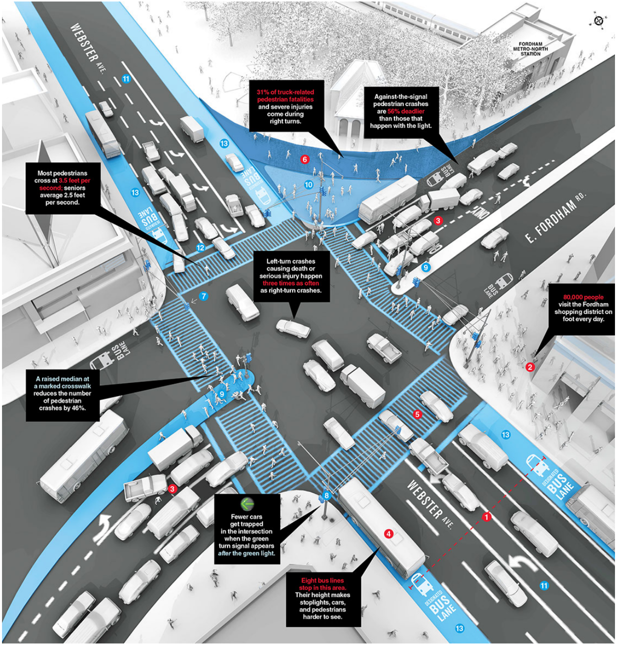
EE 144/244: Modeling Automated Intersections

Albert Ou Praagya Singh Ross Yeager

University of California, Berkeley

13 May 2013



Source: http://nymag.com/news/features/worst-nyc-traffic-intersections-2012-12/

Autonomous Intersections

- Future roadways face congestion issues due to increasing volume and population density, which would likely lead to greater accident rates
- Autonomous cars will become much more prevalent in future

Objective

Model an intelligent intersection system with a centralized controller (as opposed to a peer-to-peer protocol) to route autonomous vehicles

Original Vehicle Model

Originally, we assumed an intersection environment that contained both human and autonomously controlled vehicles where:

- Human controlled vehicles respond only to `START` and `STOP` commands from the controller
- Autonomous vehicles respond to `START`, `STOP`, `CHNG`, and `SLWDN`
- Controller analyzes all vehicles in system and sends out commands to optimize throughput.
- The intersection is a hybrid system containing both discrete and continuous components.

Controller Knowledge

Vehicles are equipped with embedded telemetry units that continually notify the controller with certain information while within the intersection environment:

- Current location
- Current speed
- Planned route
- Entrance lane

Deviations from Original Intent

Changes in intersection system model:

- Shift of focus from optimization towards correct modeling
- Constant velocity: $a(t) = 0$
- Changes in velocity can occur instantaneously
- System contains autonomous vehicles only
- Vehicle reaction to controller command is instantaneous
- Intersection turns have no effect on velocity

Rationale for Changes

- Optimization of the system may be intractable for application
- Human-controlled vehicles would be a rarity in the described system
- Constant velocity model reasonable within limited intersection distances
- Limitations with dynamic actor instantiation in Ptolemy modeling
- Learning curve involved with modeling tool (Ptolemy)

Modeling Objectives

Let c denote the event in which a collision event has occurred at time t, and let e denote the event in which car v has entered a route within the intersection.

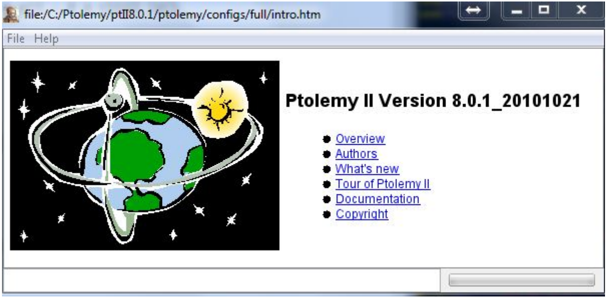
- Safety: $\forall t : \neg Gc$
- Fairness: $\forall v : Fe$

