**ATOC5860 – Application Lab #1**

**Significance Testing Using Bootstrapping and Z/T-tests**

**in class Thursday January 20 and Tuesday January 25, 2020**

**Notebook #1 – Statistical significance using Bootstrapping**

**ATOC7500\_applicationlab1\_bootstrapping.ipynb**

**LEARNING GOALS:**

1) Use an ipython notebook to read in csv file, print variables, calculate basic statistics, do a bootstrap, make histogram plot

2) Hypothesis testing and statistical significance testing using bootstrapping

**DATA and UNDERLYING SCIENCE:**

In this notebook, you will analyze the relationship between Tropical Pacific Sea Surface Temperature (SST) anomalies and Colorado snowpack. Specifically, you will test the hypothesis that December Pacific SST anomalies driven by the El Nino Southern Oscillation affect the total wintertime snow accumulation at Loveland Pass, Colorado. When SSTs in the central Pacific are anomalously warm/cold, the position of the mid-latitude jet shifts and precipitation in the United States shifts. This notebook will guide you through an analysis to investigate the connections between December Nino3.4 SST anomalies (in units of °C) and the following April 1 Loveland Pass, Colorado Snow Water Equivalence (in units of inches). Note that SWE is a measure of the amount of water contained in the snowpack. To convert to snow depth, you multiply by ~5 (the exact value depends on the snow density).

The Loveland Pass SWE data are from:

<https://www.wcc.nrcs.usda.gov/snow/>

The Nino3.4 data are from:

<https://www.esrl.noaa.gov/psd/gcos_wgsp/Timeseries/Nino34/>

**Questions to guide your analysis of Notebook #1:**

For full credit: write answers to the questions and then upload this document to your github along with notebook #1 (including any edits that you make).

1) Composite Loveland Pass, Colorado snowpack. Fill out the following table showing the April 1 SWE in all years, in El Nino years (conditioned on Nino3.4 being 1 degree C warmer than average), and in La Nina years (condition on Nino3.4 being 1 degree C cooler than average).

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Mean SWE** | **Std. Dev. SWE** | **N (# years)** |
| **All years** | **16.33** | **4.22** | **81** |
| **El Nino Years** | **15.29** | **4** | **16** |
| **La Nina Years** | **17.78** | **4.11** | **15** |

2) Use hypothesis testing to assess if the differences in snowpack are statistically significant. Write the 5 steps. Test your hypothesis using bootstrapping.

1. We want 95% confidence so
2. H0: Snowfall was not different from the typical mean

H1: Snowfall was different from the typical mean

1. We will use the z-statistic since 81 years of data is enough to accurately estimate the population mean and standard deviation.
2. We will assume 95% confidence:

From the z table, equals at .

Since we don’t have good prior knowledge of how ENSO affects Colorado precipitation, we will use a two-sided test.

1. No conclusions can be made yet.

Instructions for bootstrap: Say there are N years with El Nino conditions. Instead of averaging the Loveland SWE in those N years, randomly grab N Loveland SWE values and take their average. Then do this again, and again, and again 1000 times. In the end you will end up with a distribution of SWE averages in the case of random sampling, i.e., the distribution you would expect if there was no physical relationship between Nino3.4 SST anomalies and Loveland Pass SWE.

1. Plot a histogram of this distribution and provide basic statistics describing this distribution (mean, standard deviation, minimum, and maximum).

Chart, histogram

Description automatically generated

The mean is 16.35 in. The standard deviation is 1.07 inches. The minimum value is 13.08 inches. Finally, the maximum value is 20.5 inches.

1. Quantify the likelihood of getting your value of mean SWE by chance alone using percentiles of this bootstrapped distribution. What is the probability that differences between the El Nino composite and all years occurred by chance? What is the probability that differences between the La Nina composite and all years occurred by chance?

The probability that differences in precipitation amount during El Nino occurred by chance is 32.3%.

The probability that differences in precipitation amount during La Nina occurred by chance: 18.4%

Our z-scores for El Nino and La Nina are -0.96 and 1.34, respectively, which are within +-1.96 critical z-score. So the precipitation amounts are not significant and we cannot reject the null hypothesis.

3) Test the sensitivity of the results obtained in 2) by changing the number of bootstraps, the statistical significance level, or the definition of El Nino/La Nina (e.g., change the temperature threshold so that El Nino is defined using a 0.5 degree C temperature anomaly or a 3 degree C temperature anomaly). In other words – play and learn something about the robustness of your conclusions.

Using 2500 samples with the Bootstrap Method, our z-scores are -0.99 and 1.37 for El Nino and La Nina, respectively, which is still inside the range of the critical z-score of +-1.96. So we still cannot reject the null hypothesis.

4) Maybe you want to see if you get the same answer when you use a t-test… Maybe you want to set up the bootstrap in another way?? Another bootstrapping approach is provided by Vineel Yettella (ATOC Ph.D. 2018). Check these out and see what you find!!

Using the t-test, we calculate a p-value of 0.35, which is not less than , or 0.025, so we still cannot reject the null hypothesis.

**Notebook #2 – Statistical significance using z/t-tests**

**ATOC7500\_applicationlab1\_ztest\_ttest.ipynb**

**LEARNING GOALS:**

1) Use an ipython notebook to read in a netcdf file, make line plots and histograms, and calculate statistics

2) Calculate statistical significance of the changes in a standardized mean using a z-statistic and a t-statistic

3) Calculate confidence intervals for model-projected global warming using z-statistic and t-statistic.

