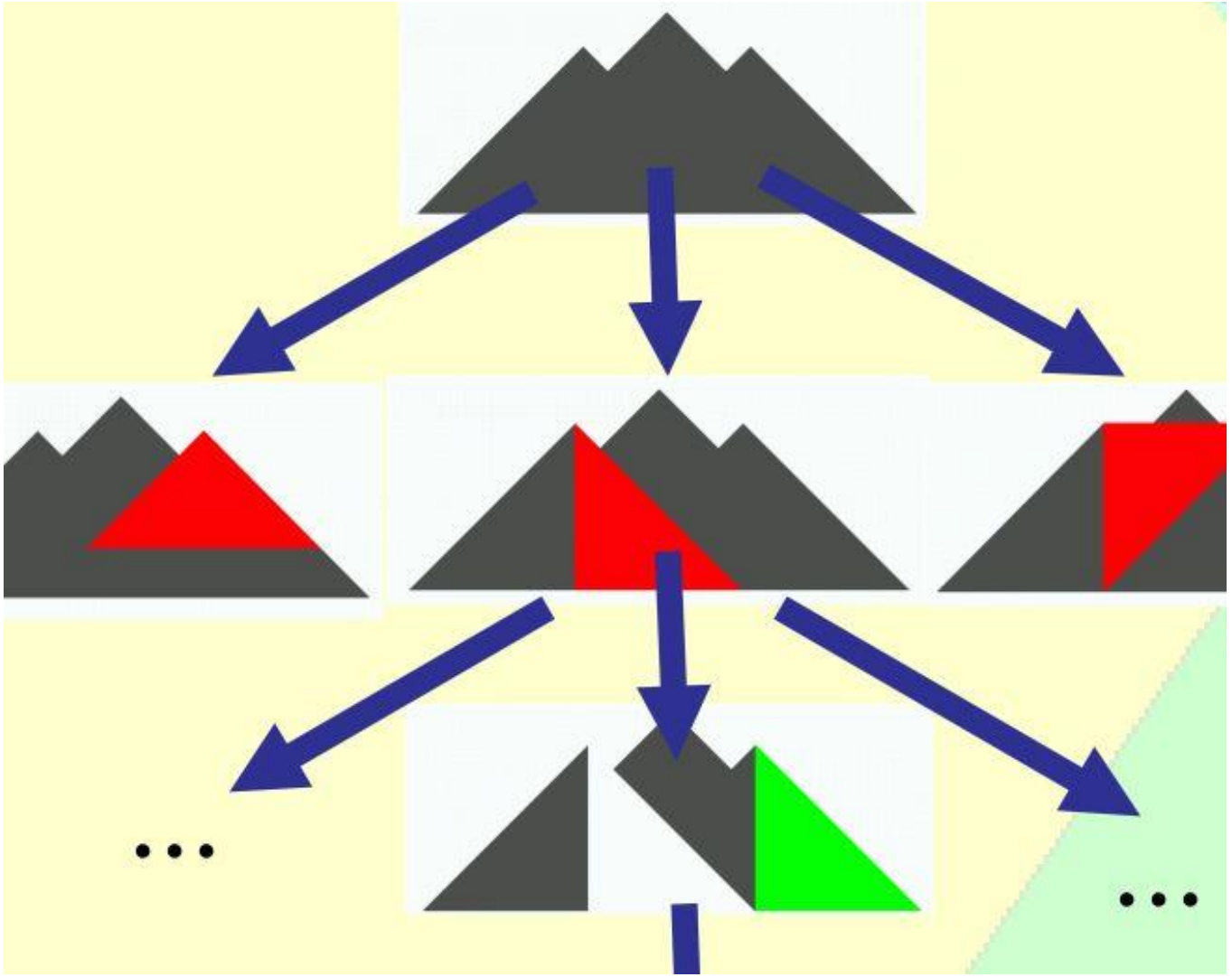
- Each tile must be used EXACTLY once
- The silhoutte / outline polygon must be partioned completely
  - no remaining space
  - no overlaps between tiles
- Tiles can be translated, rotated and reflected (flipped over)
- The reflected variants of each tile (modulo congruency) constitute a **TileType**; only ONE reflected variant of each TileType may be used



