Finding paired-dominating sets

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Definitions

- A dominating set of $G = (V, E)$ is a set $D \subseteq V$ such that each vertex not in D has at least one neighbor in D.
- A paired-dominating set is a dominating set whose induced subgraph contains at least one perfect matching.

Related works

- Haynes and Slater (1998) prove the NP-completeness of the minimum paired-dominating set problem.
- Lin and Tu (2015) design an $O(|E| + |V|)$-time algorithm for interval graphs and an $O(|E|(|E| + |V|))$-time algorithm for circular-arc graphs.

Pseudocode

$\mathcal{D} \leftarrow \emptyset$;
while $\bigcup_{v \in \mathcal{D}} N[v] \neq V$ **do**
  Among the edges in E not having an endpoint in $\mathcal{D}$, pick an edge (a, b) that maximizes $|(N[a] \cup N[b]) \cap (V \setminus \bigcup_{v \in \mathcal{D}} N[v])|$, breaking ties arbitrarily;
  $\mathcal{D} \leftarrow \mathcal{D} \cup \{a, b\}$;
end while
return $\mathcal{D}$;

Our algorithm is a modification of the famous approximation algorithm for set covering.

Analysis

Lemma

Let D^* be a dominating set of G. Whenever line 3 of our algorithm is executed, there exists $u \in D^* \setminus \mathcal{D}$ such that

$$\left| N[u] \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right) \right| \geq \frac{1}{|D^*|} \cdot \left| V \setminus \bigcup_{v \in \mathcal{D}} N[v] \right| \tag{1}$$

and that $N(u) \not\subseteq \mathcal{D}$.

Proof.

As D^* is a dominating set,

$$V \setminus \bigcup_{v \in \mathcal{D}} N[v] \subseteq \bigcup_{v \in D^*} N[v].$$

So by the averaging argument, there exists $u \in D^*$ satisfying inequality (1). We have $u \notin \mathcal{D}$ and $N(u) \not\subseteq \mathcal{D}$ for, otherwise, the left-hand side of inequality (1) would vanish. $\square$

Analysis

Corollary

Right after each execution of line 3 of our algorithm,

$$\left|(N[a] \cup N[b]) \cap \left(V \setminus \bigcup_{v \in \mathcal{D}} N[v]\right)\right| \geq \frac{1}{|D^*|} \cdot \left|V \setminus \bigcup_{v \in \mathcal{D}} N[v]\right|.$$

After $|D^*| \cdot \lceil \log |V| \rceil$ iterations,

$$\left|V \setminus \bigcup_{v \in \mathcal{D}} N[v]\right| \leq \left(1 - \frac{1}{|D^*|}\right)^{|D^*| \cdot \lceil \log |V| \rceil} |V| < 1$$

by repeatedly invoking the above corollary, implying

$$\left|V \setminus \bigcup_{v \in \mathcal{D}} N[v]\right| = 0.$$

When the algorithm halts, $|\mathcal{D}|$ is simply twice the number of iterations. As a result, the algorithm outputs a set $\mathcal{D}$ of size at most $2 \cdot |D^*| \cdot \lceil \log |V| \rceil$.

Main result

Theorem

The minimum paired-dominating set problem has a polynomial-time $(2 \cdot \lceil \log |V| \rceil)$*-approximation algorithm.*

Thank you!