

# Caching

Karthik Dantu

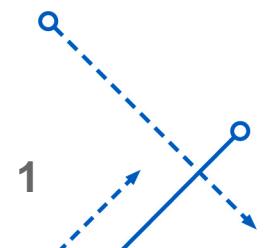
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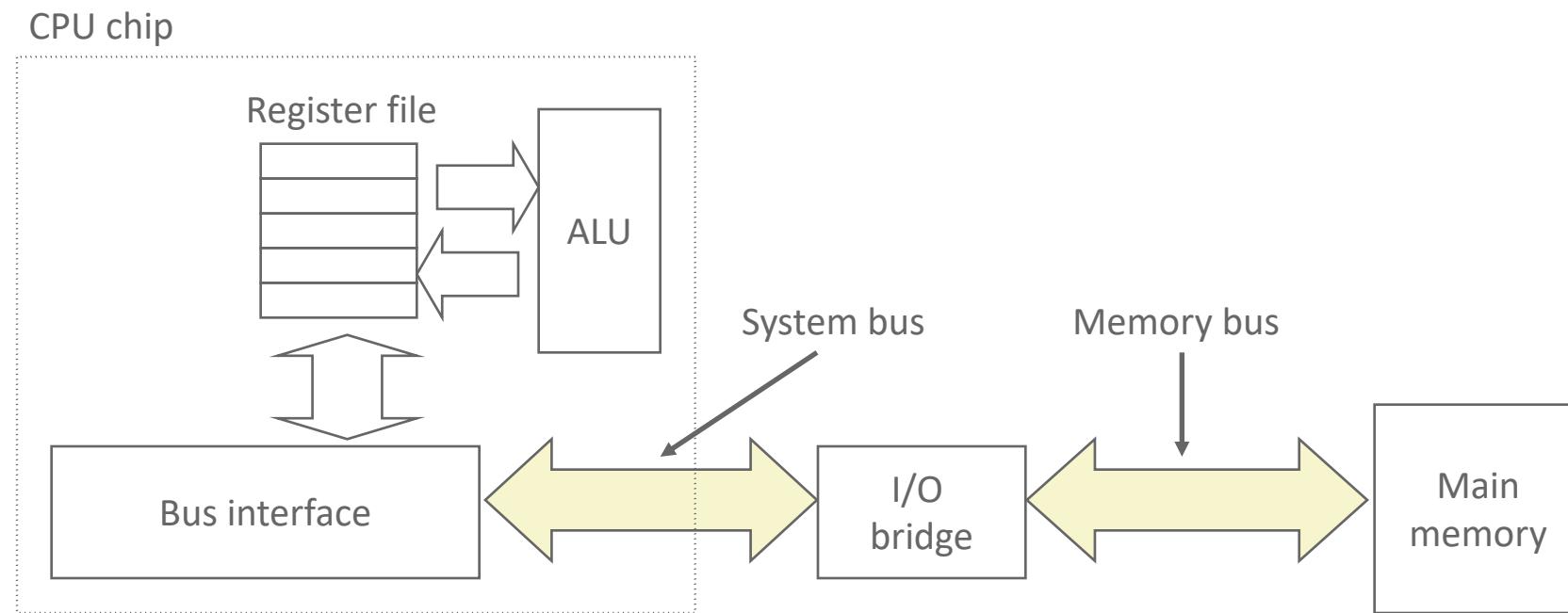


# Writing & Reading Memory

- Write
  - Transfer data from CPU to memory  
`movq 8(%rsp), %rax`
  - “Store” operation
- Read
  - Transfer data from memory to CPU  
`movq %rax, 8(%rsp)`
  - “Load” operation

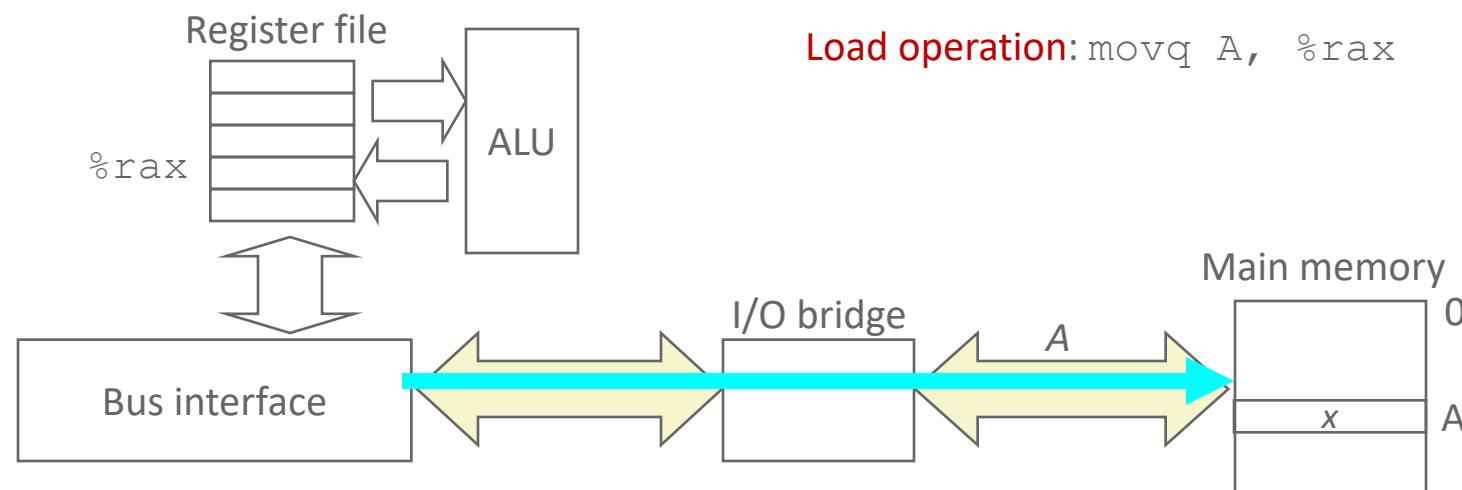
# Traditional Bus Structure Connecting CPU and Memory

- A **bus** is a collection of parallel wires that carry address, data, and control signals.
- Buses are typically shared by multiple devices.



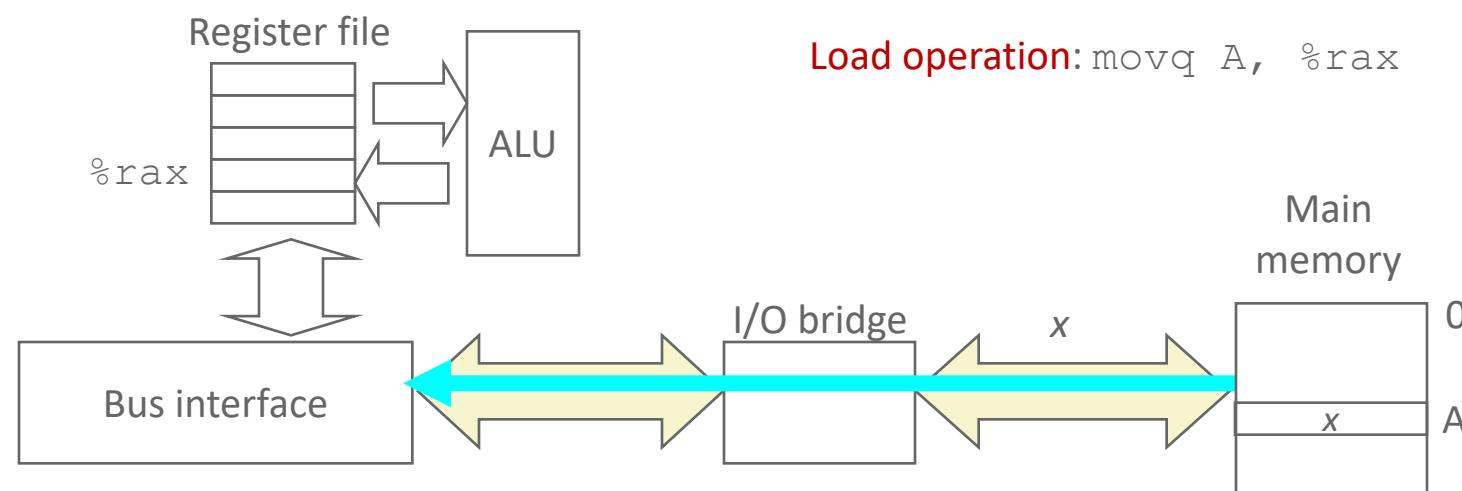
# Memory Read Transaction (1)

- CPU places address A on the memory bus.



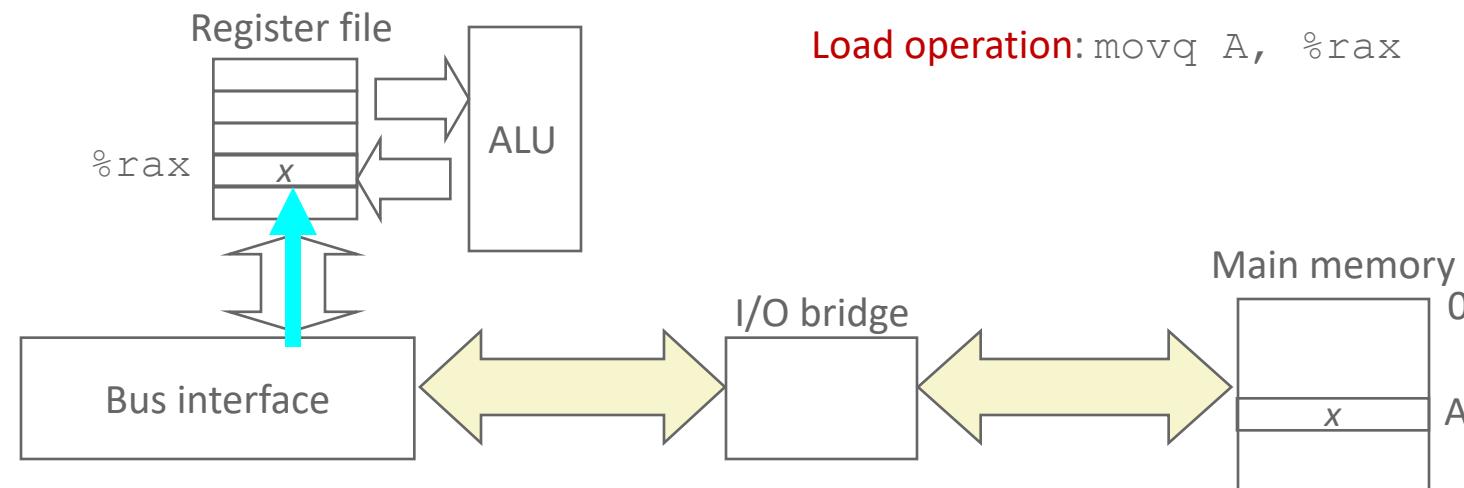
# Memory Read Transaction (2)

- Main memory reads A from the memory bus, retrieves word x, and places it on the bus.