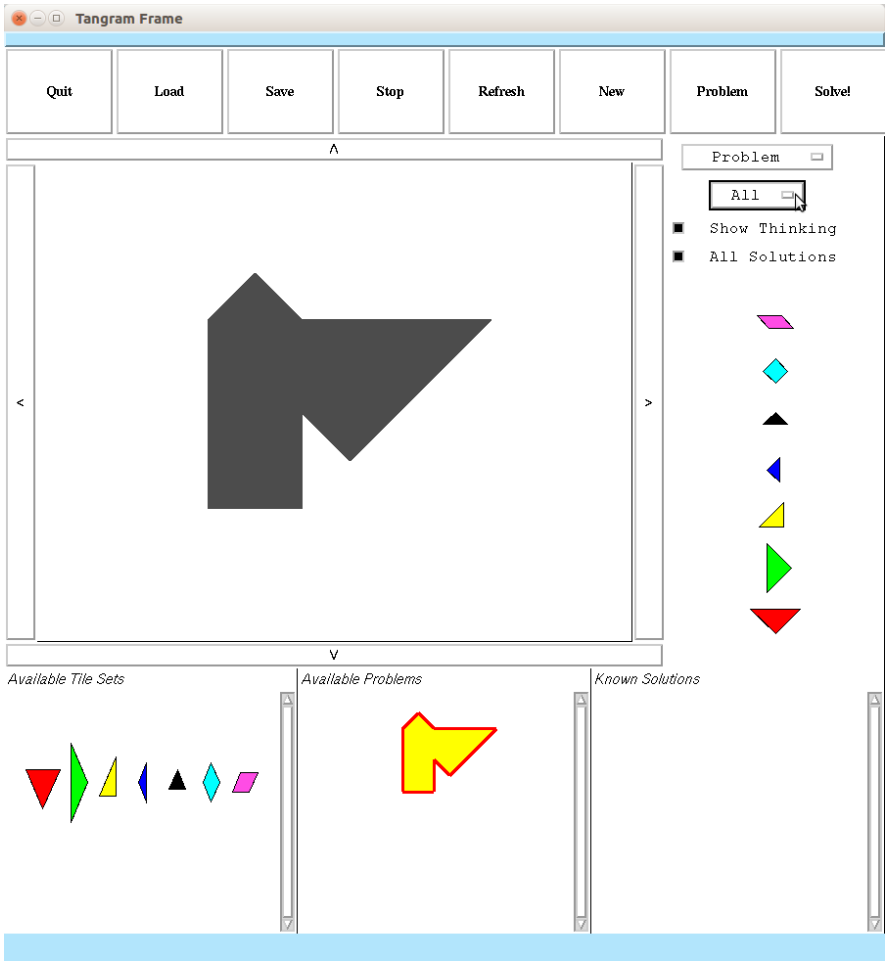
Rotations by 45 °

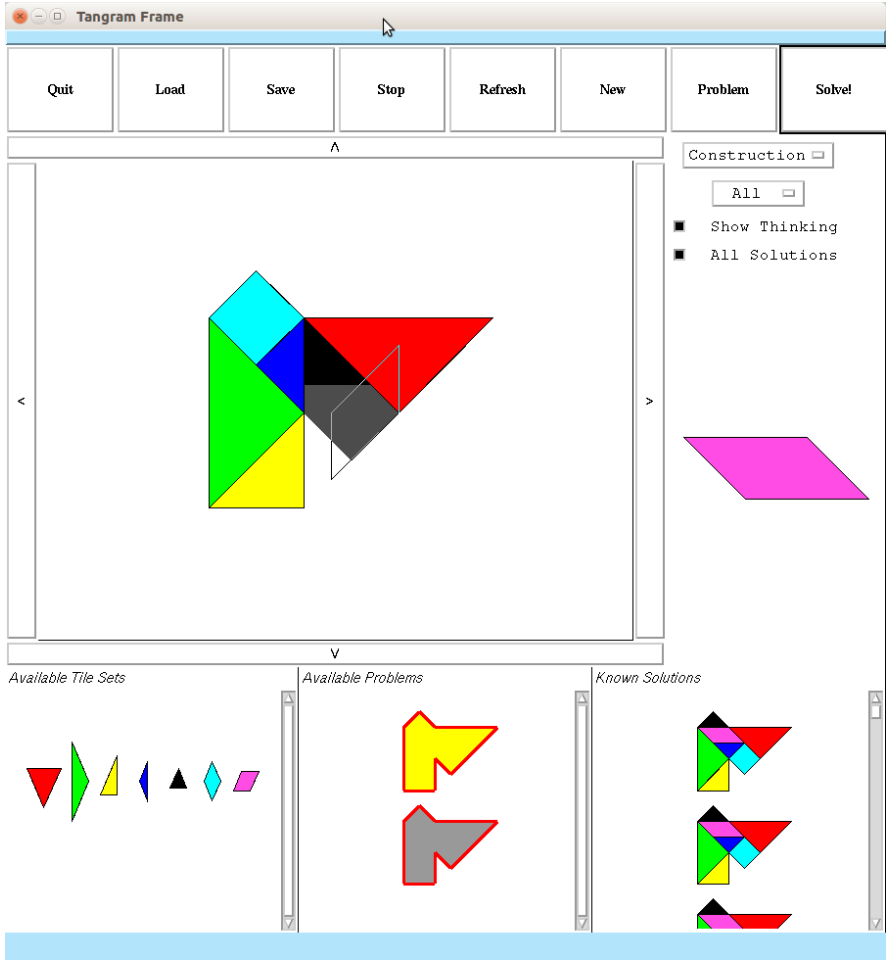


Solving Tiling Problems by Geometric Search



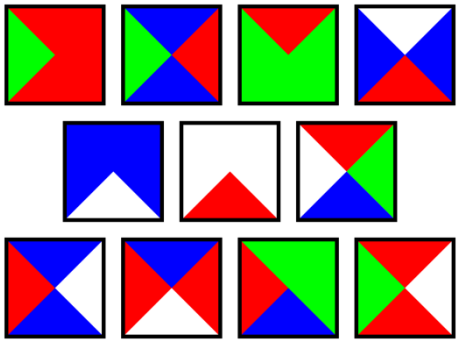
Solver Demo Video



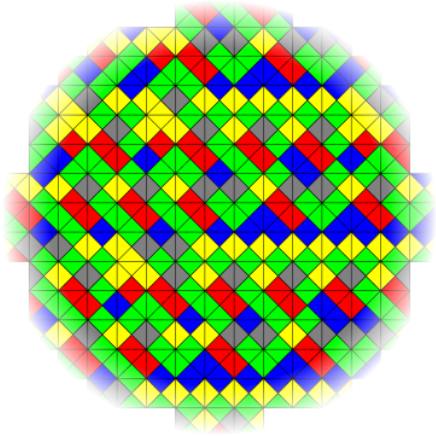


Comments on Problem Complexity

- It is undecidable if an arbitary set of (Wang domino) tiles tiles the (infinte) plane
  - reduction to Halting Problem of Turing Machines
- Wang 1961:
  - Conjecture: if a set of finite tiles tiles the (infinite) plane, then there also exists a **periodic** tiling
  - If the Conjecture is true, then it implies the existence of a decision algorithm
- Berger 1966 (Wang‘s student):
  - That algorithm does not exist – hence, the conjecture is wrong
  - What‘s wrong with it? There also exist **aperiodic** tilings
  - See 11 Wang Dominos (Jeandel and Rao 2015)
  - Penrose tilings
- Tangram is NP-hard
- Certain variants are NP-complete (e.g., symmetric puzzles)



