

$\partial_t \psi + \frac{M}{\epsilon} \int_{\Omega} \frac{|u(x,t)|^2}{2} \psi \Delta \psi + \int_{\Omega} p = 0, \quad \nabla \psi = 0, \quad \psi(x,0) = \psi_0(x), \quad \psi(x,t) = \psi_0(x)$

Exploiting Imprecise Information Sources in Sequential Decision Making Problems under Uncertainty

N.Drougard

under D.Dubois, J-L.Farges and F.Teichteil-Königsbuch supervision

doctoral school: EDSYS institution: ISAE-SUPAERO

laboratory: ONERA-The French Aerospace Lab



retour sur innovation

Context

Autonomous robotics

Onera, System Control & Flight Dynamics Department

Control Engineering, Artificial intelligence, Cognitive Sciences

Context

Autonomous robotics

Onera, System Control & Flight Dynamics Department

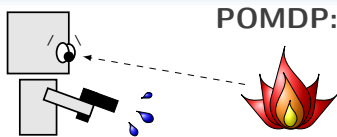
Control Engineering, Artificial intelligence, Cognitive Sciences

- autonomy and human factors
- decision making, planning
- experimental/industrial applications: UAVs, exploration robots, human-machine interaction



Context

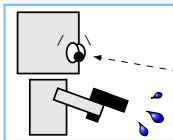
Partially Observable Markov Decision Processes (POMDPs)



POMDP: model for autonomous robotics

Context

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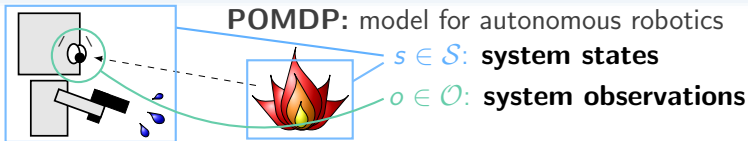
POMDP: model for autonomous robotics



$s \in \mathcal{S}$: **system states**

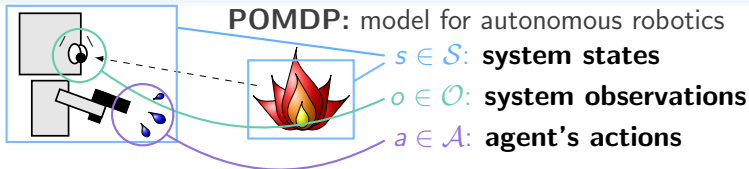
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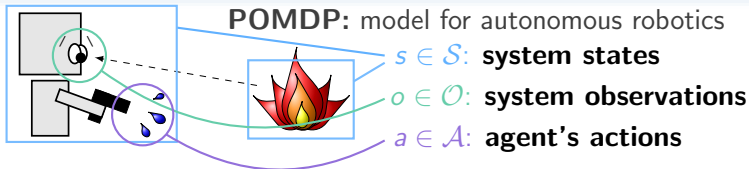
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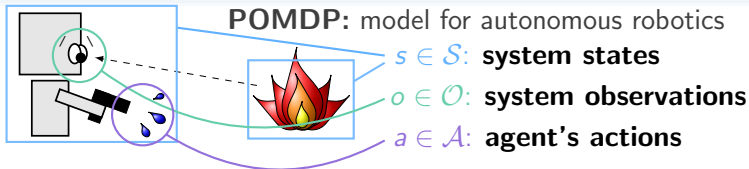
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s_t

