Homework #4

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Problem 1 Values for t_s and t_w were obtained on the mc cluster by sending point to point messages of increasing size between two machines. Each message was sent 50 times and the average return trip was used for computation. To compute t_s I used the value obtained for the smaller message (1, 10, and 50 bytes). This value turned out to be approximately $60\mu s$. From this t_w was obtained by examining the times for larger messages (10MB, 20MB) and computed by $(T - t_s)/m$. Where T was the average roundtrip time. This value turned out to be approximately $0.0085\mu s$.

Problem 2 To test these operations a similar strategy to that of problem 1 was instituted. The main difference here was that I also varied the number of nodes involved in the communication. This range was [2,4,8,12,16]. However, results for 2 and 4 weren't revealing so only 8, 12, and 16 are reported in the graphs. Each graph is broken up by number of processors on the x-axis and inside each group of processors the times for increasing message sizes is reported. These are the values obtained empirically using MPI and theoretically using the equations found in the text.

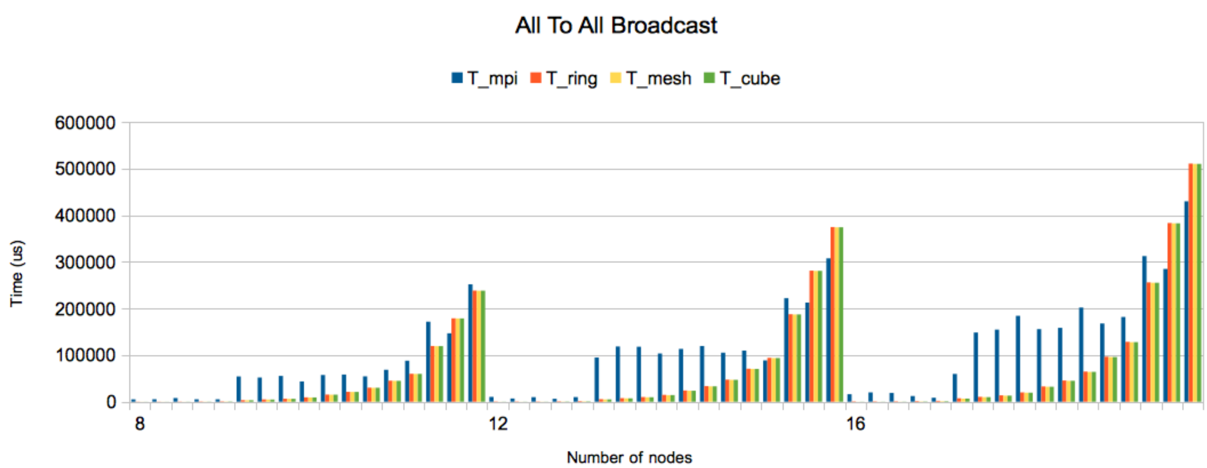


Figure 1: *All to all broadcast times for Problem 2*

5.1 (Amdahl's law [Amd67]) If a problem of size W has a serial component W_s, prove that W/W_s is an upper bound on its speedup, no matter how many processing elements are used.

We define speedup as $S = \frac{T_s}{T_p}$, the sequential time as $T_s = W$ and the parallel time as $T_p = W_s + \frac{W + T_o(W,p)}{p}$. This last term, the parallel time, is taken from the parallel time discussed in the text and adding a W_s component to is. The reason for this is that W_s is a serial component and the time must be added to the parallel run time. This time is not related to the number of processors used because it must always be executed in

