

Project 4

Dmytro Romaniv

Student ID: 151958

```
# Importing the Qiskit standard library and setting up an account
import math
import numpy as np
from math import sqrt
from numpy import pi
from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit,
transpile
from qiskit.quantum_info import Statevector, Operator
from qiskit.circuit.library import XGate
from qiskit.visualization import plot_histogram, plot_distribution
from qiskit_aer import Aer
from time import process_time
```

```
# qiskit 2.2.1
backend = Aer.get_backend('unitary_simulator')
```

```
# Construction of the Uf matrix
```

```
nn=3
oracle=np.identity(2**nn)
oracle[2,2]=-1
print(oracle)
Uf=Operator(oracle)
Operator.is_unitary(Uf)

[[ 1.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0. -1.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  1.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  1.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  1.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  1.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  1.]]
```

True

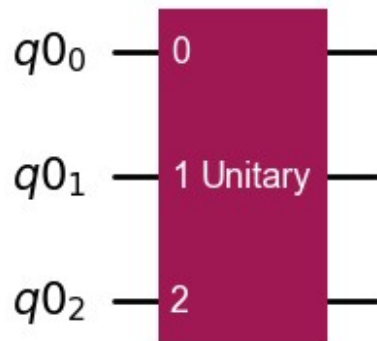
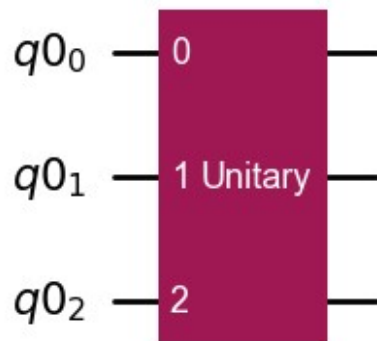
```
# Creating quantum registers,
# classical registers and a quantum circuit
# representing the created Uf operator
# Number of qubits and bits
n0=nn
# Quantum Register
q0 = QuantumRegister(n0)
```

```

# "Empty" quantum circuit
# for a gate named 'Uf'
CircuitUf = QuantumCircuit(q0,name='Uf')

# Attaching the Uf operator to the circuit
# representing the Uf gate
CircuitUf.append(Uf,[q0[0],q0[1],q0[2]])
# Sketch of a quantum circuit
CircuitUf.draw(output='mpl')

```



```

# Transforming the Uf operator
# into a uf quantum gate
# denoted as Uf
uf=CircuitUf.to_gate()

# Creating an "empty" warp
# quantum circuit of Grover's algorithm
# Number of qubits and bits
n=nn
# Quantum Register
q = QuantumRegister(n)

```

```

# Classical Register
c = ClassicalRegister(n)
# "Empty" quantum circuit -
# - the core of Grover's algorithm
Circuit = QuantumCircuit(q,c)
# Sketch of a quantum circuit
Circuit.draw(output='mpl')

```

$q1_0$ —

$q1_1$ —

$q1_2$ —

$c0$ ³ ~~—~~

$q1_0$ —

$q1_1$ —

$q1_2$ —

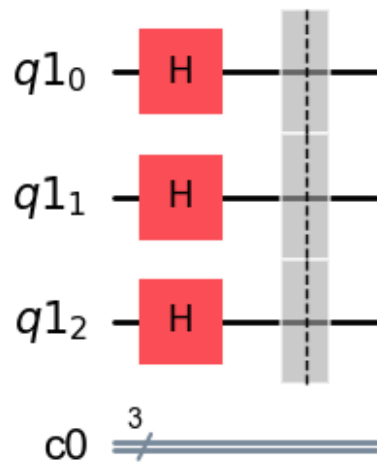
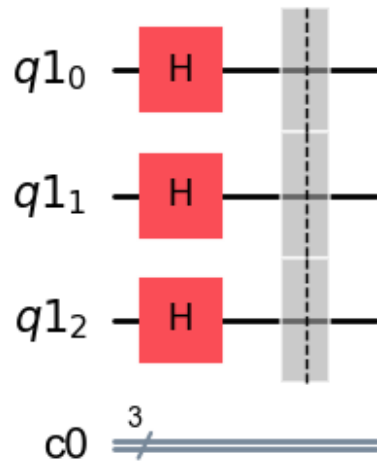
$c0$ ³ ~~—~~

```

# |fi> State initiation
Circuit.h(q[0])
Circuit.h(q[1])
Circuit.h(q[2])
Circuit.barrier()