Vehicle Properties

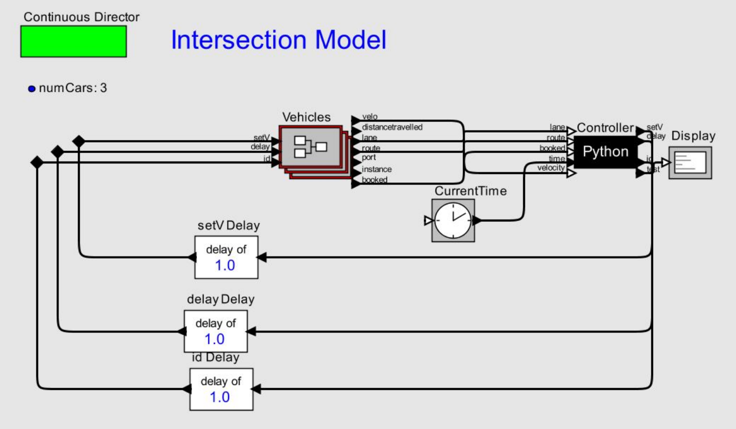
- Instantaneous change in velocity
- Constant velocity within intersection
- Velocity (MPH): $V \to V \in [20, 40]$
- Route intention constant throughout intersection traversal
- Randomized vehicle entry times
- Receives velocity change or delayed start time from controller

Modeling Tool

Ptolemy II Version 8.0.1



Intersection Model



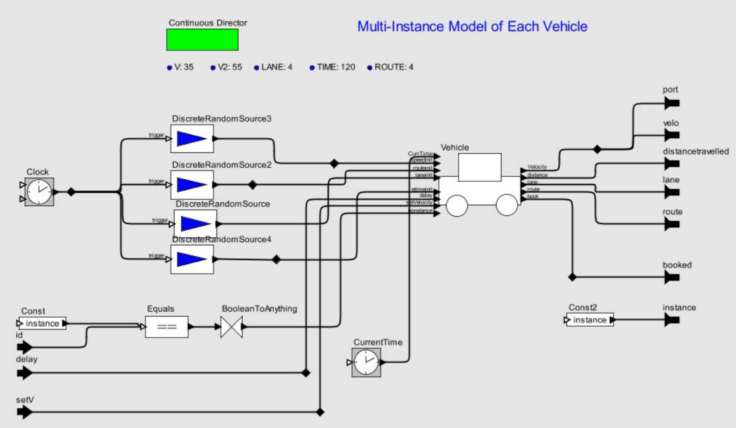
Modeling Continuous Vehicle Flow

Ptolemy limitations prevent dynamic instantiation of actors.

Multi-Instance Composite Actor

- Vehicle instances accessed by ID encoding
- Create N vehicle Modal Model instances
- Generate random:
  - Intersection entry time: $1 - 45$ seconds
  - Initial velocity (m/s): $V_0 \in [20, 40]$
  - Direction: N, S, E, W
  - Route: $L, S1, S2, R$
- Implement as many instances of the car model as desired
- Interacts with Python-based controller
- Car can "re-enter" system only after it exits (re-use vehicles vs. generating new vehicles)