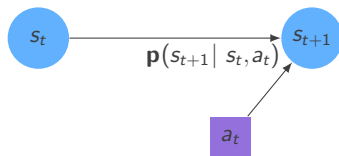
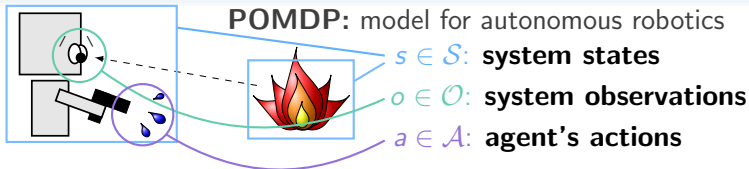
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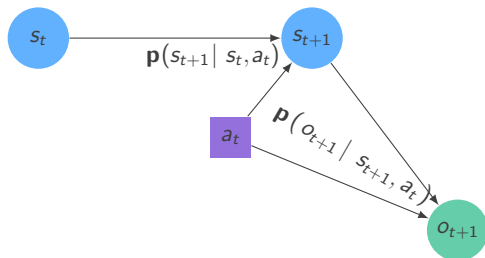
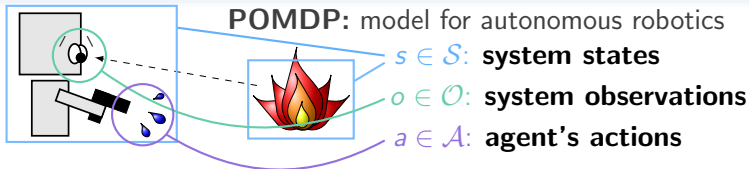
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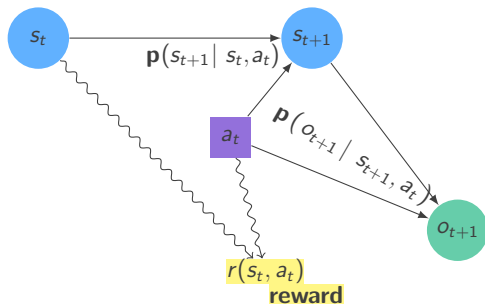
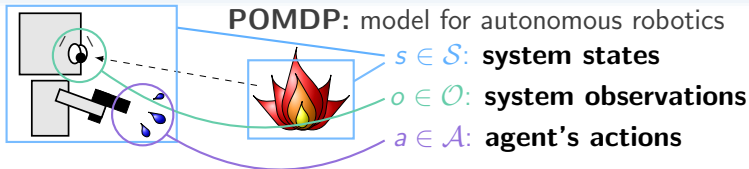
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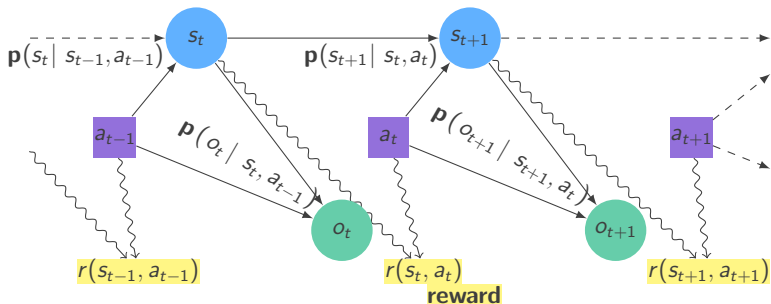
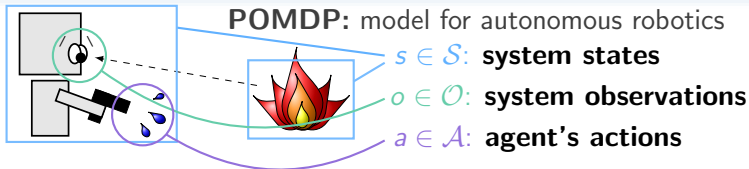
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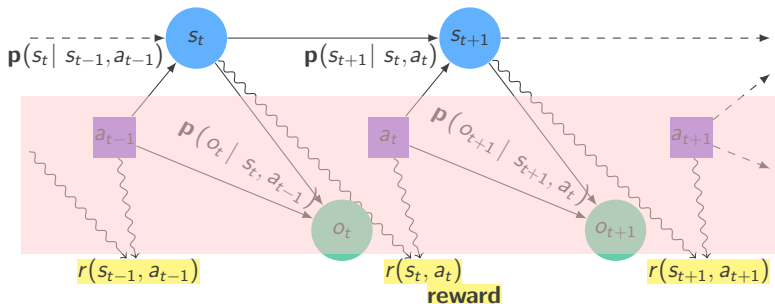
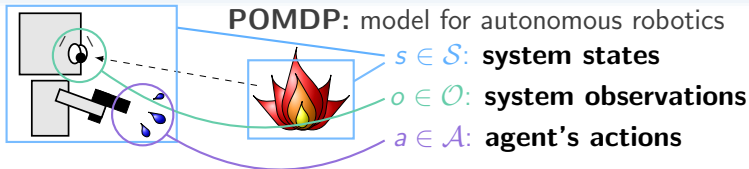
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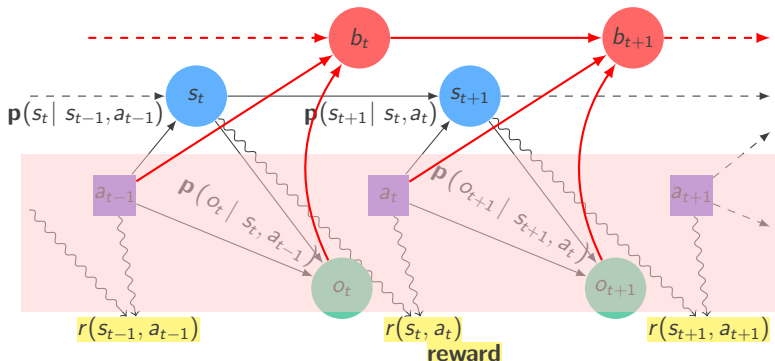
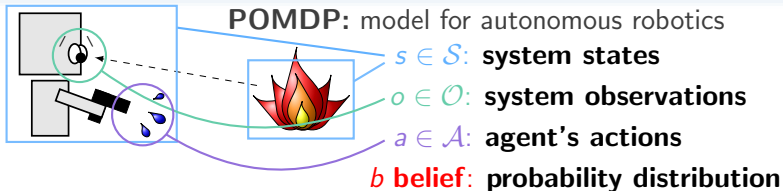
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Context

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Context

belief state, strategy, criterion

POMDP: $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$ (*Smallwood et al. 1973*)

- **transition** function $T(s, a, s') = \mathbf{p}(s' \mid s, a)$
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Context

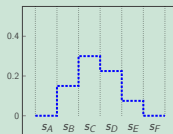
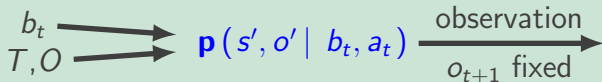
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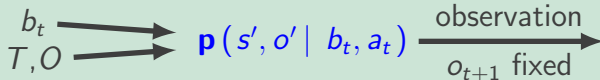
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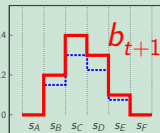
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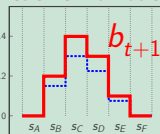
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$$\begin{array}{c} b_t \\ T, O \end{array} \Rightarrow \mathbf{p}(s', o' \mid b_t, a_t)$$

received
observation
 $\xrightarrow{o_{t+1} \text{ fixed}}$

probabilistic normalization



strategy $d_t : b_t \mapsto a_t \in \mathcal{A}$

$$\text{maximizing } \mathbb{E}_{s_0 \sim b_0} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot r(s_t, \delta(b_t)) \right], \quad 0 < \gamma < 1$$

Flaws of the POMDP model

POMDPs in practice

- optimal strategy computation **PSPACE-hard**
(*Papadimitriou et al., 1987*)
- probabilities are **imprecisely known** in practice
- **prior ignorance** semantic/management?

