Università
della
Svizzera
italiana

**Facoltà
di scienze
informatiche**

Introduction to PDEs (LAB) **Academic Year 2015/2016**

Instructor: Prof. Rolf Krause Dr. Drosos Kourounis

Assignment 7 - Iterative Krylov-based methods

Due date: Thursday 12 November 2015, 10:30

Solution of Poisson's equation using iterative Krylov methods

Solve the same equation you solved in LAB6 replacing the backslash with each one of the methods provided in the MATLAB folder of our repository. Enforcing Dirichlet boundary conditions renders the system non-symmetric. For the non-symmetric version of the system you should use Krylov-space methods designed for non-symmetric systems of equations, nameley CGS, BICGSTAB and GMRES. These can be used with both the incomplete Cholesky (ichol) and incomplete LU (ilu) preconditioners. Report iterations and residual for each choice of method and preconditioner in a table where the horizontal row is $N = 50, 100, 200$, and the vertical row of the table is the Krylov method. In order for this to work as expected, you need to write a function that returns the application of the preconditioner on a vector, and instead of providing the preconditioner matrix M on each one of these methods you should provide a pointer to function. Read MATLAB documentation to understand how to do this.

	N		
Krylov method	50	100	200
GMRES(ilu) GMRES(ichol)	iters (residual)	iters (residual)	iters (residual)
CGS(ilu) CGS(ichol)			
BICGSTAB(ilu) BICGSTAB(ichol)			

Table 1. Performance of non-symmetric Krylov methods with ilu0 and ichol preconditioners on a grid of size $N \times N \times N$.

Implement the symmetric version of Dirichlet conditions. Now you can use ichol with conjugate gradient CG and QMR as well. Create another table You should try both the preconditioners provided from MATLAB Plot the

	N		
Krylov method	50	100	200
CG	iters (residual)	iters (residual)	iters (residual)
QMR			
GMRES			
CGS			
BICGSTAB			

Table 2. Performance of symmetric and non-symmetric Krylov methods with ichol preconditioners with Dirichlet conditions preserving the symmetry of the original stiffness matrix, on a grid of size $N \times N \times N$.

log of the residual norm (y-axis) as a function of the iterations (x-axis) for all the methods of Table 2, for two cases. One plot for $N = 50$ and one for $N = 200$ side-by-side. Use different colors for each method and make sure you add a legend in your plots.

Using FEniCS

Solve the same problem using FEniCS. Choose the Krylov solvers you see below and the hypre_amg or petsc_amg and icc preconditioners. Create a similar table with Table 2. Finally make a table of the running times for both FEniCS and your code.

	N		
Krylov method	50	100	200
CG (HYPRE)			3 (1.067e-8)
CG (KSP)			7 (3.008e-8)
CG (ICC)			68 (4.739e-7)
GMRES (HYPRE)			3 (1.016e-4)
GMRES (KSP)			7 (1.016e-4)
GMRES (ICC)			60 (2.2e-4)
BICGSTAB (HYPRE)			3 (1.512e-8)
BICGSTAB (KSP)			7 (7.126e-8)
BICGSTAB (ICC)			48 (1.769e-7)

Table 3. Performance of symmetric and non-symmetric Krylov methods with icc, petsc_amg and hypre_amg preconditioners with Dirichlet conditions preserving the symmetry of the original stiffness matrix, on a grid of size $N \times N \times N$.