```

```
# Sketch of a quantum circuit
Circuit.draw(output='mpl')
```



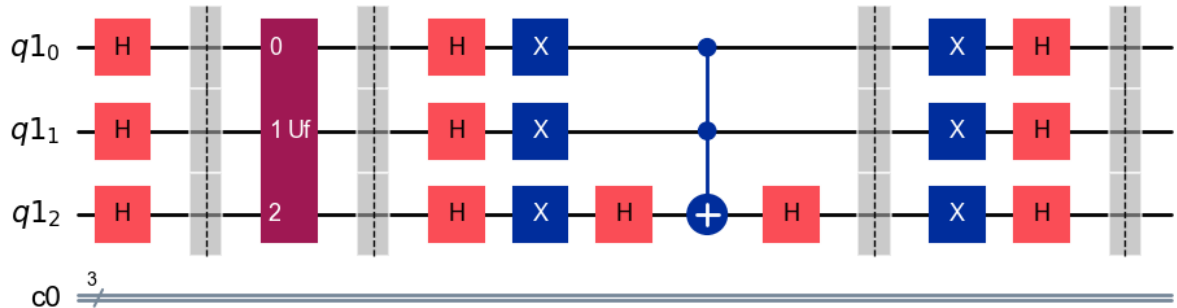
```
repeat=math.floor((pi/4)*sqrt(2**n))
print(repeat)
2
mccx=XGate().control(n0-1)
for ii in range(repeat):
    Circuit.append(uf,[0,1,2])
    Circuit.barrier()
    # Beginning of the implementation of the W diffusion operator
```

```

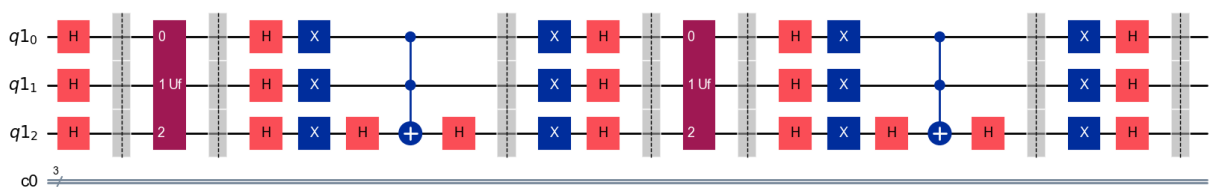
Circuit.h(q[0])
Circuit.h(q[1])
Circuit.h(q[2])
Circuit.x(q[0])
Circuit.x(q[1])
Circuit.x(q[2])
Circuit.h(q[2])
Circuit.append(mccx, [q[0],q[1],q[2]])
Circuit.h(q[2])
Circuit.barrier()
Circuit.x(q[0])
Circuit.x(q[1])
Circuit.x(q[2])
Circuit.h(q[0])
Circuit.h(q[1])
Circuit.h(q[2])
# The end of the implementation of the W diffusion operator
Circuit.barrier()
print("N=",ii+1)
# print(Circuit)
display(Circuit.draw(output='mpl'))

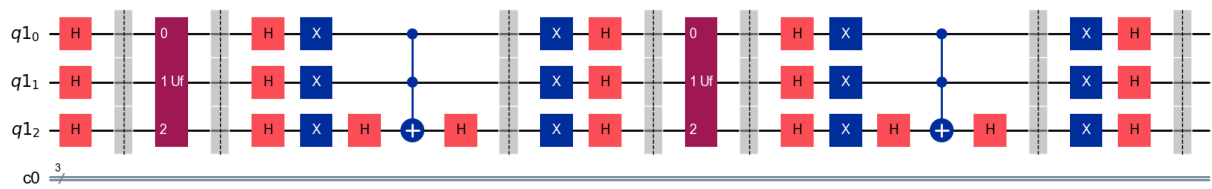
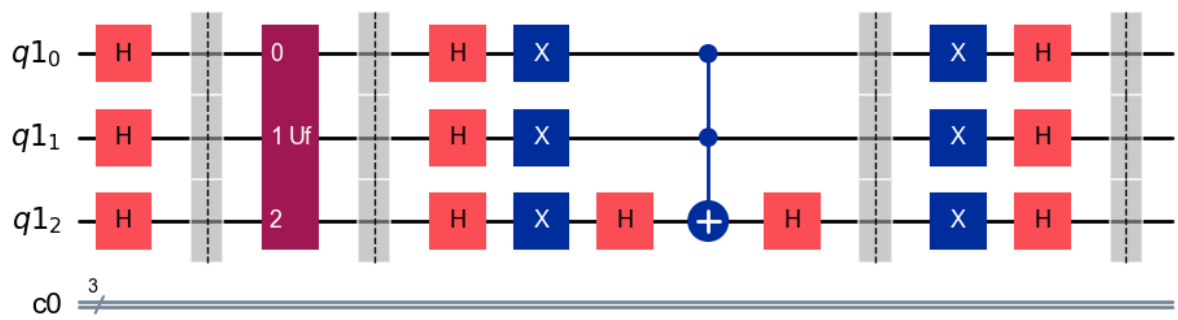
```

N= 1



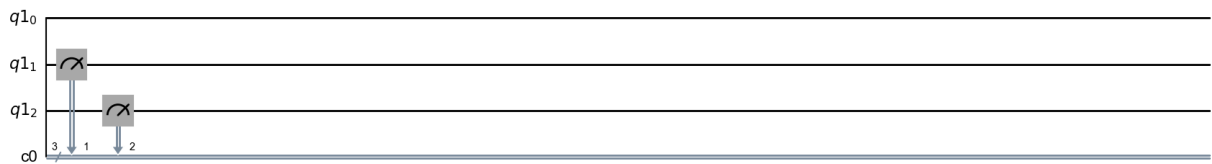
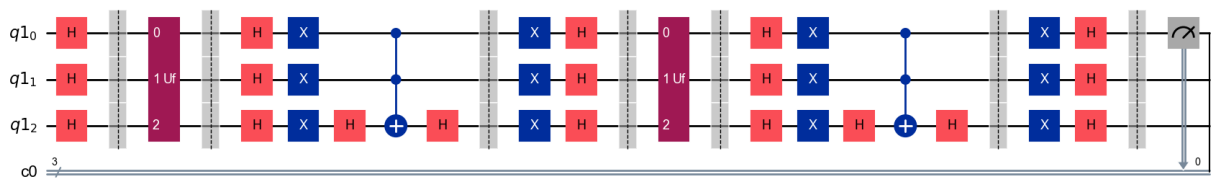
N= 2

