Exploiting Imprecise Information Sources in Sequential Decision Making Problems under Uncertainty

N.Drougard

under **D.Dubois**, **J-L.Farges** and **F.Teichteil-Königsbuch** supervision

doctoral school: EDSYS **institution:** ISAE–SUPAERO

laboratory: ONERA–The French Aerospace Lab

retour sur innovation

Context

Autonomous robotics

Onera, System Control & Flight Dynamics Department

Control Engineering, Artificial intelligence, Cognitive Sciences

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- autonomy and human factors
- decision making, planning
- experimental/industrial applications: UAVs, exploration robots, human-machine interaction

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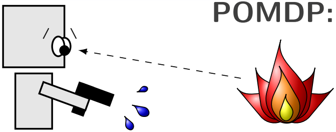
Partially Observable Markov Decision Processes (POMDPs)



POMDP: model for autonomous robotics

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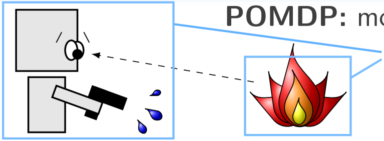

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$s \in \mathcal{S}$: **system states**

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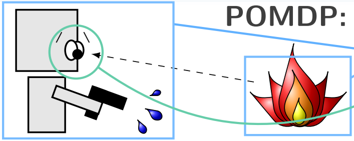

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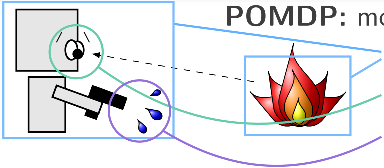

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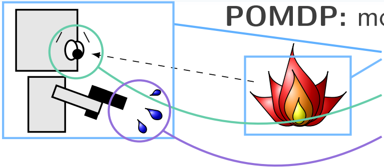

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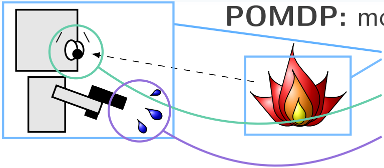

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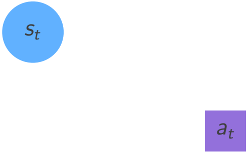

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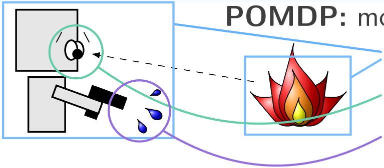

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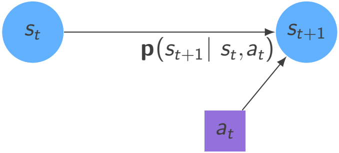

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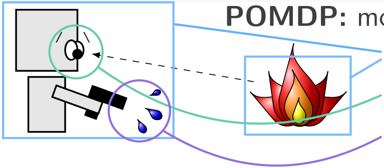

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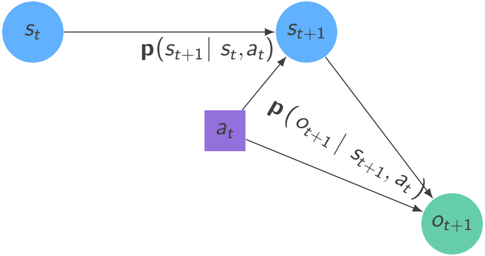

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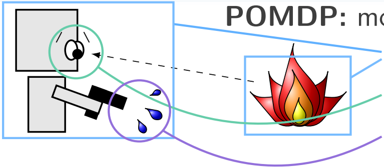

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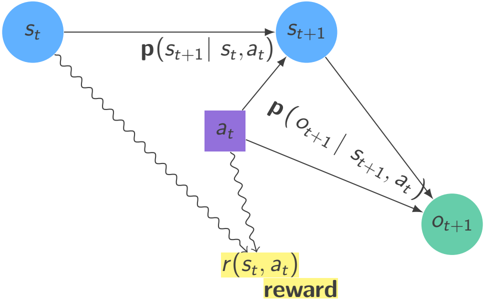

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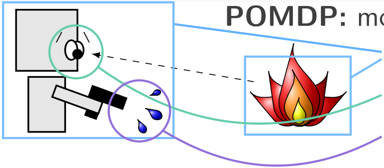