Context

practical issues: Complexity, Vision and Initial Belief

- **POMDP optimal strategy computation undecidable**
in infinite horizon (*Madani et al. 1999*)

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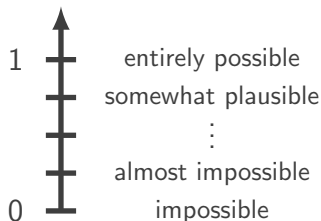
→ uniform probability distribution \neq **ignorance!**

Qualitative Possibility Theory

presentation – (max,min) “tropical” algebra

finite scale \mathcal{L}

usually $\{0, \frac{1}{k}, \frac{2}{k}, \dots, 1\}$



events $e \subset \Omega$ (universe)

sorted using possibility **degrees** $\pi(e) \in \mathcal{L}$

\neq

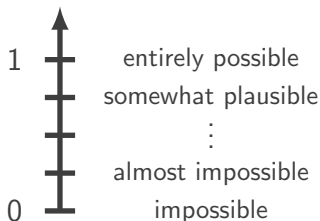
quantified with **frequencies** $p(e) \in [0, 1]$ (probabilities)

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$e_1 \neq e_2$, 2 events $\subset \Omega$

■ $\pi(e_1) < \pi(e_2) \Leftrightarrow$ “ e_1 is less plausible than e_2 ”

Qualitative Possibility Theory

Criteria from Sugeno integral

Probability / Qualitative Possibility Theories

$+$	\max
\times	\min
$\sum_x \mathbf{p}(x) = 1$	$\max_x \pi(x) = 1$
$X \in \mathbb{R}$	$X \in \mathcal{L}$
$\mathbb{E}[X] = \sum_x x \cdot \mathbf{p}(x)$	<p>optimistic:</p> $\mathbb{S}_{\Pi}[X] = \max_{x \in X} \min \{x, \pi(x)\}$ <p>pessimistic:</p> $\mathbb{S}_{\mathcal{N}}[X] = \min_{x \in X} \max \{x, 1 - \pi(x)\}$

Qualitative Possibility Theory

qualitative possibilistic POMDP (π -POMDP)

Sabbadin (UAI-98) introduces

the qualitative possibilistic POMDP

π -POMDP: $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T^\pi, O^\pi, \rho \rangle$

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-
- belief space trick: POMDP \rightarrow MDP with **infinite** space
 π -POMDP $\rightarrow \pi$ -MDP with **finite** space
 - problem becomes **decidable**
 - $\forall s \in \mathcal{S}, \pi(s) = 1 \Leftrightarrow$ total ignorance about s

A possibilistic belief state

finite belief space

$$\Pi_{\mathcal{L}}^{\mathcal{S}} = \left\{ \text{possibility distributions} \right\}: \# \Pi_{\mathcal{L}}^{\mathcal{S}} \sim \# \mathcal{L}^{\# \mathcal{S}} < +\infty$$

\rightarrow *i.e.* **finite belief space**

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possibilistic belief update

$$\beta_t$$
$$T^{\pi}, O^{\pi}$$

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$$\begin{array}{c} \beta_t \\ T^\pi, O^\pi \end{array} \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \pi(s', o' \mid \beta_t, a_t)$$

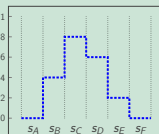
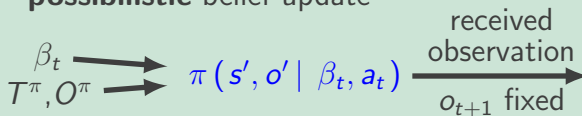
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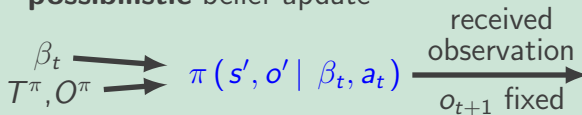
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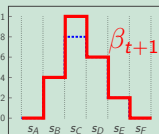
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possibilistic normalization



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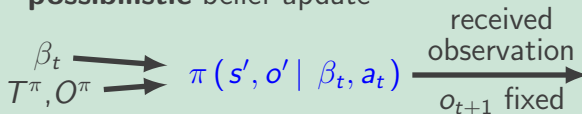
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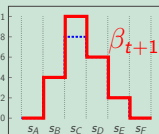
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possibilistic belief update



possibilistic normalization



- **Markovian update:** only depends on o_{t+1} , a_t and b_t^π

Overview

Qualitative Possibility Theory:

→ simplification, imprecision/prior ignorance modeling

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Qualitative Possibility Theory:

→ simplification, imprecision/prior ignorance modeling

- context

- 1 introductory example: qualitative **possibilistic modeling**
→ *human-machine interaction (HMI)*
with **Sergio Pizziol**

- 2 **advancements** in π -POMDP:
→ *mixed-observability & indefinite horizon*

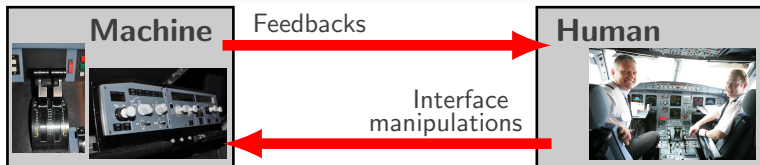
- 3 **simplifying computations**:
→ *ADD-based solver & factorization*