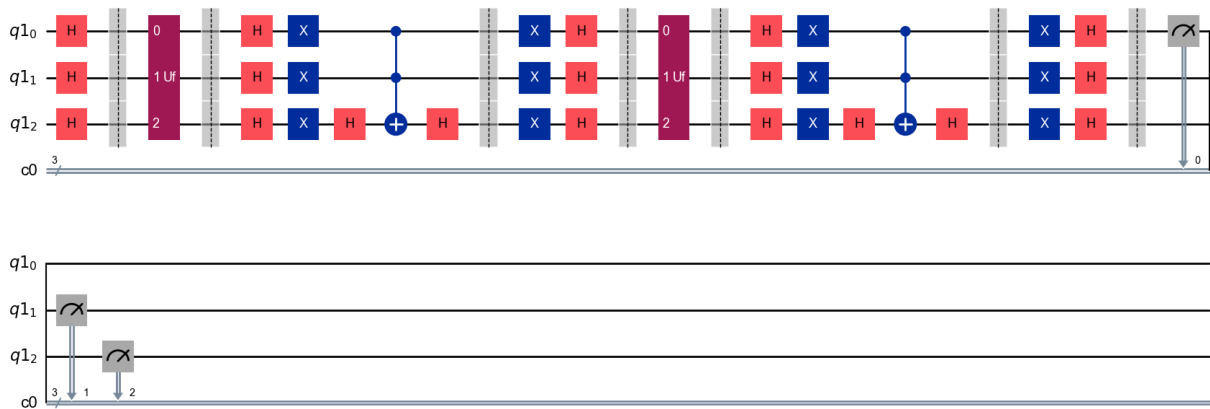




Adding measurement operations to the quantum circuit

```
Circuit.measure(q[0],c[0])
Circuit.measure(q[1],c[1])
Circuit.measure(q[2],c[2])
Circuit.draw(output='mpl') #
```





```
# Start the stopwatch / counter
t1_start = process_time()

# Select a quantum simulator (or processor).
backend_sim = Aer.get_backend('qasm_simulator')

# Transpile the circuit for the specific backend (needed in qiskit 1.x)
transpiled_circuit = transpile(Circuit, backend_sim)

# Perform quantum calculations
job_sim0 = backend_sim.run(transpiled_circuit, shots=1024)
sim_result0 = job_sim0.result()

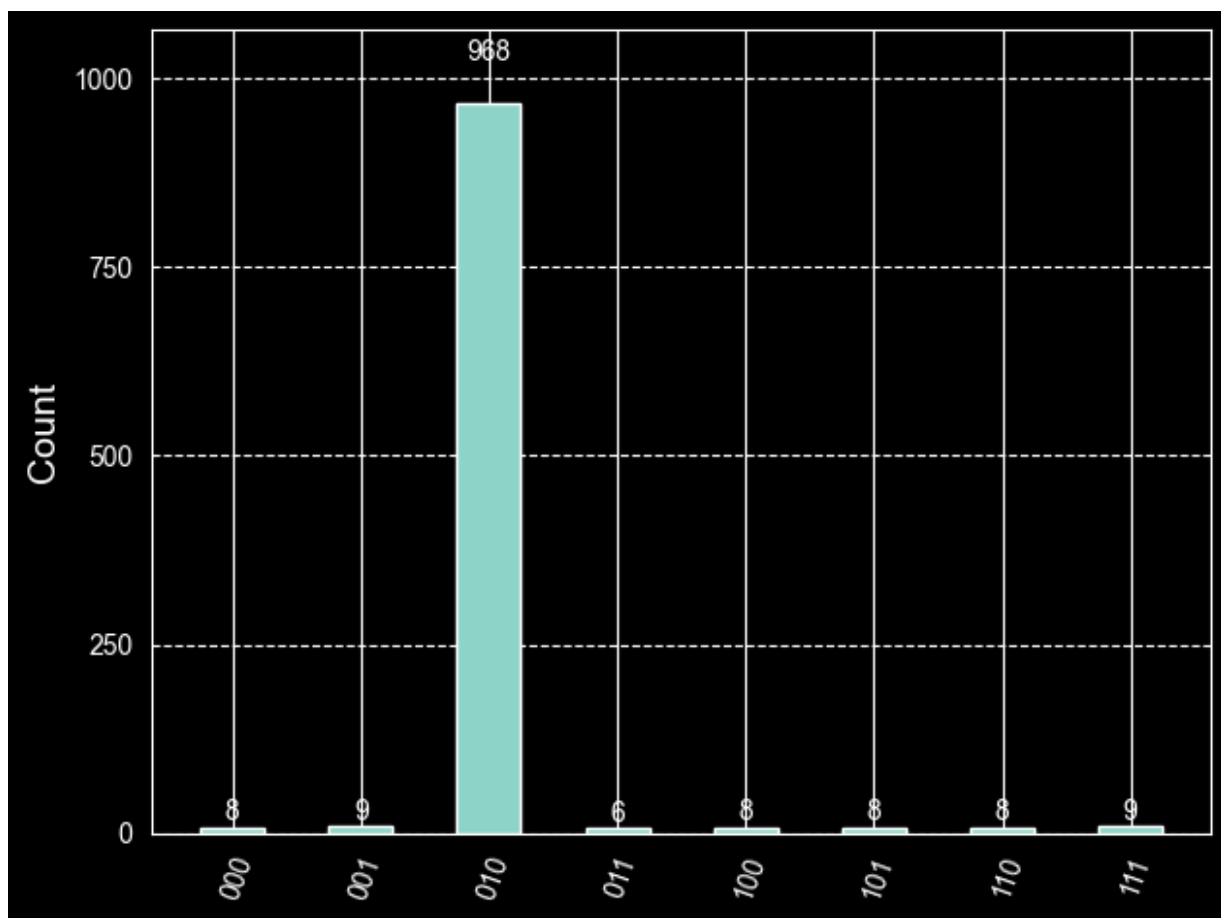