Wang Dominos (Aperiodic)
Source: Wikipedia CC



Wang Tesselation with 13 Tiles
Source: Wikipedia CC

Search Function – 1 / 3

```
(defun find-covering (polygon-set tile-types &key (remove-used-tiles-p t)
                                  frame
                                  all-solutions-p
                                  allow-less-tiles-p
                                  show-thinking-fn)

  (let ((solutions nil))

    (labels ((goal-p (polygons tile-types)
               (and (not polygons)
                    (or allow-less-tiles-p
                        (not tile-types))))

             (do-it (polygons tile-types history)

               (incf *inc-counter*)

               (cond ((goal-p polygons tile-types)

                      (push
                       (list polygons tile-types history)
                       solutions)

                      (when show-thinking-fn
                        (funcall show-thinking-fn polygons tile-types history t))

                      (unless all-solutions-p
                        (return-from find-covering solutions)))
```

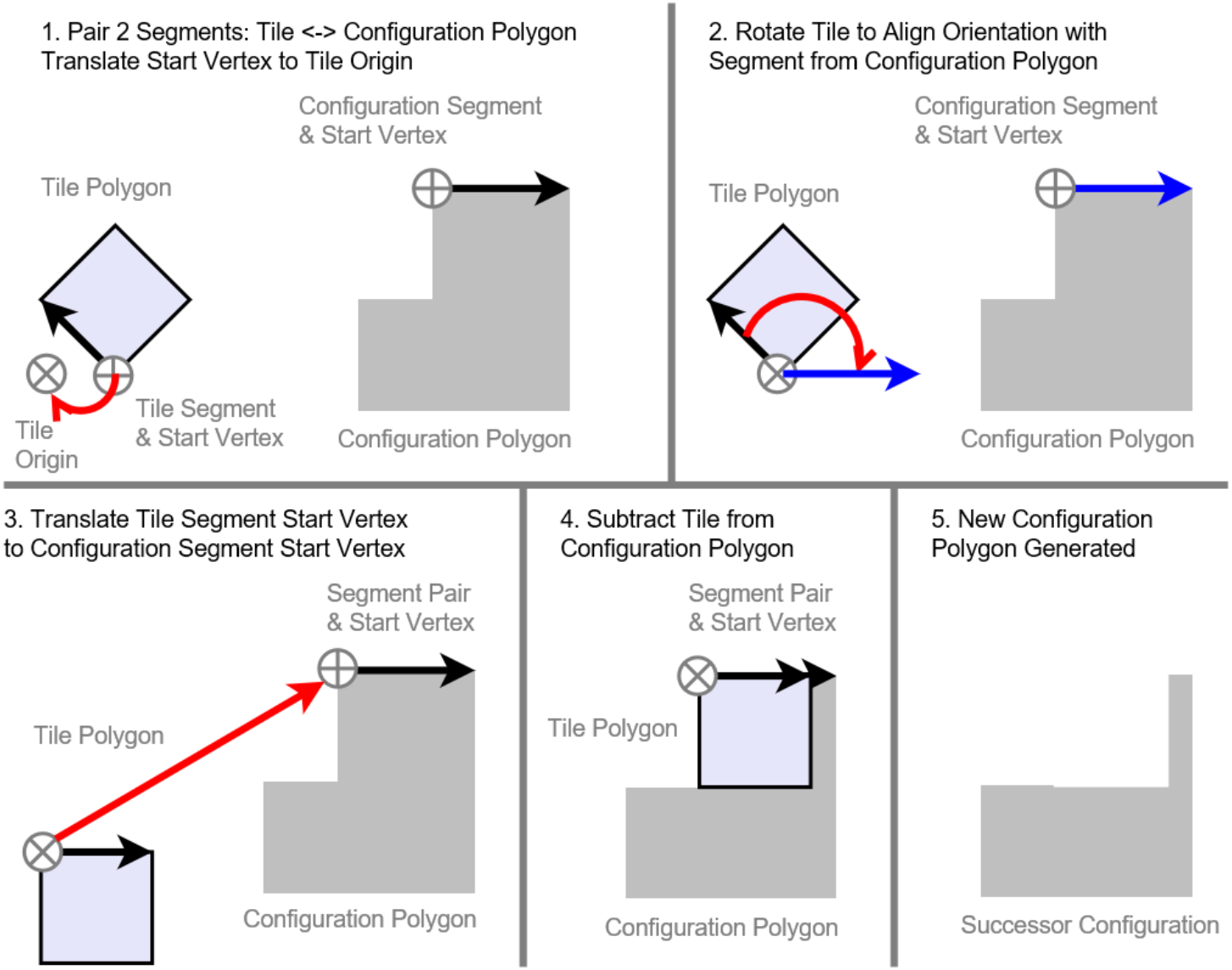
Search Function – 2 / 3

```
(t

 (when show-thinking-fn (funcall show-thinking-fn polygons tile-types history nil))

 #+:ignore
 (sleep 1)

 (let* ((alignments nil)
        (perfect-alignments nil))

   (dolist (polygon polygons)
     (dolist (tile-type tile-types)
       ;; tile-type is the set of reflected variants of that tile
       (let ((rem-tile-types
              (if remove-used-tiles-p
                  (remove tile-type tile-types)
                tile-types)))
         (dolist (tile tile-type)
           (dolist (succ-conf
                    (find-possible-alignments polygon (shape tile)
                                              :frame frame
                                              :color (color tile)
                                              :score-p t
                                              :rem-tile-types rem-tile-types))
             ;; succ-conf = (list h aligned-poly succ-conf orig-poly rem-tile-types)
             (if (eq (third succ-conf) :match)
                 (push (list succ-conf tile ) perfect-alignments)
               (push (list succ-conf tile ) alignments))))))))


   (let ((sorted (append (sort perfect-alignments #'tuple-> :key #'caar)
                         (subseq (sort alignments #'tuple-> :key #'caar)
                                 ;; best n only ?
                                 0 (length alignments)))))
```

