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1 Introduction

A graph is an ordered pair $G = (V, E)$ consisting of a finite nonempty set V of vertices and a set E of edges, where each edge is an unordered pair of vertices. A dominating set of $G = (V, E)$ is a set $D \subseteq V$ such that each vertex not in D has at least one neighbor in D. A paired-dominating set is a dominating set whose induced subgraph contains at least one perfect matching [1].

Raz and Safra prove that the dominating set problem has no polynomial-time $(c \log |V|)$-approximation algorithms for some $c{>}0$ unless P = NP [3]. Lin and Tu design an $O(|E| + |V|)$-time algorithm for interval graphs and an $O(|E|(|E| + |V|))$-time algorithm for circular-arc graphs for the minimum paired dominating set problem [2].

Let $\{ \mathrm{f}{:}N \to \mathrm{N} \}$ be any function. If, given any graph $G = (V, E)$, an algorithm A outputs a paired dominating set of G whose size is at most $f(|V|)$ times the minimum, then A is said to be $f(|V|)$-approximate for the minimum paired dominating set problem.

2 Pseudocode

$\mathcal{D} \leftarrow \emptyset$;
while $\bigcup_{v \in \mathcal{D}} N[v] \neq V$ **do**
  Among the edges in E not having an endpoint in $\mathcal{D}$, pick an edge (a, b) that maximizes $|(N[a] \cup N[b]) \cap (V \setminus \bigcup_{v \in \mathcal{D}} N[v])|$, breaking ties arbitrarily;
  $\mathcal{D} \leftarrow \mathcal{D} \cup \{a, b\}$;
end while
return $\mathcal{D}$;

Lemma 1. *Whenever line 3 of A is executed,*

$$\left| V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right| > 0.$$

Proof. To be written. $\square$

Let D^* be a smallest dominating set of $G = (V, E)$.

Lemma 2. *Whenever line 3 of A is executed, there exists $u \in D^* \setminus \mathcal{D}$ satisfying*

$$\left| N[u] \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right) \right| \geq \frac{1}{|D^*|} \cdot \left| V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right|$$

and that $N(u) \not\subseteq \mathcal{D}$.

Proof. To be written. $\square$

Lemma 3. *Right after each execution of line 3 of A,*

$$\left| (N[a] \cup N[b]) \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right) \right| \geq \frac{1}{|D^*|} \cdot \left| V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right|.$$

Proof. To be written. $\square$

References

[1] T. W. Haynes and P. J. Slater. Paired-domination in graphs. *Networks*, 32(3):199–206, 1998.

[2] C.-C. Lin and H.-L. Tu. A linear-time algorithm for paired-domination on circular-arc graphs. *Theoretical Computer Science*, 591(C):99–105, 2015.

[3] R. Raz and S. Safra. A sub-constant error-probability low-degree test, and a sub-constant error-probability PCP characterization of NP. In *Proceedings of the 29th Annual ACM Symposium on Theory of Computing*, pages 475–484, 1998.