Exploiting Imprecise Information Sources in Sequential Decision Making Problems under Uncertainty

N.Drougard

under **D.Dubois**, **J-L.Farges** and **F.Teichteil-Königsbuch** supervision

doctoral school: EDSYS **institution:** ISAE–SUPAERO

laboratory: ONERA–The French Aerospace Lab

retour sur innovation

Context

Autonomous robotics

Onera, System Control & Flight Dynamics Department

Control Engineering, Artificial intelligence, Cognitive Sciences

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- autonomy and human factors
- decision making, planning
- experimental/industrial applications: UAVs, exploration robots, human-machine interaction

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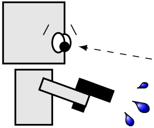
Partially Observable Markov Decision Processes (POMDPs)



POMDP: model for autonomous robotics

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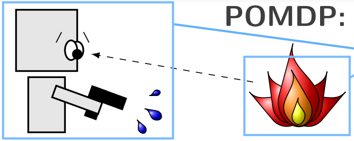

$s \in \mathcal{S}$: **system states**

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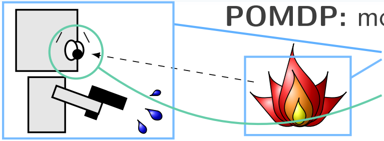

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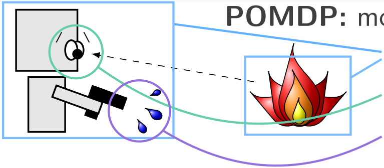

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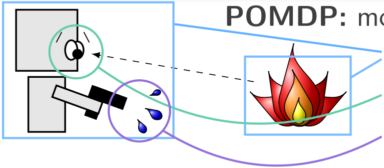

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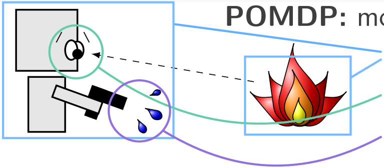

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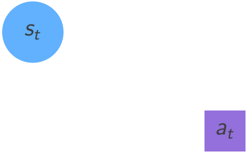

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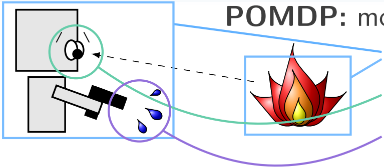

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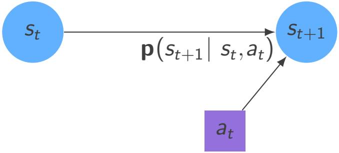

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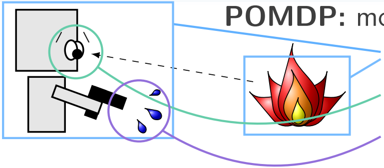

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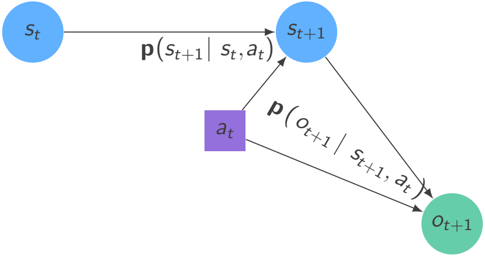

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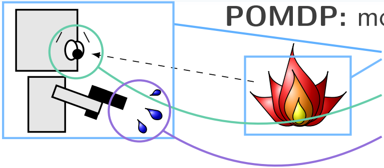

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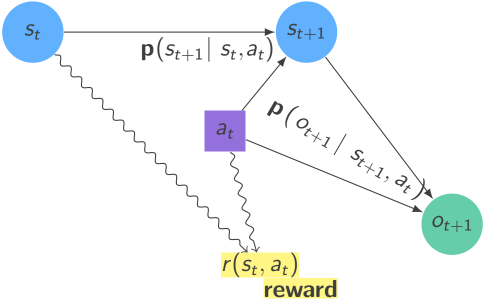

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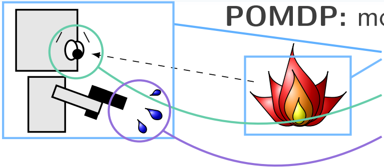

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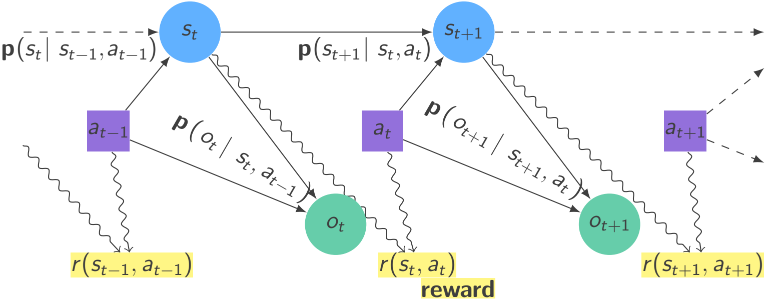

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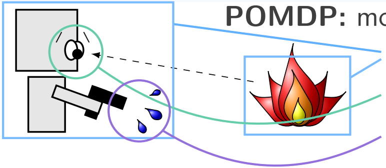