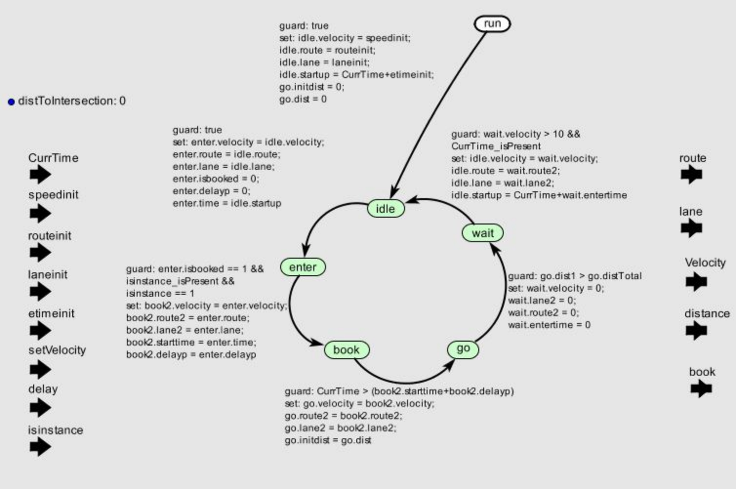
Vehicle Modeling



Vehicle States

- RUN
- $IDLE$
- $ENTER$
- $BOOK$
- GO
- $WAIT$

Vehicle FSM Actor



ENTER State

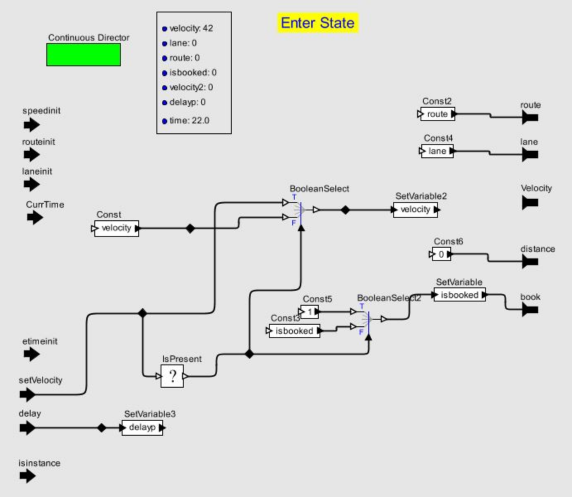
Abstraction:

- Equivalent to pre-conflict entrance approach
- Initial velocity used to calculate time until intersection
- Stops at intersection if instructions not received from controller

Guard:

- $t_{\text{GLOBAL}} > t_{\text{ENTER}}$

ENTER State


Enter State
Continuous Director
velocity: 42
lane: 0
route: 0
isbooked: 0
velocity2: 0
delayp: 0
time: 22.0
speedinit
routeinit
laneinit
CurrTime
Const
velocity
BooleanSelect
SetVariable2
velocity
Const2
route
route
Const4
lane
lane
Velocity
Const6
0
distance
SetVariable
isbooked
book
Const5
1
Const3
isbooked
BooleanSelect2
IsPresent
?
etimeinit
setVelocity
delay
SetVariable3
delayp
isinstance

GO State

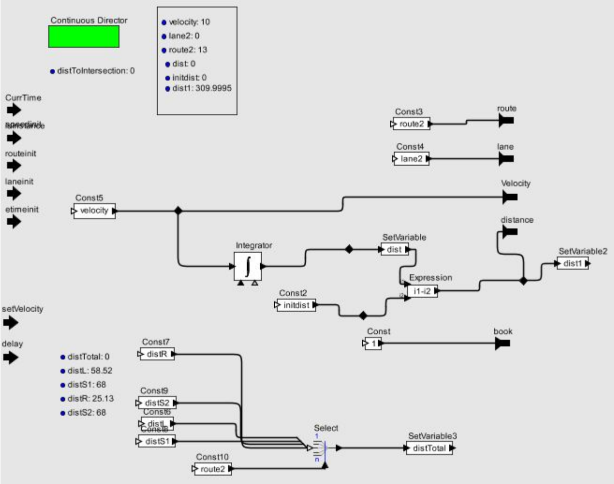
Abstraction:

- Equivalent to the approach through the intersection
- Tracks when the vehicle leaves the system
- Stops at intersection if instructions not received from controller

Guard:

- $t_{\text{GLOBAL}} > t_{\text{STRAIGHTAWAY}}$

GO State


Continuous Director
distToIntersection: 0
velocity: 10
lane2: 0
route2: 13
dist: 0
initdist: 0
dist1: 309.9995
CurrTime
routeinit
laneinit
etimeinit
setVelocity
delay
Const3
route2
route
Const4
lane2
lane
Const5
velocity
Velocity
distance
Integrator
SetVariable
dist
SetVariable2
dist1
Expression
i1-i2
Const2
initdist
Const
1
book
distTotal: 0
distL: 58.52
distS1: 68
distR: 25.13
distS2: 68
Const7
distR
Const9
distS2
distL
distS1
Select
SetVariable3
distTotal
Const10
route2

Collision Detection Schemes

The primary rationale for an automated intersection is to prevent traffic accidents. Methods to predict collisions:

- Tiling – Autonomous Intersection Management (AIM) project at UT Austin
- Euclidean distance between three-dimensional trajectories $f(x, y, t)$

We would like an alternative that is less computationally intensive and amenable to optimization.

Conflict Points

Definition

A *traffic conflict point* is the point at which multiple traffic flows may cross or merge.

- Require that all vehicles adhere to designated lanes when traversing the intersection, so that trajectories can be bounded exactly.
- The set of conflict points are determined by the intersections between curves representing all legal paths of vehicles.