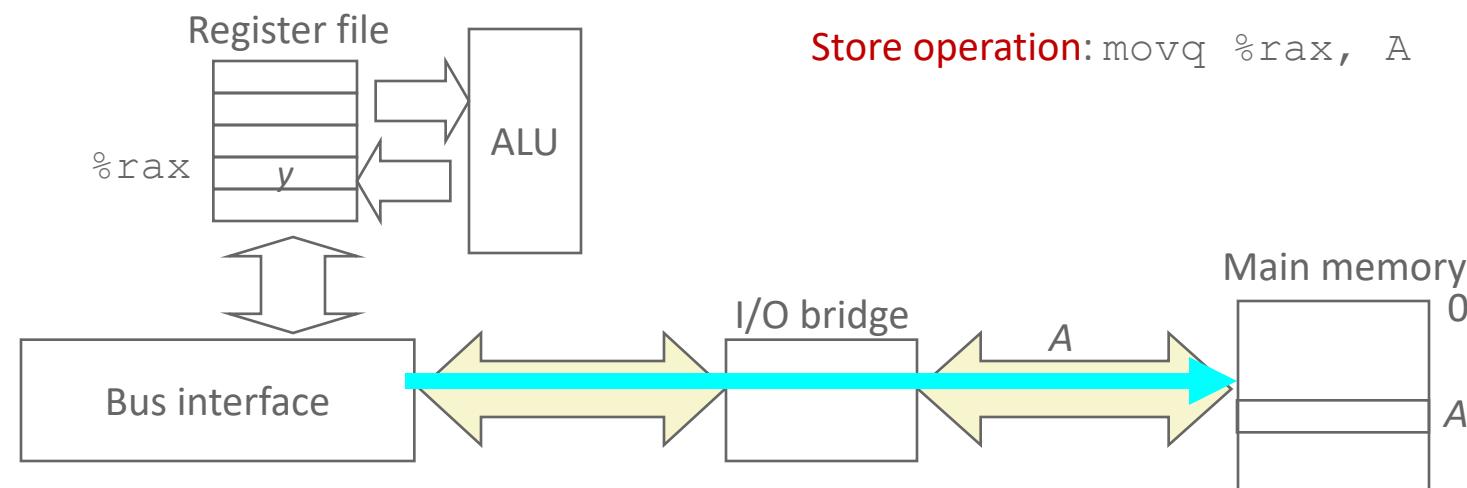
# Memory Read Transaction (3)

- CPU read word  $x$  from the bus and copies it into register  $\%rax$ .



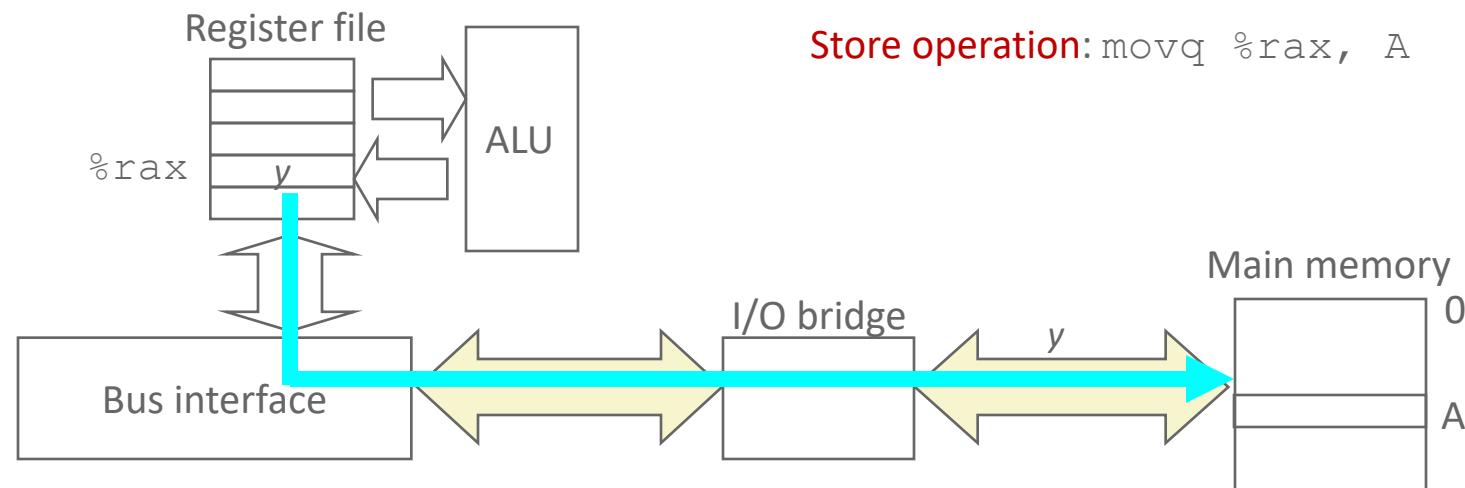
# Memory Write Transaction (1)

- CPU places address A on bus. Main memory reads it and waits for the corresponding data word to arrive.



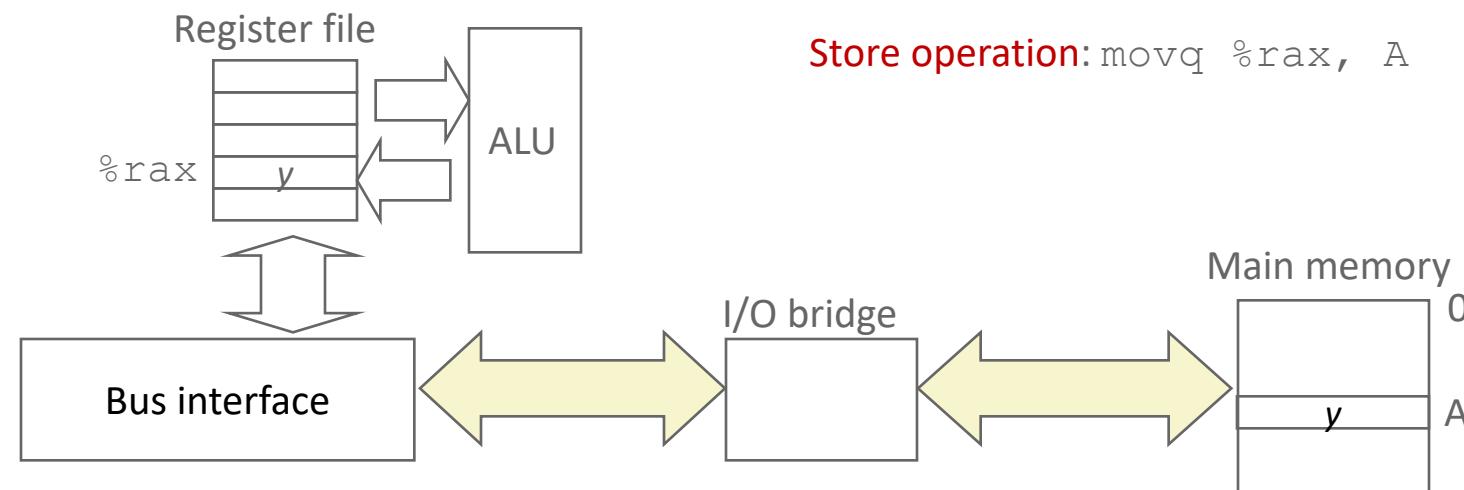
# Memory Write Transaction (2)

- CPU places data word  $y$  on the bus.



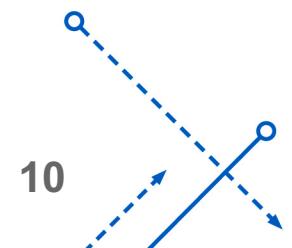
# Memory Write Transaction (3)

- Main memory reads data word  $y$  from the bus and stores it at address A.



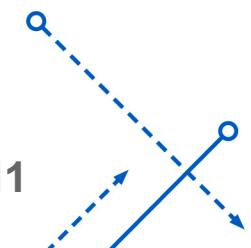
# Today

- The memory abstraction
- RAM : main memory building block
- Locality of reference
- The memory hierarchy
- Storage technologies



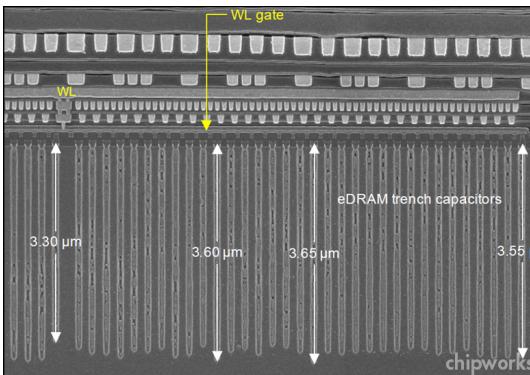
# Random-Access Memory (RAM)

- Key features
  - RAM is traditionally packaged as a chip.
    - or embedded as part of processor chip
  - Basic storage unit is normally a **cell** (one bit per cell).
  - Multiple RAM chips form a memory.
- RAM comes in two varieties:
  - SRAM (Static RAM)
  - DRAM (Dynamic RAM)



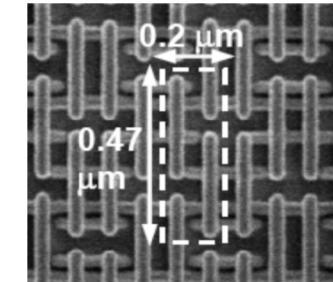
# RAM Technologies

- DRAM



- 1 Transistor + 1 capacitor / bit
  - Capacitor oriented vertically
  - Must refresh state periodically

- SRAM



- 6 transistors / bit
- Holds state indefinitely

# SRAM vs DRAM Summary

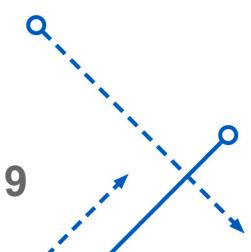
	Trans. per bit	Access time	Needs refresh?	Needs EDC?	Cost	Applications
SRAM	6 or 8	1x	No	Maybe	100x	Cache memories
DRAM	1	10x	Yes	Yes	1x	Main memories, frame buffers

EDC: Error detection and correction

- Trends
  - SRAM scales with semiconductor technology
    - Reaching its limits
  - DRAM scaling limited by need for minimum capacitance
    - Aspect ratio limits how deep can make capacitor
    - Also reaching its limits