# Numerical presentation of measurement results
print(sim_result0.get_counts())

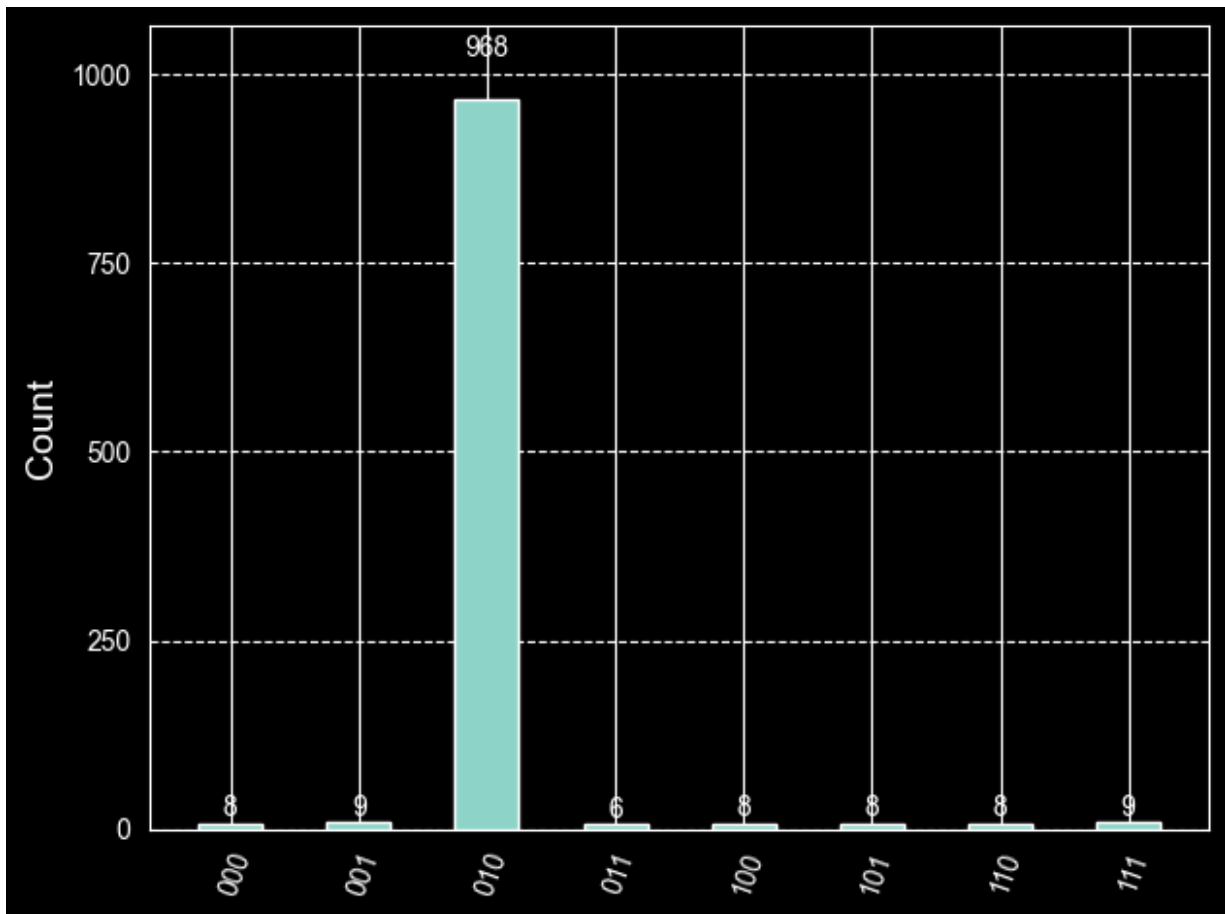
{'111': 9, '011': 6, '110': 8, '100': 8, '001': 9, '101': 8, '000': 8, '010': 968}

# Stop the stopwatch / counter
t1_stop = process_time()
print("Elapsed time:", t1_stop, t1_start)
print("Elapsed time during the whole program in seconds:", t1_stop-t1_start)

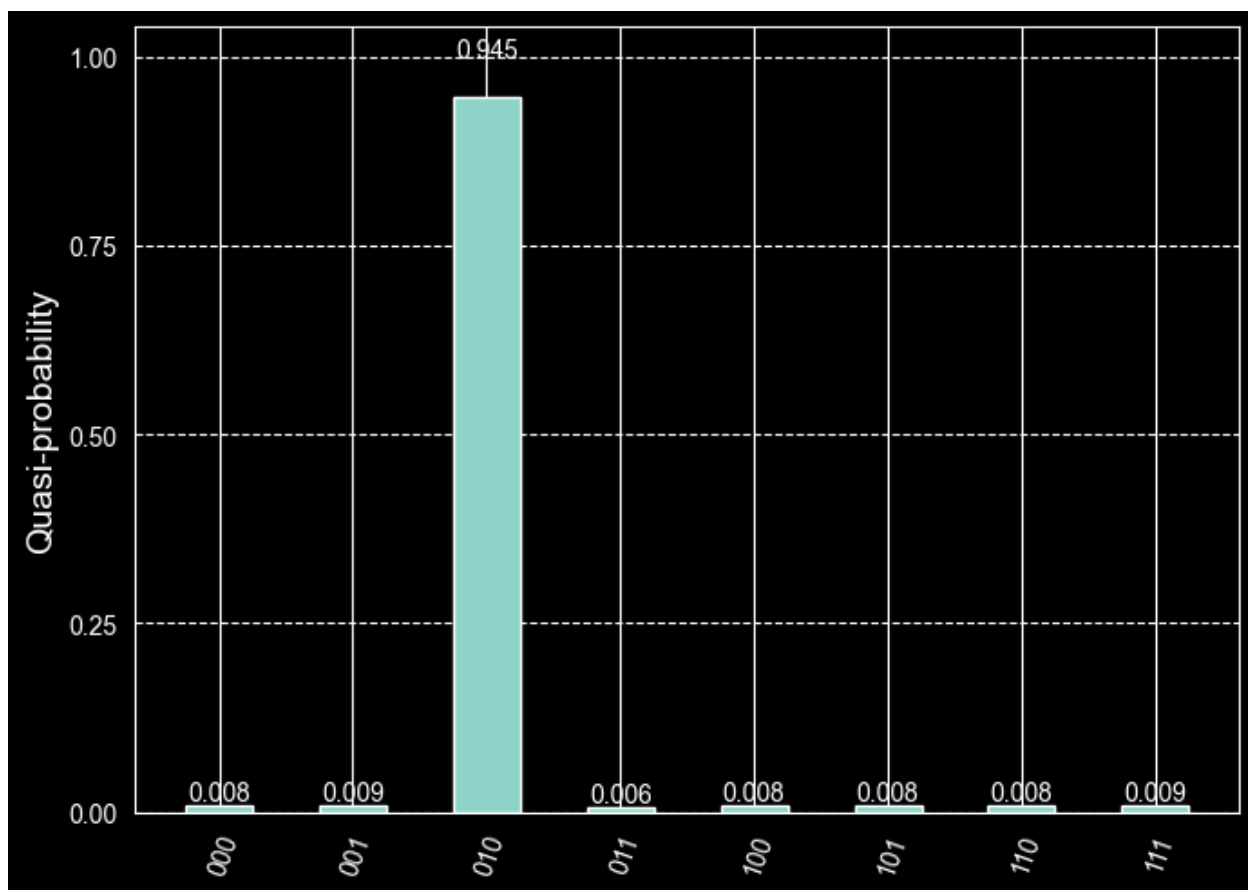
Elapsed time: 1.68023 1.557838
Elapsed time during the whole program in seconds: 0.122391999999999983

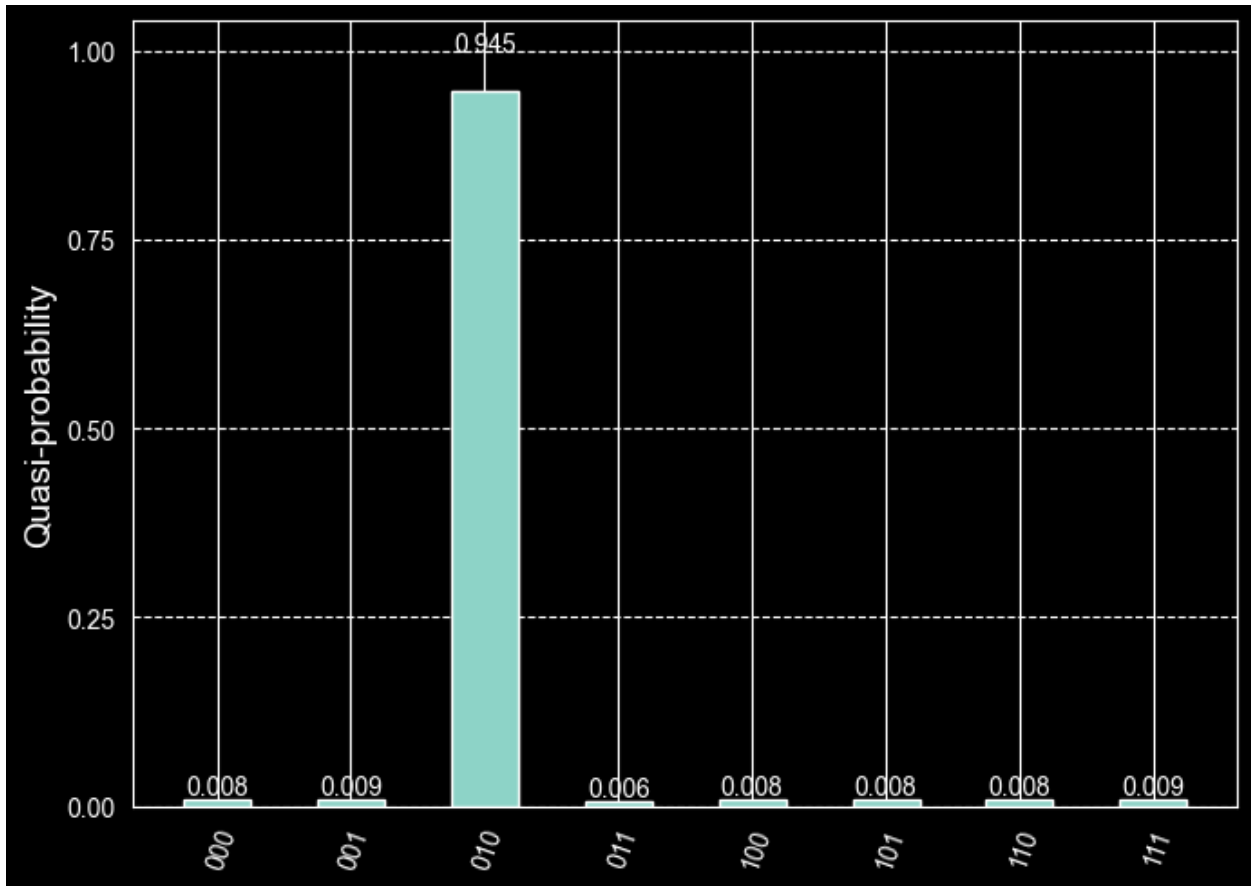
# Graphical presentation of X measurement results
plot_histogram(sim_result0.get_counts(Circuit))
```





```
# Graphical presentation of X measurement results  
plot_distribution(sim_result0.get_counts(Circuit))
```





```
import math
import numpy as np
import matplotlib.pyplot as plt
from qiskit.quantum_info import Operator

# --- TASK 1 ---
print("--- TASK 1: Optimal Iterations r ---")
print(f"{'n (Qubits)':<10} | {'N (States)':<10} | {'r (Iterations)':<15}")
print("-" * 40)

optimal_r_dict = {}

for n in range(2, 7):
    N = 2**n
    # Formula: floor( (pi/4) * sqrt(N) )
    r = math.floor((np.pi / 4) * math.sqrt(N))
    optimal_r_dict[n] = r
    print(f"{'n':<10} | {'N':<10} | {'r':<15}")

--- TASK 1: Optimal Iterations r ---
n (Qubits) | N (States) | r (Iterations)
-----
```