Intersection Geometry

- Four-way symmetric intersection with orthogonal arms and perfect alignment
- Open space without obstructions (e.g., channelization features, pedestrians)
- Total number of lanes at any exit approach equals the total number of lanes at any entrance approach
- Traffic lanes at an exit approach can only serve as the exit of lanes with the same lane number relative to the center line
- No exclusive right/left turn lanes

Path Equations

N	number of lanes per direction
M	number of left turn lanes per direction $(M < N)$
w	lane width
c	corner radius
d	minimum buffer distance between vehicles in potential conflict
k	lane number

Quantities defined for convenience:

$$\alpha_k = \left(k - \frac{1}{2}\right) w \qquad \text{offset from center line}$$

$$\beta_k = \frac{1}{\sqrt{2}} \left[\frac{d}{2} + \left(M - k + \frac{1}{2}\right) w\right] \qquad \text{projected offset from origin}$$

$$\gamma = Nw + c \qquad \text{half of intersection width}$$

Path Equations

- Straight:

$$y = \pm\alpha_k \qquad k = 1, \ldots, N \tag{1}$$

$$x = \pm\alpha_k \qquad k = 1, \ldots, N \tag{2}$$

- Right turns: Circular quadrants of radius $\frac{w}{2} + c$ $(k = M + 1, \ldots, N)$

Direction	Center	Entrance	Exit
S $\to$ E	$(\gamma, -\gamma)$	$(\alpha_k, -\gamma)$	$(\gamma, -\alpha_k)$
N $\to$ W	$(-\gamma, \gamma)$	$(-\alpha_k, \gamma)$	$(-\gamma, \alpha_k)$
E $\to$ N	(γ, γ)	(γ, α_k)	(α_k, γ)
W $\to$ S	$(-\gamma, -\gamma)$	$(-\gamma, -\alpha_k)$	$(-\alpha_k, -\gamma)$

Path Equations: Left Turns

- Circular arcs fitted to three control points ($k = 1, \ldots, M$):

Direction	Entrance	Midpoint	Exit
S → W	$(\alpha_k,\ -\gamma)$	$(-\beta_k,\ -\beta_k)$	$(-\gamma,\ \alpha_k)$
N → E	$(-\alpha_k,\ \gamma)$	$(\beta_k,\ \beta_k)$	$(\gamma,\ \alpha_k)$
E → S	$(\gamma,\ \alpha_k)$	$(\beta_k,\ -\beta_k)$	$(-\alpha_k,\ -\gamma)$
W → N	$(-\gamma,\ -\alpha_k)$	$(-\beta_k,\ \beta_k)$	$(\alpha_k,\ \gamma)$

- Outermost left turn lane is diagonally offset $\frac{d+w}{2}$ from origin to avoid conflicts with oncoming left turn traffic.
- For simplicity, U-turns are disallowed.

Circumcircle of a Triangle

$$a = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad b_x = -\begin{vmatrix} (x_1^2 + y_1^2) & y_1 & 1 \\ (x_2^2 + y_2^2) & y_2 & 1 \\ (x_3^2 + y_3^2) & y_3 & 1 \end{vmatrix}$$

$$c = -\begin{vmatrix} (x_1^2 + y_1^2) & x_1 & y_1 \\ (x_2^2 + y_2^2) & x_2 & y_2 \\ (x_3^2 + y_3^2) & x_3 & y_3 \end{vmatrix} \qquad b_y = \begin{vmatrix} (x_1^2 + y_1^2) & x_1 & 1 \\ (x_2^2 + y_2^2) & x_2 & 1 \\ (x_3^2 + y_3^2) & x_3 & 1 \end{vmatrix}$$

Circumcenter:

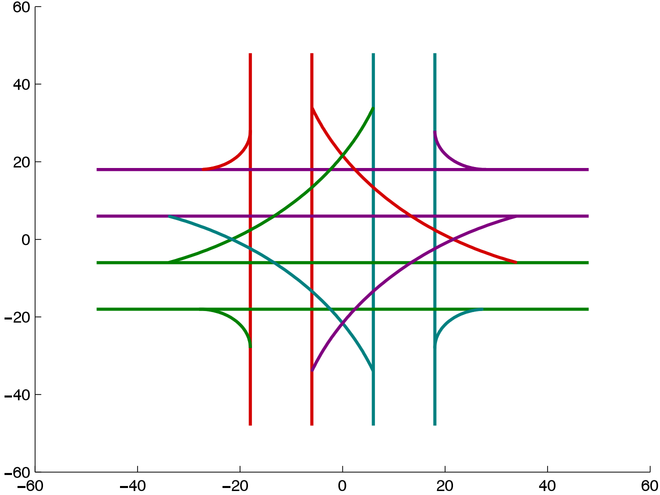
$$x_c = -\frac{b_x}{2a} \qquad (3)$$

$$y_c = -\frac{b_y}{2a} \qquad (4)$$

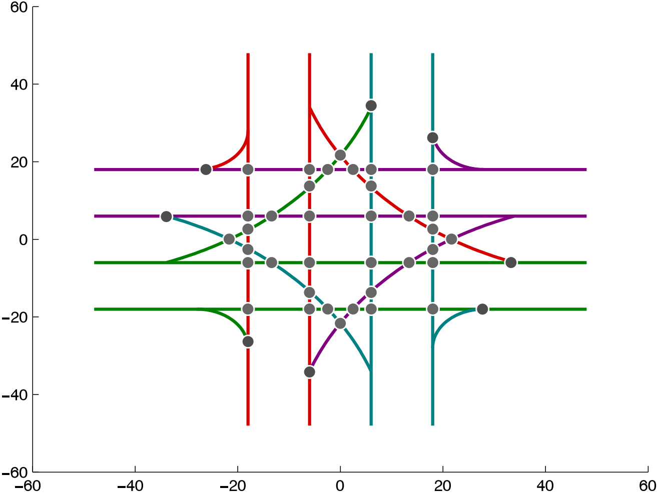
Circumradius:

$$r = \frac{\sqrt{b_x^2 + b_y^2 - 4ac}}{2|a|} \qquad (5)$$

Conflict Points of Model Intersection



Conflict Points of Model Intersection



Conflict Zones

- In reality, vehicles are not point masses – they occupy area.
- **Safety property**: Enforce a minimum separation distance d at all times between vehicles traveling on conflicting paths.

Definition

A *traffic conflict zone* is the segment of a given path within distance d of a conflict point.

- The initial and terminal points of a conflict zone can be expressed as an ordered pair of offsets/displacements along the curve, relative to the start of the path.

Position as Arc Length

- Recall that all turns are modeled as circular arcs.
- Central angle with vertex (x_c, y_c), endpoint (x, y) on the circle, and one side parallel to the x-axis:

$$\theta(x, y) = \left|\arccos\left(\frac{x - x_c}{r}\right)\right| = \left|\arcsin\left(\frac{y - y_c}{r}\right)\right| \tag{6}$$

- Arc length (displacement) from the initial point (x_0, y_0) to the terminal point (x_1, y_1):

$$s = r|\theta(x_1, y_1) - \theta(x_0, y_0)| \tag{7}$$

- Only one coordinate needs to be known for each endpoint.

Conflict Point Displacements

- $w = 12$ ft, $c = 10$ ft, $d = 15$ ft
- Merge conflict points (highlighted in red) are the exit points of turns, which also indicate the total length of each path.

	$\longrightarrow$				$\downarrow$				$\longleftarrow$			
	L	S1	S2	R	L	S1	S2	R	L	S1	S2	R
L	44.62	34.44	18.12	–	–	23.99	40.30	–	13.80	58.42	–	–
S1	–	28.00	16.00	–	47.38	–	–	–	20.62	40.00	52.00	68.00
S2	68.00	28.00	16.00	–	36.39	–	–	–	31.61	40.00	52.00	–
R	–	–	25.13	–	–	–	–	–	–	–	–	–

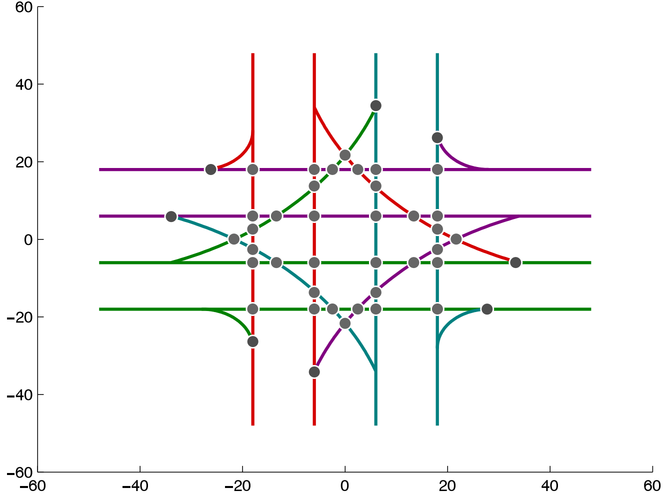
Conflict Zone Partitioning

Rigorous method is to calculate intersection points between a circle of radius d, centered at conflict point (x, y), and involved path curves – more complicated than necessary.