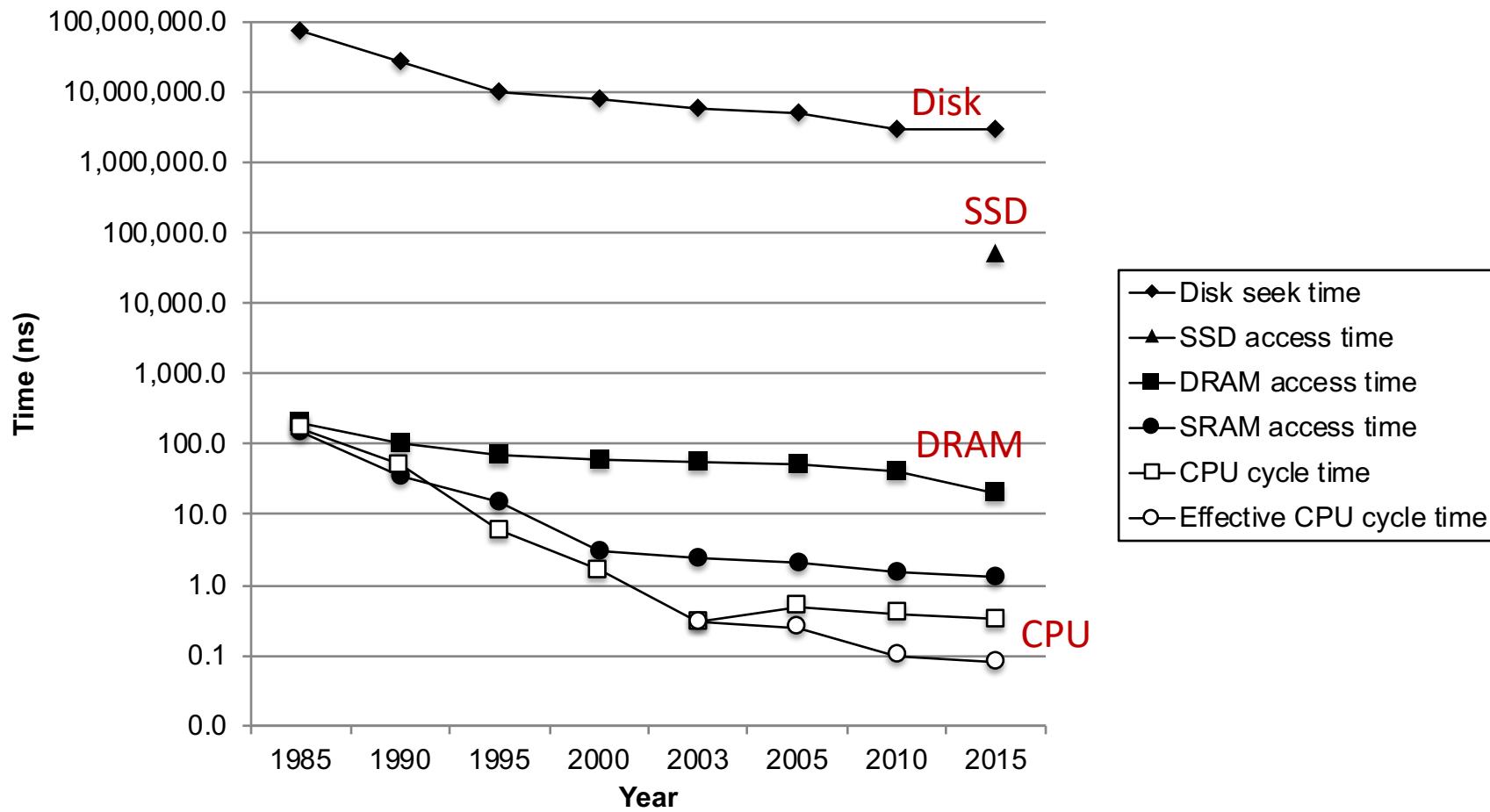
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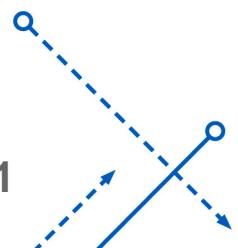
# The CPU-Memory Gap

The gap *widens* between DRAM, disk, and CPU speeds.



# Locality to the Rescue!

The key to bridging this CPU-Memory gap is a fundamental property of computer programs known as **locality**.

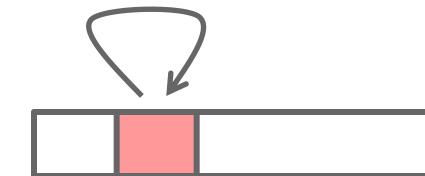


# Locality

- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently

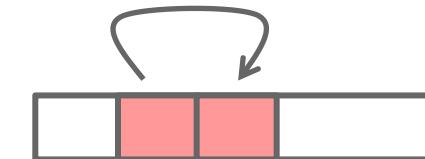
- **Temporal locality:**

- Recently referenced items are likely to be referenced again in the near future



- **Spatial locality:**

- Items with nearby addresses tend to be referenced close together in time



# Locality Example

```
sum = 0;  
for (i = 0; i < n; i++)  
    sum += a[i];  
return sum;
```

- Data references
  - Reference array elements in succession (stride-1 reference pattern). spatial
  - Reference variable **sum** each iteration. temporal
- Instruction references
  - Reference instructions in sequence. spatial
  - Cycle through loop repeatedly. temporal

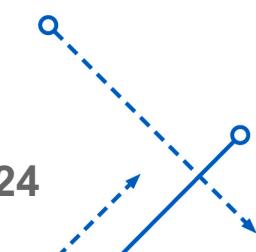
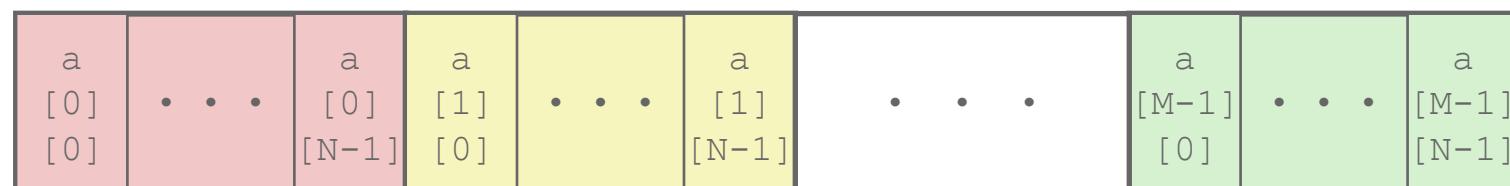
# Qualitative Estimates of Locality

- **Claim:** Being able to look at code and get a qualitative sense of its locality is a key skill for a professional programmer.
  - **Question:** Does this function exhibit good locality with respect to array access?
- Hint: array layout  
is row-major order

Answer: yes

```
int sum_array_rows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```



# Locality Example

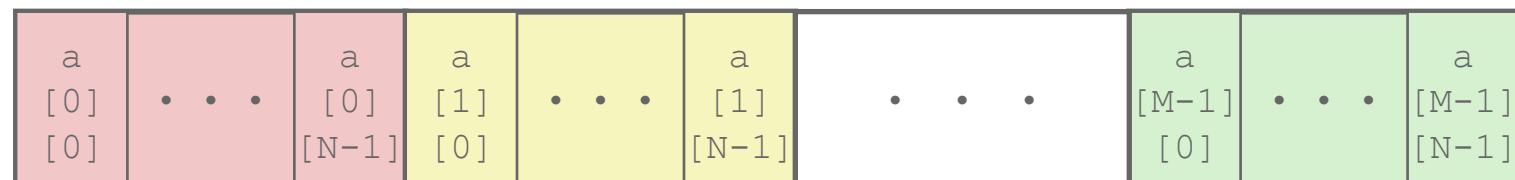
- **Question:** Does this function have good locality with respect to array  $a$ ?

```
int sum_array_cols(int a[M] [N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i] [j];
    return sum;
}
```

Answer: no, unless...

M is very small



# Locality Example

- **Question:** Can you permute the loops so that the function scans the 3-d array  $a$  with a stride-1 reference pattern (and thus has good spatial locality)?

```
int sum_array_3d(int a[M] [N] [N] )
{
    int i, j, k, sum = 0;

