Integer Encoding

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Encoding Integers - Unsigned

- Bijection

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

```
short int x =  15213;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101

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Encoding Integers - Signed

- 2s complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x =  -15213;
```

	Decimal	Hex	Binary
y	-15213	C4 93	11000100 10010011

Encoding Integers

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- Unsigned Values

  UMin = 0

  000…0

  UMax = $2^w - 1$

  111…1

- Two's Complement Values

  TMin = -2^{w-1}

  100…0

  TMax = $2^{w-1} - 1$

  011…1

Width = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Ranges - Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

$$|TMin| \quad = \qquad TMax + 1$$

Asymmetric range

$$UMax \quad = \qquad 2 * TMax + 1$$

```
#include <limits.h>
```

K&R App. B11

Declares constants, e.g.,

```
ULONG_MAX
LONG_MAX
LONG_MIN
```

Values platform-specific

Signed/Unsigned Numeric Values

- Equivalence

  Same encodings for nonnegative values

- Uniqueness

  Every bit pattern represents unique integer value

  Each representable integer has unique bit encoding

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

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Converting Signed to Unsigned

- C Allows conversions from `signed` to `unsigned`

```
short int          x =  15213;
unsigned short int ux = (unsigned short) x;
short int          y  = -15213;
unsigned short int uy = (unsigned short) y;
```

- Resulting Value

  No change in bit representation

  Nonnegative values unchanged

    ux = 15213

  Negative values change into (large) positive values

    uy = 50323

Signed and Unsigned

Weight	-15213		50323	
1	1	1	1	1
2	1	2	1	2
4	0	0	0	0
8	0	0	0	0
16	1	16	1	16
32	0	0	0	0
64	0	0	0	0
128	1	128	1	128
256	0	0	0	0
512	0	0	0	0
1024	1	1024	1	1024
2048	0	0	0	0
4096	0	0	0	0
8192	0	0	0	0
16384	1	16384	1	16384
32768	1	*-32768*	1	*32768*
Sum		**-15213**		**50323**

$uy \quad = \quad y + 2 * 32768 = \quad y + 65536$

Signed vs Unsigned in C

- Constants

  By default are considered to be signed integers

  Unsigned if have "U" as suffix

```
0U, 4294967259u
```

- Casting

  Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

C Casting Surprises

- Expression Evaluation
  - If mix unsigned and signed in single expression, signed values implicitly cast to unsigned
  - Including comparison operations <, >, ==, <=, >=
  - Examples for *W* = 32

Constant$_1$	Constant$_2$	Relation	Evaluation
`0`	`0u`		
`-1`	`0`		
`-1`	`0u`		
`2147483647`	`-2147483648`		
`2147483647u`	`-2147483648`		
`-1`	`-2`		
`(unsigned) -1`	`-2`		
`2147483647`	`2147483648u`		
`2147483647`	`(int) 2147483648u`		

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C Casting Surprises

- Expression Evaluation

  If mix unsigned and signed in single expression, signed values implicitly cast to unsigned

  Including comparison operations <, >, ==, <=, >=

  Examples for *W* = 32

- Constant$_1$ / Constant$_2$ / Relation / Evaluation

Constant$_1$	Constant$_2$	Relation	Evaluation
`0`	`0u`	==	**unsigned**
`-1`	`0`	<	**signed**
`-1`	`0u`	>	**unsigned**
`2147483647`	`-2147483648`	>	**signed**
`2147483647u`	`-2147483648`	<	**unsigned**
`-1`	`-2`	>	**signed**
`(unsigned) -1`	`-2`	>	**unsigned**
`2147483647`	`2147483648u`	<	**unsigned**
`2147483647`	`(int) 2147483648u`	>	**signed**

