Open Problems

Computable Structure Theory and Interactions

Question 1 **by Dino Rossegger**

Suppose $R \subseteq \mathbb{N}^k$ is definable by a $\Sigma_1(X)$ formula in $\mathcal{L}_{\omega_1\omega}$ for some comeager set X.
Questions:

- Is R $\Sigma_1(\emptyset)$-definable?
- What about the same question as above replacing "comeager" by "co-null"?

Question 2 **by Russell Miller**

Notations:

- TFAB_n denotes the class of torsion free abelian group of rank n;
- FTD_n denotes the class of fields of trascendence degree n;
- TD_n denotes the class of fields of characteristic 0 and transcendence degree n.

Known results:

- (Hjorth, Thomas) $\mathsf{TFAB}_n <_B \mathsf{TFAB}_{n+1}$ (here $<_B$ denotes Borel reducibility). In contrast,
- (Thomas, Velikovic) For every n, $\mathsf{FTD}_{13} \equiv_B \mathsf{FTD}_n$.

The following is known in the context of Turing computable embeddings (denoted by $\leq_{tc}$).

- (Ho, Knight, Miller) For every n, $\mathsf{TFAB}_n \leq_{tc} \mathsf{FTD}_n$.

Questions:

- $\mathsf{TD}_n \geq_B \mathsf{FTD}_{n+1}$?
- $\mathsf{TD}_n \leq_B \mathsf{TFAB}_n$? The answer is no for $n \geq 13$.
- Same question as above but replacing $\leq_B$ with $\leq_{tc}$. Again, the answer is no for $n \geq 13$.

Question 3 **by Stefan Vatev**

It is known that for all limit ordinals α, β, $(\alpha, \alpha^*) \equiv_{tc} (\beta, \beta^*)$.
Question:

- What happens if we replace Turing computable embedding with enumeration reducibility?

Definition 1 (Enumeration reducibility) *Let $A, B \subseteq \mathbb{N}$. We say that A is enumeration reducible to B (notation, $A \leq_e B$) if:*

$$(\exists e)(x \in A \iff (\exists \textit{ finite } D \subseteq B)(\langle x, D\rangle \in W_e)).$$

For example, $(\omega \cdot n, \omega^* \cdot n) \leq_c (\omega \cdot k, \omega^* \cdot k)$ if and only if $n | k$ (here $\leq_c$ denotes computable embedding).

Question 4 **by Gianluca Paolini**

- Is there a computable first-order theory T such that the isomorphism relation for models of T with domain ω is as complicated as graph isomorphism with respect to Borel reducibility?
- Consider classes as linear orders, Trees, Boolean algebras: is Elementary embeddability restricted to these classesa a complete analytic quasi-order? (In general, the anwer is yes).

Question 5 **by Mateusz Łełik**

Let the Scott function of a theory be the function defined as

$$S_T(\alpha) := |\{\mathcal{M} \models T : SR(\mathcal{M}) = \alpha\}|.$$

Question:

- Is there a non-trivial first-order theory T such that S_T is not monotone or has interesting behavior?

Question 6 **by David Gonzalez**

What are the possible Scott spectra of first-order theories?