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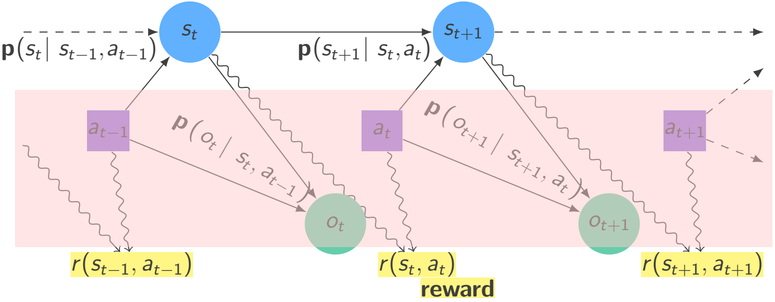

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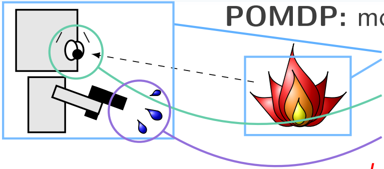

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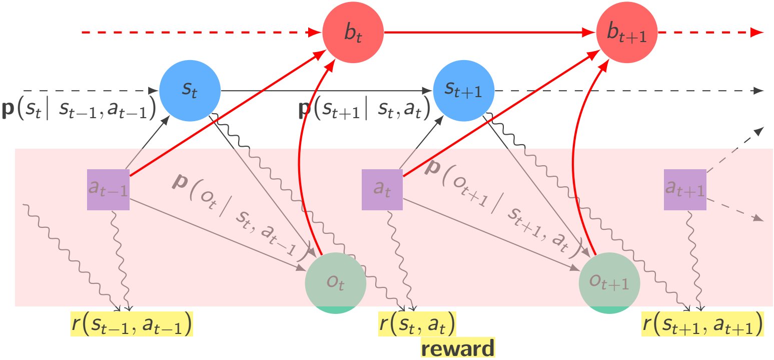
- b **belief**: **probability distribution**



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belief state, strategy, criterion

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- **transition** function $T(s,a,s') = \mathbf{p}\left(s' \mid s, a \right)$
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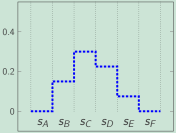

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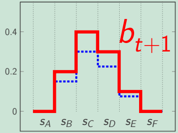

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b_t, T,O → $\mathbf{p}\left(s', o' \mid b_t, a_t\right)$ —received observation / o_{t+1} fixed→

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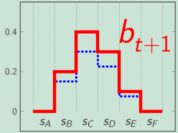

strategy $d_t : b_t \mapsto a_t \in \mathcal{A}$

maximizing $\mathbb{E}_{s_0 \sim b_0}\left[\sum_{t=0}^{+\infty} \gamma^t \cdot r\Big(s_t, \delta(b_t)\Big)\right]$, $0 < \gamma < 1$

Flaws of the POMDP model

POMDPs in practice

- optimal strategy computation **PSPACE-hard**
  (*Papadimitriou et al., 1987*)
- probabilities are **imprecisely known** in practice
- **prior ignorance** semantic/management?

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practical issues: Complexity, Vision and Initial Belief

- **POMDP optimal strategy computation undecidable** in infinite horizon (*Madani et al. 1999*)

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$\rightarrow$ uniform probability distribution $\neq$ ignorance!

Qualitative Possibility Theory

presentation – (max,min) “tropical” algebra

finite scale $\mathcal{L}$

usually $\{0, \frac{1}{k}, \frac{2}{k}, \dots, 1\}$

1 — entirely possible
somewhat plausible
$\vdots$
almost impossible
0 — impossible

events $e \subset \Omega$ (universe)

sorted using possibility **degrees** $\pi(e) \in \mathcal{L}$

$\neq$

quantified with **frequencies p**$(e) \in [0, 1]$ (probabilities)

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$e_1 \neq e_2$, 2 events $\subset \Omega$

- $\pi(e_1) < \pi(e_2) \Leftrightarrow$ "*e_1 is less plausible than e_2*"

Qualitative Possibility Theory

Criteria from Sugeno integral

Probability	/ Qualitative Possibility Theories
$+$	max
$\times$	min
$\sum_x \mathbf{p}(x) = 1$	$\max_x \pi(x) = 1$
$X \in \mathbb{R}$	$X \in \mathcal{L}$
$\mathbb{P}(A) = 1 - \mathbb{P}(\overline{A})$	$\mathcal{N}(A) = 1 - \Pi(\overline{A})$ (necessity)
$\mathbb{E}[X] = \sum_x x \cdot \mathbf{p}(x)$	**optimistic:** $\mathbb{S}_\Pi[X] = \max_{x \in X} \min \{ x, \pi(x) \}$ **pessimistic:** $\mathbb{S}_\mathcal{N}[X] = \min_{x \in X} \max \{ x, 1 - \pi(x) \}$

Qualitative Possibility Theory

qualitative possibilistic POMDP (π-POMDP)

Sabbadin (UAI-98) introduces

the qualitative possibilistic POMDP

π-**POMDP:** $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T^{\pi}, O^{\pi}, \rho \rangle$

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$\forall s \in \mathcal{S},\ \pi(s) = 1 \Leftrightarrow$ total ignorance about s
each state possible, **none necessary**

A possibilistic belief state

finite belief space

$$\Pi_{\mathcal{L}}^{\mathcal{S}} = \left\{ \text{possibility distributions} \right\}: \#\Pi_{\mathcal{L}}^{\mathcal{S}} \sim \#\mathcal{L}^{\#\mathcal{S}} < +\infty$$

