An algorithm to be analyzed

1 Pseudocode

We modify the famous greedy algorithm for the set-covering problem to give the following algorithm, called ALG, for the minimum paired-dominating set problem.

1: $\mathcal{D} \leftarrow \emptyset$;
2: **while** $\bigcup_{v\in\mathcal{D}} N[v] \neq V$ **do**
3: Among the edges in E not having an endpoint in $\mathcal{D}$, pick an edge (a, b) that maximizes $|(N[a] \cup N[b]) \cap (V \setminus \bigcup_{v\in\mathcal{D}} N[v])|$, breaking ties arbitrarily;
4: $\mathcal{D} \leftarrow \mathcal{D} \cup \{a, b\}$;
5: **end while**
6: **return** $\mathcal{D}$;

Lemma 1. *Whenever line 3 of A is executed,*

$$\left| V \setminus \bigcup_{v\in\mathcal{D}} N[v] \right| > 0.$$

Proof. To be written. $\square$

Let D^* be a smallest dominating set of $G = (V, E)$.

Lemma 2. *Whenever line 3 of A is executed, there exists $u \in D^* \setminus \mathcal{D}$ satisfying*

$$\left| N[u] \cap \left(V \setminus \bigcup_{v\in\mathcal{D}} N[v] \right) \right| \geq \frac{1}{|D^*|} \cdot \left| V \setminus \bigcup_{v\in\mathcal{D}} N[v] \right|$$

and that $N(u) \not\subseteq \mathcal{D}$.

Proof. To be written. □

Lemma 3. *Right after each execution of line 3 of A,*

$$\left|(N[a] \cup N[b]) \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v]\right)\right| \geq \frac{1}{|D^*|} \cdot \left|V \setminus \bigcup_{v \in \mathcal{D}} N[v]\right|.$$

Proof. To be written. □