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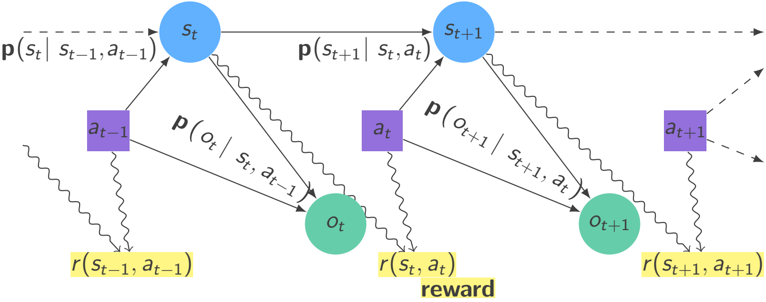

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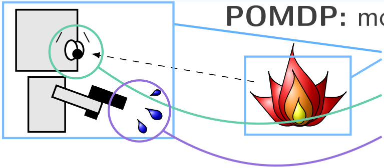

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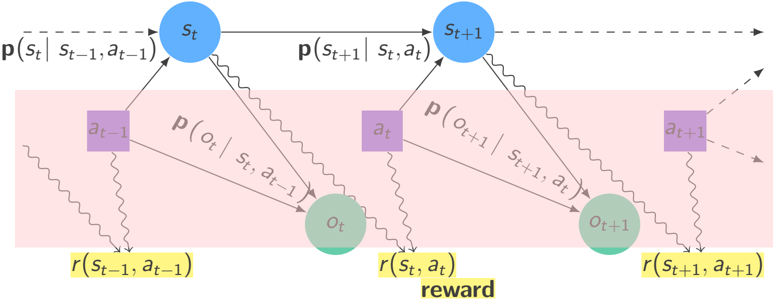

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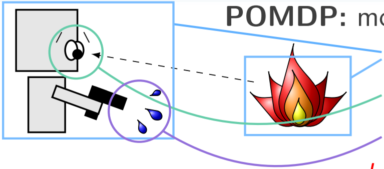

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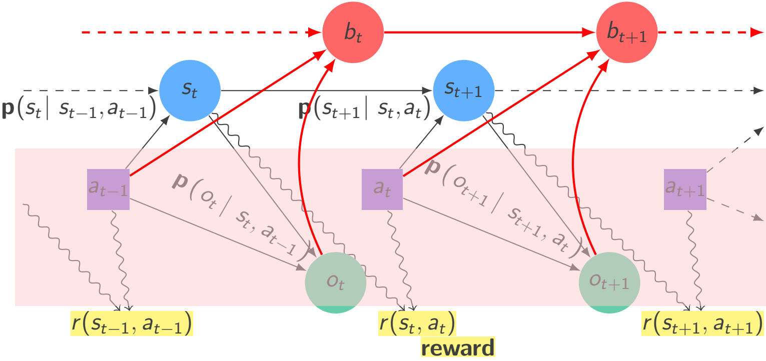
- b **belief**: **probability distribution**



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belief state, strategy, criterion

POMDP: $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$ (*Smallwood et al. 1973*)

- **transition** function $T(s,a,s') = \mathbf{p}\left(s' \mid s, a \right)$
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probabilistic belief update

b_t

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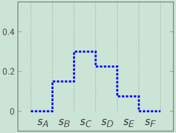

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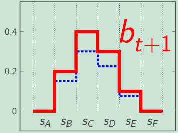
b_t, T,O $\longrightarrow$ $\mathbf{p}\left(s', o' \mid b_t, a_t\right)$ $\xrightarrow[o_{t+1} \text{ fixed}]{\text{received observation}}$ **probabilistic normalization**



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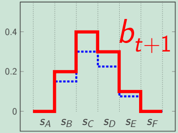

strategy $d_t : b_t \mapsto a_t \in \mathcal{A}$

$$\text{maximizing } \mathbb{E}_{s_0 \sim b_0}\left[\sum_{t=0}^{+\infty} \gamma^t \cdot r\Big(s_t, \delta(b_t)\Big)\right], \; 0 < \gamma < 1$$

Flaws of the POMDP model

POMDPs in practice

- optimal strategy computation **PSPACE-hard**
  (*Papadimitriou et al., 1987*)

- conditional probabilities are **imprecisely known**

- **prior ignorance** semantic/management?

