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Tutorial No. 2 (Two)

Q.1 If $\vec{A} = \hat{i} + 4\hat{j} + 3\hat{k}$, $\vec{B} = 4\hat{i} + 2\hat{j} + \lambda\hat{k}$. Find value of λ so that vector A and B may be $\perp$ar

$\vec{A} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + 2\hat{j} + \lambda\hat{k}$

We know that $\vec{A}$ and $\vec{B}$ are perpendicular

$\vec{A} \cdot \vec{B} = 0$

$= 4 + 8 + 3\lambda = 0$

$12 + 3\lambda = 0$

$3\lambda = -12$

$\lambda = \frac{-\cancel{12}}{\cancel{3}} = -4$

$\boxed{\lambda = -4}$

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Q.2 For what value of λ vector $2\hat{i}+\lambda\hat{j}+\hat{k}$ and $4\hat{i}-2\hat{j}-2\hat{k}$ are perpendicular.

$$\vec{a}\cdot\vec{b}=0$$

$$= 8-\lambda 2-2 = 0$$

$$= 6-\lambda 2 = 0$$

$$-\lambda 2 = -6$$

$$\lambda = \frac{\cancel{-}6}{\cancel{-}2} = 3$$

Q.3 Find the dot product between two vectors $\vec{A}=7\hat{i}+2\hat{j}+\hat{k}$, $\vec{B}=2\hat{i}-4\hat{j}+3\hat{k}$. Also find the Cosine of $\angle$ between them.

$$\cos\theta = \frac{a\cdot b}{|a||b|}$$

$$\vec{A}\cdot\vec{B} = 14-8+3$$

$$= 9$$

$$|A| = \sqrt{49+4+1} = \sqrt{54}$$

$$|B| = \sqrt{4+16+9} = \sqrt{29}$$

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$$\cos\theta = \frac{9}{\sqrt{54}\sqrt{29}}$$

Q.4 Find the Sine ∠ between vector

$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{B} = 3\hat{i} + 2\hat{j} + \hat{k}$

$$\sin\theta = \frac{a \times b}{|a \times b|}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(2-6) - \hat{j}(\overset{1-9}{\cancel{1-9}}) + \hat{k}(2-6)$$

$$= -4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$|A \times B| = \sqrt{16 + 64 + 16}$$

$$= \sqrt{96}$$

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$$\sin\theta = \frac{1}{\sqrt{96}} \cdot \vartheta - 4\hat{i} + 8\hat{j} - 4\hat{k}$$

Q.5 Prove that the following four vectors coplanner

$4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$

$5\hat{i} + 8\hat{j} + 5\hat{k}$

let

$$\vec{A} = 4\hat{i} + 8\hat{j} + 12\hat{k}$$
$$\vec{B} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$
$$\vec{C} = 3\hat{i} + 5\hat{j} + 4\hat{k}$$
$$\vec{D} = 5\hat{i} + 8\hat{j} + 5\hat{k}$$

$AB = B - A$

$AC = C - A$

$AD = D - A$

$$AB = \begin{array}{r} 2\hat{i} + 4\hat{j} + 6\hat{k} \\ 4\hat{i} + 8\hat{j} + 12\hat{k} \\ - \quad - \quad - \\ \hline -2\hat{i} - 4\hat{j} - 6\hat{k} \end{array}$$

$$AC = \begin{array}{r} 3\hat{i} + 5\hat{j} + 4\hat{k} \\ 4\hat{i} + 8\hat{j} + 12\hat{k} \\ - \quad - \quad - \\ \hline -1\hat{i} - 3\hat{j} - 8\hat{k} \end{array}$$

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$$AD = \begin{array}{r} 5\hat{i} + 3\hat{j} + 5\hat{k} \\ 4\hat{i} + 3\hat{j} + 12\hat{k} \\ - \quad - \quad - \\ \hline \cancel{2}\,\hat{i} - 7\hat{k} \end{array}$$

$$\begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix}$$

$\cancel{-2(-3x}$

$-2(-3\times-7+8\times0)+4(-1\times-7+8\times1)-6(-1\times0+3\times1)$

$-2\times21+4\times15-6\times3$

$=-42+60-18$

$=0$

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Q.6 Prove that vector $-2\hat{i}-2\hat{j}+4\hat{k}$, $-2\hat{i}+4\hat{j}-2\hat{k}$ and $\hat{i}-2\hat{j}-2\hat{k}$ are coplanner.

