An algorithm to be analyzed

1 A sketch of proof of the key lemma

Let D^* be a smallest dominating set of $G = (V, E)$.

Lemma 1. *Whenever line 3 of A is executed, there exists $u \in D^* \setminus \mathcal{D}$ satisfying*

$$\left| N[u] \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right) \right| \geq \frac{1}{|D^*|} \cdot \left| V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right|$$

and that $N(u) \not\subseteq \mathcal{D}$.

Sketch of proof. Denote the elements of D^* by v_1, v_2, $\ldots$, v_ℓ. As D^* is a dominating set,

$$\bigcup_{i=1}^{\ell} N[v_\ell] = V.$$

Consequently,

$$V \setminus \bigcup_{v \in \mathcal{D}} N[v] \subseteq \bigcup_{i=1}^{\ell} N[v_\ell].$$

Use this to show the existence of $j \in \{1, 2, \ldots, \ell\}$ satisfying

$$N[v_j] \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right) \geq \frac{1}{\ell} \cdot \left| V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right|.$$

$\square$