- 4 **probabilistic-possibilistic** (*hybrid*) approach

- conclusion

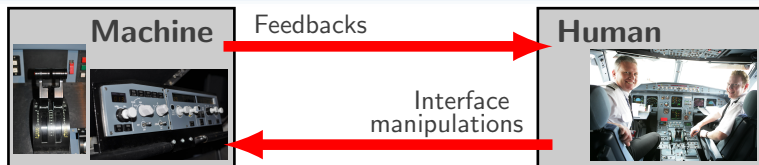
Example: Human-Machine Interaction (HMI)

joint work with **Sergio Pizziol** – Context



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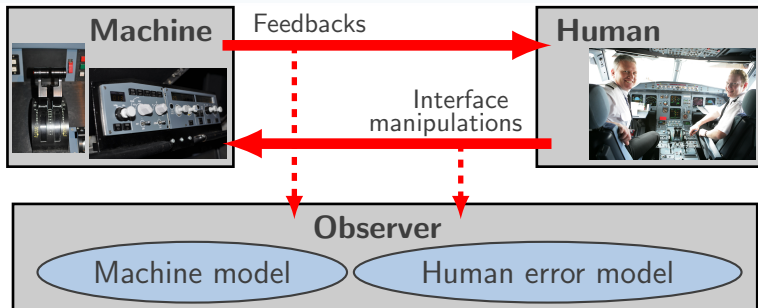
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Issue: incorrect human assessment of the machine state
→ **accident risk**

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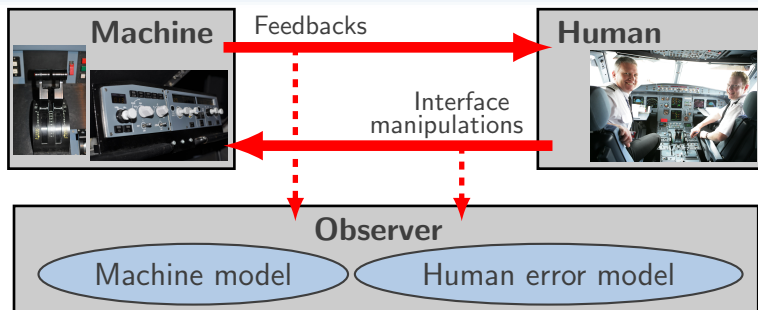
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joint work with **Sergio Pizziol** – Context



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π -POMDP without actions: π -Hidden Markov Process

- **system space** \mathcal{S} : set of human assessments → **hidden**
- **observation space** \mathcal{O} : feedbacks/human manipulations

Example: Human-Machine Interaction (HMI)

Human error model from expert knowledge

Machine with states A, B, C, \dots

state $s_A \in \mathcal{S}$: “human thinks machine state is A ”

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Machine with states A, B, C, \dots

state $s_A \in \mathcal{S}$: “human thinks machine state is A ”

Machine state transition $A \rightarrow B$

■ observation: **machine feedback** $o'_f \in \mathcal{O}$:

“human usually aware of feedbacks” $\rightarrow \pi(s'_B, o'_f \mid s_A) = 1$

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Example: Human-Machine Interaction (HMI)

Human error model from expert knowledge

Machine with states A, B, C, \dots

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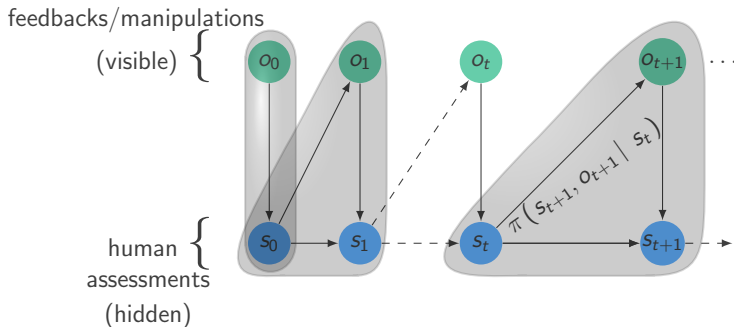
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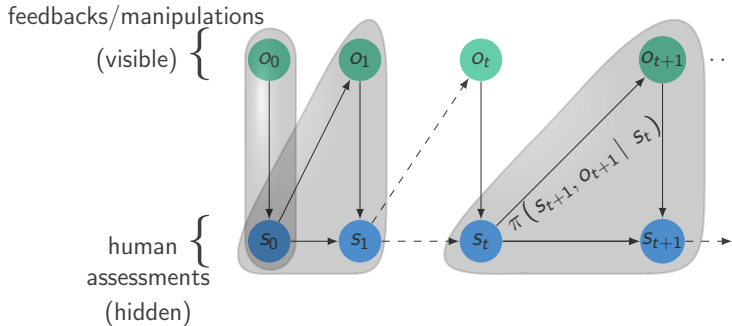
Qualitative Possibilistic Hidden Markov Process:

π -HMP, detection & diagnosis tool for HMI (with Sergio Pizziol)



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- **estimation** of the human assessment
 \Leftrightarrow **possibilistic belief state**
- **detection** of human assessment errors + **diagnosis**
- validated with pilots on flight simulator missions

Applicability of the π -POMDPs

three advancements

- lack of proof of optimality in indefinite horizon settings
- criterion/algorithm/proof
- curse of dimensionality:
 - belief space size of a π -POMDP: exponential in $\#\mathcal{S}$
- in practice, part of $s \in \mathcal{S}$ is visible
 - ⇒ complexity reduction
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Indefinite Horizon, Mixed-Observability, Simulations
contribution UAI 2013

Indefinite Horizon

criterion, DP scheme, optimal strategy

indefinite horizon criterion $\Psi : \mathcal{S} \rightarrow \mathcal{L}$ terminal pref. func.

$$\forall s \in \mathcal{S}, \text{ maximizing } \mathbb{S}_{\Pi} \left[\Psi(S_{\# \delta}) \mid S_0 = s \right]$$