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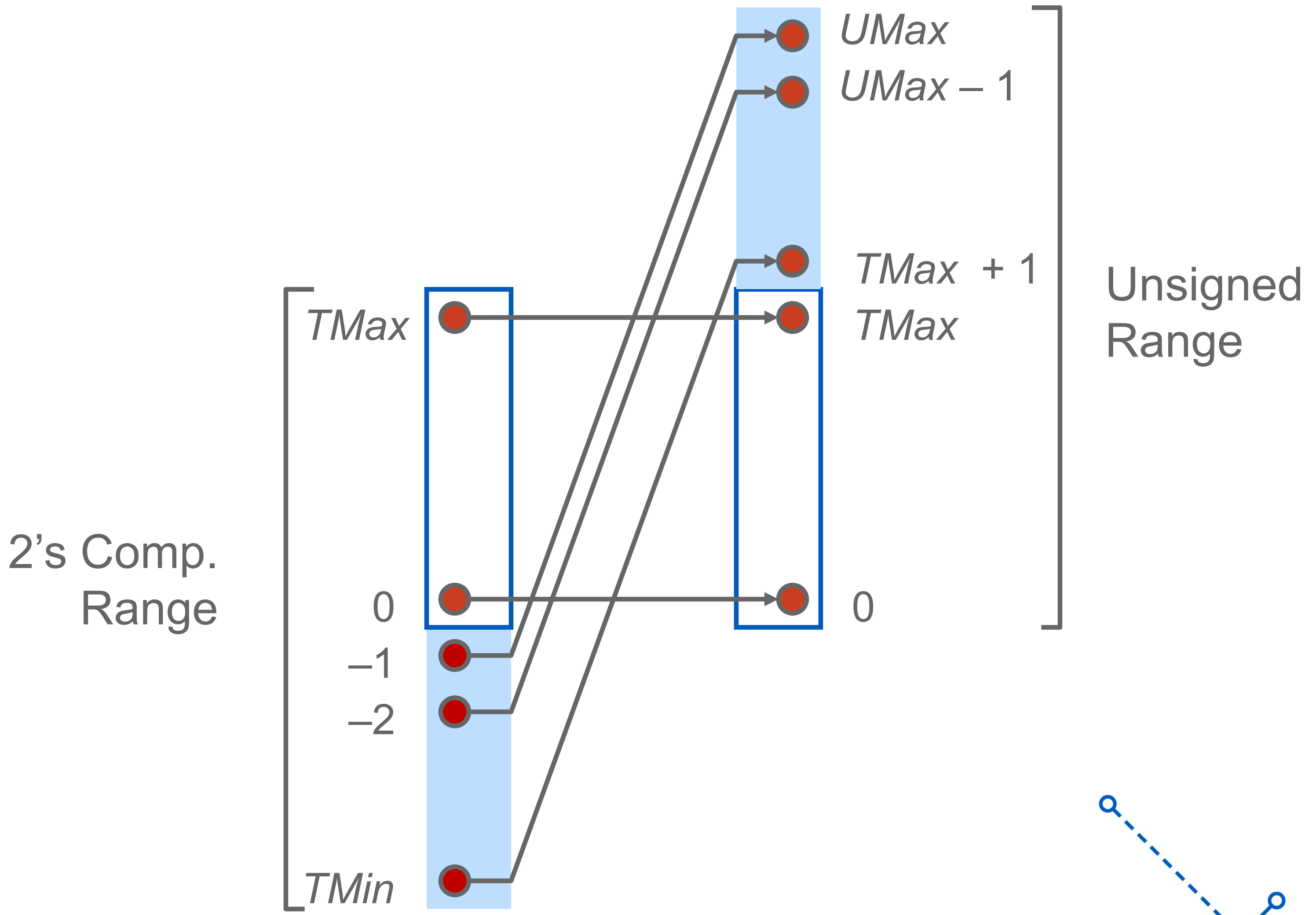
Casting Visualization