Search Function – 3 / 3

```
(let ((sorted (append (sort perfect-alignments #'tuple-> :key #'caar)
                      (subseq (sort alignments #'tuple-> :key #'caar)
                              ;; best n only ?
                              0 (length alignments)))))

  (push (list *inc-counter* (length tile-types) (length sorted)) *statistics*)

  (loop as ((h aligned-poly succ-conf orig-poly rem-tile-types) tile) in sorted do

        (let* ((rem-polygons (remove orig-poly polygons)))
          (do-it (if (eq succ-conf :match)
                     rem-polygons
                   (append succ-conf rem-polygons))
                 rem-tile-types
                 (cons (list aligned-poly tile) history)))))))))

(do-it polygon-set tile-types nil)

solutions)))
```

Successor Generator Function

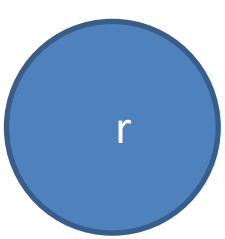


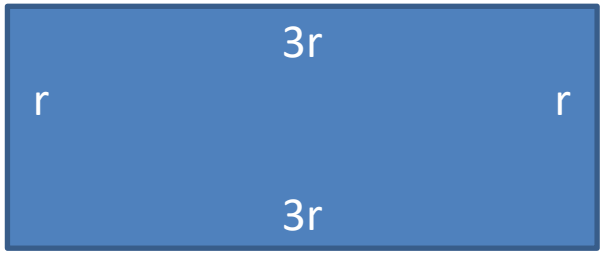
Geometric Heuristics

- **Tile area** (biggest first)
- **Compactness** of successor polygon *P*

$$compactness(P) = \frac{circumference(P)}{\sqrt{area(P)}}$$

```
(defun compactness (poly)
  (with-no-matrix-at-all
    (/
     (loop as s in (segments poly)
           sum (length-of-line s))
     (sqrt (calculate-area poly)))))
```

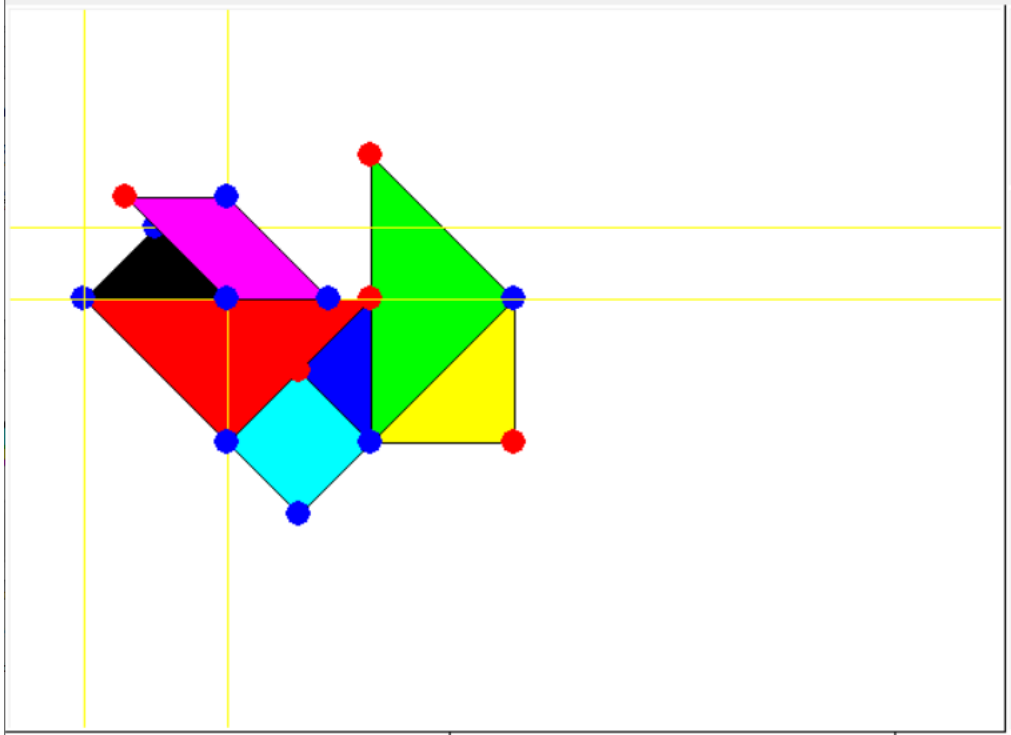




$\frac{2\pi r}{\sqrt{\pi r^2}} = 2\sqrt{\pi} \sim 3.5$ $\frac{4r}{\sqrt{r^2}} = 4$ $\frac{8r}{\sqrt{3r^2}} \sim 4.6$ $\frac{20}{\sqrt{14}} \sim 5.3$

Tangram Frame

Quit | Load | Save | Stop | Refresh | New | Problem | Solve!