    for (i = 0; i < N; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < M; k++)
                sum += a[k] [i] [j];
    return sum;
}
```

Answer: make  $j$  the inner loop

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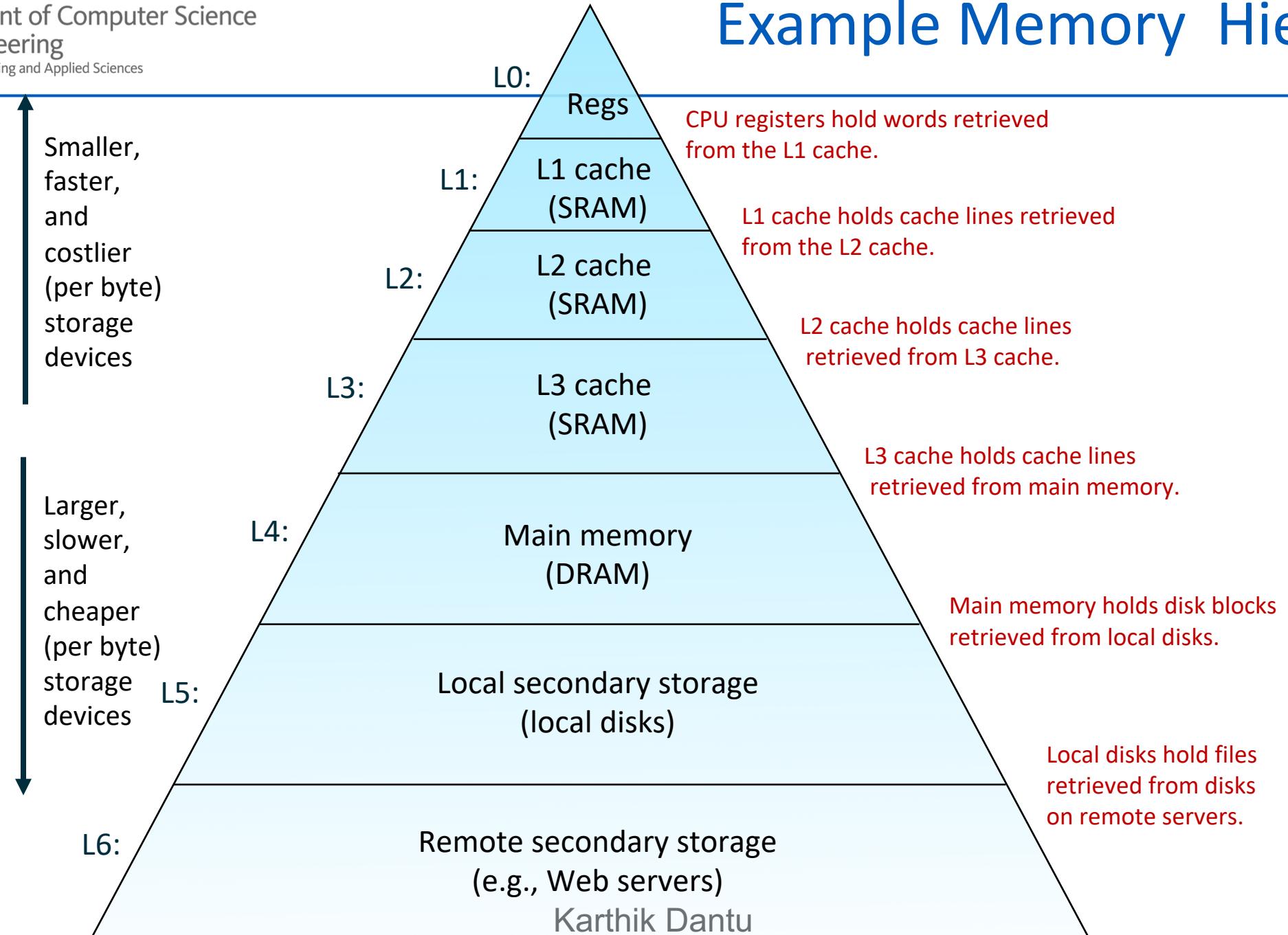


# Memory Hierarchies

- Some fundamental and enduring properties of hardware and software:
  - Fast storage technologies cost more per byte, have less capacity, and require more power (heat!).
  - The gap between CPU and main memory speed is widening.
  - Well-written programs tend to exhibit good locality.
- These fundamental properties complement each other beautifully.
- They suggest an approach for organizing memory and storage systems known as a **memory hierarchy**.



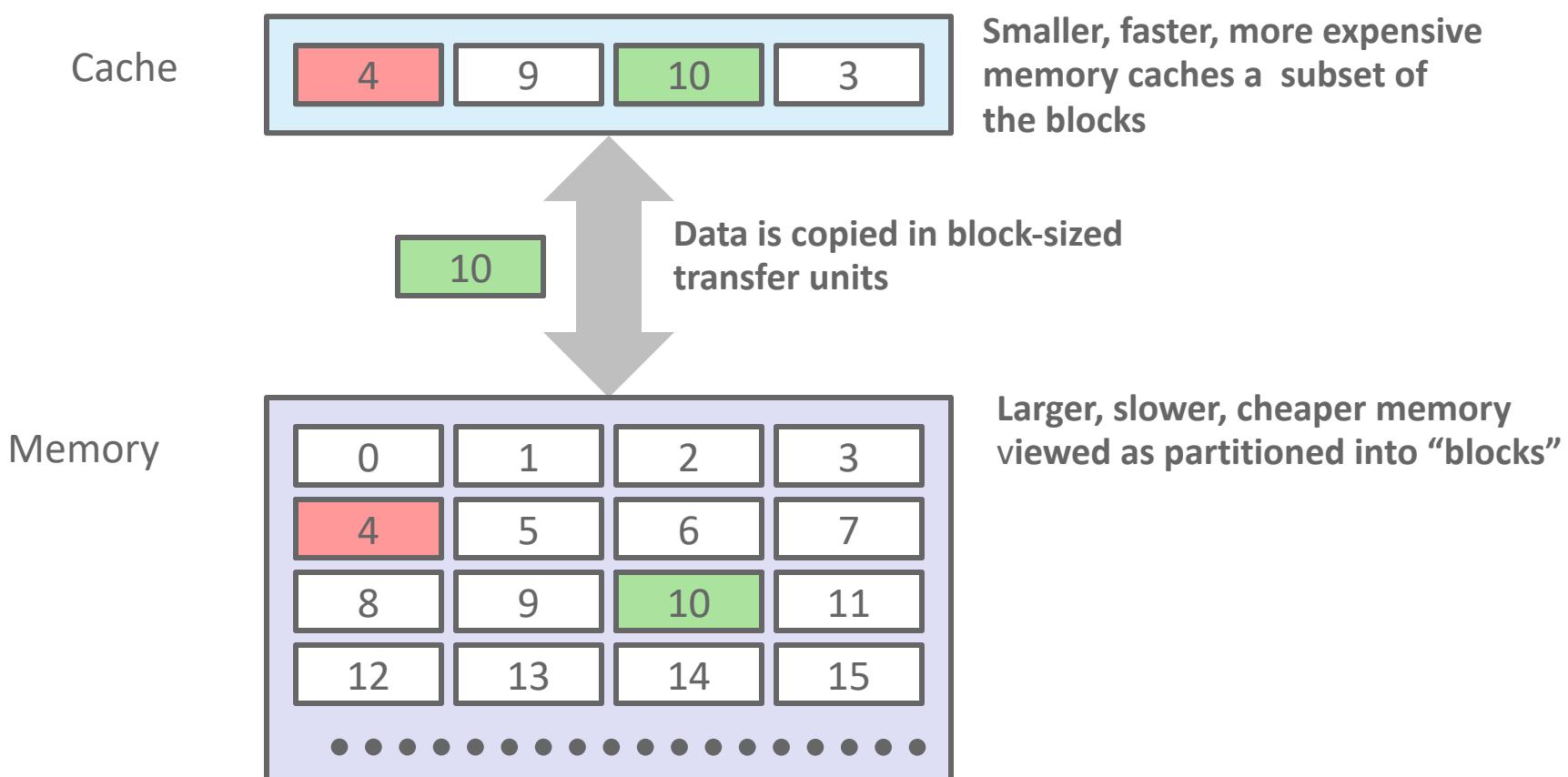
# Example Memory Hierarchy



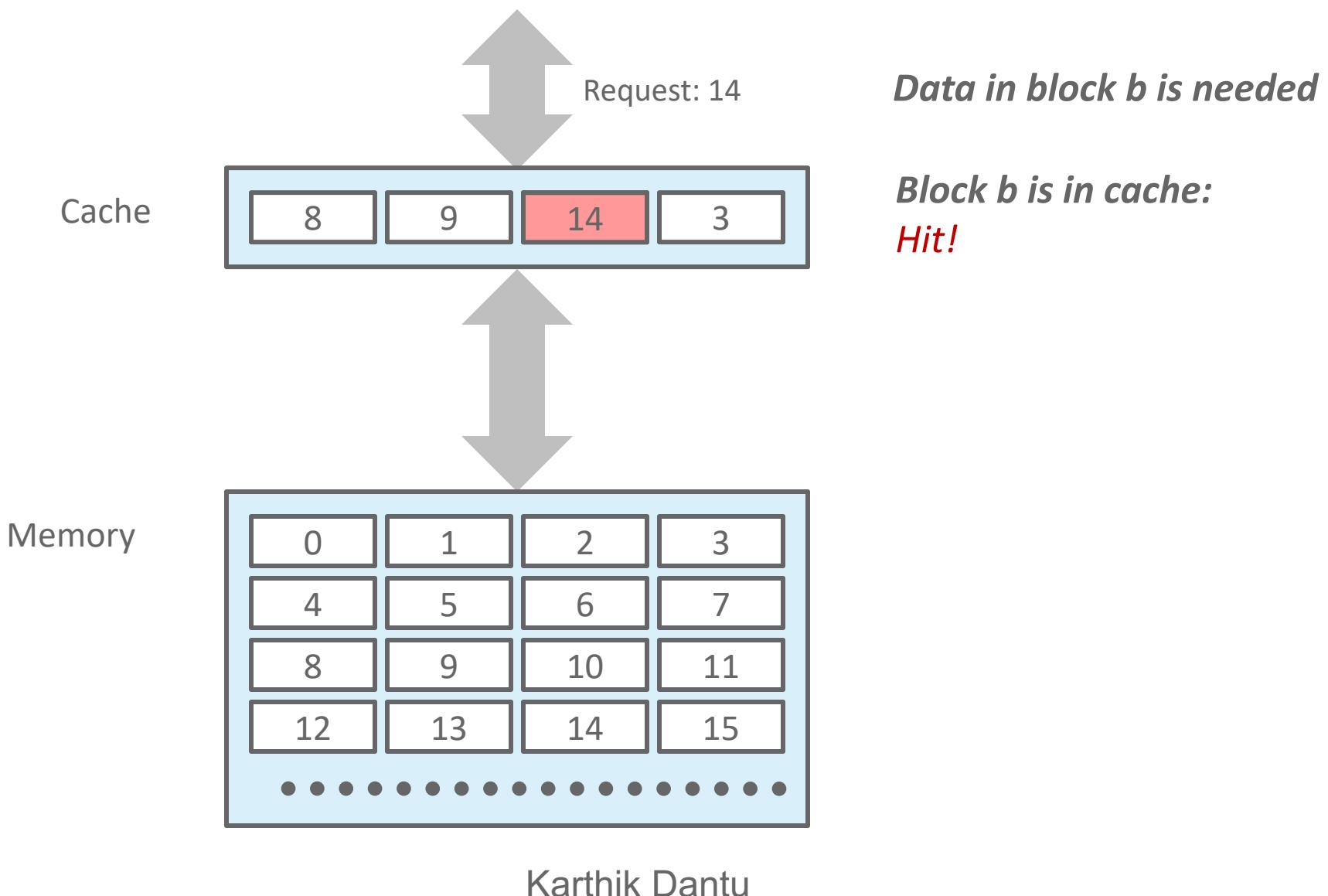
# Caches

- **Cache:** A smaller, faster storage device that acts as a staging area for a subset of the data in a larger, slower device.
- Fundamental idea of a memory hierarchy:
  - For each  $k$ , the faster, smaller device at level  $k$  serves as a cache for the larger, slower device at level  $k+1$ .
- Why do memory hierarchies work?
  - Because of locality, programs tend to access the data at level  $k$  more often than they access the data at level  $k+1$ .
  - Thus, the storage at level  $k+1$  can be slower, and thus larger and cheaper per bit.
- **Big Idea (Ideal):** The memory hierarchy creates a large pool of storage that costs as much as the cheap storage near the bottom, but that serves data to programs at the rate of the fast storage near the top.

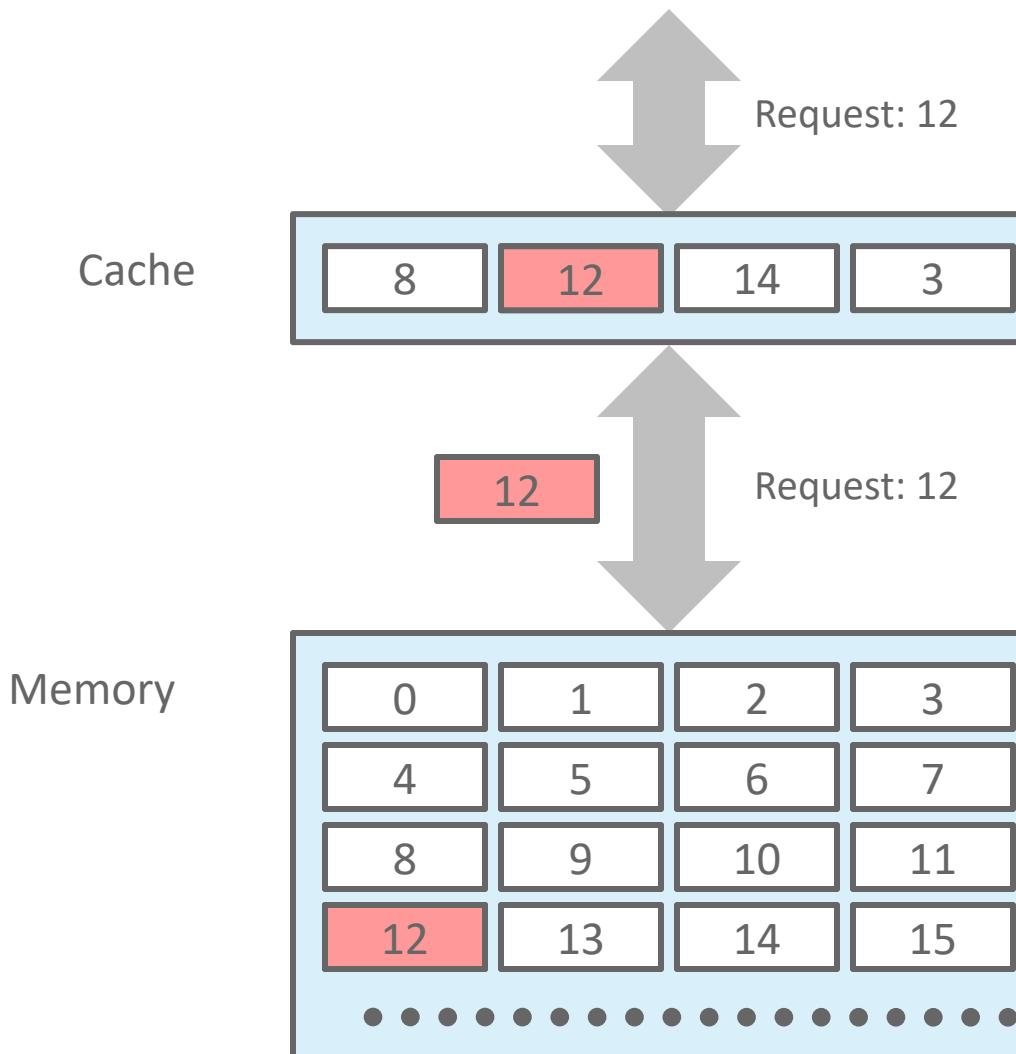
# General Cache Concepts



# General Cache Concepts: Hit



# General Cache Concepts: Miss



*Data in block b is needed*

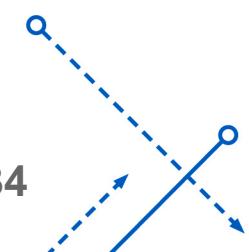
*Block b is not in cache:  
**Miss!***

*Block b is fetched from  
memory*

*Block b is stored in cache*

- **Placement policy:**  
determines where b goes
- **Replacement policy:**  
determines which block gets evicted (victim)

- **Cold (compulsory) miss**
  - Cold misses occur because the cache starts empty and this is the first reference to the block.
- **Capacity miss**
  - Occurs when the set of active cache blocks (**working set**) is larger than the cache.
- **Conflict miss**
  - Most caches limit blocks at level  $k+1$  to a small subset (sometimes a singleton) of the block positions at level  $k$ .
    - E.g. Block  $i$  at level  $k+1$  must be placed in block  $(i \bmod 4)$  at level  $k$ .
  - Conflict misses occur when the level  $k$  cache is large enough, but multiple data objects all map to the same level  $k$  block.
    - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.

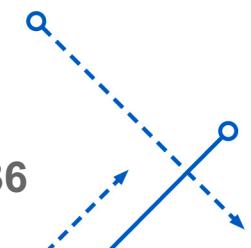


# Examples of Caching in the Mem. Hierarchy

Cache Type	What is Cached?	Where is it Cached?	Latency (cycles)	Managed By
Registers	4-8 byte words	CPU core	0	Compiler
TLB	Address translations	On-Chip TLB	0	Hardware MMU
L1 cache	64-byte blocks	On-Chip L1	4	Hardware
L2 cache	64-byte blocks	On-Chip L2	10	Hardware
Virtual Memory	4-KB pages	Main memory	100	Hardware + OS
Buffer cache	Parts of files	Main memory	100	OS
Disk cache	Disk sectors	Disk controller	100,000	Disk firmware
Network buffer cache	Parts of files	Local disk	10,000,000	NFS client
Browser cache	Web pages	Local disk	10,000,000	Web browser
Web cache	Web pages	Remote server disks	1,000,000,000	Web proxy server

# Today

- The memory abstraction
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- Locality of reference
- The memory hierarchy
- Storage technologies and trends



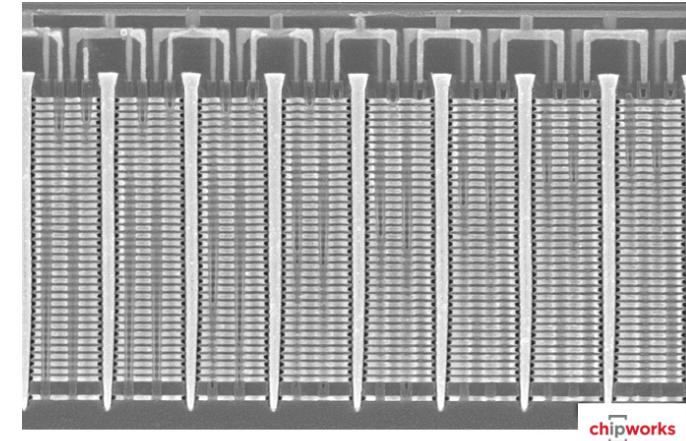
# Storage Technologies

- Magnetic Disks



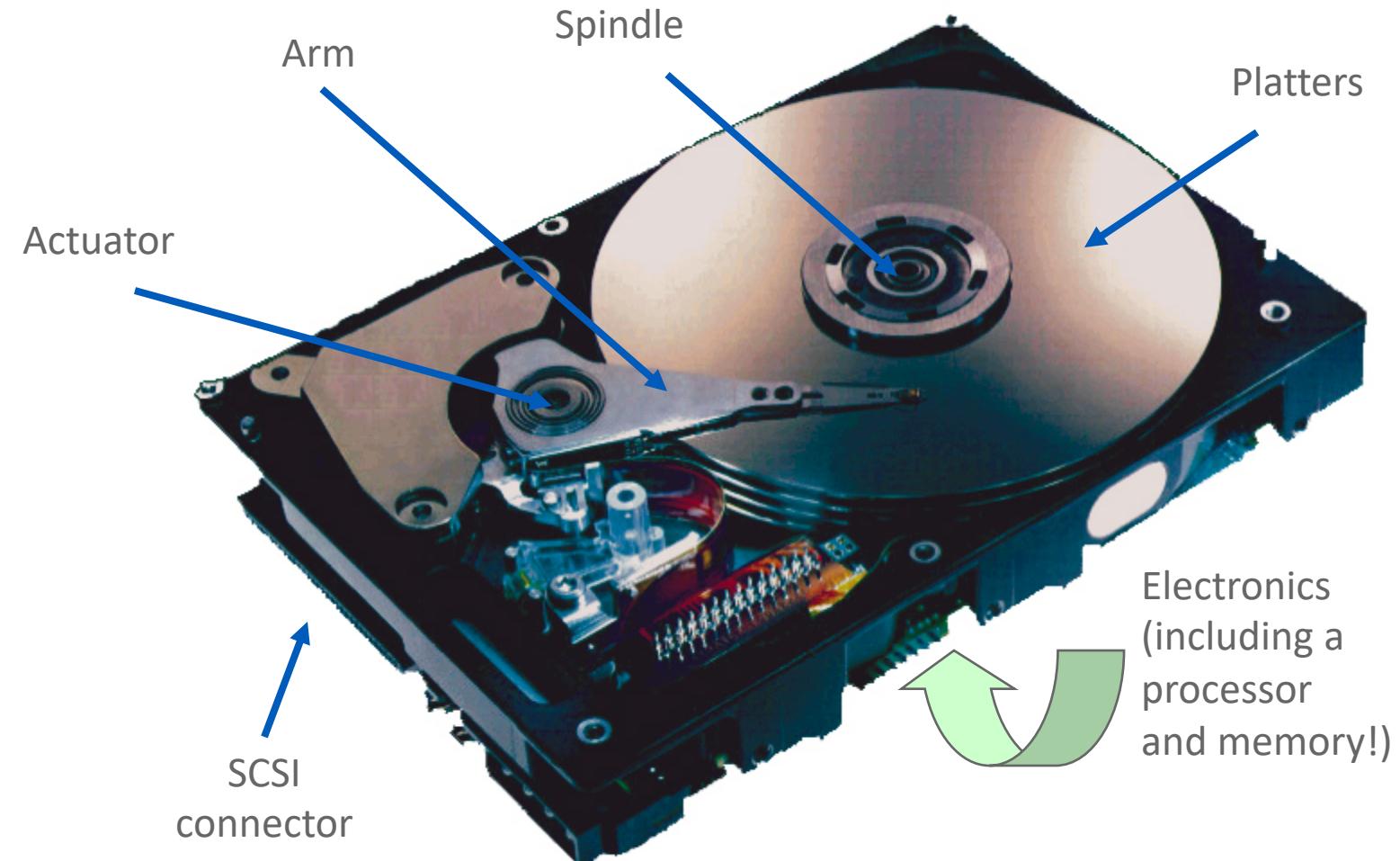
- Store on magnetic medium
- Electromechanical access

- Nonvolatile (Flash) Memory



- Store as persistent charge
- Implemented with 3-D structure
  - 100+ levels of cells
  - 3 bits data per cell

# What's Inside A Disk Drive?



*Image courtesy of Seagate Technology*

Karthik Dantu

# Disk Access Time

- Average time to access some target sector approximated by:
  - $T_{\text{access}} = T_{\text{avg seek}} + T_{\text{avg rotation}} + T_{\text{avg transfer}}$
- **Seek time ( $T_{\text{avg seek}}$ )**
