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Tutorial No. 1 (One)

Q.1 If $\begin{vmatrix} a & b \\ 2 & 3 \end{vmatrix} = 5b$, then the value of $\frac{a}{b}$ is ?

$$\begin{vmatrix} a & b \\ 2 & 3 \end{vmatrix} = 5b$$

$$3a - 2b = 5b$$

$$3a = 5b + 2b$$

$$3a = 7b$$

$$\boxed{\frac{a}{b} = \frac{7}{3}}$$ Ans.

Q.2. If $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then $|A|$ will be ?

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= (5 \times 5 - 0 \times 0)$$

$$= 25 - 0$$

$$\boxed{|A| = 25}$$

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Q.3 If $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$, then Prove that A is Orthogonal Matri

$$A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$A^T = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$AA^T = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \times \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & -2-2+4 & -2+4-2 \\ -2-2+4 & 4+1+4 & 4-2-2 \\ -2+4-2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

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Matri

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Q.4 Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + ab + bc + ca$

Multiply it by $\frac{abc}{abc}$

$$\frac{abc}{abc}\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$R_1 \rightarrow \frac{1}{a}$, $R_2 \rightarrow \frac{1}{b}$, $R_3 \rightarrow \frac{1}{c}$

$R_1 = R_1 + R_2 + R_3$, $R_2 = R_1 + R_2 + R_3$, $R_3 = R_1 + R_2 + R_3$

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$$abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & \frac{1}{b} \\ 0 & -1 & \frac{1}{c}+1 \end{vmatrix}$$

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$$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(-1\times -1 - 0\times 1)$$

$$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)1$$

$$\cancel{abc\left(abc + abc\right.}$$

$$\cancel{abc}\left(\frac{abc+bc+ac+ab}{\cancel{abc}}\right)$$

$$= \boxed{abc+bc+ac+ab}$$

Hence Proved.

Q.5 $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Prove that $A^2-4a-5I=0$.

$$A^2=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1\times1+2\times2+2\times2 & 1\times2+2\times1+2\times2 & 1\times2+2\times2+2\times1 \\ 2\times1+1\times2+2\times2 & 2\times2+1\times1+2\times2 & 2\times2+1\times2+2\times1 \\ 2\times1+2\times2+1\times2 & 2\times2+2\times1+1\times2 & 2\times2+2\times2+1\times1 \end{bmatrix}$$

$$\begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} = A^2$$

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$$4a = 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$4a = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

Proving : $A^2 - 4a - 5I = 0$

$$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 0$$

Hence Proved

Q.6 $(x \quad 2y \quad 3z) - 2(y \quad z \quad -x) + 3(-z \quad x \quad y) = [-12 \quad 1 \quad 17]$

$(x \quad 2y \quad 3z)(-2y \quad -2z \quad 2x)(-3z \quad 3x \quad 3y) = [-12 \quad 1 \quad 17]$

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$$x - 2y - 3z = -12 \quad \text{———} \quad (1)$$

$$3x + 3y - 2z = 1 \quad \text{———} \quad (2)$$

$$2x + 3y + 3z = 17 \quad \text{———} \quad (3)$$

- From eq. (1) & (2) :-

$$x \quad \cancel{-2y} \quad -3z = -12$$

$$3x \quad \cancel{2y} \quad -2z = 1$$

$$\boxed{4x - 5z = -11 \quad \text{———} \quad (4)}$$

- From eq. (1) & (3) :-

$$3x \quad \cancel{-6y} \quad -9z = -36$$

$$4x \quad \cancel{6y} \quad 6z = 34$$

$$\boxed{7x - 3z = -2 \quad \text{———} \quad (5)}$$

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$$12x - 15z = -33 \quad — ④ \times 3$$
$$35x - 15z = -10 \quad — ⑤ \times 5$$
$$- \quad + \quad\quad +$$

$$-23x = -23$$

$$\boxed{x = \frac{\cancel{-}\cancel{23}}{\cancel{-}\cancel{23}} = 1}$$

- Putting the value of x in equation ⑤

$$7 - 3z = -2$$

$$-3z = -2 - 7$$

$$-3z = -9$$

$$\boxed{z = \frac{\cancel{-}\overset{3}{\cancel{9}}}{\underset{1}{\cancel{-}\cancel{3}}} = 3}$$

- Putting the value of x & z in eq. ①

$$1 - 2y - 3 \times 3 = -12$$

$$= -2y = -12 + 8$$

$$\boxed{y = \frac{\cancel{8}}{2} \quad \frac{\cancel{-}\cancel{4}}{\cancel{-}\cancel{2}} = 2}$$

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07 If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^2 - (a+d)A = (bc-ad)I$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

A^2

$$\begin{bmatrix} a^2+bc & ab+bd \\ ca+dc & cb+d^2 \end{bmatrix} - (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} \cancel{a^2}+bc & \cancel{ab+bd} \\ \cancel{ca+dc} & cb+\cancel{d^2} \end{bmatrix} - \begin{bmatrix} \cancel{a^2}+ad & \cancel{ab+bd} \\ \cancel{ac+cd} & ad+\cancel{d^2} \end{bmatrix}$$

$$\begin{bmatrix} bc-ad & 0 \\ 0 & bc-ad \end{bmatrix} = (bc-ad)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(bc-ad)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (bc-ad)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence proved

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I Q.8 $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$, Prove that

i. $A + A^T$ is Symatric Matrix

ii $A - A^T$ is skew symatric Matrix

iii AA^T is symmetrix

(i) Symatric Matrix = $\boxed{A = A^T}$

$$A + A^T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & 3 \\ -2 & 3 & 4 \end{bmatrix}$$

$$A + A^T = \left[A + A^T \right]^T$$

Hence Proved.