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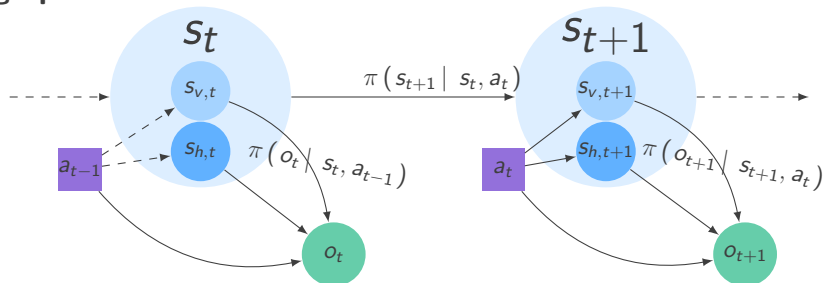
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- action update for states increasing the criterion
- **proof of optimality** of the resulting **stationary** strategy

Mixed-Observability (MOMDP, *Ong et al., 2005*)

π -Mixed-Observable Markov Decision Process (π -MOMDP)

graphical model of a π -MOMDP:



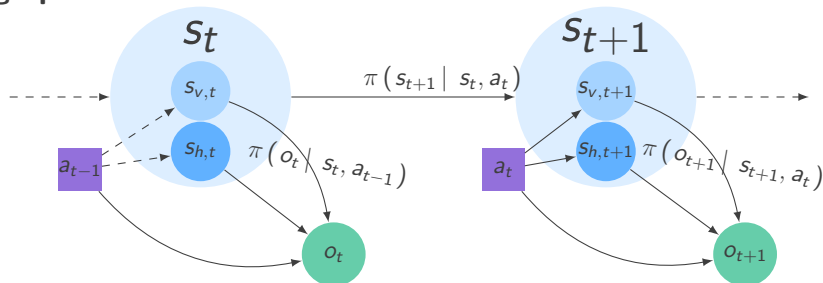
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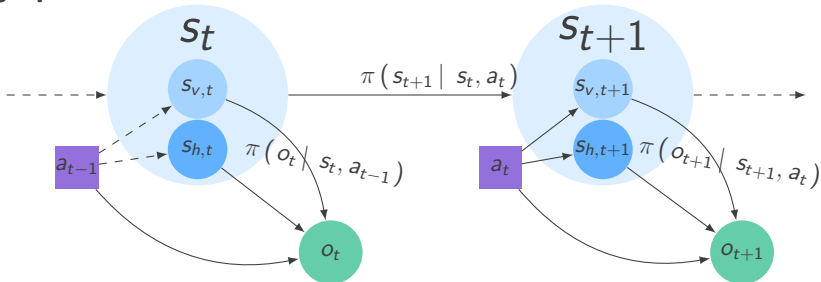
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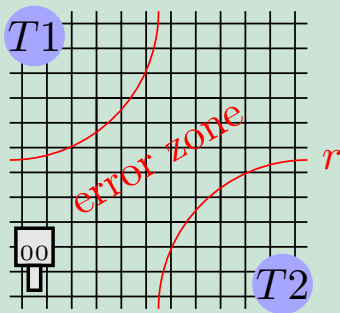
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 - $\rightarrow \pi$ -MOMDP: computations on $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$
- $\#\mathcal{X} \sim \#\mathcal{S}_v \cdot \#\mathcal{L}^{\#\mathcal{S}_h} \ll \#\Pi_{\mathcal{L}}^{\mathcal{S}}$

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simulations with machine vision behavior imprecisely known

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- noisy observations of the location of the object A

Recognition mission: robot on a grid, targets $T1$ & $T2$

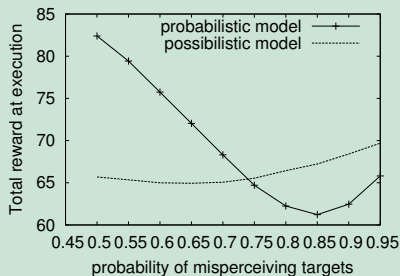
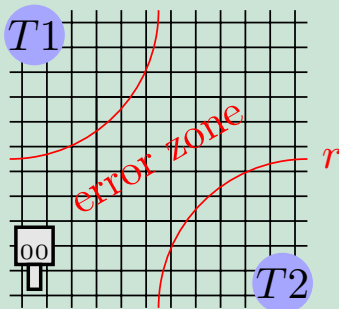


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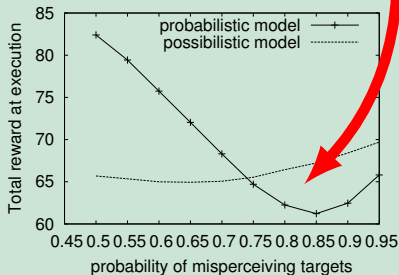
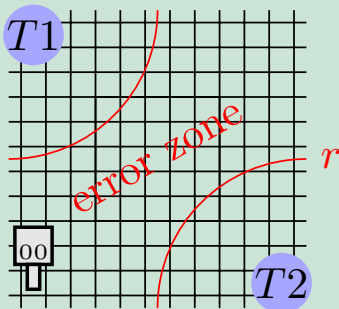
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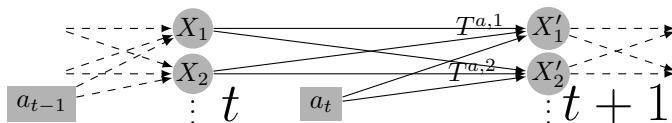
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Factored π -MOMDP and computations with ADDs

qualitative possibilistic models to reduce complexity

contribution (AAAI-14): factored π -MOMDP

\Leftrightarrow state space $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h} = \text{Boolean variables } (X_1, \dots, X_n)$
 + independence assumptions \Leftarrow graphical model