$$\begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{vmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 1 & -2 & -2 \end{vmatrix}$$

$$-2(4\times-2-2\times2)+2(-2\times-2+2\times1)+4(-2\times-2-1\times4)$$

$$-2(-8-4)+2(-4+2)+4(4-4)$$

$$= -2\times-12+2\times-2$$

$$= 24+(-4)$$

$$= 20$$

Q.7 For what value of λ vector

$a = 2\hat{i}-\hat{j}+\hat{k}$
$b = \hat{i}+2\hat{j}-3\hat{k}$
$c = 3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanner.

$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \end{array}$$
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix}$$

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~~$7\lambda + 28 = 0$~~

$2(2\times5 + 3\times\lambda) + 1(1\times5 + 3\times3) + 1(1\times\lambda - 2\times3)$

$= 20 + 6\lambda + 14 + \lambda - 6$

$= 7\lambda + 28 = 0$

$\lambda = \frac{-28}{7}$

$\boxed{\lambda = -4}$

Q8 A Force $3\hat{i} + \hat{k}$ is passing through point $2\hat{i} - \hat{j} + 3\hat{k}$.
Find the Moment of Force with respect to point $\hat{i} + 2\hat{j} - k$

$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$
$\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{A} - \vec{B} = 2\hat{i} - \hat{j} + 3\hat{k}$
$\hat{i} + 2\hat{j} - \hat{k}$
$- \quad - \quad +$
$\hat{i} - 3\hat{j} + 4\hat{k}$

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$F = 3\hat{i} + \hat{k}$

$$(\vec{F} \times \vec{A} - \vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 4 \\ 3 & 0 & 1 \end{vmatrix}$$

$$= i(-3-0) - j(1-12) \cancel{+ \hat{k}(0+19)} + \hat{k}(0+19)$$

$$= -3\hat{i} + 11\hat{j} + 9\hat{k}$$

Q4 Find the projection of Vector

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$ $\qquad$ $P = \frac{A.B}{|a||b|}$

$\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{A}.\vec{B} = 2 + 2 + 1 = 5$

$|A| = \sqrt{4+1+1} = \sqrt{6}$

$|B| = \sqrt{1+4+1} = \sqrt{6}$

projection of Vector $= \frac{5}{6}$

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Q.10 Prove that the following 4 points are collinear.
(4,5,1) , (0,-1,-1) , (3,9,4) , (-4,4,4)

$\vec{A} = 4\hat{i} + 5\hat{j} + k$

$\vec{B} = -\hat{j} - \hat{k}$

$\vec{C} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$\vec{D} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

$$\begin{array}{rrrl} A.B = & & -\hat{j} & -\hat{k} \\ & 4i & +5\hat{j} & +k \\ & - & - & - \\ \hline & -4\hat{i} & -6\hat{j} & -2\hat{k} \end{array}$$

$$\begin{array}{rrrl} A.C = & 3\hat{i} & +9\hat{j} & +4\hat{k} \\ & 4\hat{i} & +5j & +k \\ & - & - & - \\ \hline & -i & +4j & +3k \end{array}$$

$$\begin{array}{rrrl} AD = & -4\hat{i} & +4\hat{j} & +4\hat{k} \\ & 4\hat{i} & +5\hat{j} & +k \\ & - & - & - \\ \hline & -8i & -\hat{j} & +3\hat{k} \end{array}$$

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$$\begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$$-4(4\times 3 + 3\times 1) + 6(-1\times 3 + 3\times 8) - 2(-1\times -1 + 4\times 8)$$

$$= -4(12+3) + 6(-3 + 24) - 2(1+32)$$

$$= -4\times 15 + 6\times 21 - 2\times 33$$

$$= -60 + 126 - 66 = 0$$

Hence Proved

Q.12 If vector $2\hat{i} + P\hat{j} + 2\hat{k}$ and $4\hat{i} + 2\hat{j} - \hat{k}$ are $\perp$ar to each other than P.

$$\vec{A}\cdot\vec{B} = \cancel{8+2P=0} \quad 8+2P-2 = 0$$

$$= 6+2P = 0$$

$$= 2P = -6$$

$$P = \frac{-6}{2} = -3$$