Recovering output from Dynare

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Dynare is currently one of the most used packages to solve and analyze rational expectations models in macroeconomics. In this note, I summarize how to recover the output from Dynare in order to work without the need of running every time the mod file.

1 Policy function

Let s_t be the $m \times 1$ vector of state variables and x_t the $n \times 1$ vector of control variables in the model and consider a first order approximation around the steady state. Then, the state space representation of the system can be written as

$$s_t = As_{t-1} + B\varepsilon_t \quad (1)$$
$$x_t = \Phi s_t, \quad (2)$$

where ε_t is the $w \times 1$ vector of structural shocks and A, B and Φ are $m \times m$, $m \times w$, and $n \times m$ matrices respectively. The last of these matrices is the policy function of the model. By replacing the first equation in the second, the latter can be written as $x_t = Cs_{t-1} + D\varepsilon_t$, with $C = \Phi A$ and $D = \Phi B$. Moreover, the whole system can be written as

$$Y_t = \Psi s_{t-1} + \Omega\varepsilon_t, \quad (3)$$

with $Y_t = [s_t \ x_t]'$, $\Psi = [A \ C]'$ and $\Omega = [B \ D]'$.
To get this expressions from the output in Dynare, you need the following

1. In the mod file, you declare variables in a certain order. This is not necessarily the order that Dynare uses. Therefore, recover the original order of declaration of those variables you are interested in (can be the whole set of variables) and call it `obs_var`. To get the index of these variables in Dynare, use `oo_.dr.inv_order_var(obs_var)`.

2. Repeat the procedure for the index of state variables, which are ordered as you declare them. This can be done with `ipred = M_.nstatic+(1:M_.nspred)'`. Alternatively, this can be done as in the case of control variables indicating the indexes of states.

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3. Get the the transition equation matrices A and B with
   `A = oo_.dr.ghx(ipred,:)` and `B = oo_.dr.ghu(ipred,:)`.

   Also get the observation equation matrices with
   `C = oo_.dr.ghx(obs_var,:)` and `D = oo_.dr.ghu(obs_var,:)`.

4. Once you have A, B, C and D, you can construct the policy function for all the variables of interest.

2 Impulse-response function

In the case of first order approximations, there is a particularly simple way of obtaining the IRFs of the model. Let h be the horizon of the IRF, σ the size of the shock and n_s and n_x the number of state and control variables. Let $SIRF$ and $XIRF$ be the $m \times h$ and $n \times h$ matrices with the responses to the shock. The impact response of the state and control variables is just $SIRF_0 = \sigma \times B$ and $XIRF_0 = \sigma \times D$, respectively. The rest of responses can be computed iteratively as $SIRF_t = A \times SIRF_{t-1}$ and $XIRF_t = C \times SIRF_{t-1}$ up to $t = h$.[1]

3 Theoretical moments

To compute theoretical moments, first note that the mean is just the steady state of the model. To compute second order moments you need the following

1. Compute the covariance matrix of the state variables. Let Σ_s and Σ_ε be the covariance matrices of the state variables and the shocks, respectively. From (1), we get $\Sigma_s = A\Sigma_s A' + B\Sigma_\varepsilon B'$, because the process is stationary. One way to solve for Σ_s is to use the doubling algorithm. Let $\Sigma_{s,0} = I$, $A_0 = A$ and $\Sigma_{\varepsilon,0} = \Sigma_\varepsilon$, where I is the identity matrix. Then iterate over the following system of equations until convergence

$$\begin{aligned}
\Sigma_{s,t+1} &= A_t \Sigma_{s,t} A_t' + B\Sigma_{\varepsilon,t} B' \\
\Sigma_{\varepsilon,t+1} &= A_t \Sigma_{\varepsilon,t} A_t' + \Sigma_{\varepsilon,t} \\
A_{t+1} &= A_t A_t.
\end{aligned}$$

2. Obtain the (auto-)covariance matrix of all relevant variables. From (3), note that $E(Y_t Y_t') = E[(\Psi s_{t-1} + \Omega\varepsilon_t)(\Psi s_{t-1} + \Omega\varepsilon_t)'] = \Psi\Sigma_s\Psi' + \Omega\Sigma_\varepsilon\Omega'$. For the first auto-covariance we have $E(Y_t Y_{t-1}') = E[(\Psi s_{t-1} + \Omega\varepsilon_t)(\Psi s_{t-2} + \Omega\varepsilon_{t-1})]$. Replacing (1) in the previous expression and developing we get $E(Y_t Y_{t-1}') = \Psi(A\Sigma_s\Psi' + B\Sigma_\varepsilon\Omega')$. In the same way, $E(Y_t Y_{t-2}') = \Psi A(A\Sigma_s\Psi' + B\Sigma_\varepsilon\Omega')$. Therefore, for any $j = 0, 1, \ldots$

$$E(Y_t Y_{t-j}') = \Psi A^{j-1}(A\Sigma_s\Psi' + B\Sigma_\varepsilon\Omega'),$$

[1] Importantly: if you define the stochastic process in Dynare multiplying the shock with the standard deviation and set the variance of the shock as 1 in the shock's section, all responses are to a unitary shock and not to a one standard deviation shock.

where is direct to note that this equation holds for $j = 0$ given the definition of Ψ, Ω, C and D.