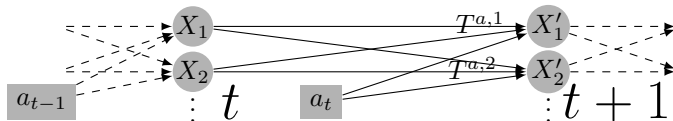


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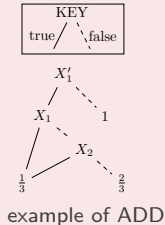
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- **factorization:** transition functions $T_i^a = \pi(X'_i \mid \text{parents}(X'_i), a)$ stored as **Algebraic Decision Diagrams (ADD)**

probabilistic case:

SPUDD (Hoey et al., 1999)



Simplify computations with π -MOMDPs

Resulting π -MOMDP solver: PPUDD

- probabilistic model: $+$ and $\times \Rightarrow$ new values created
 \Rightarrow number of ADDs leaves **potentially huge**
- possibilistic model: \min and $\max \Rightarrow$ values $\in \mathcal{L}$ finite
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PPUDD: Possibilistic Planning Using Decision Diagrams

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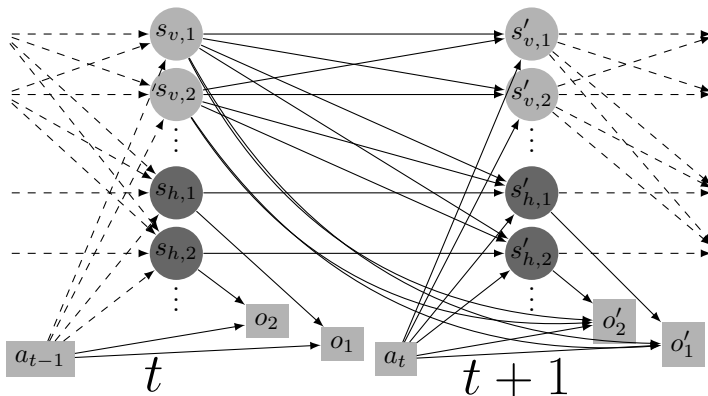
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- computations on trees: *CU Decision Diagram Package*.

Simplifying computations with π -MOMDPs

Natural factorization: belief independence

contribution (AAAI-14):

independent sensors, hidden states, ... \Rightarrow graphical model



Simplifying computations with π -MOMDPs

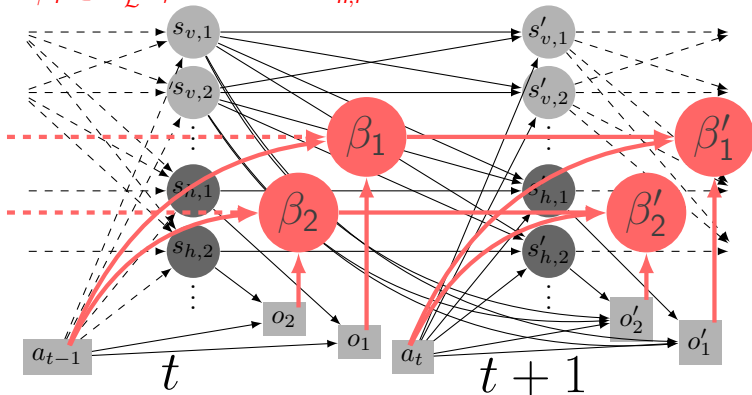
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d-Separation $\Rightarrow (s_v, \beta) = (s_{v,1}, \dots, s_{v,m}, \beta_1, \dots, \beta_l)$

$\beta_i \in \Pi_{\mathcal{L}}^{s_{h,i}}$, belief over $s_{h,i}$



Simplifying computations with π -MOMDPs

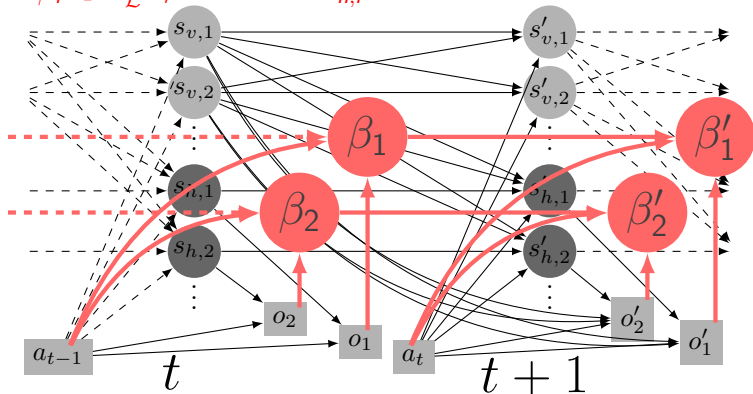
Natural factorization: belief independence

$\perp\!\!\!\perp$ assumptions on state & observation variables

→ belief variable factorization

→ **additional** computation savings

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Simplify computations with π -MOMDPs

Experiments – perfect sensing: Navigation problem

PPUDD vs SPUDD (*Hoey et al.*, 1999)

Navigation benchmark: reach a goal – spots with accident risk
M1 (resp. M2) optimistic (resp. pessimistic) criterion