2	4	1
3	8	2
4	16	3
5	32	4
6	64	6

--- **TASK 2: Generic Grover Implementation** ---

```
def create_phase_oracle(n, target_index):
    """Creates a diagonal Phase Oracle Operator for a specific target
    index."""
    matrix = np.eye(2**n, dtype=complex)
    matrix[target_index, target_index] = -1
    return Operator(matrix)

def add_diffuser(circuit, n):
    """Appends the Grover Diffuser (W operator) to the circuit."""
    # 1. Apply H to all
    circuit.h(range(n))
    # 2. Apply X to all
    circuit.x(range(n))

    # 3. Multi-controlled Z gate (implemented as H -> MCX -> H on the
    last qubit)
    circuit.h(n-1)

    # Create a Multi-Controlled X (Toffoli with n-1 controls)
    if n > 1:
        mcx_gate = XGate().control(n - 1)
        circuit.append(mcx_gate, list(range(n)))
    else:
        circuit.x(0)

    circuit.h(n-1)

    # 4. Apply X to all
    circuit.x(range(n))
    # 5. Apply H to all
    circuit.h(range(n))

def build_grover_circuit(n, target_index, iterations):
    """Builds the complete Grover circuit for n qubits and r
    iterations."""
    q_reg = QuantumRegister(n, 'q')
    c_reg = ClassicalRegister(n, 'c')
    qc = QuantumCircuit(q_reg, c_reg)

    # 1. Initialization |s> (Hadamard on all)
    qc.h(q_reg)
```

```

# Prepare Oracle Operator
oracle_op = create_phase_oracle(n, target_index)

# 2. Grover Iterations ( $G = W * V$ )
for _ in range(iterations):
    qc.barrier()
    # Apply Oracle ( $V$ )
    qc.append(oracle_op, range(n))

    qc.barrier()
    # Apply Diffuser ( $W$ )
    add_diffuser(qc, n)

qc.barrier()
# 3. Measurement
qc.measure(q_reg, c_reg)

return qc

print("Generic Grover functions defined successfully.")
Generic Grover functions defined successfully.

# --- TASKS 3 & 4: Probability vs Iterations for n=6 ---

STUDENT_ID = 151958
n_sim = 6
N_sim = 2**n_sim
target_a = STUDENT_ID % N_sim

# Calculate range for Task 4 ( $\pi/2 * \sqrt{N}$  is roughly  $2 * \text{optimal}_r$ )
max_s = math.floor((np.pi / 2) * math.sqrt(N_sim))
print(f"Student ID: {STUDENT_ID}")
print(f"Target state 'a' (decimal): {target_a}")
print(f"Target state 'a' (binary): {format(target_a, '06b')}")
print(f"Simulating up to s = {max_s} iterations...")

probabilities = []
iterations_range = range(1, max_s + 1)

# Backend setup
backend_sim = Aer.get_backend('qasm_simulator')

for s in iterations_range:
    # Build circuit
    qc = build_grover_circuit(n_sim, target_a, s)

    # Transpile
    t_qc = transpile(qc, backend_sim)

# Run

