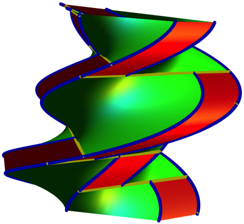

On the Parameterization and the Geometry of the Configuration Space of a Single Planar Robot

Computational Geometric Learning supported by FET-Open grant

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Freie Universität Berlin

June 27th 2013

Outline

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Introduction

Configuration Space

Contact Surfaces Parameterization

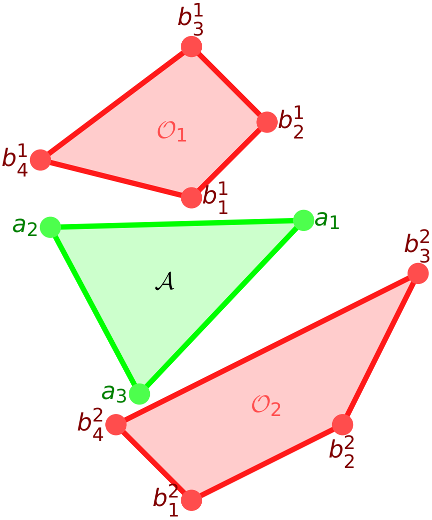
Geometrical Properties

Summary

A planar polygonal (convex) robot $\mathcal{A}$ moving amid polygonal (convex) obstacles $\{\mathcal{O}_j\}_{j\in J}$.

Free. . .

Can be either in a *free configuration* $\mathcal{C}_{free}$.



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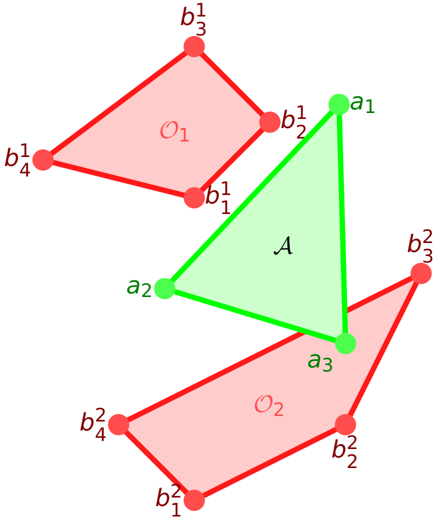
Can be either in a *free configuration* $\mathcal{C}_{free}$.

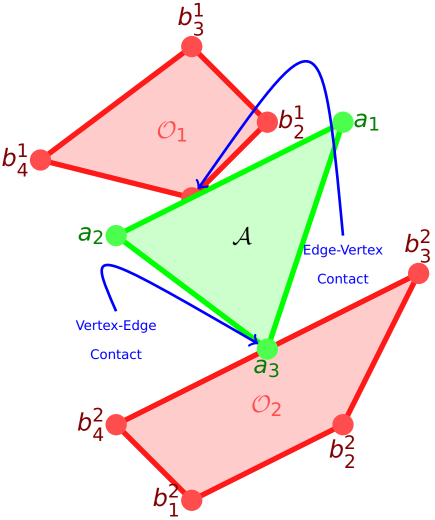
Forbidden...

Or, forbidden one $\mathcal{C}_{forb} = \mathcal{C} \setminus \mathcal{C}_{free}$.



We consider two *main* types of contacts:

- Vertex-Edge
- Edge-Vertex

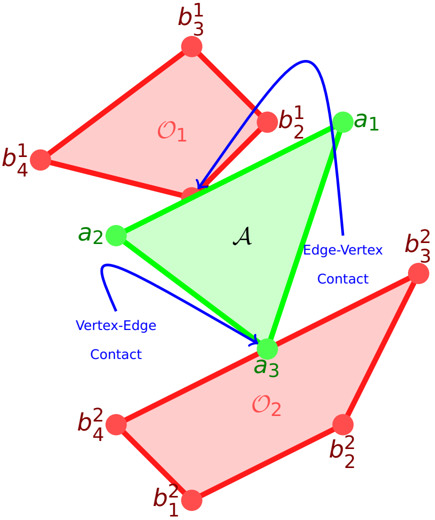


We consider two *main* types of contacts:

- Vertex-Edge
- Edge-Vertex

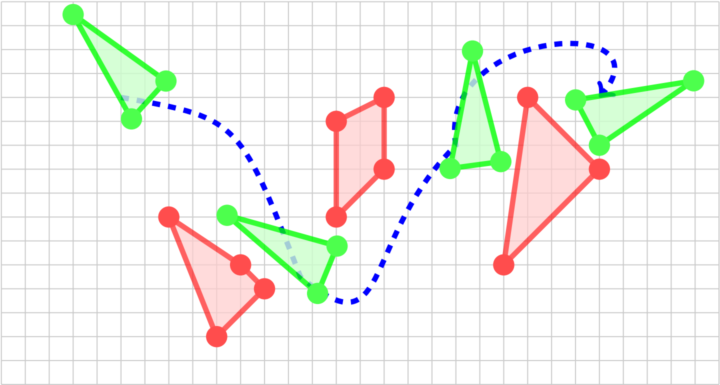
We can have two additional types of contact:

- Vertex-Vertex
- Edge-Edge



Ultimate Goal

Find a *collision free* path for $\mathcal{A}$ to move from a start point to a target point.



Outline

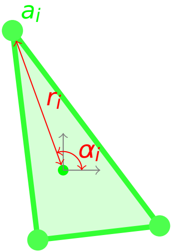
A robot's configuration (pose) is determined by a *translation vector* $\vec{r} = (x, y) \in \mathbb{R}^2$ and an *orientation angle* $\theta \in S^1$.

Definition (Configuration Space)

The space $\mathcal{C} = \left(\mathbb{R}^2 \times S^1\right)$ is called the *configuration space* of a given robot $\mathcal{A}$.

- $\mathcal{C}_{forb} = \left\{q \in \mathcal{C} \mid \operatorname{int}(\mathcal{A}(q)) \cap \left(\bigcup_j \operatorname{int}(\mathcal{O}_j)\right) \neq \emptyset\right\}$
- $\mathcal{C}_{free} = \mathcal{C} \setminus \mathcal{C}_{forb}$

Given a point $q = (x, y, \theta) \in \mathcal{C}$, the corresponding configuration of the robot in the *work space* is given by: $a_i(q) = \binom{x}{y} + r_i \binom{\cos(\alpha_i+\theta)}{\sin(\alpha_i+\theta)}$



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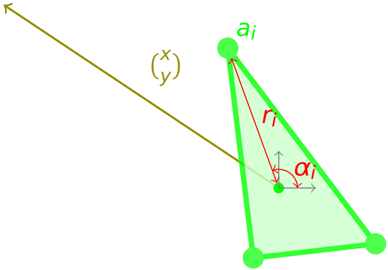

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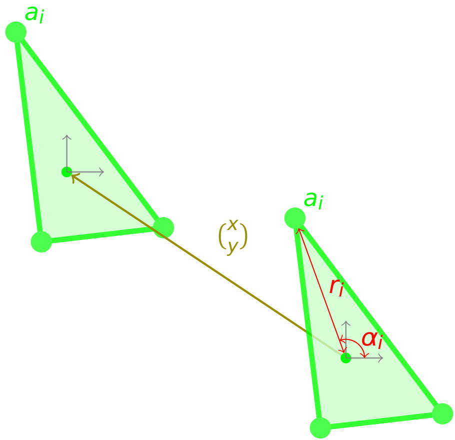

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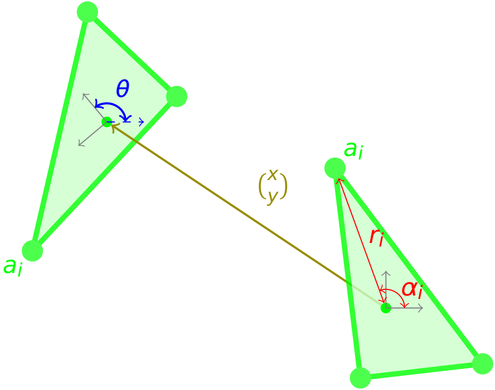
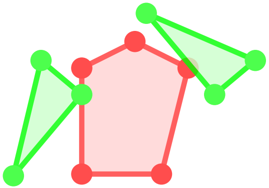


In the *configuration space*, we consider the following sets:

- **Vertex-Edge Contact**: $\left\{q \in \mathcal{C} \mid a_i \in \partial\mathcal{O}_j\right\}$
- **Edge-Vertex Contact**: $\left\{q \in \mathcal{C} \mid b_j \in \partial\mathcal{A}\right\}$



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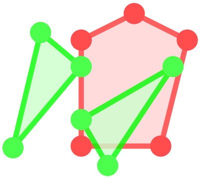

Remark

- Each of these sets intersect both $\mathcal{C}_{free}$ and $\mathcal{C}_{forb}$
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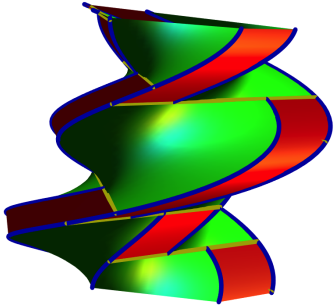
Goal

Parameterize and study the geometry of these sets.



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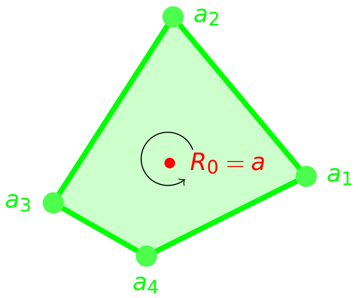
We consider the rotation of the robot about a fixed point $P \in \mathcal{W}$ such that $a \in \mathcal{A}$ is fixed to P.

$$P_a = \{q \in \mathcal{C} \mid a(q) = P\}$$



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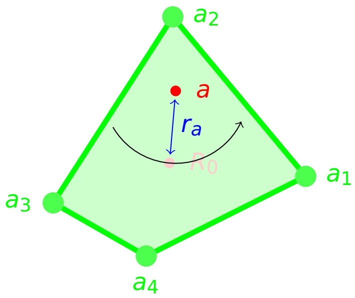
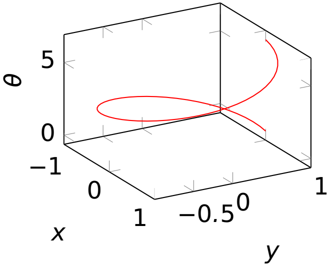


If a is some arbitrary point in $\mathcal{A}$ then P_a is a *helix* with axis in the θ direction.



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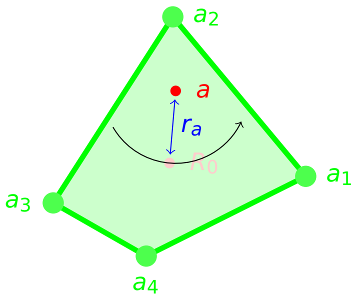

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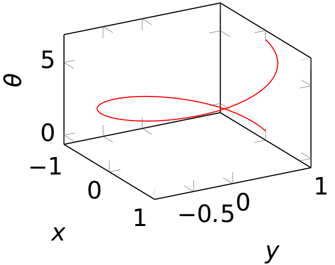

Remark

Helices cover $\mathcal{C}$.

Input

- A robot $\mathcal{A}$ and a point $a \in \mathcal{A}$.
- A fixed point $P \in \mathcal{W}$ in the work space.

The parameterization

$$q_a^P(\phi) = \begin{pmatrix} \vec{r}(\phi) \\ \theta(\phi) \end{pmatrix} \in \mathcal{C}$$

where:

$$\vec{r}(\phi) = P - R^{\phi} a$$
$$\theta(\phi) = \phi \mod 2\pi$$

Added Value

Currently, if the robot and the obstacles are convex then *exact bounds* of the free part of the contact surface can be found.

- Take the parameterization of rotation $q_a^P(\phi) = \binom{P - R^\phi a}{\phi \mod 2\pi}$

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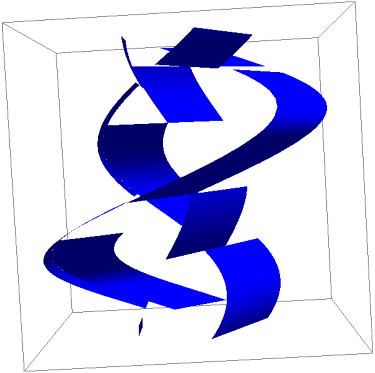
- Then
$$S(t, \phi) = q_{a_i}^{P(t)}(\phi)$$
is a *developable surface* which parameterizes a *vertex-edge* contact surface.