Simplify computations with π -MOMDPs

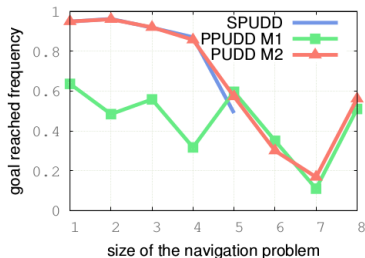
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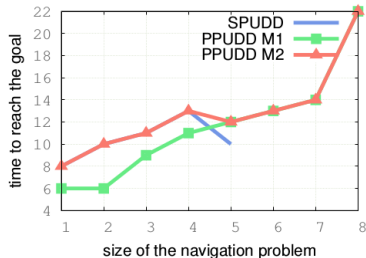
Performances, function of the problem index

reached goal frequency



the higher the better

steps to reach the goal

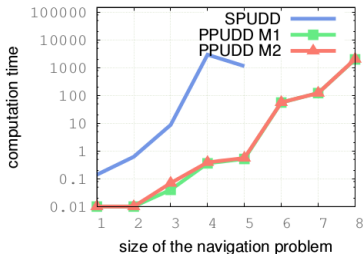


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Simplify computations with π -MOMDPs

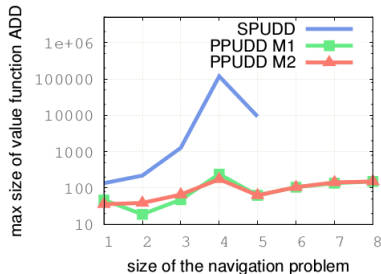
Experiments – perfect sensing: Navigation problem

computation time



the lower the better

max size of ADDs



the lower the better

- PPUDD + M2 (pessimistic criterion)
faster with same performances as SPUDD
- SPUDD only solves the first 5 instances
- verified intuition: ADDs are smaller

Simplify computations with π -MOMDPs

Experiments – imperfect sensing: RockSample problem

PPUDD vs APPL (*Kurniawati et al.*, 2008, solver MOMDP)

symbolic HSVI (*Sim et al.*, 2008, solver POMDP)

RockSample benchmark: recognize and sample “good” rocks

Simplify computations with π -MOMDPs

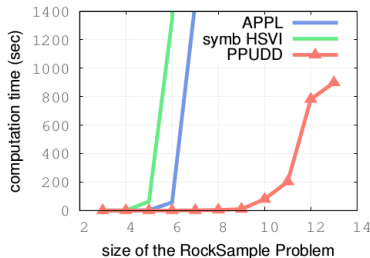
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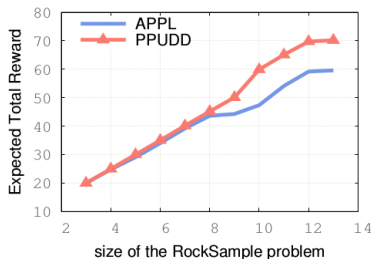
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computation time:



the lower the better

average of rewards

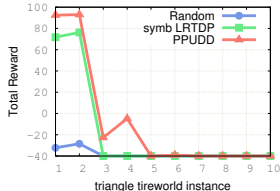
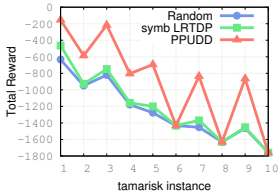
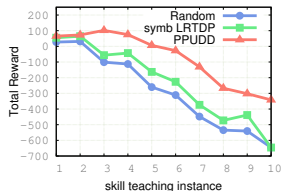
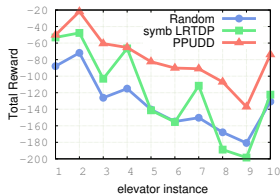
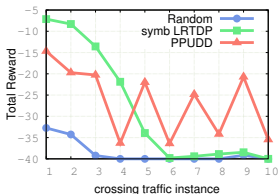
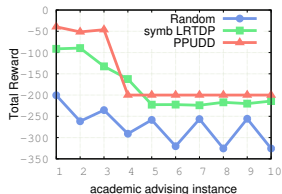


the higher the better

- approximate model + exact resolution solver can be **better than** exact model + approximate resolution solver

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

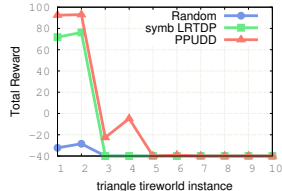
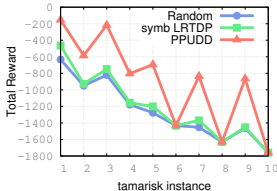
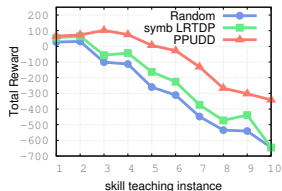
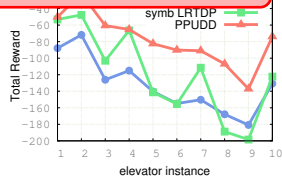
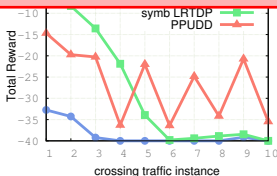
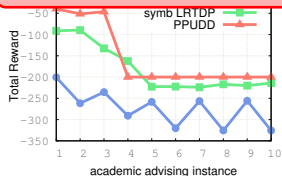
PPUDD + BDD mask over reachable states.



average of rewards over simulations — the higher the better

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

PPUDD outperforms its probabilistic counterpart



average of rewards over simulations — the higher the better

Qualitative possibilistic approach: benefits/drawbacks towards a hybrid POMDP

- **granulated** belief space (discrete)
- efficient problem **simplification** (PPUDD 2× better than LRTDP with ADDs)
- **ignorance and imprecision** modeling

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-
- ADD methods \prec state space search methods
→ winners of IPPC 2014: 2× better than PPUDD
 - choice of the qualitative criterion (optimistic/pessimistic)
 - preference → non additive degrees
→ same scale as possibility degrees (commensurability)
 - coarse approximation of probabilistic model
→ no frequentist information