  - Time to position heads over cylinder containing target sector.
  - Typical  $T_{\text{avg seek}}$  is 3–9 ms
- **Rotational latency ( $T_{\text{avg rotation}}$ )**
  - Time waiting for first bit of target sector to pass under r/w head.
  - $T_{\text{avg rotation}} = 1/2 \times 1/\text{RPMs} \times 60 \text{ sec}/1 \text{ min}$
  - Typical rotational rate = 7,200 RPMs
- **Transfer time ( $T_{\text{avg transfer}}$ )**
  - Time to read the bits in the target sector.
  - $T_{\text{avg transfer}} = 1/\text{RPM} \times 1/(\text{avg # sectors/track}) \times 60 \text{ secs}/1 \text{ min}$

time for one rotation (in minutes)

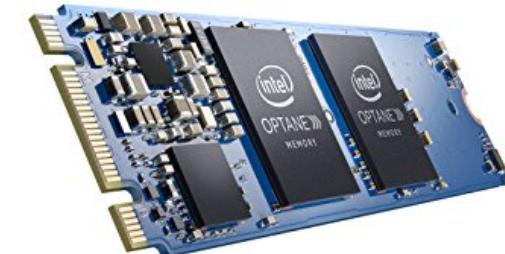
fraction of a rotation to be read

# Disk Access Time Example

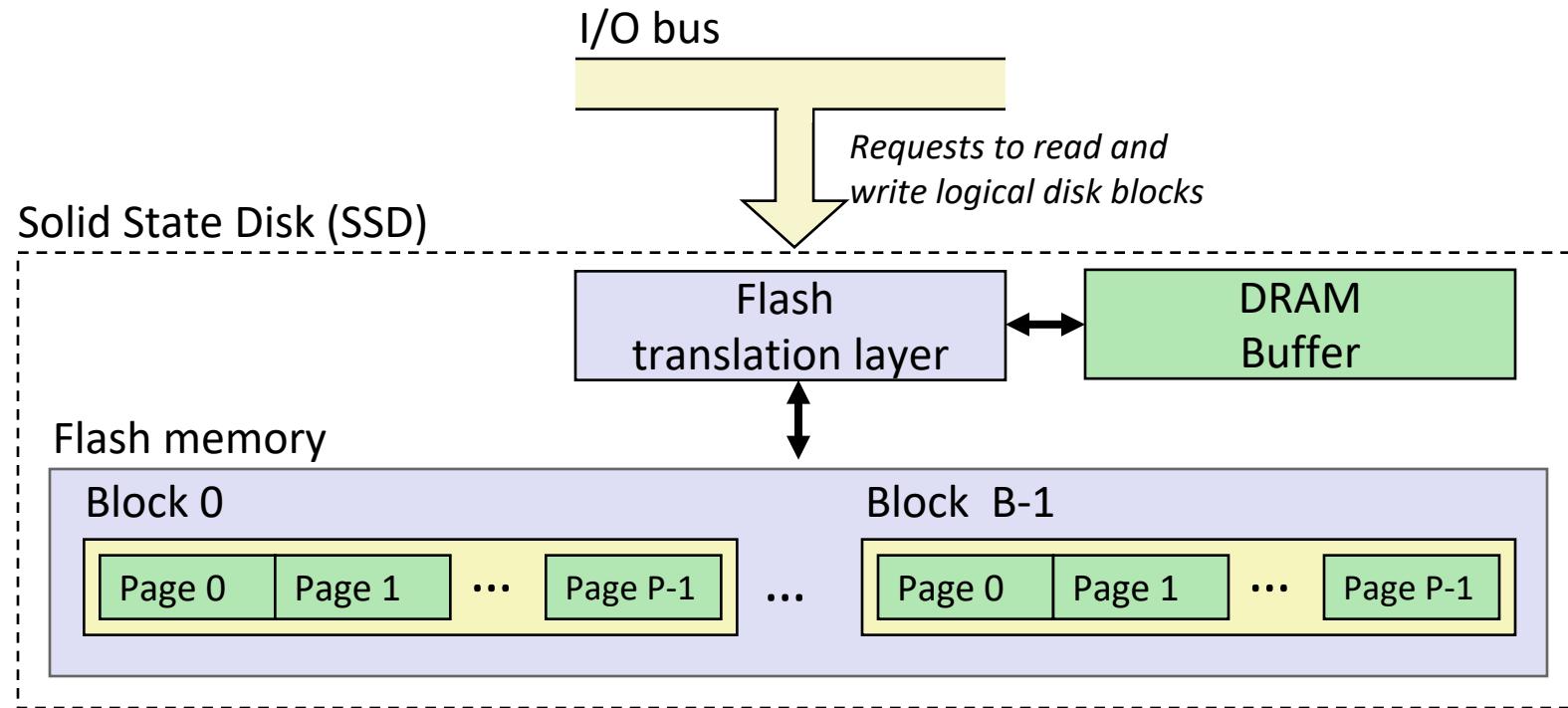
- Given:
  - Rotational rate = 7,200 RPM
  - Average seek time = **9 ms**
  - Avg # sectors/track = 400
- Derived:
  - $T_{\text{avg rotation}} = 1/2 \times (60 \text{ secs}/7200 \text{ RPM}) \times 1000 \text{ ms/sec} = 4 \text{ ms}$
  - $T_{\text{avg transfer}} = 60/7200 \times 1/400 \times 1000 \text{ ms/sec} = 0.02 \text{ ms}$
  - $T_{\text{access}} = 9 \text{ ms} + 4 \text{ ms} + 0.02 \text{ ms}$
- Important points:
  - Access time dominated by seek time and rotational latency.
  - First bit in a sector is the most expensive, the rest are free.
  - ***SRAM access time is about 4 ns/doubleword, DRAM about 60 ns***
    - *Disk is about 40,000 times slower than SRAM,*
    - *2,500 times slower than DRAM.*

# Nonvolatile Memories

- DRAM and SRAM are volatile memories
  - Lose information if powered off.
- Nonvolatile memories retain value even if powered off
  - Read-only memory (**ROM**): programmed during production
  - Electrically erasable PROM (**EEPROM**): electronic erase capability
  - Flash memory: EEPROMs, with partial (block-level) erase capability
    - Wears out after about 100,000 erasings
  - 3D XPoint (Intel Optane) & emerging NVMs
    - New materials
- Uses for Nonvolatile Memories
  - Firmware programs stored in a ROM (BIOS, controllers for disks, network cards, graphics accelerators, security subsystems,...)
  - Solid state disks (replacing rotating disks)
  - Disk caches



# Solid State Disks (SSDs)



- Pages: 512KB to 4KB, Blocks: 32 to 128 pages
- Data read/written in units of pages.
- Page can be written only after its block has been erased.
- A block wears out after about 100,000 repeated writes.

# SSD Performance Characteristics

- Benchmark of Samsung 940 EVO Plus

<https://ssd.userbenchmark.com/SpeedTest/711305/Samsung-SSD-970-EVO-Plus-250GB>

Sequential read throughput	2,126 MB/s	Sequential write tput	1,880 MB/s
Random read throughput	140 MB/s	Random write tput	59 MB/s

- Sequential access faster than random access
  - Common theme in the memory hierarchy
- Random writes are somewhat slower
  - Erasing a block takes a long time (~1 ms).
  - Modifying a block page requires all other pages to be copied to new block.
  - Flash translation layer allows accumulating series of small writes before doing block write.

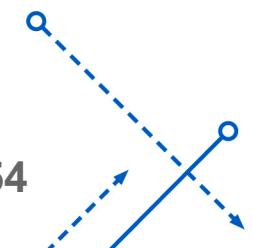
# SSD Tradeoffs vs Rotating Disks

- Advantages
  - No moving parts → faster, less power, more rugged
- Disadvantages
  - Have the potential to wear out
    - Mitigated by “wear leveling logic” in flash translation layer
    - E.g. Samsung 940 EVO Plus guarantees 600 writes/byte of writes before they wear out
    - Controller migrates data to minimize wear level
  - In 2019, about 4 times more expensive per byte
    - And, relative cost will keep dropping
- Applications
  - MP3 players, smart phones, laptops
  - Increasingly common in desktops and servers



# Summary

- The speed gap between CPU, memory and mass storage continues to widen.
- Well-written programs exhibit a property called *locality*.
- Memory hierarchies based on *caching* close the gap by exploiting locality.
- Flash memory progress outpacing all other memory and storage technologies (DRAM, SRAM, magnetic disk)
  - Able to stack cells in three dimensions



# The Memory Mountain

- **Read throughput** (read bandwidth)
  - Number of bytes read from memory per second (MB/s)
- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

# Memory Mountain Test Function

