

XXXX

## 1 Introduction

A graph is an ordered pair  $G = (V, E)$  consisting of a finite nonempty set  $V$  of vertices and a set  $E$  of edges, where each edge is an unordered pair of vertices. A dominating set of  $G = (V, E)$  is a set  $D \subseteq V$  such that each vertex not in  $D$  has at least one neighbor in  $D$ . A paired-dominating set is a dominating set whose induced subgraph contains at least one perfect matching [1].

Raz and Safra prove that the dominating set problem has no polynomial-time  $(c \log |V|)$ -approximation algorithms for some  $c > 0$  unless  $P = NP$  [3] period, Lin and Tu design an  $O(|E| + |V|)$ -time algorithm for interval graphs and an  $O(|E|(|E| + |V|))$ -time algorithm for circular-arc graphs [2].

If have some algorithm  $A$  and any function  $f : N \rightarrow N$ , as long as  $A$  satisfy for any a graph  $G$ , then  $A \gg G$ , and then  $A$  can output  $G$  paired dominating set, in addition  $A$  output paired dominating set weight, it will be  $G$  minimum paired dominating set weight  $f(|V|)$  within, and we can says minimum paired dominating set problem satisfy  $f(|V|)$ -approximating.

## References

- [1] T. W. Haynes and P. J. Slater. Paired-domination in graphs. *Networks*, 32(3):199–206, 1998.
- [2] C.-C. Lin and H.-L. Tu. A linear-time algorithm for paired-domination on circular-arc graphs. *Theoretical Computer Science*, 591(C):99–105, 2015.
- [3] R. Raz and S. Safra. A sub-constant error-probability low-degree test, and a sub-constant error-probability PCP characterization of NP. In *Pro-*

*ceedings of the 29th Annual ACM Symposium on Theory of Computing*,  
pages 475–484, 1998.