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β_t

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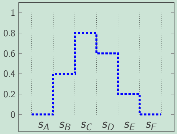

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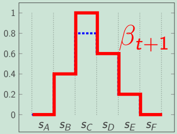


A possibilistic belief state

finite belief space

$$\Pi^{\mathcal{S}}_{\mathcal{L}} = \Big\{ \text{ possibility distributions } \Big\} : \ \#\Pi^{\mathcal{S}}_{\mathcal{L}} \sim \#\mathcal{L}^{\#\mathcal{S}} < +\infty$$

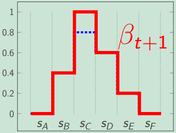
$\rightarrow$ *i.e.* **finite belief space**

$$\beta_t(s) = \pi \left(s_t = s \mid a_0, o_1, \ldots, a_{t-1}, o_t \right)$$

possibilistic belief update

β_t, T^{π}, O^{π} $\rightarrow$ $\pi \left(s', o' \mid \beta_t, a_t \right)$ $\xrightarrow[o_{t+1} \text{ fixed}]{\text{received observation}}$

possibilistic normalization



- **Markovian update:** only depends on o_{t+1}, a_t and b^{π}_t

Overview

Qualitative Possibility Theory:

$\rightarrow$ simplification, imprecision/prior ignorance modeling

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$\rightarrow$ simplification, imprecision/prior ignorance modeling

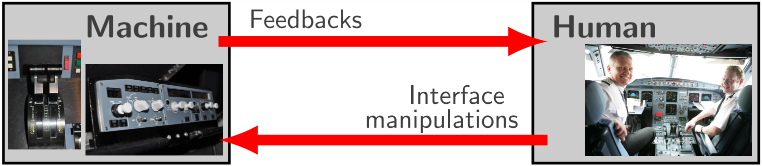
- context

1. introductory example: qualitative **possibilistic modeling**
   $\rightarrow$ *human-machine interaction (HMI)*
   with **Sergio Pizziol**
2. **advancements** in π-POMDP:
   $\rightarrow$ *mixed-observability & indefinite horizon*
3. **simplifying computations**:
   $\rightarrow$ *ADD-based solver & factorization*
4. **probabilistic-possibilistic** (*hybrid*) approach

- conclusion

Example: Human-Machine Interaction (HMI)

joint work with **Sergio Pizziol** – Context



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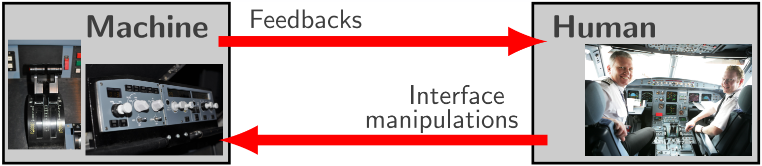

Issue: incorrect human assessment of the machine state
$\rightarrow$ **accident risk**

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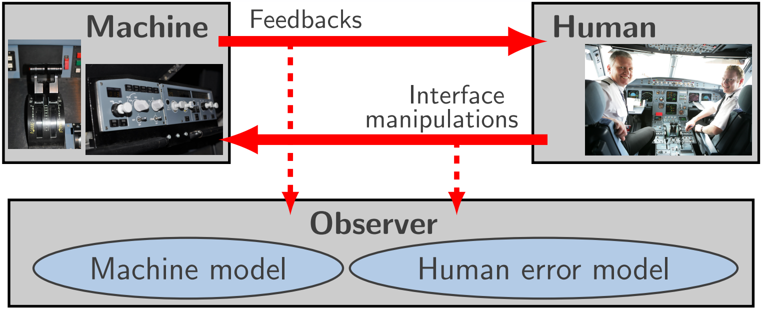

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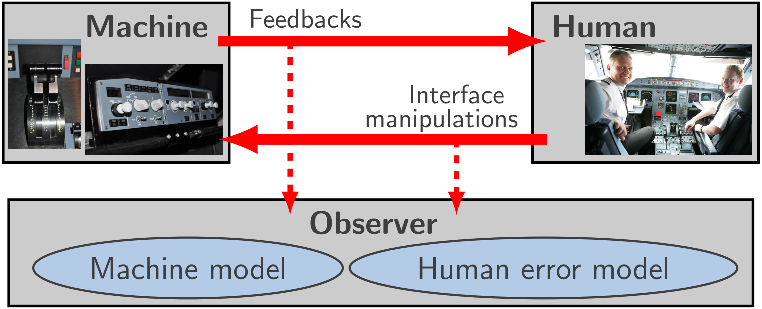

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π-POMDP without actions: π-Hidden Markov Process

- **system space** $\mathcal{S}$: set of human assessments $\rightarrow$ **hidden**
- **observation space** $\mathcal{O}$: feedbacks/human manipulations

Example: Human-Machine Interaction (HMI)

Human error model from expert knowledge

Machine with states A, B, C, ...

state $s_A \in \mathcal{S}$: "human thinks machine state is A"