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practical issues: Complexity, Vision and Initial Belief

- **POMDP optimal strategy computation undecidable** in infinite horizon (*Madani et al. 1999*)

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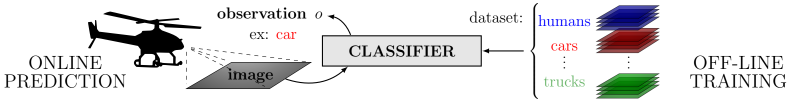

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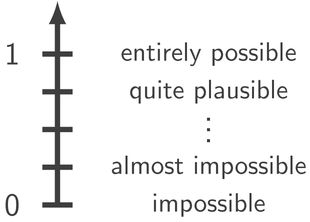
$\rightarrow$ uniform probability distribution $\neq$ ignorance!

Qualitative Possibility Theory

presentation – (max,min) “tropical” algebra

finite scale $\mathcal{L}$

usually $\{0, \frac{1}{k}, \frac{2}{k}, \dots, 1\}$



events $E \subset \Omega$ (universe)

sorted using possibility **degrees** $\Pi(E) \in \mathcal{L}$

$\neq$

quantified with **frequencies** $\mathbb{P}(E) \in [0,1]$ (probabilities)

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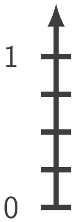

entirely possible

quite plausible

$\vdots$

almost impossible

impossible

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$$\Pi(E) = \max_{e \in E} \Pi(\{ e \}) = \max_{e \in E} \pi(e)$$

Qualitative Possibility Theory

Criteria from special cases of Sugeno integral

Probability	/ Qualitative Possibility Theories
$+$	$\max$
$\times$	$\min$
$\sum_x \mathbf{p}(x) = 1$	$\max\limits_x \pi(x) = 1$
$X \in \mathbb{R}$	$X \in \mathcal{L}$
$\mathbb{P}(A) = 1 - \mathbb{P}(\overline{A})$	$\mathcal{N}(A) = 1 - \Pi(\overline{A})$ (necessity)
$\mathbb{E}[X] = \sum_x x \cdot \mathbf{p}(x)$	**optimistic:** $\mathbb{S}_{\Pi}[X] = \max\limits_{x \in X} \min \{ x, \pi(x) \}$ **pessimistic:** $\mathbb{S}_{\mathcal{N}}[X] = \min\limits_{x \in X} \max \{ x, 1 - \pi(x) \}$

Qualitative Possibility Theory

qualitative possibilistic POMDP (π-POMDP)

Sabbadin (UAI-98) introduces

the qualitative possibilistic POMDP

π-**POMDP:** $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T^{\pi}, O^{\pi}, \rho \rangle$

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$\forall s \in \mathcal{S}$, $\pi(s) = 1 \Leftrightarrow$ total ignorance about s
each state possible, **none necessary**

A possibilistic belief state

finite belief space

$\Pi^{\mathcal{S}}_{\mathcal{L}} = \Big\{ \text{ possibility distributions } \Big\}$: $\#\Pi^{\mathcal{S}}_{\mathcal{L}} \sim \#\mathcal{L}^{\#\mathcal{S}} < +\infty$

$\rightarrow$ *i.e.* **finite belief space**

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$$\begin{matrix} \beta_t \longrightarrow \\ T^{\pi}, O^{\pi} \longrightarrow \end{matrix} \ \pi\left(s', o' \mid \beta_t, a_t \right)$$

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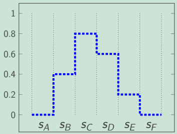