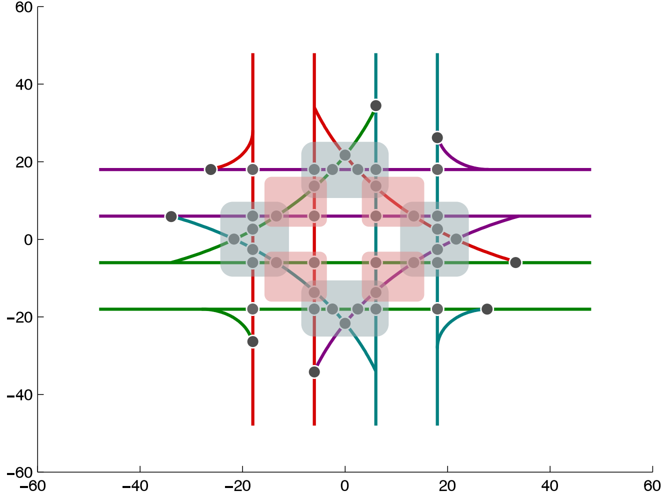
- Conflict zones for straight paths are simply $x \pm d$ for E $\rightleftarrows$ W and $y \pm d$ for N $\rightleftarrows$ S.
- The chord length approximates the circular arc length when the subtended angle remains relatively small. For left turns where $d \ll r$, the extent of a conflict zone can be treated as $s \pm d$.
- The entire right turn path is considered a conflict zone.

Aggregation: Group conflict points into multiple overlapping zones for coverage and flexibility

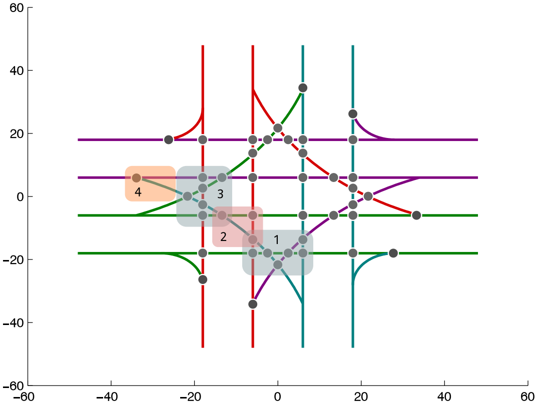
Conflict Zone Aggregation



Conflict Zone Aggregation

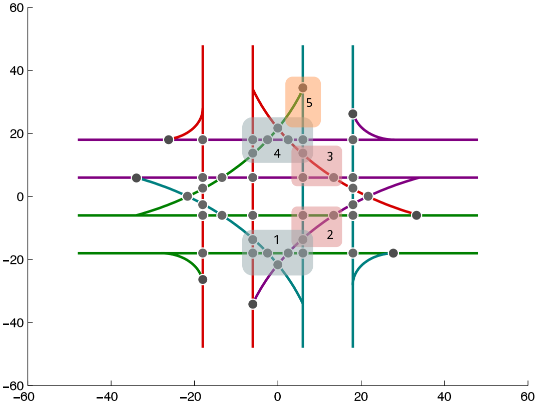


Left Turn Path



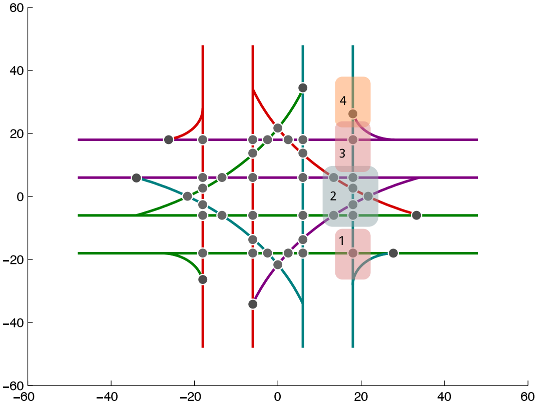
1	8.80	28.99
2	18.88	39.44
3	29.44	49.62
4	43.52	58.52

Straight Path (Inner)



1	8.50	25.62
2	15.62	34.00
3	34.00	52.38
4	42.38	59.50
5	53.00	68.00

Straight Path (Outer)



1	8.50	23.50
2	20.50	47.50
3	44.50	59.50
4	53.00	68.00

Collision Detection Algorithm

Given: Intended route and lane assignment, initial velocity v_0 at the intersection entrance, planned acceleration function $a(t)$, dimensions of vehicle

1. Determine the set of conflict zones along the path.
2. Numerically integrate to determine when endpoints of conflict zones are crossed, yielding a set of time intervals.
3. For each conflict zone, check whether the corresponding time interval overlaps with that of any other active vehicle.