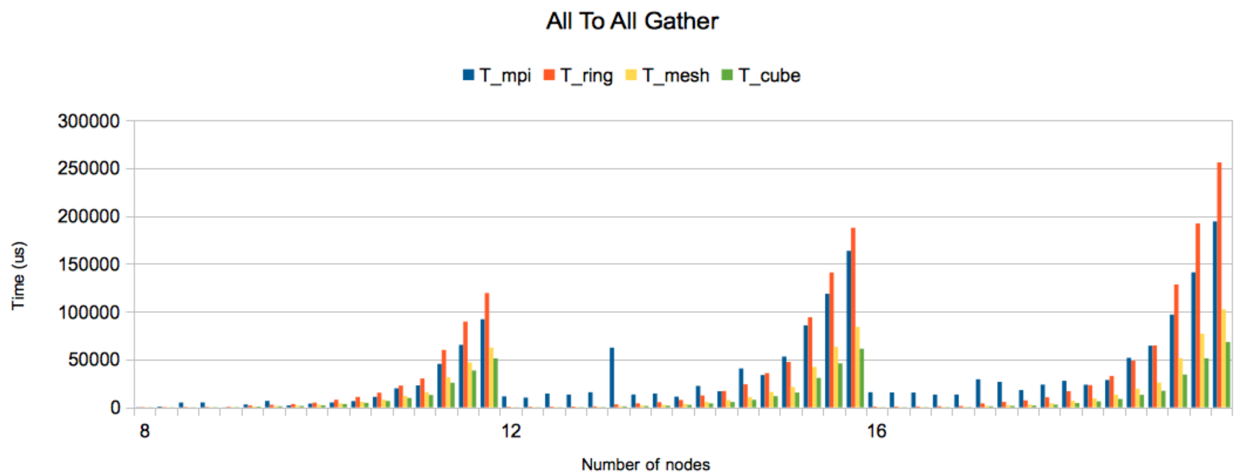


Figure 2: *All to all gather times for Problem 2*

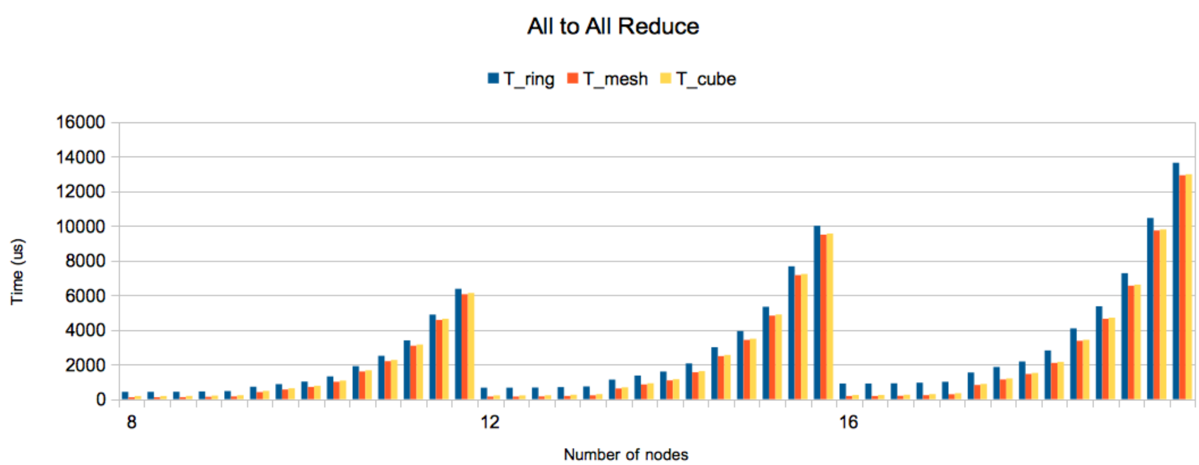


Figure 3: *All to all reduce times for Problem 2*

serial. With these definitions we can formulate an equation for the speedup as:

$$
\begin{aligned}
S &= \frac{T_s}{T_p} \\
&= \frac{W}{W_s + \frac{W + T_O(W,p)}{p}}
\end{aligned}
$$

From this we can see that even if p grows to infinity and the second term vanishes from the denominator we are still left with $\frac{W}{W_s}$ as the speedup. Thus giving us an upper bound.

5.2 (Superlinear speedup) Consider the search tree shown in Figure 5.10(a), in which the dark node represents the solution.

a. If a sequential search of the tree is performed using the standard depth-first search (DFS) algorithm (Section 11.2.1), how much time does it take to find the solution if traversing each arc of the tree takes one unit of time?

b. Assume that the tree is partitioned between two processing elements that are assigned to do the search job, as shown in Figure 5.10(b). If both processing elements perform a DFS on their respective halves of the tree, how much time does it take for the solution to be found? What is the speedup? Is there a speedup anomaly? If so, can you explain the anomaly?

a. If one processor executes the DFS on this tree this will take 11 units of time ($T_s = 11$).
b. With 2 processing units split in the fashion described this will take 4 unites of time ($T_p = 4$). This will give us a speedup of $\frac{11}{4} = 2.75$! This anomaly is a superlinear speedup because we have a speedup larger than p. This anomaly is produced because each processing units had to do less work in order to find the solution. We say a superlinear speedup is possible if each element spends less than time T_s/p solving the problem. In this case $T_s/p = 11/2 = 5.5$ and each element only had to spend a time of 4 to solve the problem.

5.4 Consider a parallel system containing p processing elements solving a problem consisting of W units of work. Prove that if the isoefficiency function of the system is worse (greater) than $\Theta(p)$, then the problem cannot be solved cost-optimally with $p = \Theta(W)$. Also prove the converse that if the problem can be solved cost-optimally only for $p < \Theta(W)$, then the isoefficiency function of the parallel system is worse than linear.