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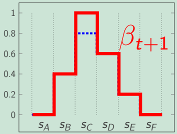
$$\beta_t(s) = \pi\left(s_t = s \mid a_0, o_1, \dots, a_{t-1}, o_t \right)$$

possibilistic belief update

possibilistic normalization

$$\begin{matrix} \beta_t \\ T^{\pi}, O^{\pi} \end{matrix} \longrightarrow \pi\left(s', o' \mid \beta_t, a_t \right) \xrightarrow[o_{t+1} \text{ fixed}]{\text{received observation}}$$



A possibilistic belief state

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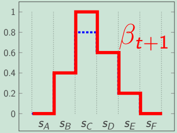


- **Markovian update:** only depends on o_{t+1}, a_t and b_t^π

Overview

Qualitative Possibility Theory:
→ simplification, imprecision/prior ignorance modeling

Overview

Qualitative Possibility Theory:
$\rightarrow$ simplification, imprecision/prior ignorance modeling

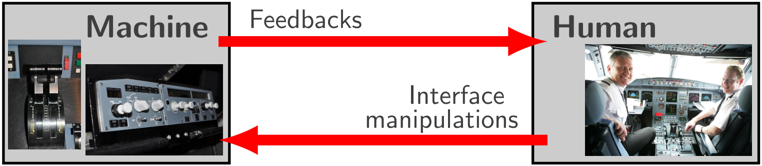
- context

1. introductory example: qualitative **possibilistic modeling**
   $\rightarrow$ *human-machine interaction (HMI)*
   with **Sergio Pizziol**
2. **advances** in π-POMDP:
   $\rightarrow$ *mixed-observability & indefinite horizon*
3. **simplifying computations**:
   $\rightarrow$ *ADD-based solver & factorization*
4. **probabilistic-possibilistic** (*hybrid*) approach

- conclusion

Example: Human-Machine Interaction (HMI)

joint work with **Sergio Pizziol** – Context



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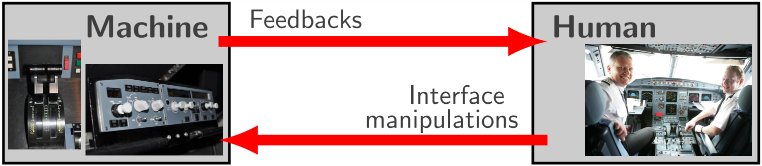

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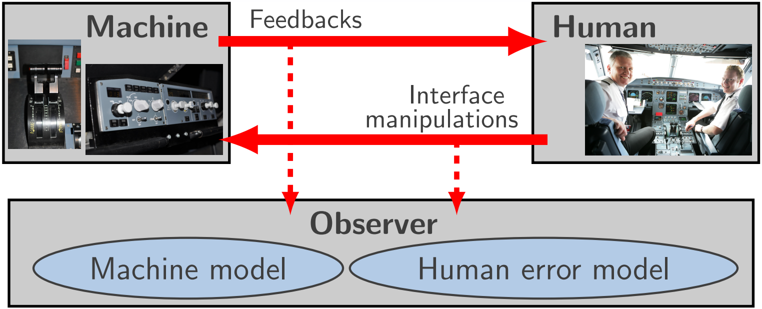

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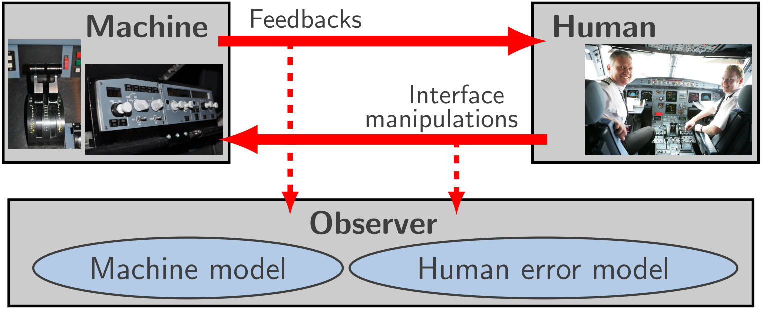

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π-POMDP without actions: π-Hidden Markov Process

- **system space** $\mathcal{S}$: set of human assessments $\rightarrow$ **hidden**
- **observation space** $\mathcal{O}$: feedbacks/human manipulations

Example: Human-Machine Interaction (HMI)