```

```

# Using enough shots to get a decent probability estimate
job = backend_sim.run(t_qc, shots=2048)
result = job.result()
counts = result.get_counts()

# Get count for target state (convert decimal target to binary
string key)
target_bin = format(target_a, f'0{n_sim}b')

# Extract count, default to 0 if state not found
target_count = counts.get(target_bin, 0)
prob = target_count / 2048
probabilities.append(prob)

# Plotting Task 3 & 4
plt.figure(figsize=(10, 6))
plt.plot(iterations_range, probabilities, 'o-', label=f'Prob of state
|{target_a}>')
plt.axvline(x=optimal_r_dict[n_sim], color='r', linestyle='--',
label=f'Optimal r={optimal_r_dict[n_sim]}')
plt.title(f'Grover Algorithm: Probability vs Iterations (n={n_sim})')
plt.xlabel('Iterations (s)')
plt.ylabel('Probability P_a')
plt.xticks(iterations_range)
plt.grid(True)
plt.legend()
plt.show()

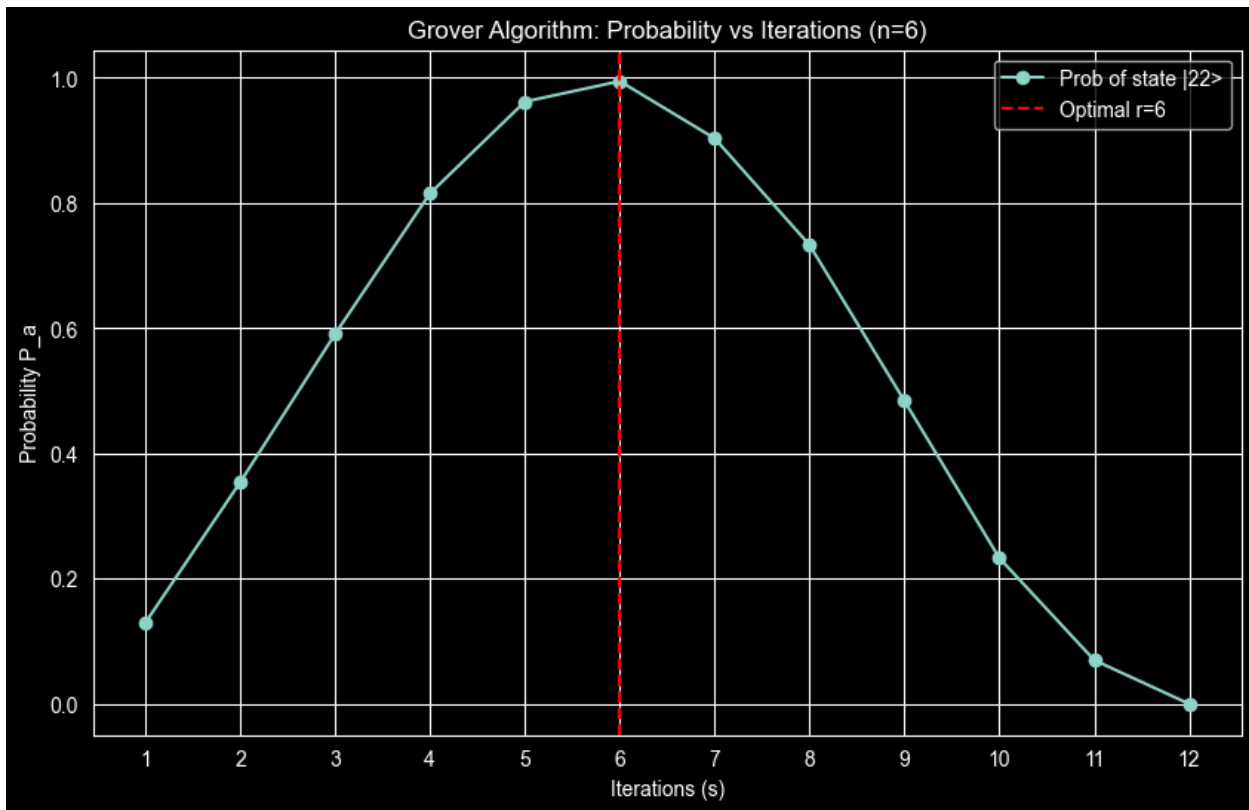
```

