Article

Violation of the CHSH Inequality on the Bell State $|\Psi^-\rangle$

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Abstract: Bell tests check whether two quantum systems show correlations that no classical local model can explain. The CHSH inequality bounds such correlations by $|S| \leq 2$, while quantum mechanics allows up to $|S| = 2\sqrt{2}$. I prepared the Bell singlet $|\Psi^-\rangle$ on two qubits and measured four settings: Z or X on qubit A, and $W = (Z + X)/\sqrt{2}$ or $V = (Z - X)/\sqrt{2}$ on qubit B. Outcomes $0, 1$ were mapped to $+1, -1$ and used to estimate correlators and S. Circuits were implemented in Qiskit and executed on the `qasm_simulator` with $N = 1024$ shots per setting; the full experiment was repeated five times. Averaging across the five repeats gave the correlators $E_{ZW} = -0.7066 \pm 0.0255$, $E_{ZV} = -0.6984 \pm 0.0179$, $E_{XW} = -0.6938 \pm 0.0225$, $E_{XV} = +0.7117 \pm 0.0281$. The five $|S|$ values were $\{2.8418, 2.8301, 2.6836, 2.8535, 2.8438\}$ with mean 2.8105 and standard deviation 0.0715. These results violate the classical bound and agree with the quantum prediction within statistical uncertainty.

Keywords: Bell inequality; CHSH; entanglement; quantum circuits; simulator

1. Introduction

Bell's theorem shows that quantum mechanics can produce correlations that cannot be reproduced by any local hidden variable model. A practical form is the CHSH inequality, which combines four correlation terms into a single number S and satisfies $|S| \leq 2$ for any local model. For a maximally entangled two-qubit state and suitable measurement directions, quantum theory predicts $|S| = 2\sqrt{2}$ (Tsirelson's bound). This report reproduces the CHSH test on the Bell state $|\Psi^-\rangle$ using Qiskit and provides an uncertainty estimate from repeated runs.

2. Materials and Methods

2.1. State Preparation

The singlet $|\Psi^-\rangle = (|10\rangle - |01\rangle)/\sqrt{2}$ was prepared from $|00\rangle$ using X on qubit 1, H on qubit 0, a CNOT (0→1), and Z on qubit 0. This matches the state preparation shown in the course slides.

2.2. Measurement Settings

Alice (qubit 0) measured Z or X; Bob (qubit 1) measured $W = (Z + X)/\sqrt{2}$ or $V = (Z - X)/\sqrt{2}$. In the circuit model, measurements are performed in the computational (Z) basis. To realize the required observables, I used pre-measurement rotations so that a Z-basis readout implements the desired operator: no rotation for Z, H for X, $R_y(-\pi/4)$ for W, and $R_y(+\pi/4)$ for V. Bits were mapped $0 \to +1$, $1 \to -1$, and correlators were computed as averages of the product of the two outcomes.

2.3. Execution and Repetition

Each setting (ZW, ZV, XW, XV) was executed with $N = 1024$ shots on the Qiskit `qasm_simulator`, and the full set was repeated five times to estimate run-to-run spread. All figures shown below were rendered from these runs, and numerical tables were exported directly from the simulation output.

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3. Results

3.1. Circuit Diagrams

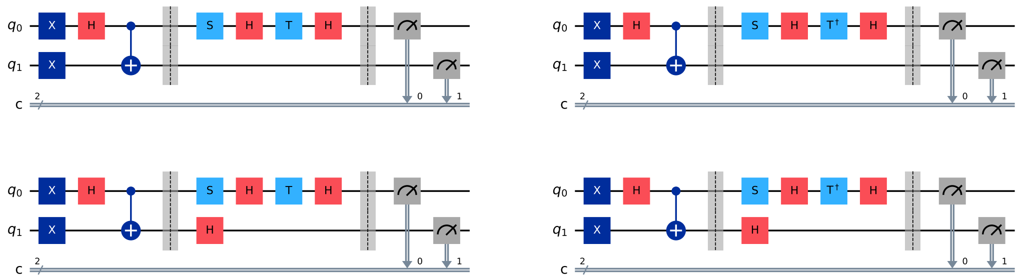

Figure 1. Circuits for the four CHSH measurement settings. The left part of each circuit prepares the Bell state $|\Psi^-\rangle$; the right part implements the local rotations corresponding to Z, X, $W = (Z+X)/\sqrt{2}$, and $V = (Z-X)/\sqrt{2}$, followed by computational-basis measurements.

3.2. Joint Outcome Probabilities

The joint probabilities $p(x, y)$ for a representative run (Run 1) are given in Table 1. The bitstrings are `ab`, where a is Alice's outcome and b is Bob's.

Table 1. Joint outcome probabilities $p(x, y)$ for Run 1.