Human error model from expert knowledge

Machine with states A, B, C, . . .

state $s_A \in \mathcal{S}$: "human thinks machine state is A"

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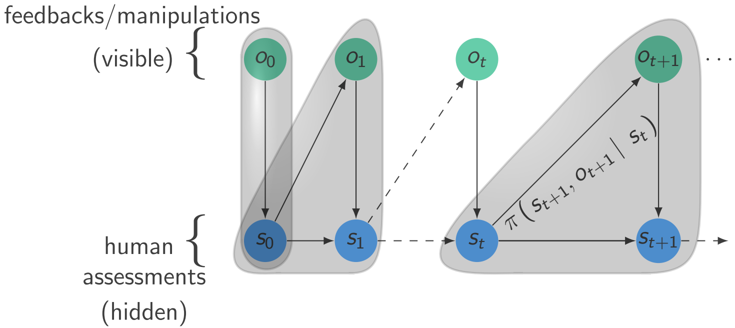
- impossible cases: possibility degree 0

Qualitative Possibilistic Hidden Markov Process:

π-HMP, detection & diagnosis tool for HMI (with **Sergio Pizziol**)



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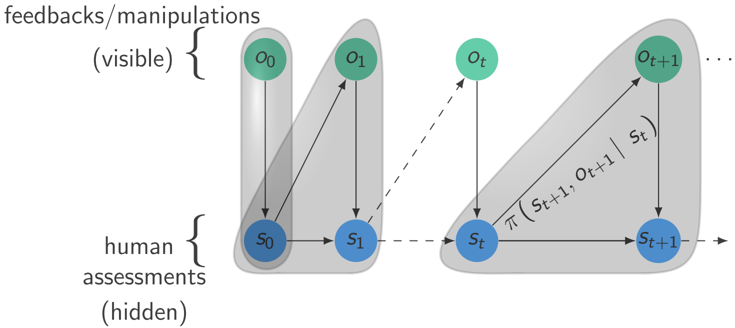

- **estimation** of the human assessment
  $\Leftrightarrow$ **possibilistic belief state**
- **detection** of human assessment errors + **diagnosis**
- validated with pilots on flight simulator missions

Applicability of the π-POMDPs

advances

- **lack of proof of optimality in indefinite horizon settings**
- criterion/proof

- **curse of dimensionality:**
  $\rightarrow$ belief space size of a π-POMDP: exponential in $\#\mathcal{S}$
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Indefinite Horizon, Mixed-Observability, Simulations
contribution UAI 2013

Proof of optimality under Indefinite Horizon

criterion, DP scheme, optimal strategy

indefinite horizon criterion $\Psi : \mathcal{S} \rightarrow \mathcal{L}$ terminal pref. func.

$$\forall s \in \mathcal{S}, \text{ maximizing } \mathbb{S}_\Pi \left[\Psi(S_{\#\delta}) \middle| S_0 = s \right]$$

with respect to the strategy $\delta : (t, s) \mapsto a_t \in \mathcal{A}$.

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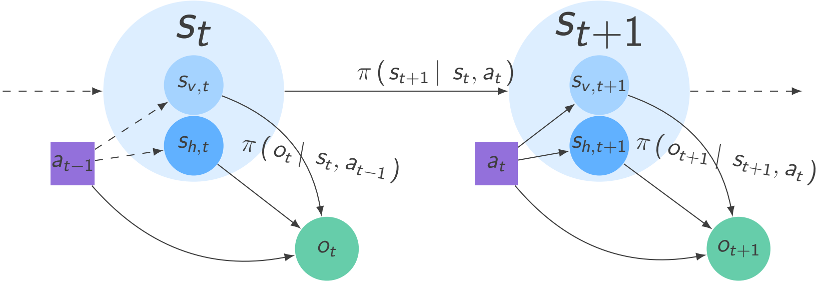
- criterion value **non decreasing** with iterations
- action update for states increasing the criterion
- **proof of optimality** of the resulting **stationary** strategy

Scalability capabilities with Mixed-Observability

π-Mixed-Observable Markov Decision Process (π-MOMDP)

graphical model of a π-MOMDP:



Mixed-Observability (*Ong et al., 2005*): $s \in \mathcal{S} = \mathcal{S}_v \times \mathcal{S}_h$

i.e. state $s =$ visible component s_v & hidden component s_h

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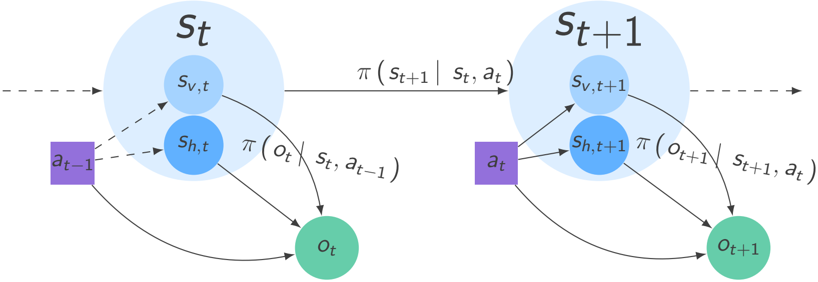

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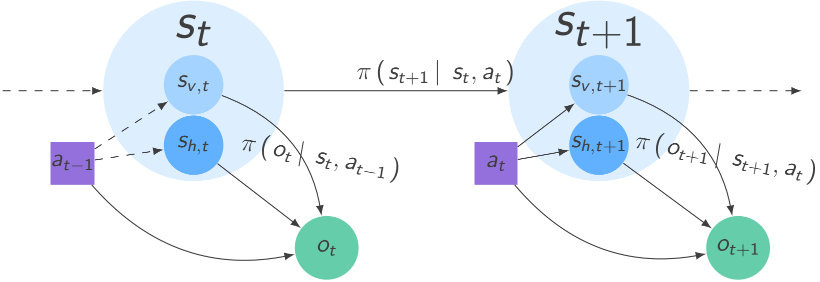

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- $\rightarrow$ π-POMDP: belief space $\Pi^{\mathcal{S}}_{\mathcal{L}}$ $\quad \#\Pi^{\mathcal{S}}_{\mathcal{L}} \sim \#\mathcal{L}^{\#\mathcal{S}}$

  $\rightarrow$ π-MOMDP: computations on $\mathcal{X} = \mathcal{S}_v \times \Pi^{\mathcal{S}_h}_{\mathcal{L}}$

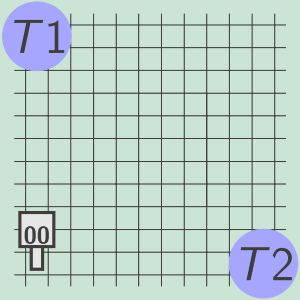
  $\#\mathcal{X} \sim \#\mathcal{S}_v \cdot \#\mathcal{L}^{\#\mathcal{S}_h} \ll \#\Pi^{\mathcal{S}}_{\mathcal{L}}$

Experimental results

π-MOMDP for robotics with imprecise probabilities

- **goal:** reach the object $A = T1$ or $T2$
- noisy observations of the location of the object A

Recognition mission: robot on a grid, targets $T1$ & $T2$



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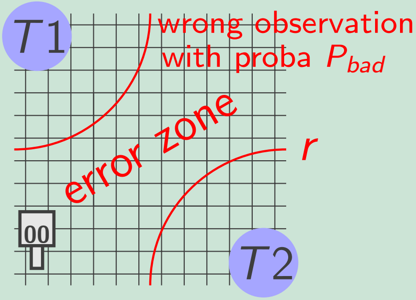

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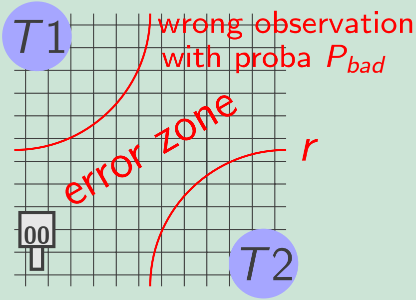