Construction

All

Show Thinking

All Solutions

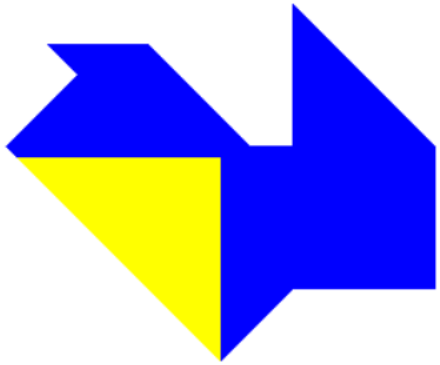
Available Tile Sets

Available Problems

Known Solutions

Tangram Frame

Quit | Load | Save | Stop | Refresh | New | Problem | Solve!



Construction

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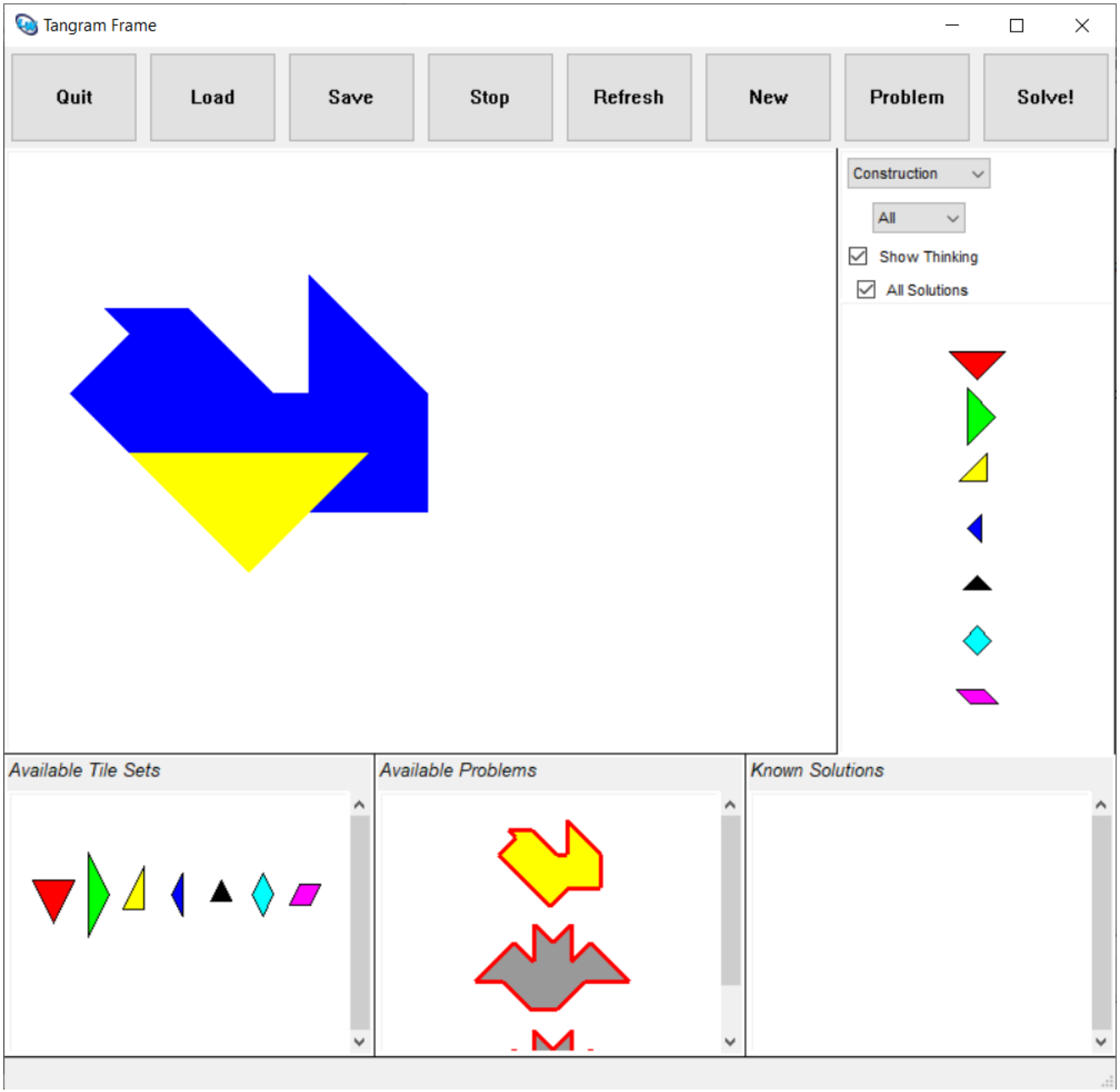
Tangram Frame
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Problem
Solve!
Construction
All
Show Thinking
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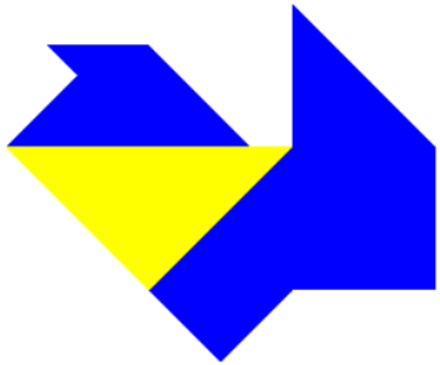

Construction

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Geometric Heuristics Continued

```
(let ((sorted (append (sort perfect-alignments #'tuple-> :key #'caar)
                      (subseq (sort alignments #'tuple-> :key #'caar)
                              ;; best n only ?
                              0 (length alignments)
                              ))))
```