**DATA and UNDERLYING SCIENCE:**

You will be plotting *munged* climate model output from the Community Earth System Model (CESM) Large Ensemble Project. The Large Ensemble Project includes a 42-member ensemble of fully coupled climate model simulations for the period 1920-2100 (*note: only the original 30 are provided here*). Each individual ensemble member is subject to the same radiative forcing scenario (historical up to 2005 and high greenhouse gas emission scenario (RCP8.5) thereafter), but begins from a slightly different initial atmospheric state (created by randomly perturbing temperatures at the level of round-off error). In the notebook, you will compare the ensemble remembers with a 2600-year-long model simulation having constant pre-industrial (1850) radiative forcing conditions (perpetual 1850). By comparing the ensemble members to each other and to the 1850 control, you can assess the climate change in the presence of internal climate variability.

**More information on the CESM Large Ensemble Project can be found at:**

<http://www.cesm.ucar.edu/projects/community-projects/LENS/>

**Questions to guide your analysis of Notebook #2:**

For full credit: write answers to the questions and then upload this document to your github along with notebook #1 (including any edits that you make).

1) Use the 2600-year long 1850 control run to calculate population statistics with constant forcing (in the absence of climate change). Find the population mean and population standard deviation for CESM1 global annual mean surface temperature. Standardize the data and again find the population mean and population standard deviation. Plot a histogram of the standardized data. Is the distribution Gaussian?

Using raw data:

* The mean global surface temperature for the 1850 control run (population) is 287.11 K.
* The standard deviation of global surface temperature for the 1850 control run (population) is 0.1 K.

Using standardized data:

* The mean global surface temperature for tehg1850 control run (population) is 1.
* The standard deviation of global surface temperature for the 1850 control run (population) is 0.1.

Chart, histogram

Description automatically generated

This distribution is gaussian because the standardized mean is 0 and the standardized standard deviation is 1.

2) Calculate global warming in the first ensemble member over a given time period defined by the startyear and endyear variables. Compare the warming in this first ensemble member with the 1850 control run statistics and assess if the warming is statistically significant. Use hypothesis testing and state the 5 steps. What is your null hypothesis? Try using a z-statistic (appropriate for N>30) and a t-statistic (appropriate for N<30). What is the probability that the warming in the first ensemble member occurred by chance? Change the startyear and endyear variables – When does global warming become statistically significant in the first ensemble member?

For the period 1900 to 2030, ensemble 1 has a standardized mean of 2.67. The standardized population mean from the 1850 control run is 0. Is this warming statistically significant?

1. We use a 95% confidence level so
2. H0: Average temperature from ensemble 1 is not different from the average temperature in the 1850 control run.
3. Since we have 130 independent observation and the underlying population is Normally distributed, we use the z-score. Using the z-table with 130 observations and a 95% confidence level, the critical z-score is 1.96.
4. From Barnes Eq. 83:

Since 30.46 is greater than 1.96, we can reject the null hypothesis that the average temperatures are the same.

For the period 2000 to 2030, ensemble 1 has a standardized mean of 8.27. The standardized population mean from the 1850 control run is 0. Is this warming statistically significant?

1. We use a 95% confidence interval so
2. H0: The average temperature from 1000 to 2030 is not different than the population mean.
3. We use the t-score since we have 30 independent data points and the underlying population is Normally distributed.
4. From Barnes Eq. 96:
5. Since 15.92 is greater than the critical value of 1.96, we cannot reject the null hypothesis.

3) Many climate modeling centers run only a handful of ensemble members for climate change projections. Given that the CESM Large Ensemble has lots of members, you can calculate the warming over the 21st century and place confidence intervals in that warming by assessing the spread across ensemble members. Calculate confidence intervals using both a z-statistic and a t-statistic. How different are they? Plot a histogram of global warming in the ensemble members – Is a normal distribution a good approximation? Re-do your confidence interval analysis by assuming that you only had 6 ensemble members or 3 ensemble members. How many members do you need? Look at the difference between a 95% confidence interval and a 99% confidence interval.

Ensemble 1 has a mean of 3.7 while the global ensemble mean is 3.63. Is this warming significant?

The 95% confidence interval using a t-score is 3.6 to 3.68.

The 99% confidence interval using a t-score is 3.57 to 3.71.

The 95% confidence interval using a z-score is 3.61 to 3.67.

The 99% confidence interval using a z-score is 3.6 to 3.68.

The interval using a 95% confidence limit with a t-score is the same as using a 99% confidence interval using a z-score. In general, increasing the confidence level increasing the range of possible values. The lowest value in the range is 3.57 and the highest value in the range is 3.68.

Chart, histogram

Description automatically generated

Yes, this data can be assumed Gaussian because the standardized data has a mean of and a standard deviation of .

Using only 6 members:

The confidence interval at a 95% limit using the t-score is 3.59 to 3.68.

The confidence interval at a 99% limit using the t-score is 3.57 to 3.70.

Using only 3 members:

The confidence interval at a 95% limit using the t-score is 3.58 to 3.73.

The confidence interval at a 99% limit using the t-score is 3.48 to 3.83.

Both 3 and 6 ensemble members correctly contain the true population mean of 3.63 so either amount works.