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$$\text{"human usually aware of feedbacks"} \rightarrow \pi\left(s'_B, o'_f \mid s_A\right) = 1$$
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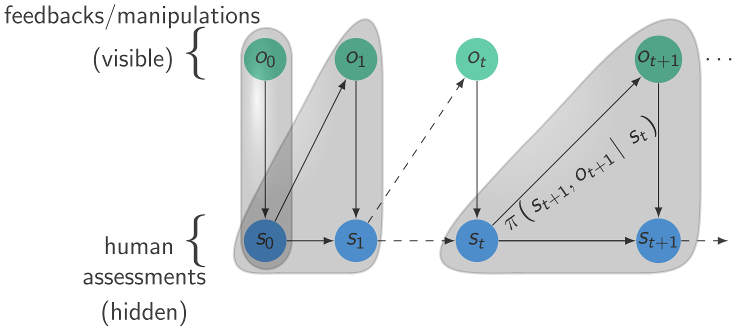
- impossible cases: possibility degree 0

Qualitative Possibilistic Hidden Markov Process:

π-HMP, detection & diagnosis tool for HMI (with **Sergio Pizziol**)



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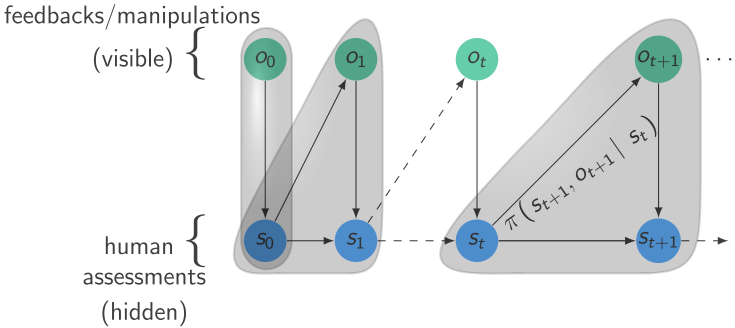

- **estimation** of the human assessment
  $\Leftrightarrow$ **possibilistic belief state**
- **detection** of human assessment errors + **diagnosis**
- validated with pilots on flight simulator missions

Applicability of the π-POMDPs
advancements

- **lack of proof of optimality in indefinite horizon settings**
- criterion/proof

- **curse of dimensionality:**
  $\rightarrow$ belief space size of a π-POMDP: exponential in $\#\mathcal{S}$
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Indefinite Horizon, Mixed-Observability, Simulations
contribution UAI 2013

Proof of optimality under Indefinite Horizon

criterion, DP scheme, optimal strategy

indefinite horizon criterion $\Psi : \mathcal{S} \rightarrow \mathcal{L}$ terminal pref. func.

$$\forall s \in \mathcal{S}, \text{ maximizing } \mathbb{S}_\Pi \left[\Psi(S_{\#\delta}) \middle| S_0 = s \right]$$

with respect to the strategy $\delta : (t,s) \mapsto a_t \in \mathcal{A}$.

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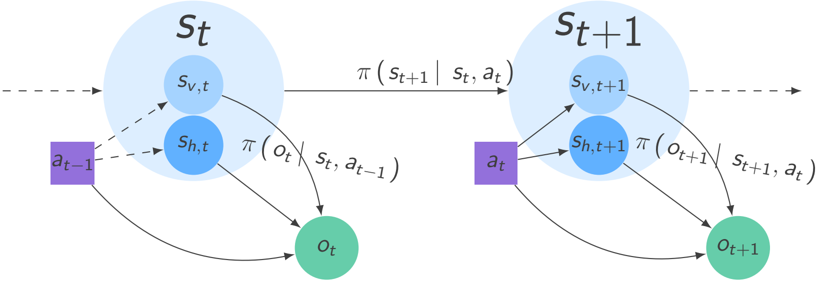
  as in classical planning / γ counterpart
- criterion **non decreasing** with iterations
- action update for states increasing the criterion
- **proof of optimality** of the resulting **stationary** strategy

Scalability capabilities with Mixed-Observability

π-Mixed-Observable Markov Decision Process (π-MOMDP)

graphical model of a π-MOMDP:



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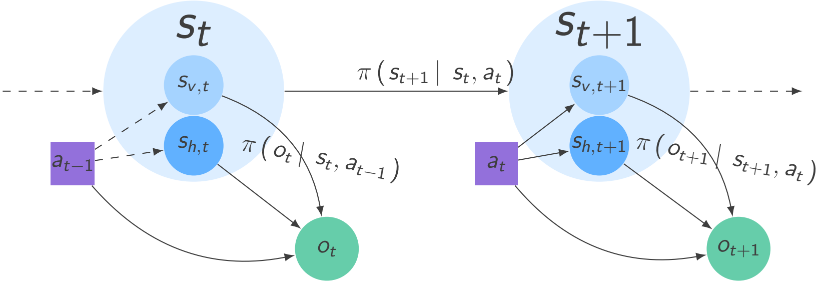

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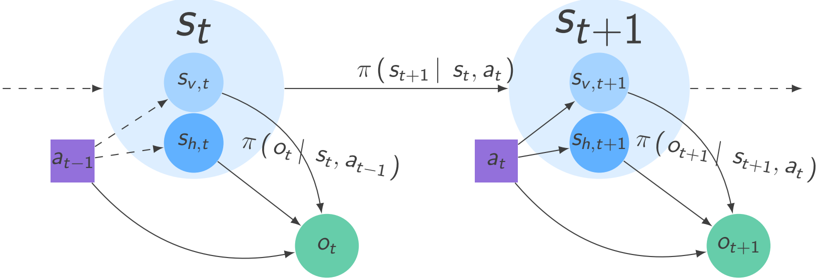