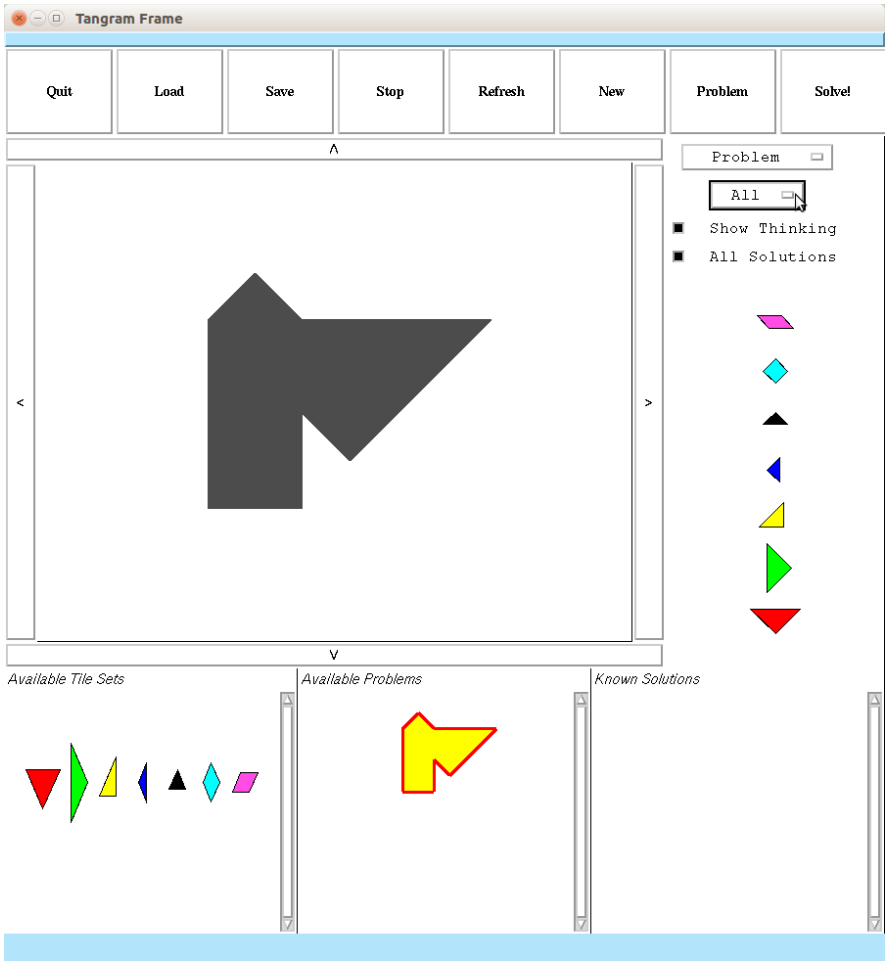
- How to compare two successor configurations wrt. different criteria?
  - Comparing **Apples with Pears**
  - Combine into one score
    - Weighted sum
    - Difficult, not normalized
  - Use **ordered list of criteria instead lexicographic ordering (see tuple->)**
  - **Score = (tile size, -compactness, - #interior vertices, …)**

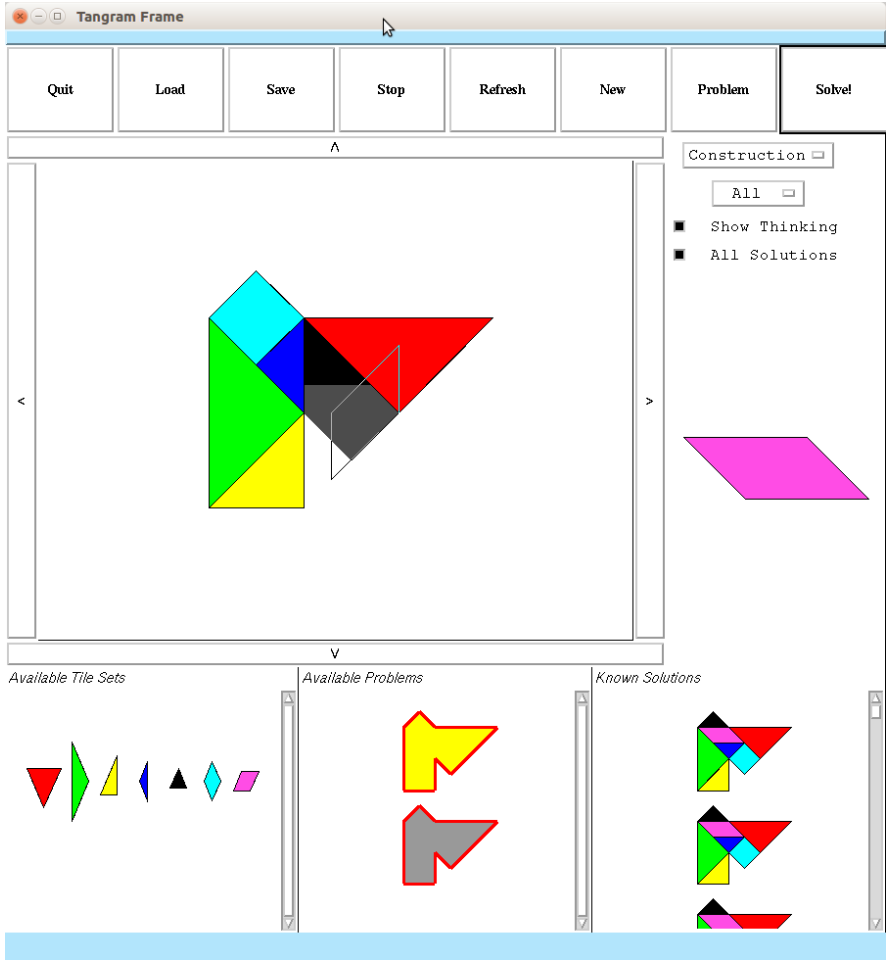
```
(defun tuple-> (a b)
  (when (and a b)
    (cond ((> (first a) (first b))
           t)
          ((= (first a) (first b))
           (tuple-> (rest a) (rest b)))
          (t nil))))
```