```
long data[MAXELEMS]; /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *         array "data" with stride of "stride",
 *         using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

*mountain/mountain.c*

Call `test()` with many combinations of `elems` and `stride`.

For each `elems` and `stride`:

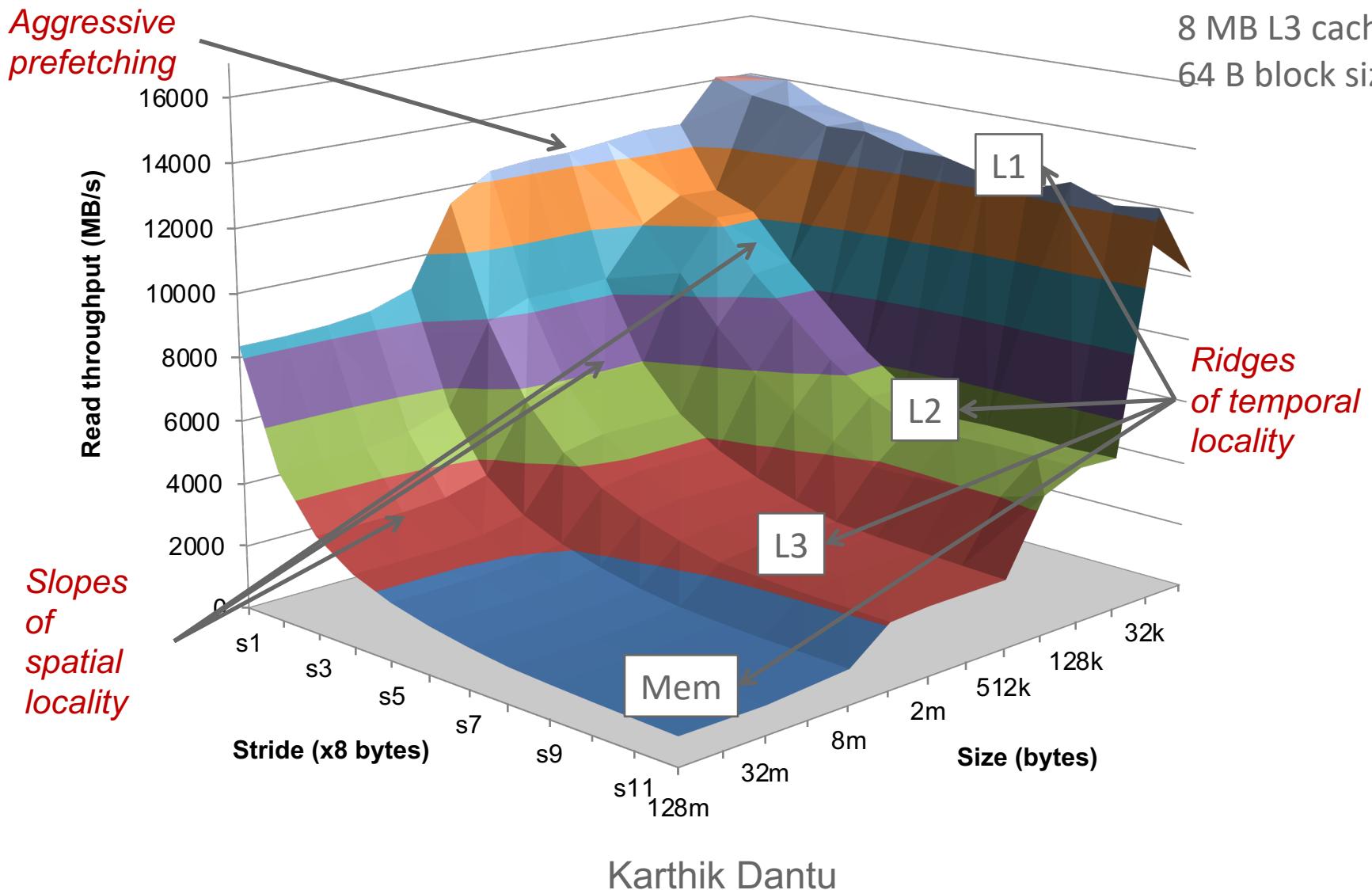
1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput(MB/s)

# The Memory Mountain

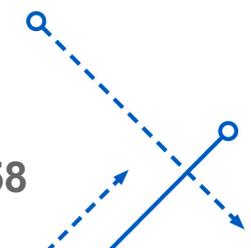
Core i7 Haswell  
2.1 GHz

32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size



# Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality



# Matrix Multiplication Example

- Description:
  - Multiply  $N \times N$  matrices
  - Matrix elements are doubles (8 bytes)
  - $O(N^3)$  total operations
  - $N$  reads per source element
  - $N$  values summed per destination
    - but may be able to hold in register

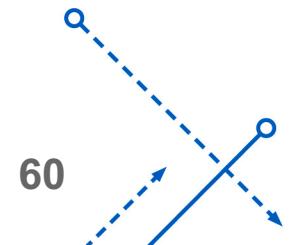
```
/* ijk */  
for (i=0; i<n; i++) {  
    for (j=0; j<n; j++) {  
        sum = 0.0; ← Variable sum  
held in register  
        for (k=0; k<n; k++)  
            sum += a[i][k] * b[k][j];  
        c[i][j] = sum;  
    }  
}
```