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- $\rightarrow$ π-POMDP: belief space $\Pi_{\mathcal{L}}^{\mathcal{S}}$ $\quad \#\Pi_{\mathcal{L}}^{\mathcal{S}} \sim \#\mathcal{L}^{\#\mathcal{S}}$

  $\rightarrow$ π-MOMDP: computations on $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$

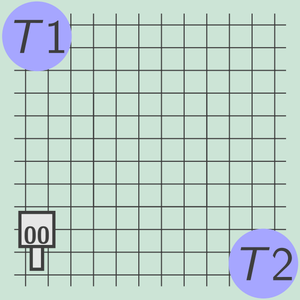
  $\#\mathcal{X} \sim \#\mathcal{S}_v \cdot \#\mathcal{L}^{\#\mathcal{S}_h} \ll \#\Pi_{\mathcal{L}}^{\mathcal{S}}$

Experimental results

π-MOMDP for robotics with imprecise probabilities

- **goal:** reach the object $A = T1$ or $T2$
- noisy observations of the location of the object A

Recognition mission: robot on a grid, targets $T1$ & $T2$



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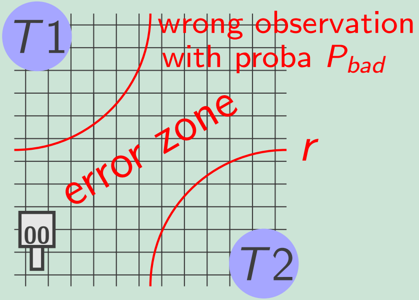

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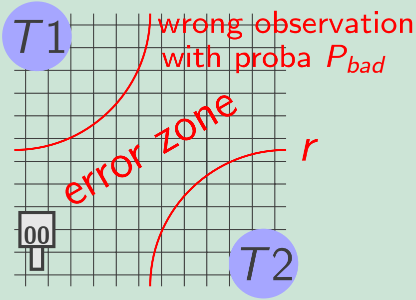

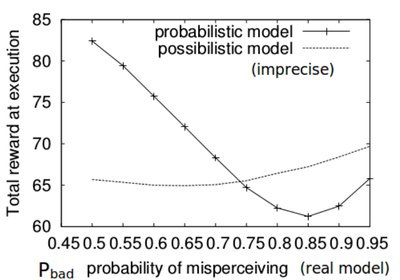

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probabilistic model inappropriate when probabilities too imprecise

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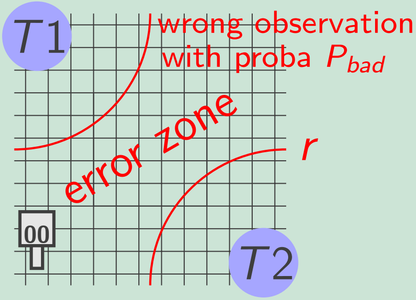

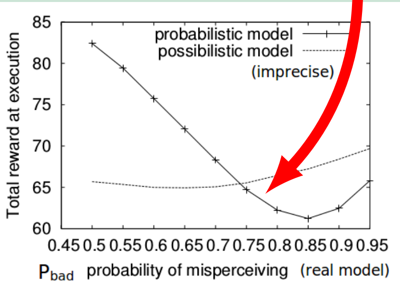

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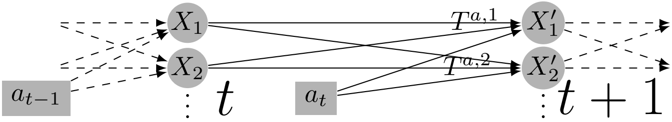
Factored π-MOMDP and computations with ADDs

qualitative possibilistic models to reduce complexity

contribution (AAAI-14): factored π-MOMDP
$\Leftrightarrow$ state space $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$ = Boolean variables $(X_1, \dots, X_n)$
+ independence assumptions $\Leftarrow$ graphical model



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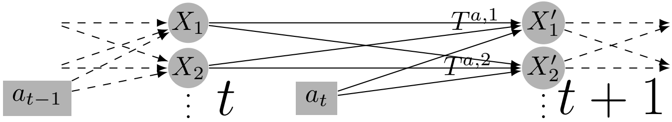
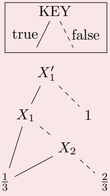


- **factorization:** transition functions $T_i^a = \pi \left(X'_i \mid parents(X'_i), a \right)$ stored as **Algebraic Decision Diagrams (ADD)**

  probabilistic case:
  SPUDD (*Hoey et al., 1999*)



example of ADD

Simplify computations with π-MOMDPs

Resulting π-MOMDP solver: PPUDD

- probabilistic model: $+$ and $\times$ $\Rightarrow$ new values created
  $\Rightarrow$ number of ADDs leaves **potentially huge**
- possibilistic model: min and max $\Rightarrow$ values $\in \mathcal{L}$ finite
  $\Rightarrow$ number of leaves bounded, **ADDs smaller**.

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PPUDD: Possibilistic Planning Using Decision Diagrams

- factorization $\Rightarrow$ each DP steps divided into n stages
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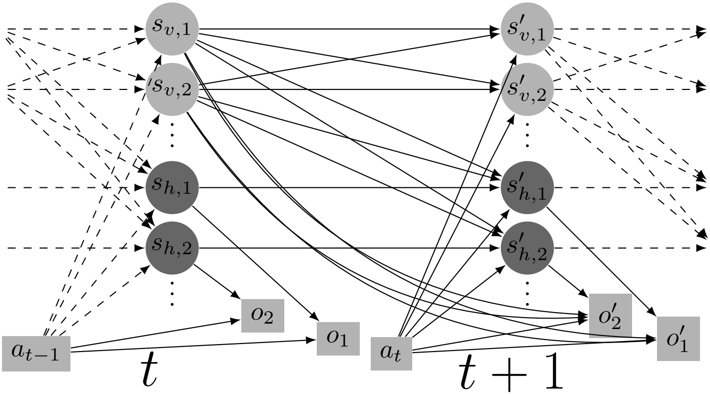
- computations on trees: *CU Decision Diagram Package*.