Outcome	XW	XV	ZW	ZV
00	0.0801	0.4180	0.0684	0.0879
01	0.4277	0.0625	0.4326	0.4033
10	0.4219	0.0625	0.4248	0.4355
11	0.0703	0.4570	0.0742	0.0732

These probabilities show the expected pattern for the singlet: for ZW, ZV, and XW the anti-correlated outcomes `01` and `10` dominate, whereas for XV the correlated outcomes `00` and `11` are most likely.

3.3. Correlators and CHSH Values

Averaging across the five repeats gave the following correlators (mean $\pm$ sample standard deviation):

$$E_{ZW} = -0.7066 \pm 0.0255, \quad E_{ZV} = -0.6984 \pm 0.0179,$$
$$E_{XW} = -0.6938 \pm 0.0225, \quad E_{XV} = +0.7117 \pm 0.0281.$$

The absolute CHSH values over the five runs were

$$|S| = \{2.8418,\ 2.8301,\ 2.6836,\ 2.8535,\ 2.8438\},$$

with mean

$$\langle |S| \rangle = 2.8105 \pm 0.0715,$$

where the quoted uncertainty is the sample standard deviation. These values are consistent with the ideal correlators $\pm 1/\sqrt{2} \approx \pm 0.7071$ and the Tsirelson bound $|S| = 2\sqrt{2} \approx 2.8284$ within statistical error.

3.4. Per-Setting Probability Plots

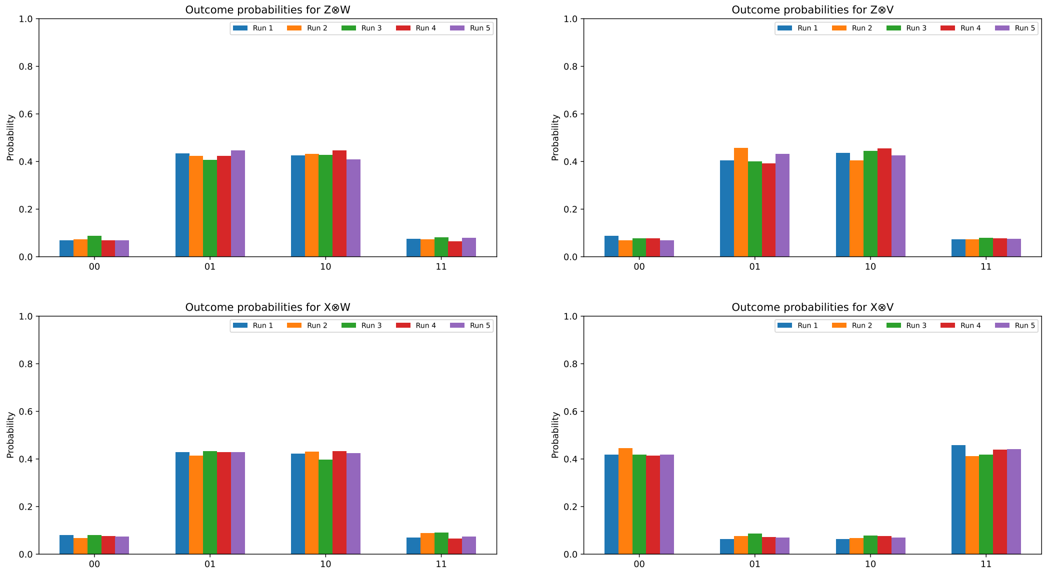

Figure 2. Outcome probabilities per setting for each of the five runs (N=1024 shots per run). The strong anti-correlation in ZW, ZV, and XW, and the strong correlation in XV, combine to produce a large $|S|$.

4. Discussion

For the singlet, $\langle(\hat{a}\cdot\vec{\sigma})\otimes(\hat{b}\cdot\vec{\sigma})\rangle = -\hat{a}\cdot\hat{b}$. With the 45° choices for V and W, three inner products are $+1/\sqrt{2}$ and one is $-1/\sqrt{2}$, leading to three negative correlators and one positive, as observed in the data. The spread across runs is consistent with shot noise for $N = 1024$. The small deviations from the ideal Tsirelson value can be attributed to finite-sample statistics and the compiled rotation sequence used to implement the W and V measurements.

5. Conclusions

A compact CHSH test on $|\Psi^-\rangle$ was executed in Qiskit and repeated five times on the `qasm_simulator`. The mean $|S| = 2.8105$ with standard deviation 0.0715 clearly exceeds the classical limit and agrees with quantum predictions within uncertainty. The figures and tables included here allow straightforward reproduction or extension of the experiment.

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Abbreviations

CHSH	Clauser–Horne–Shimony–Holt
CNOT	Controlled-NOT quantum gate
MDPI	Multidisciplinary Digital Publishing Institute

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