Scoring Function

```
(defun alignment-result-heuristic-for-polygon (aligned-polygon conf orig-polygon rem-tile-types)
  (cond ((symbolp conf) ; :match!

         (list (tangram-geometry:calculate-area aligned-polygon) 0))

        ((some #'(lambda (poly)
                   (every #'(lambda (tile-type)
                              (let ((tile (first tile-type))) ; use first reflected variant
                                (< (+ (tangram-geometry:calculate-area poly) 2)
                                   ;; account for inaccuracies...
                                   (tangram-geometry:calculate-area (shape tile)))))
                          rem-tile-types))
               conf)
         :bad )

        (t

         ;; score list for lexicographic tuple->-p sorting

         (list
          (tangram-geometry:calculate-area aligned-polygon)
          (- (count-if #'(lambda (p)
                           (inside-p p orig-polygon))
                       (point-list aligned-polygon)))
          (apply #'min (mapcar #'(lambda (x) (- (compactness x))) conf))
          (- (reduce #'+ (mapcar #'length (mapcar #'segments conf))))))))
```

Demo Video Heuristics





Further Essential Tricks

- Remove congruent successor configurations
  - Congruent configurations (the same configuration being generated from different input) occur frequently
  - Tiles have rotational symmetries
  - There are congruent tile types
  - Don‘t explore the same search space regions repeatedly
  - Reduce branching factor / search space
- Remove configurations that have polygons which are smaller than the smallest (remaining) tile
  - Backtrack early
  - Avoid trashing

Performance & Heuristics

Problem	Optimizations A & B	Optimizations A & ! B	Optimizations ! A & ! B
Standard Problem Solution Found at Branching Factor First 3 Levels	~ 3 mins **2705** **104, 76, 44**	~ 6 mins **4230** **104, 76, 44**	~ 27 mins **22788** **148, 112, 65**
Bat Problem	~ 1 min	~ 2.5 mins	**~ 45 mins (!)**
Other Test Problem	~ 10 seconds		

- **Without heuristics, none of these problems can be solved**
- ~15 Configurations / Second
- These **additional heuristics** are hightly effective as well
  Optimization A Remove congruent successor configurations
  Optimization B Remove successor configuration with polygons that are too small for any tile to cover / fit in

About Lisp – 1 / 3

- CLOS is used extensively
  - Geometric classes
  - Compositional hierachy
    - Segments from Points, Polygons from Segments, …
  - Dynamically affected by transformation matrix
  - Reader methods for `x`, `y`

```
(defmethod x ((point geom-point))
  (with-slots (x y affected-by-matrix-p) point
    (let ((val
           (if (and *matrix* affected-by-matrix-p)
               (with-slots (a b tx) *matrix*
                 (+ (* a x) (* b y) tx))
             x)))
      (my-round val))))

(defmethod y ((point geom-point))
  (with-slots (x y affected-by-matrix-p) point
    (let ((val
           (if (and *matrix* affected-by-matrix-p)
               (with-slots (c d ty) *matrix*
                 (+ (* c x) (* d y) ty))
             y)))
      (my-round val))))
```

=> all geometric functions are automatically aware of active transformations (e.g., `calculate-rcc-relation` next slide)

About Lisp – 2 / 3

- Transformations similar to CLIM

```
(setf (affected-by-matrix-p a) nil)
(setf (affected-by-matrix-p b) t)

(with-translation ( (- xf) (- yf) )
  (with-rotation ( phi )
    (with-translation ( xt yt )

      (when (and frame (show-thinking-p frame))
        (show-polygons (list b)
                       :clear-p nil
                       :inks (list +flipping-ink+)
                       :filled-p nil))

      (let* ((rel (calculate-rcc-relation a b)))
```

About Lisp – 3 / 3

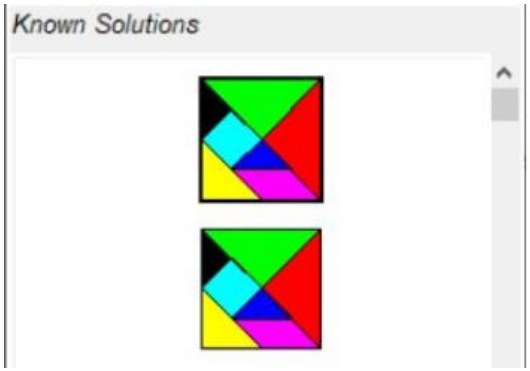
- Multi-Methods are nice for **Case Analysis**

```
(defmethod touches-p ((line geom-line) (poly geom-polygon))
  (and (intersects-p line poly)
       (or (lies-on-p line poly)
           (not (one-part-is-inside-p line poly)))))

(defmethod touches-p ((poly geom-polygon) (line geom-line))
  (touches-p line poly))

(defmethod touches-p ((chain1 geom-chain) (chain2 geom-chain))
  ;;; (error "TOUCHES-P ~A ~A: Not well-defined!" chain1 chain2))
  )

(defmethod-memo touches-p ((obj geom-chain) (poly geom-polygon))
    ((obj poly (trafo-id *matrix*)))
  (and (intersects-p obj poly)
       (every #'(lambda (segment)
                  (=> (intersects-p segment poly)
                      (touches-p segment poly)))
              (segments obj))))

(defmethod touches-p ((poly geom-polygon) (obj geom-chain))
  (touches-p obj poly))

(defmethod-memo touches-p ((poly1 geom-polygon) (poly2 geom-polygon))
    ((poly1 poly2 (trafo-id *matrix*)))
  (and (not (eq poly1 poly2))
       (intersects-p poly1 poly2)
       (every #'(lambda (segment)
                  (=> (intersects-p segment poly2)
                      (touches-p segment poly2)))
              (segments poly1))))
```