Simplifying computations with π-MOMDPs

Natural factorization: belief independence

contribution (AAAI-14):

independent sensors, hidden states, ... $\Rightarrow$ graphical model



Simplifying computations with π-MOMDPs

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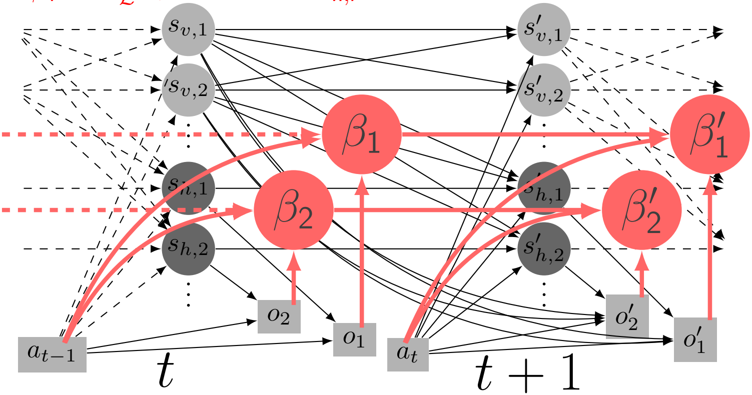
d-Separation $\Rightarrow$ $(s_v, \beta) = (s_{v,1}, \dots, s_{v,m}, \beta_1, \dots, \beta_l)$

$\beta_i \in \Pi_{\mathcal{L}}^{\mathcal{S}_{h,i}}$, belief over $s_{h,i}$

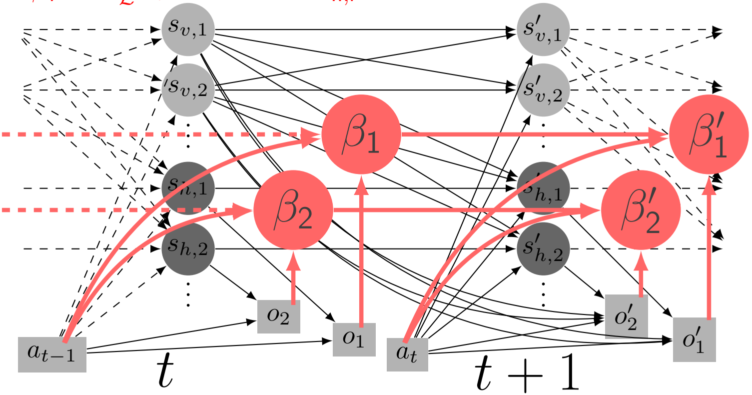


Simplifying computations with π-MOMDPs

Natural factorization: belief independence

$\perp\!\!\!\perp$ assumptions on state & observation variables
$\rightarrow$ belief variable factorization
$\rightarrow$ **additional** computation savings

$\beta_i \in \Pi_{\mathcal{L}}^{\mathcal{S}_{h,i}}$, belief over $s_{h,i}$



Simplify computations with π-MOMDPs

Experiments – perfect sensing: Navigation problem

PPUDD vs SPUDD (*Hoey et al.*, 1999)

Navigation benchmark: reach a goal – spots with accident risk

M1 (resp. M2) optimistic (resp. pessimistic) criterion

Simplify computations with π-MOMDPs

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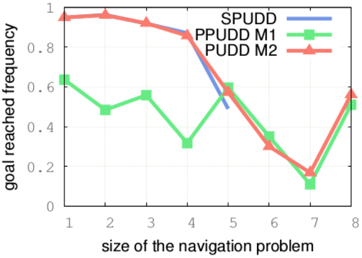
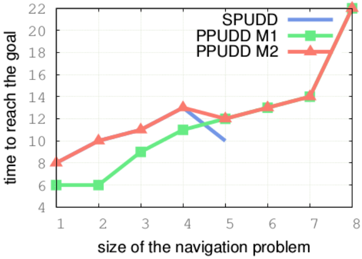
Performances, function of the problem index

reached goal frequency



higher is better

steps to reach the goal

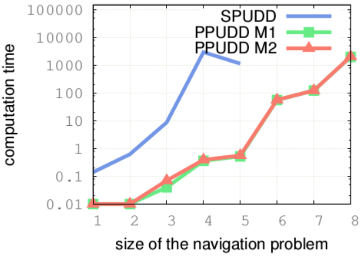


lower is better

Simplify computations with π-MOMDPs

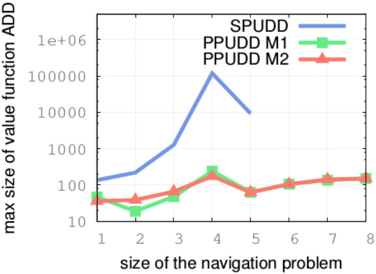
Experiments – perfect sensing: Navigation problem

computation time



lower is better

max size of ADDs



lower is better

- PPUDD + M2 (pessimistic criterion) **faster with same performances** as SPUDD
- SPUDD only solves the first 5 instances
- verified intuition: ADDs are smaller