Student ID: 151958

Target state 'a' (decimal): 22

Target state 'a' (binary): 010110

Simulating up to s = 12 iterations...



```
# --- TASK 5: Probability vs Number of Qubits (n) ---

print("--- TASK 5: Simulation for n=2..6 with optimal r ---")

n_values = range(2, 7)
success_probabilities = []

for n_curr in n_values:
    # 1. Determine Target
    N_curr = 2**n_curr
    target_curr = STUDENT_ID % N_curr
    target_bin_curr = format(target_curr, f'0{n_curr}b')

    # 2. Determine Optimal Iterations
    r_curr = optimal_r_dict[n_curr]

    # 3. Build and Run
    qc = build_grover_circuit(n_curr, target_curr, r_curr)
    t_qc = transpile(qc, backend_sim)

    # Run with high shots for precision
    shots = 4096
    job = backend_sim.run(t_qc, shots=shots)
    counts = job.result().get_counts()

    # 4. Calculate Probability
```

```

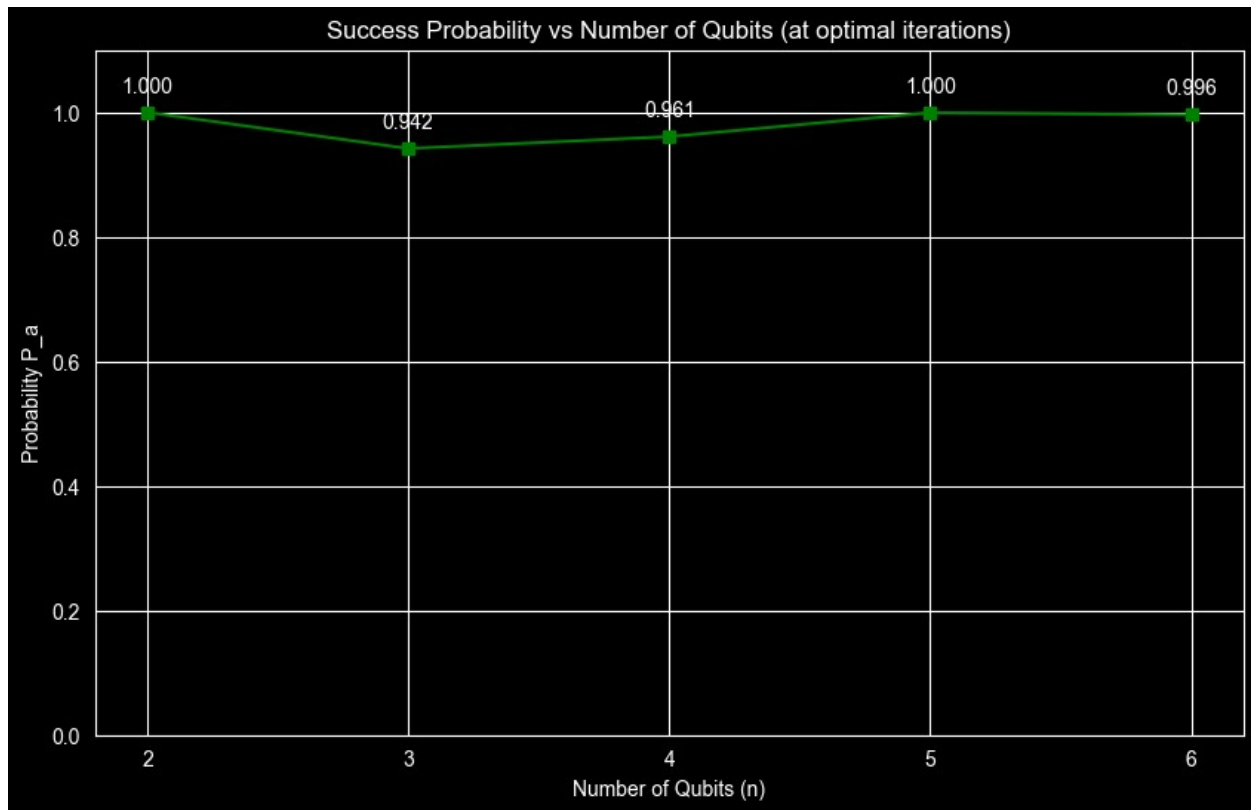
count_hit = counts.get(target_bin_curr, 0)
prob = count_hit / shots
success_probabilities.append(prob)

print(f"n={n_curr}, Target={target_curr} ({target_bin_curr}),
r={r_curr}, Probability={prob:.4f}")

# Plotting Task 5
plt.figure(figsize=(10, 6))
plt.plot(n_values, success_probabilities, 's-', color='green')
plt.title('Success Probability vs Number of Qubits (at optimal
iterations)')
plt.xlabel('Number of Qubits (n)')
plt.ylabel('Probability Pa')
plt.ylim(0, 1.1)
plt.grid(True)
plt.xticks(n_values)
for i, txt in enumerate(success_probabilities):
    plt.annotate(f"{txt:.3f}", (n_values[i],
success_probabilities[i]),
                textcoords="offset points", xytext=(0,10),
ha='center')
plt.show()

--- TASK 5: Simulation for n=2..6 with optimal r ---
n=2, Target=2 (10), r=1, Probability=1.0000
n=3, Target=6 (110), r=2, Probability=0.9419
n=4, Target=6 (0110), r=3, Probability=0.9609
n=5, Target=22 (10110), r=4, Probability=0.9995
n=6, Target=22 (010110), r=6, Probability=0.9958

```

```
# Visualize the circuit for n=6 and r=1 (the first iteration)

n_vis = 6
# Recalculate target to ensure consistency
target_a_vis = STUDENT_ID % (2**n_vis)
iterations_vis = 1

print(f"Generating circuit diagram for n={n_vis}, Target State=|
{target_a_vis}>, Iterations={iterations_vis}")

# Build the circuit using the previously defined function
qc_vis = build_grover_circuit(n_vis, target_a_vis, iterations_vis)

# Draw the circuit
display(qc_vis.draw(output='mpl', style='iqp', fold=-1))

Generating circuit diagram for n=6, Target State=|22>, Iterations=1
```