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ii $A - A^T$

Skew symatric matrix = $\boxed{A = -A^T}$

$$A - A^T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$[A - A^T]^T = \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & -3 \\ 0 & 3 & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}$$

$A - A^T = [A - A^T]^T$ Hence Proved

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iii AA^T

$$AA^T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+2\times2+1 & 1\times0+2\times1+-1\times3 & -1\times1+2\times0+-1\times2 \\ 0\times1+1\times2+3\times-1 & 1\times1+3\times3 & 0\times-1+1\times0+3\times2 \\ -1\times1+0\times2+2\times-1 & -1\times0+0\times1+2\times3 & -1\times-1+0\times0+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+1 & 2+(-3) & -1+(-2) \\ 2+(-3) & 1+9 & 6 \\ -1+(-2) & 6 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -1 & -3 \\ -1 & 10 & 6 \\ -3 & 6 & 5 \end{bmatrix}$$

$$AA^T = [AA^T]^T$$

Hence Proved

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Q.9 $$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

$$abc \begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix}$$

$$(abc)(abc) \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$R_1 \longrightarrow R_1 + R_2$

$R_2 \longrightarrow R_2 - R_3$

$$a^2b^2c^2 \begin{vmatrix} 0 & 0 & 2 \\ 0 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix}$$

$$(a^2b^2c^2)(2)(0+2)$$

$$= 4a^2b^2c^2 = 4a^2b^2c^2$$

Hence proved

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Q.10 Solve the following using Cramer rule

$x + 2y + 3z = 6$

$2x + 4y + z = 7$

$3x + 2y + 9z = 14$

$$\begin{array}{cccc} x & y & z & 1 \\ \left|\begin{matrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 2 & 9 \end{matrix}\right| & & & \begin{matrix} 6 \\ 7 \\ 14 \end{matrix} \end{array}$$

$\Delta = 1(36-2) - 2(18-3) + 3(4-12)$

$\Delta = 34 - 30 - 24$

$\Delta = -20$

$$\Delta x = \begin{vmatrix} 6 & 2 & 3 \\ 7 & 4 & 1 \\ 14 & 2 & 9 \end{vmatrix}$$

$=$

$= 6(36-2) - 2(63-14) + 3(14 - \cancel{48}\,56)$

$= \Delta x = 204 - 98 - 126$

$\Delta x = -20$

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$$\Delta y = \begin{vmatrix} 1 & 6 & 3 \\ 2 & 7 & 1 \\ 3 & 14 & 9 \end{vmatrix}$$

$$\Delta y = 1(63-14) - 6(18-3) + 3(28-21)$$

$$\Delta y = 49 - 90 + 21$$

$$\Delta y = -20$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 4 & 7 \\ 3 & 2 & 14 \end{vmatrix}$$

$$\Delta z = 1(56-14) - 2(28-21) + 6(4-12)$$

$$\Delta z = 42 - 14 - 48$$

$$\Delta z = -20$$

$$\boxed{\frac{\Delta x}{\Delta} = \frac{\cancel{-20}}{\cancel{-20}} = 1}$$

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$$\boxed{\frac{\Delta y}{\Delta} = \frac{\cancel{-20}}{\cancel{-20}} = 1}$$

$$\boxed{\frac{\Delta z}{\Delta} = \frac{\cancel{-20}}{\cancel{-20}} = 1}$$

Q.11 Solve the Following eq: using Inverse Matrix Method

$x + y + z = 9$ ——— (1)

$2x + y - z = 0$ ——— ~~(2)~~ (3)

$2x + 5y + 7z = \cancel{5z}\ 52$ ——— ~~(3)~~ (2)

$x + y + z = 9$

$2x + 5y + 7z = 52$

$2x + y - z = 0$

$$\begin{array}{ccc} x & y & z \end{array}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 9 \\ 52 \\ 0 \end{vmatrix}$$

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$$|A| = 1(-5-7) - 1(-2-14) + 1(2-10)$$

$$= -12 + 16 - 8$$

$$= -4$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix}$$

$$= (-5-7)$$

$$= -12$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix}$$

$$= -(-2-14)$$

$$= 16$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$$= (2-10)$$

$$= -8$$

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$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -(-1-1)$$

$$= 2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= (-1-2)$$
$$= -3$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= -(1-2)$$
$$= 1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 5 & 7 \end{vmatrix}$$

$$= (7-5)$$
$$= 2$$

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$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 7 \end{vmatrix}$$

$$= -(7-2)$$

$$= \cancel{-9} -5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix}$$

$$= (5-2)$$
$$= 3$$

$$A = \begin{bmatrix} -12 & 16 & -8 \\ 2 & -3 & 1 \\ 2 & -5 & 3 \end{bmatrix}$$

$$\text{Adjoiming} = \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

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$$A^{-1} = \frac{1}{|A|} \text{ Adjoint of } A$$

$$= \frac{1}{-4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -108 & 104 & 0 \\ 144 & -156 & 0 \\ -72 & 52 & 0 \end{bmatrix}$$

$$= \frac{-1}{4}\begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} = \boxed{x = 1}, \boxed{y = 3}, \boxed{z = 5}$$