*matmult/mm.c*

# Miss Rate Analysis for Matrix Multiply

- Assume:
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate  $1/N$  as 0.0
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop

$$\begin{matrix} & \xrightarrow{j} \\ \downarrow i & \boxed{\text{C}} \end{matrix} = \begin{matrix} & \xrightarrow{k} \\ \downarrow i & \boxed{\text{A}} \end{matrix} \times \begin{matrix} & \xrightarrow{j} \\ \downarrow k & \boxed{\text{B}} \end{matrix}$$



# Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:
  - `for (i = 0; i < N; i++)`  
 `sum += a[0][i];`
  - accesses successive elements
  - if block size ( $B$ ) > `sizeof(aij)` bytes, exploit spatial locality
    - miss rate = `sizeof(aij) / B`
- Stepping through rows in one column:
  - `for (i = 0; i < n; i++)`  
 `sum += a[i][0];`
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

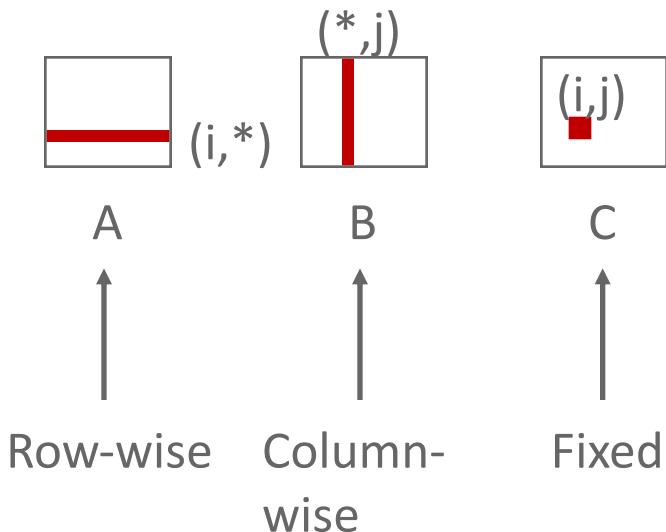


# Matrix Multiplication (i j k)

```
/* ijk */  
for (i=0; i<n; i++) {  
    for (j=0; j<n; j++) {  
        sum = 0.0;  
        for (k=0; k<n; k++)  
            sum += a[i][k] * b[k][j];  
        c[i][j] = sum;  
    }  
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A  
0.25

B  
1.0

C  
0.0

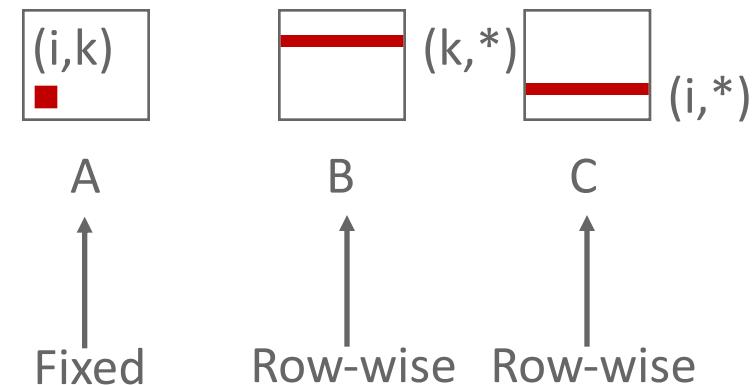
Block size = 32B (four doubles)

# Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A  
0.0

B  
0.25

C  
0.25

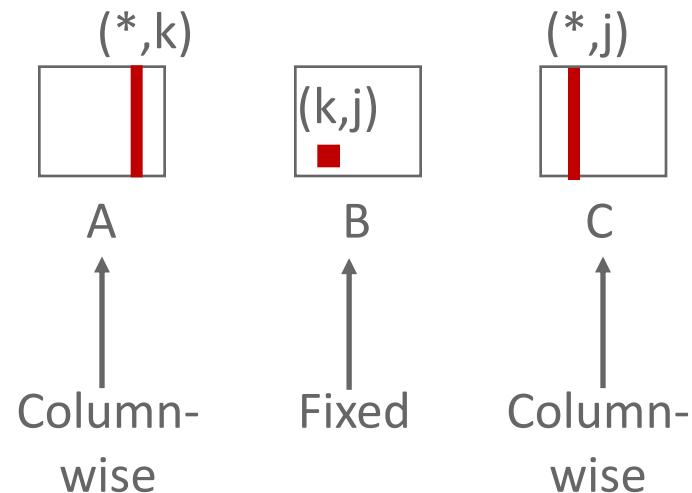
Block size = 32B (four doubles)

# Matrix Multiplication (j k i)

```
/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

*matmult/mm.c*

Inner loop:



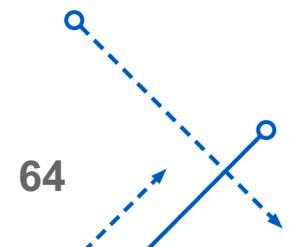
Miss rate for inner loop iterations:

A  
1.0

B  
0.0

C  
1.0

Block size = 32B (four doubles)



# Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {  
    for (j=0; j<n; j++) {  
        sum = 0.0;  
        for (k=0; k<n; k++)  
            sum += a[i][k] * b[k][j];  
        c[i][j] = sum;  
    }  
}
```

```
for (k=0; k<n; k++) {  
    for (i=0; i<n; i++) {  
        r = a[i][k];  
        for (j=0; j<n; j++)  
            c[i][j] += r * b[k][j];  
    }  
}
```

```
for (j=0; j<n; j++) {  
    for (k=0; k<n; k++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```

ijk (& jik):

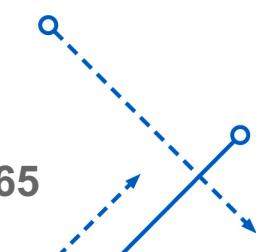
- 2 loads, 0 stores
- avg misses/iter = 1.25

kij (& ikj):

- 2 loads, 1 store
- avg misses/iter = 0.5

jki (& kji):

- 2 loads, 1 store
- avg misses/iter = 2.0



## Core i7 Matrix Multiply Performance

Cycles per inner loop iteration

100

- jki
- kji
- ijk
- jik
- kij
- ikj

10

jki / kji (2.0)

ijk / jik (1.25)

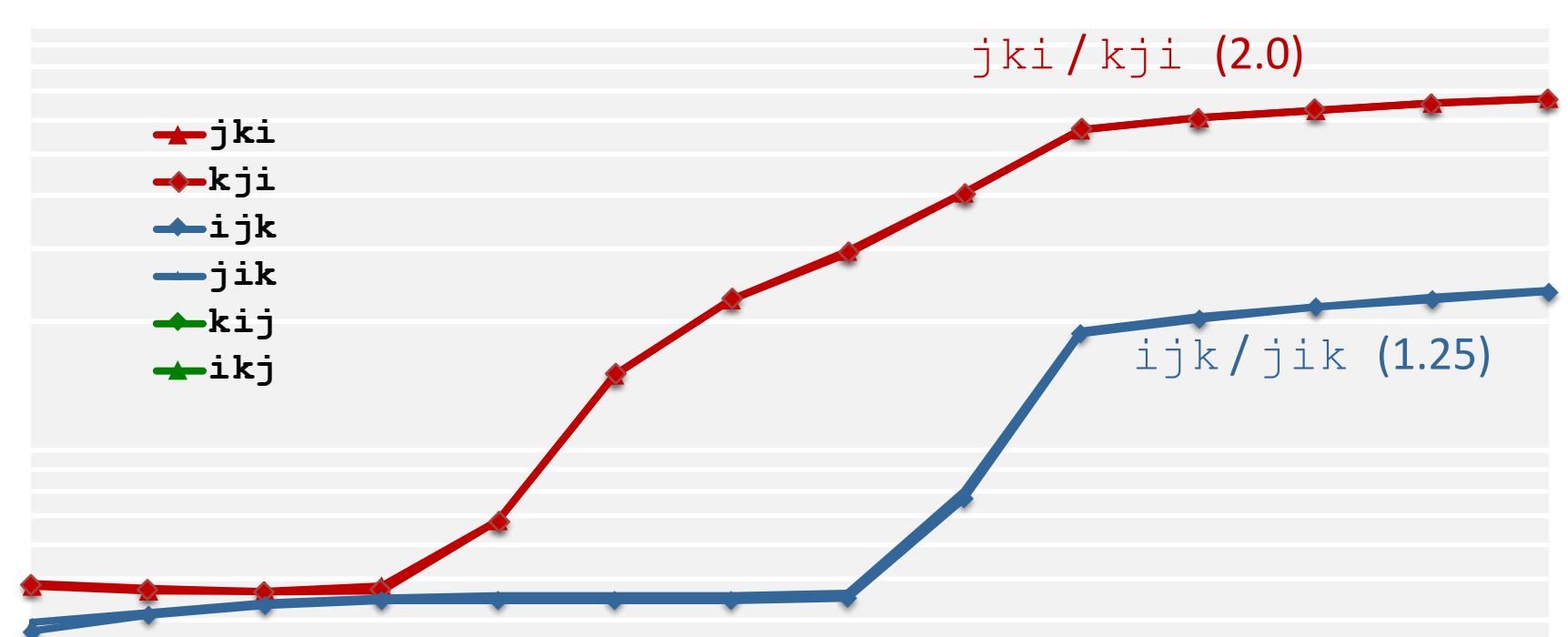
1

50 100 150 200 250 300 350 400 450 500 550 600 650 700

Array size (n)  
Karthik Dantu

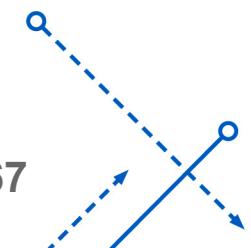
66

66



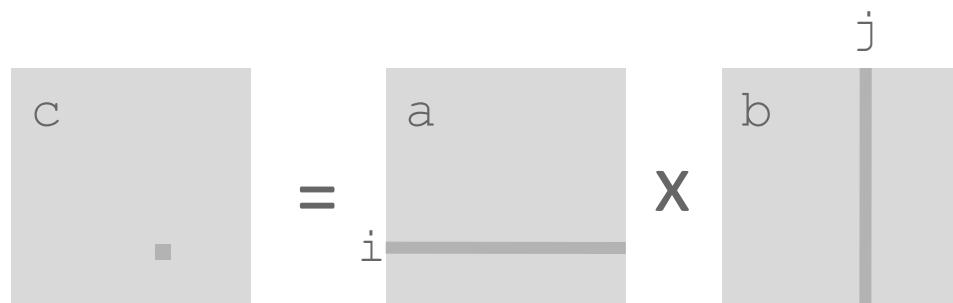
# Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality



# Example: Matrix Multiplication

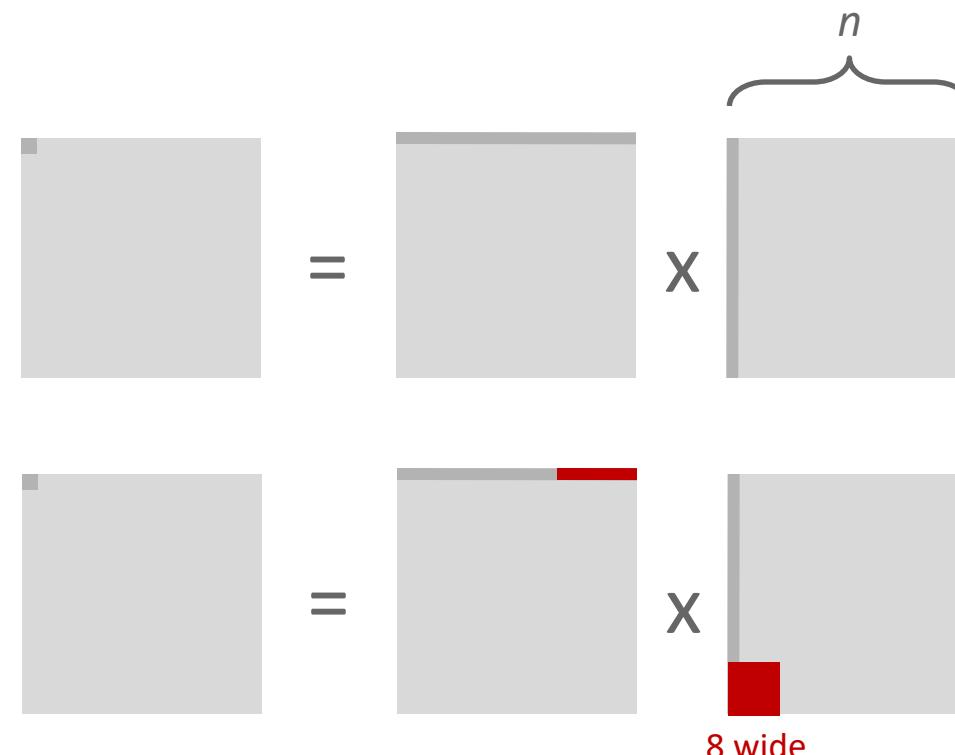
```
c = (double *) calloc(sizeof(double), n*n);  
  
/* Multiply n x n matrices a and b */  
void mmm(double *a, double *b, double *c, int n) {  
    int i, j, k;  
    for (i = 0; i < n; i++)  
        for (j = 0; j < n; j++)  
            for (k = 0; k < n; k++)  
                c[i*n + j] += a[i*n + k] * b[k*n + j];  
}
```



# Cache Miss Analysis

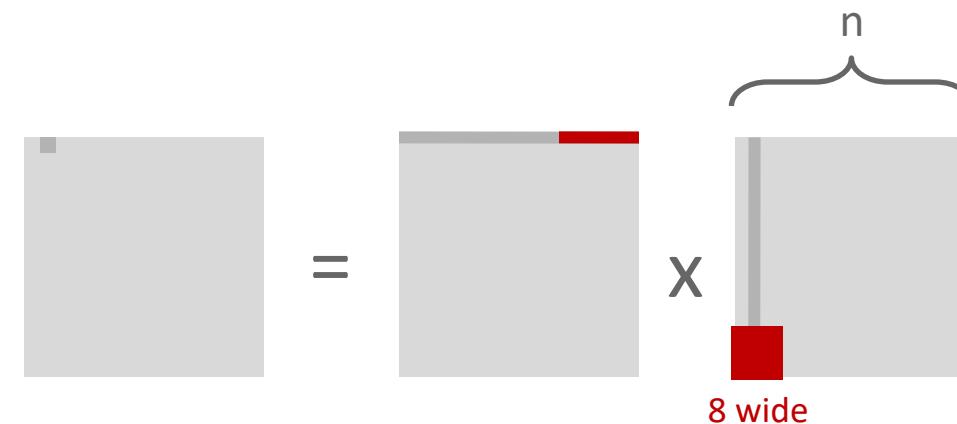
- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size  $C \ll n$  (much smaller than  $n$ )
- First iteration:
  - $n/8 + n = 9n/8$  misses

- Afterwards **in cache:**  
(schematic)



# Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size  $C \ll n$  (much smaller than  $n$ )
- Second iteration:
  - Again:
$$n/8 + n = 9n/8$$
 misses
- Total misses:
  - $$9n/8 n^2 = (9/8) n^3$$

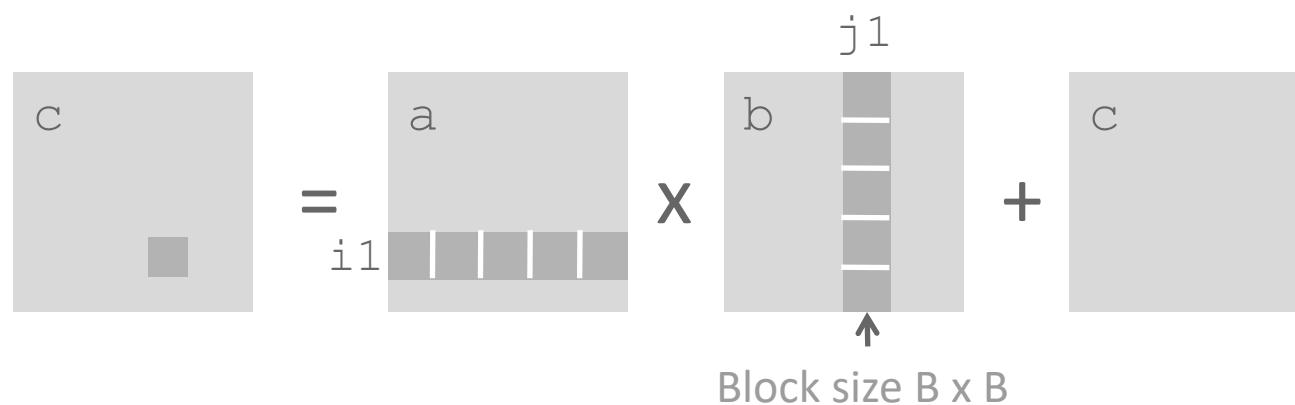


# Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i1++)
                    for (j1 = j; j1 < j+B; j1++)
                        for (k1 = k; k1 < k+B; k1++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

*matmult/bmm.c*



# Cache Miss Analysis

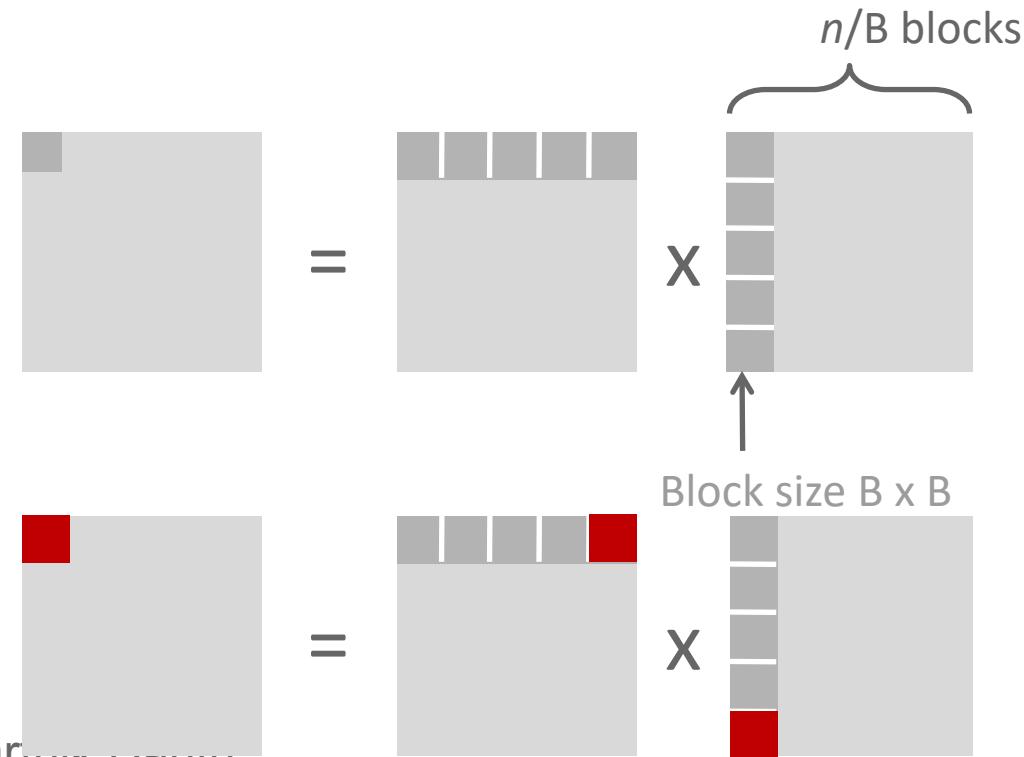
- Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks fit into cache:  $3B^2 < C$

- First (block) iteration:

- $B^2/8$  misses for each block
- $2n/B \times B^2/8 = nB/4$   
(omitting matrix c)

- Afterwards in cache  
(schematic)



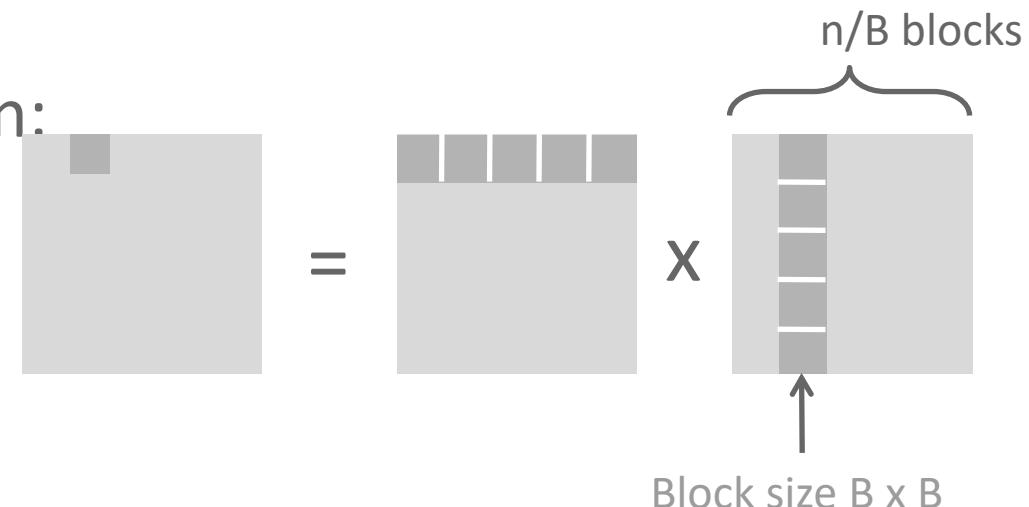
# Cache Miss Analysis

- Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks fit into cache:  $3B^2 < C$

- Second (block) iteration:

- Same as first iteration
- $2n/B \times B^2/8 = nB/4$

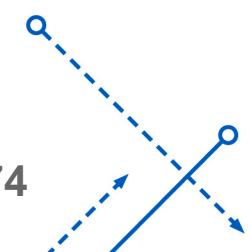


- Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$

# Blocking Summary

- No blocking:  $(9/8) n^3$  misses
- Blocking:  $(1/(4B)) n^3$  misses
- Use largest block size  $B$ , such that  $B$  satisfies  $3B^2 < C$ 
  - Fit three blocks in cache! Two input, one output.
- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data:  $3n^2$ , computation  $2n^3$
    - Every array elements used  $O(n)$  times!
  - But program has to be written properly



# Cache Summary

- Cache memories can have significant performance impact
- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.