When looking at the isoefficiency of a parallel system we examine the equation: $W = KT_o(W, p)$ where W is work to be done, K is a constant proportional to the efficiency, and $T_o(W, p)$ is the overhead in the parallel algorithm. If we assume the isoefficiency of the system is worse than $\Theta(p)$ this means that $T_o > \Theta(p)$ and therefore $T_o > W$. Plugging this into our equation for efficieny yields

$$\frac{1}{1 + T_o(W, p)/W}$$

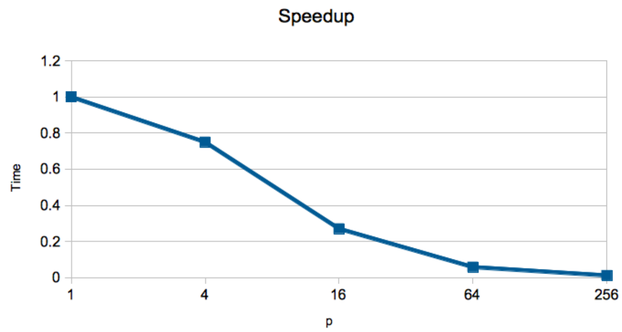
We know that in order to be cost efficient this equation must be equal to 1. However, in this case we can see it will never be the case because T_o grows faster than W so the denominator will always be 1 + something greater than 1. In other words if the isoefficiency is greater than $\Theta(p)$ this means the overhead will be growing at a rate larger than p so we will never be able to be cost optimal if $p = \Theta(p)$ because the overhead in the algorithm is growing faster than that. The converse argument of this is very similar. Since T_o is a function of both W and p if $p < \Theta(W)$ this means the work will need to grow faster than the rate of processing elements which means that the isoefficiency will have to be greater than linear (as we will need to increase the work greater than a linear amount when adding processing elements).

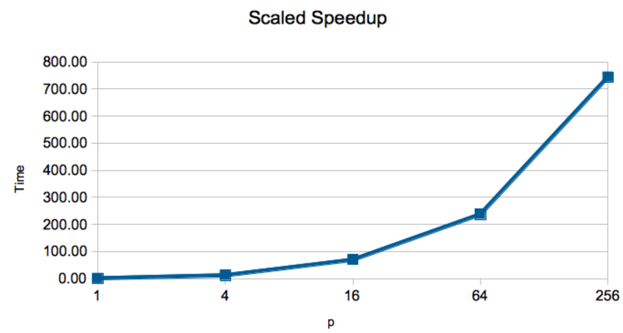
5.5 (Scaled speedup) Scaled speedup is defined as the speedup obtained when the problem size is increased linearly with the number of processing elements; that is, if W is chosen as a base problem size for a single processing element, then

$$SS = \frac{pW}{T_p(pW, p)}$$

For the problem of adding n numbers on p processing elements (Example 5.1), plot the speedup curves, assuming that the base problem for $p = 1$ is that of adding 256 numbers. Use p = 1, 4, 16, 64, and 256. Assume that it takes 10 time units to communicate a number between two processing elements, and that it takes one unit of time to add two numbers. Now plot the standard speedup curve for the base problem size and compare it with the scaled speedup curve. Hint: The parallel runtime is $(n/p - 1) + 11logp$.

We can see from these figures that by using the scaled speedup shows a significantly larger speedup as p is





increased while the standard speedup reports a speedup of less than 1 as the number of processing elements is increased.

5.10 (Prefix sums) Consider the problem of computing the prefix sums (Example 5.1) of n numbers on n processing elements. What is the parallel runtime, speedup, and efficiency of this algorithm? Assume that adding two numbers takes one unit of time and that communicating one number between two processing elements takes 10 units of time. Is the algorithm cost-optimal?

The parallel runtime of this algorithm will be $(n/p - 1) + 11logp$. From here we can see the speedup is $\frac{n}{(n/p-1)+11logp}$ and the efficiency will be $\frac{n}{n-p+11plogp}$. This algorithm will be cost optimal so long as $n = \Omega(plogp)$.

5.13 The parallel runtime of a parallel implementation of the FFT algorithm with p processing elements is given by $T_p = (n/p)logn + t_w(n/p)logp$ for an input sequence of length n (Equation 13.4 with $t_s = 0$). The maximum number of processing elements that the algorithm can use for an n-point FFT is n. What are the values of p_0 (the value of p that satisfies Equation 5.21) and T_P^{min} for $t_w = 10$?

Equating the derivative with respect to p of the right hand side of T_p to zero we can solve for p as follows:

$$\begin{aligned}
-\frac{nlogn}{p^2} + t_w n\frac{1-logp}{p^2} &= 0 \\
-nlogn + t_w n(1-logp) &= 0 \\
t_w n(1-logp) &= nlogn \\
t_w(1-logp) &= logn \\
1 - \frac{logn}{t_w} &= logp \\
logn &= logp \qquad \text{dropping 1 and } t_w \\
n &= p
\end{aligned}$$

This gives us a p_0 values of n. Substituting this into T_p we get and $t_w = 10$:

$$\begin{aligned}
T_p^{min} &= \frac{nlogn}{n} + 10\frac{nlogn}{n} \\
&= logn + 10logn \\
&= 10logn
\end{aligned}$$