Simplify computations with π-MOMDPs

Experiments – imperfect sensing: RockSample problem

PPUDD vs APPL (*Kurniawati et al.*, 2008, solver MOMDP)
symbolic HSVI (*Sim et al.*, 2008, solver POMDP)
RockSample benchmark: recognize and sample "good" rocks

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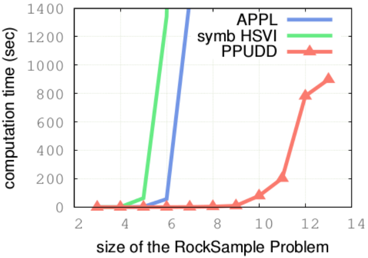
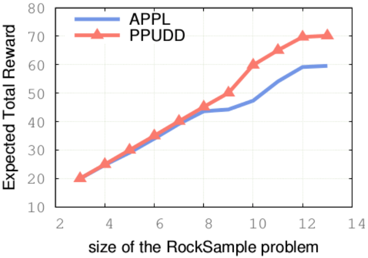


lower is better

average of rewards

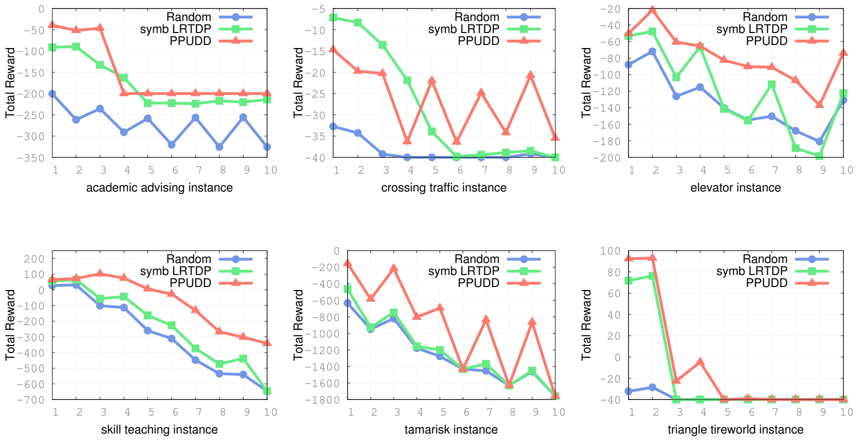


higher is better

- approximate model + exact resolution solver can be **better than** exact model + approximate resolution solver

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

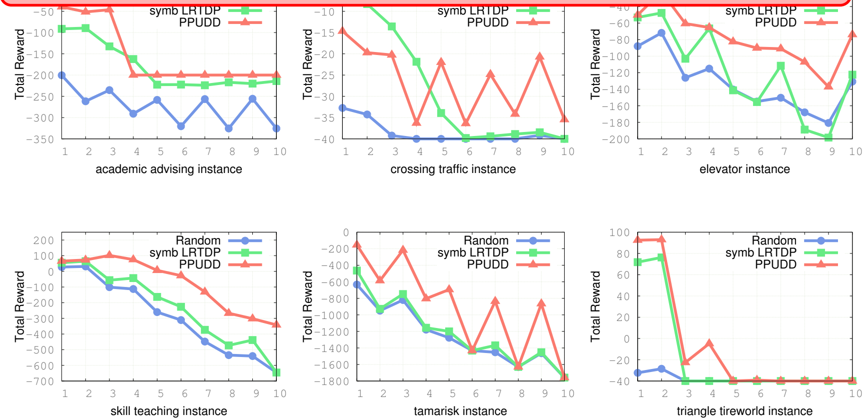
PPUDD + BDD mask over reachable states.



average of rewards over simulations – higher is better

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

PPUDD outperforms its probabilistic counterpart



average of rewards over simulations – higher is better

Qualitative possibilistic approach: benefits/drawbacks

towards a hybrid POMDP

- **granulated** belief space (discrete)
- efficient problem **simplification** (PPUDD 2$\times$ better than LRTDP with ADDs)
- **ignorance and imprecision** modeling

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- ADD methods $\prec$ state space search methods
  $\rightarrow$ winners of IPPC 2014: $2\times$ better than PPUDD
- choice of the qualitative criterion (optimistic/pessimistic)
- preference $\rightarrow$ non additive degrees
  $\rightarrow$ same scale as possibility degrees (commensurability)
- coarse approximation of probabilistic model
  $\rightarrow$ no frequentist information

A hybrid model

a probabilistic POMDP with possibilistic belief states

hybrid approach

- agent knowledge = possibilistic belief states
- probabilistic dynamics & quantitative rewards

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Usefullness

→ **heuristic** for solving POMDPs:

results in a standard (finite state space) MDP

→ problem with **qualitative** & **quantitative** uncertainty

Transitions and rewards

belief-based transition and reward functions

- possibility distribution $\beta \rightarrow$ probability distribution $\overline{\beta}$ using poss-prob tranformations *(Dubois et al., FSS-92)*

Transition function on belief states

$$\Rightarrow \mathbf{p}\left(\beta' \middle| \overline{\beta}, a\right) = \sum_{\substack{o' \text{ t.q.} \\ update(\beta,a,o')=\beta'}} \mathbf{p}\left(o' \mid \overline{\beta}, a\right)$$