- 2’s Comp. → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



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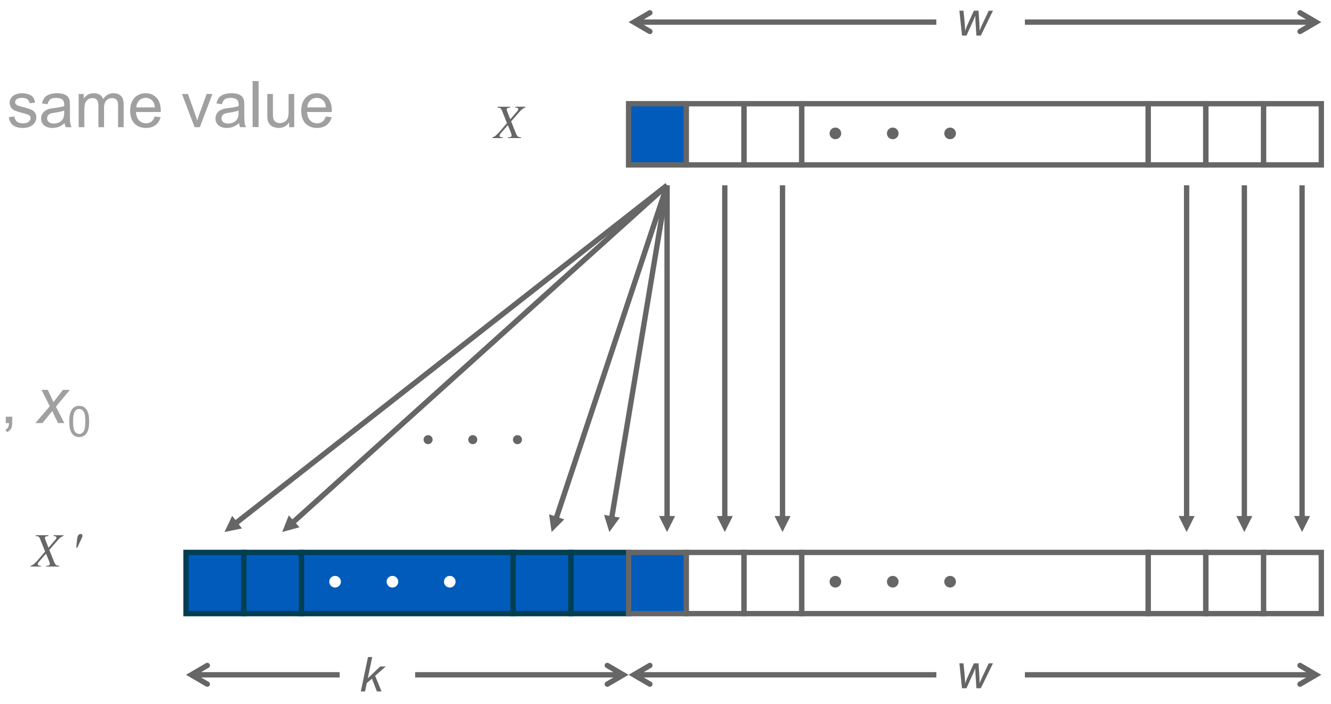
Sign Extension

- Task:
  Given *w*-bit signed integer *x*
  Convert it to *w*+*k*-bit integer with same value
- Rule:
  Make *k* copies of sign bit:

$$X' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \ldots, x_0$$



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Sign Extension Example

```
short int x =  15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

Converting from smaller to larger integer data type

C automatically performs sign extension

Homework (Optional)

For each of the following C expressions, either:

- Argue that it is true for all argument values
- Give example where it is not true

Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

- `x < 0` ⇒ `((x*2) < 0)`
- `ux >= 0`
- `x & 7 == 7` ⇒ `(x<<30) < 0`
- `ux > -1`
- `x > y` ⇒ `-x < -y`
- `x * x >= 0`
- `x > 0 && y > 0` ⇒ `x + y > 0`
- `x >= 0` ⇒ `-x <= 0`
- `x <= 0` ⇒ `-x >= 0`

Homework 2 (Optional)

- Write your own `dump_mem()` method to read memory byte-by-byte from a given pointer and display it in hex
- Syntax: `void dump_mem(void *ptr, int size);`
- Hint: `char` is a data type of size 1 byte. Casting to `char` allows you to read any data byte-by-byte

Required readings

- B&O 2.2