Edge-Vertex Contact

- Take the parameterization of rotation $q_a^P(\phi) = \binom{P - R^{\phi} a}{\phi \mod 2\pi}$

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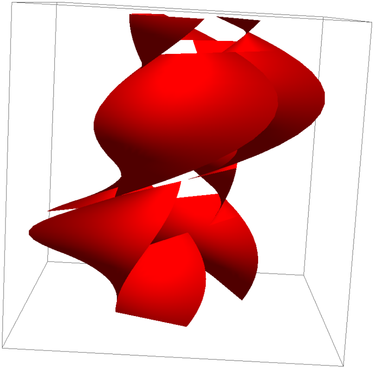
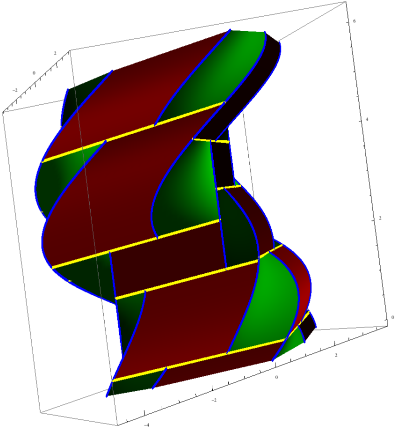
- Then

$$S(t, \phi) = q_{a(t)}^{b_j}(\phi)$$

is a *non cylindrical ruled surface* which parameterizes an *edge-vertex* contact surface.



- For all possible vertex-edge and edge-vertex combinations we obtain the object to the right.
- In the convex case, we can easily compute the exact contact patches.



Drawback

In the introduced parameterization, non algebraic computations are needed. Thus, its accuracy in practical applications is limited.

Rational Workaround

Set $\psi = \tan\frac{\phi}{2}$ and $M^\psi = \frac{1}{1+\psi^2}\begin{pmatrix} 1-\psi^2 & -2\psi \\ 2\psi & 1-\psi^2 \end{pmatrix}$ we obtain a *rational parameterization*

$$k_a^P(\psi) = (\vec{r}(\psi), \tau(\psi))$$

where

$$\begin{aligned} \vec{r}(\psi) =& P - M^\psi a \\ \tau(\psi) =& \psi \end{aligned}$$

for $\psi \in (-\infty, +\infty)$.

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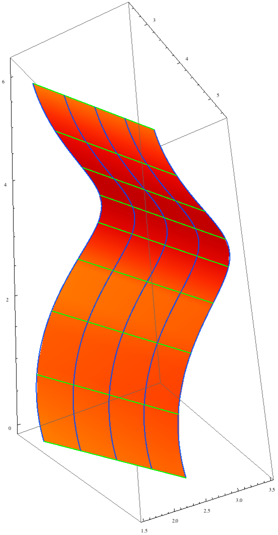
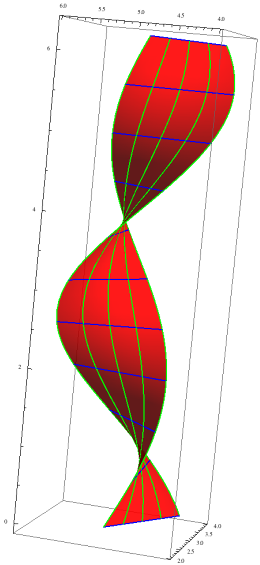
The contact surface consists of translated copies of *congruent* helix, therefore it is:

- Cylindrical
- Developable
- Has vanishing *Gaussian curvature*
- The *mean curvature* is identically 0 if and only if $a_i = \vec{0}$. In this case, the surface is a vertical plane.



- Has negative *Gaussian curvature*.
- The *mean curvature* is identically 0 if the reference point is on the edge.
- Both curvatures depend *only* on t
- The curvatures extrema are attained along the *same* and *unique* helix.



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Recap

We considered:

- Parameterization of contact surfaces for polygonal robot in the plan.
- Geometrical analysis of the contact surfaces.

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Thank you!

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