A hybrid model

a probabilistic POMDP with possibilistic belief states

hybrid approach

- agent knowledge = possibilistic belief states
- probabilistic dynamics & quantitative rewards

A hybrid model

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Usefulness

- **heuristic** for solving POMDPs:
 results in a standard (finite state space) MDP
- problem with **qualitative** & **quantitative** uncertainty

Transitions and rewards

belief-based transition and reward functions

- possibility distribution $\beta \rightarrow$ probability distribution $\bar{\beta}$
using poss-prob transformations (*Dubois et al., FSS-92*)

Transition function on belief states

$$\Rightarrow \mathbf{p}(\beta' | \bar{\beta}, a) = \sum_{\substack{o' \text{ t.q.} \\ \text{update}(\beta, a, o') = \beta'}} \mathbf{p}(o' | \bar{\beta}, a)$$

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- reward cautious according to β

Pessimistic Choquet Integral

$$r(\beta, a) = \sum_{i=1}^{\#\mathcal{L}-1} (l_i - l_{i+1}) \cdot \min_{\substack{s \in \mathcal{S} \text{ s.t.} \\ \beta(s) \geq l_i}} r(s, a)$$

Resulting MDP

translation from hybrid POMDP to MDP – **contribution (SUM-15):**

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$

output: the MDP $\langle \tilde{\mathcal{S}}, \mathcal{A}, \tilde{T}, \tilde{r}, \gamma \rangle$:

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- **state space** $\tilde{\mathcal{S}} = \Pi_{\mathcal{L}}^{\mathcal{S}}$,
the set of the possibility distributions over \mathcal{S}

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Resulting MDP

translation from hybrid POMDP to MDP – **contribution (SUM-15):**

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$$\text{criterion: } \mathbb{E}_{\beta_t \sim \tilde{T}} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot \tilde{r}(\beta_t, d_t) \right].$$

Belief variable factorization

3 classes of state variables – **contribution** (SUM-15)

variable: **visible** $s'_v \in \mathbb{S}_v$

 s'_v

inferred hidden $s'_h \in \mathbb{S}_h$

 s'_h

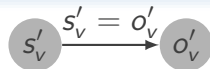
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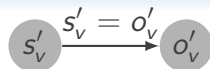
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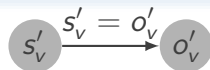
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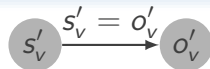
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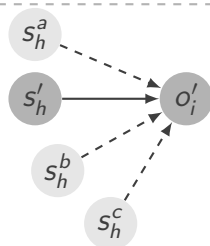
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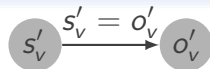
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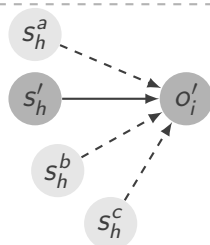
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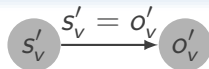
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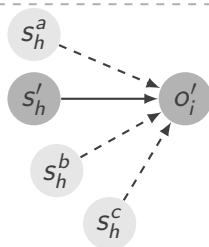
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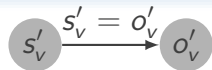
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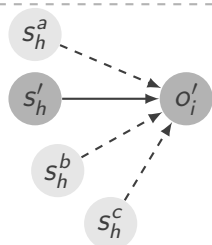
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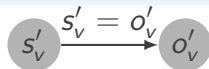
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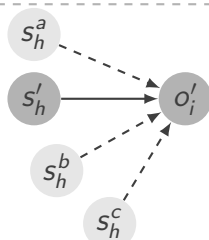
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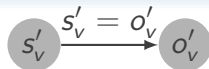
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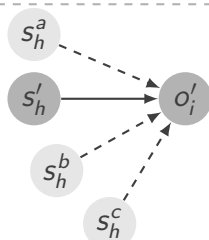
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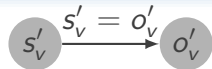
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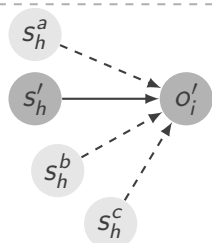
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Belief variable factorization

global belief state from marginal belief variables

bound over the global belief state

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- min of marginals = a **less informative** belief state
- computed using **marginal belief states**
 → **factorization & smaller state space**

Conclusion

contributions

- **modeling efforts:** \rightarrow human-machine interaction

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- **hybrid POMDP** $\xrightarrow{\text{translation}}$ MDP with **finite** space
 \rightarrow probabilities on possibilistic belief states
pessimistic rewards (Choquet integral)
 \rightarrow factored POMDP $\xrightarrow{\text{translation}}$ factored **finite** MPD

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perspectives

- refined criteria (*Weng 2005, Dubois et al. 2005*)
 \Rightarrow finer π -POMDP
- state space heuristic search for π -POMDPs
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hybrid work

- IPPC problems (factored POMDPs);
- tests of this approach:
 - 1 **simplification:** π distributions definition?
 - 2 **imprecision:** robust in practice?

Thank you!

produced work:

- *Qualitative Possibilistic Mixed-Observable MDPs*, **UAI-2013**
- *Structured Possibilistic Planning Using Decision Diagrams*,
AAAI-2014
- *Planning in Partially Observable Domains with Fuzzy Epistemic States and Probabilistic Dynamics*,
SUM-2015
- *Processus Décisionnels de Markov Possibilistes à Observabilité Mixte*,
Revue d'Intelligence Artificielle (**RIA french journal**)
- *A Possibilistic Estimation of Human Attentional Errors*,
submitted to **IEEE-TFS journal**