Observations

- Conflict zones are static and therefore need only to be calculated once during initialization.
- Runtime does not require knowledge about curvature of paths – only linear displacements matter.

Scheduling

Reservations for a vehicle are scheduled as time intervals during which it has exclusive access to the conflict zones along its path.

As with majority of other CS problems, finding a solution is much more difficult than checking a solution.

Questions for Validation

- Is it possible for a vehicles with different velocities in the same lane to collide without being detected?
- Are static conflict zones optimal?
- Can collisions occur at locations other than conflict points?

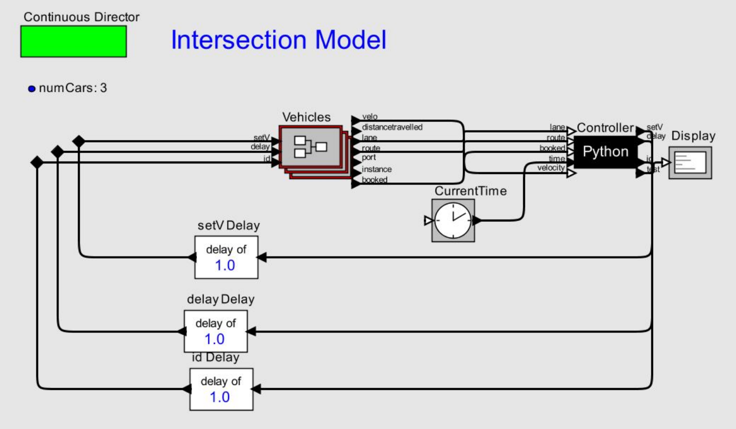
Questions for Validation

- Is it possible for a vehicles with different velocities in the same lane to collide without being detected?
  - No, if conflict zones overlap, then the two vehicles will be in the same conflict zone when the incident occurs.
- Are static conflict zones optimal?
  - No, conflict zones must consider worst-case scenarios. In reality, vehicles may be at opposite ends of the same conflict zone and be sufficiently separated.
- Can collisions occur at locations other than conflict points?
  - Yes, trajectories can conceivably pass closely together but not intersect. Intersection topology must be designed to avoid this.

Controller Properties

- Lack of multiport output (serial communication)
- Delayed feedback to vehicle
  - Prevent a zeno system
  - Models communication delay
- Receives requests and sends commands to system vehicles
- Inputs to controller: $velocity$, $booking$, $entertime$
- Python script actor

Intersection Model



Controller Algorithm

Initialize list of conflict zones and displacements along paths
for each timestep t **do**
    $current \leftarrow$ all tokens received from Vehicle modal model
    for each car in $current$ **do**
        if car needs reservation **then**
            $v_{\text{max}} \leftarrow$ maximum velocity $\in [v_{\text{current}} \pm 10]$ that ensures safety
            if v_{max} exists **then**
                $v_{\text{new}} \leftarrow v_{\text{max}}$
                $t_{\text{delay}} \leftarrow 0$
            else
                $v_{\text{new}} \leftarrow v_{\text{current}} + 10$
                $t_{\text{delay}} \leftarrow$ end of latest reservation for all conflict zones
            end if
            broadcast(v_{new}, t_{delay}, id $\leftarrow car$.id)
        end if
    end for
end for

Technical Challenges

- Continuous car flow given static number of vehicles
- Delivering controller commands to specified car instance
- Continuous time solver time step evaluation issues
- Hybrid system causality issues
- Scalability

Results

- Functional hybrid model
- Continuous vehicle flow with static vehicle set
- Centralized responsive controller
- Rigorous intersection path definitions
- Multiplexed communication to vehicles

Further Work

- Investigate resource scheduling algorithms for optimization
- Introduce non-zero vehicle acceleration into the model
- Add support for vehicles with human drivers