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- reward pessimistic according to β

Pessimistic Choquet Integral

$$r(\beta, a) = \sum_{i=1}^{\#\mathcal{L}-1} (l_i - l_{i+1}) \cdot \min_{\substack{s \in \mathcal{S} \text{ s.t.} \\ \beta(s) \geqslant l_i}} r(s, a)$$

Resulting MDP

translation from hybrid POMDP to MDP – **contribution (SUM-15):**

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$
output: the MDP $\langle \tilde{\mathcal{S}}, \mathcal{A}, \tilde{T}, \tilde{r}, \gamma \rangle$:

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$$\textbf{criterion: } \mathbb{E}_{\beta_t \sim \tilde{T}} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot \tilde{r}\left(\beta_t, d_t\right) \right].$$

Belief variable factorization

3 classes of state variables – **contribution (SUM-15)**

variable: **visible** $s'_v \in \mathbb{S}_v$

s'_v

inferred hidden $s'_h \in \mathbb{S}_h$

s'_h

fully hidden $s'_f \in \mathbb{S}_f$

s'_f

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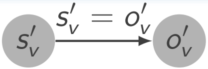

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$$s'_v \xrightarrow{s'_v = o'_v} o'_v$$

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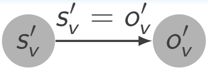

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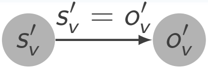

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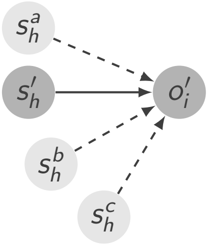

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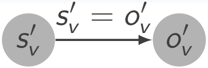

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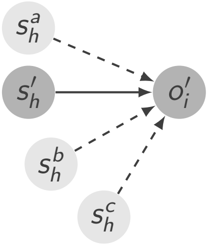

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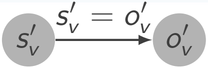

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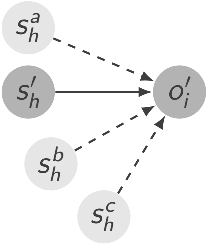

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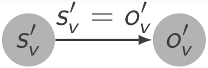

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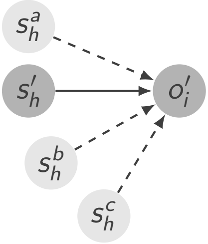

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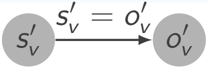

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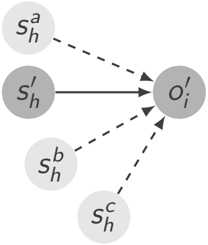

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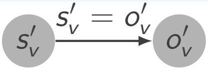

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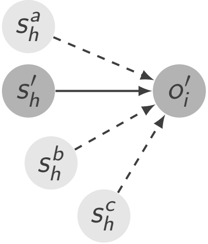

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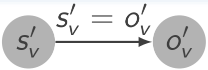

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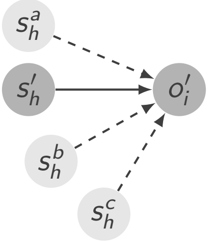

fully hidden $s'_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f.

$$\beta_{t+1}(s'_f) = \pi\,(\,s'_f \mid \beta_t, a\,)$$

Belief variable factorization

global belief state from marginal belief variables

bound over the global belief state

$$\beta_{t+1}(s'_1,\dots,s'_n) = \pi\left(s'_1,\dots,s'_n \mid a_0, o_1,\dots,a_t,o_{t+1}\right)$$

$$\leqslant \min\left\{ \min_{s'_j \in \mathbb{S}_v} \left[\mathbb{1}_{\{s'_j = o'_j\}} \right], \min_{s'_j \in \mathbb{S}_f} \left[\beta_{t+1}(s'_j) \right], \min_{o'_i \in \mathbb{O}_h} \left[\beta_{t+1}\left(parents(o'_i) \right) \right] \right\}$$

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- min of marginals = a **less informative** belief state
- computed using **marginal belief states**
  → factorization & smaller state space

Conclusion

contributions

- **modeling efforts**: $\rightarrow$ human-machine interaction

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- **hybrid POMDP** $\xrightarrow{\text{translation}}$ MDP with **finite** space
  → probabilities on possibilistic belief states
  pessimistic rewards (Choquet integral)
  → factored POMDP $\xrightarrow{\text{translation}}$ factored **finite** MPD

Conclusion

perspectives

- refined criteria *(Weng 2005, Dubois et al. 2005)*
  $\Rightarrow$ finer π-POMDP
- state space heuristic search for π-POMDPs
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hybrid work

- IPPC problems (factored POMDPs);
- tests of this approach:
  1. **simplification:** π distributions definition?
  2. **imprecision:** robust in practice?

Thank you!

produced work:

- *Qualitative Possibilistic Mixed-Observable MDPs*, **UAI-2013**
- *Structured Possibilistic Planning Using Decision Diagrams*, **AAAI-2014**
- *Planning in Partially Observable Domains with Fuzzy Epistemic States and Probabilistic Dynamics*, **SUM-2015**
- *Processus Décisionnels de Markov Possibilistes à Observabilité Mixte*, Revue d'Intelligence Artificielle (**RIA french journal**)
- *A Possibilistic Estimation of Human Attentional Errors*, submitted to **IEEE-TFS journal**