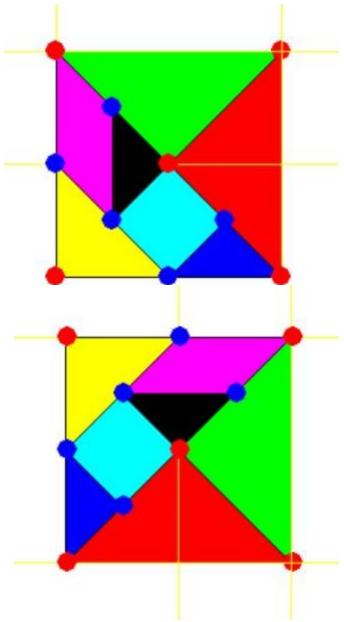
About CLIM

- CLIM is great for building prototypes quickly
- CLOS classes are presentations
  - Active objects that are alive on the screen (highlighting)
  - Automatically generated context menus etc.
  - You get A LOT of functionallity with minimal effort

Observations



- Frequently finds interesting solutions
- Is it **superhuman**? I guess it depends:
  - Search frequently is, not only since DNNs (e.g., Checkers, Chess / DeepBlue, …)
  - Compared to my tangram puzzle abilities, yes!
  - Not sure how fast people are
- Are other programs faster?
  - There are few available online… and they appeared only recently
  - Most of them are special-purpose and Tangram-specific
  - Whereas ours is generic and can (in principle) solve arbirtary tiling problems
  - Our program is complete (probably not the case for some of the others)
  - Sometimes, completeness has to be sacrificied for performance
- **Show Thinking** checkbox is useful for checking the progress
  - However, 60 % slower
  - The heuristics were devised by watching the search program and giving the program some „common sense“ and human insight

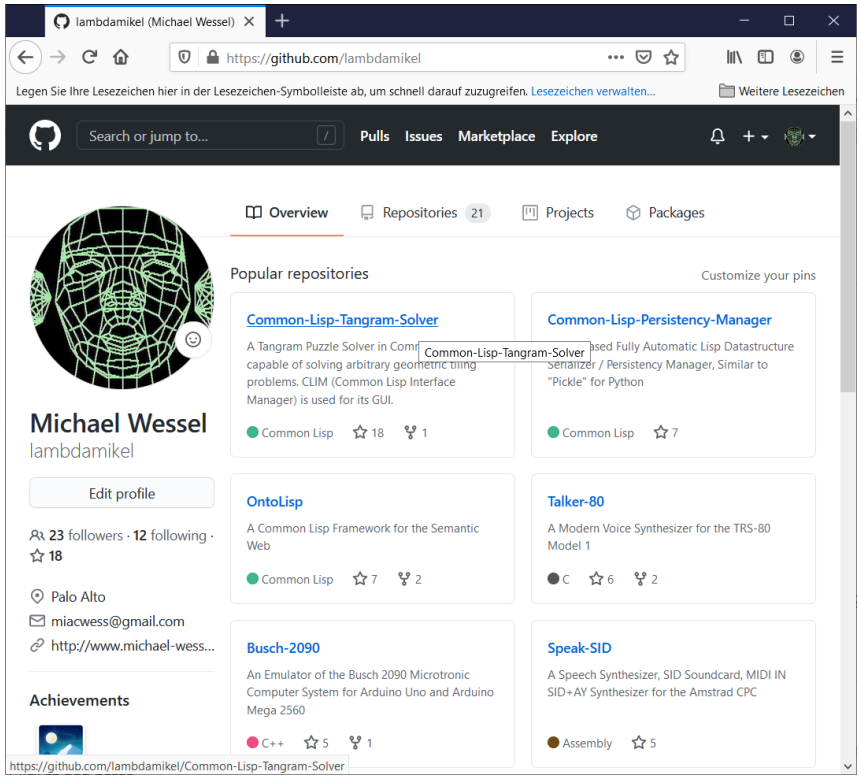


Wrap Up & Future Work

- **Q: Would I still be able to maintain and extent my Lisp code from almost 20 years ago?**
- **A: YES**
  - Lisp was fun to work with again after many years of absence
  - Lisp has good code maintainability
  - Concise, precise, semi-formal, small computational units (functions & methods)
  - The program is running much better than in 2003; I feel that, for now, **I finished what was started**
- The search space is very large
  - Getting the initial positions of the first two tiles right is essential
  - Else backtracking to them won‘t happen for a long time
  - Without heuristics no solutions can be found within 24 hours
- Future Work
  - SBCL / McCLIM, Penrose Tilings, …
  - More optimizations (Random Restarts, …)

Some Resources – Parens for You!

- Github repositories
  `https://github.com/lambdamikel`
  - Geometric substrate
  - Persistence module
  - Some reusable functions there for you
- Sourcecode and executables for Tangram
  - Try tweaking it, change heuristics! It‘s easy and inutitive
- Uses Lispworks (6.1) + CLIM
  - Linux, Windows
  - M1 Silicon Mac – Lispworks 7.1
  - **Thanks to Rainer Joswig (Lispm)!**



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Thank You!

Questions?