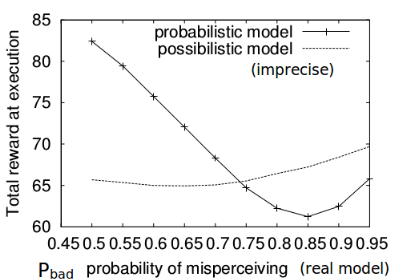

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probabilistic model inappropriate
when probabilities too imprecise

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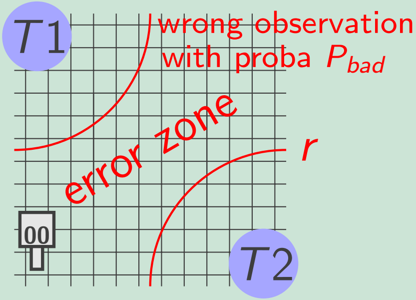

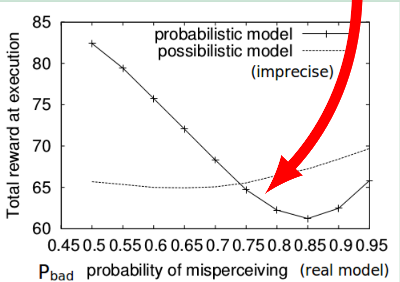

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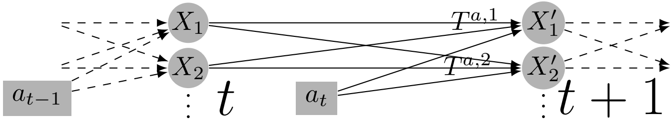
Factored π-MOMDP and computations with ADDs

qualitative possibilistic models to reduce complexity

contribution (AAAI-14): factored π-MOMDP

$\Leftrightarrow$ state space $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$ = Boolean variables $(X_1, \ldots, X_n)$

+ independence assumptions $\Leftarrow$ graphical model



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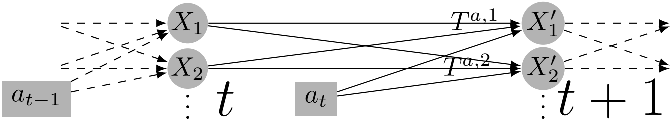
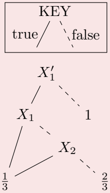


- **factorization:** transition functions $T_i^a = \pi\left(X'_i \mid parents(X'_i), a\right)$ stored as **Algebraic Decision Diagrams (ADD)**

  probabilistic case:
  SPUDD (*Hoey et al., 1999*)



example of ADD

Simplify computations with π-MOMDPs

Resulting π-MOMDP solver: PPUDD

- probabilistic model: $+$ and $\times$ $\Rightarrow$ new values created
  $\Rightarrow$ number of ADDs leaves **potentially huge**
- possibilistic model: min and max $\Rightarrow$ values $\in \mathcal{L}$ finite
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PPUDD: Possibilistic Planning Using Decision Diagrams

- factorization $\Rightarrow$ each DP step divided into n stages
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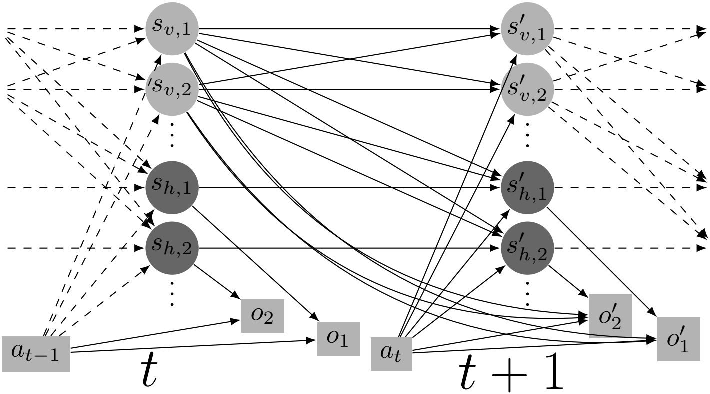
- computations on trees: *CU Decision Diagram Package*.

Simplifying computations with π-MOMDPs

Natural factorization: belief independence

contribution (AAAI-14):

independent sensors, hidden states, ... $\Rightarrow$ graphical model



Simplifying computations with π-MOMDPs

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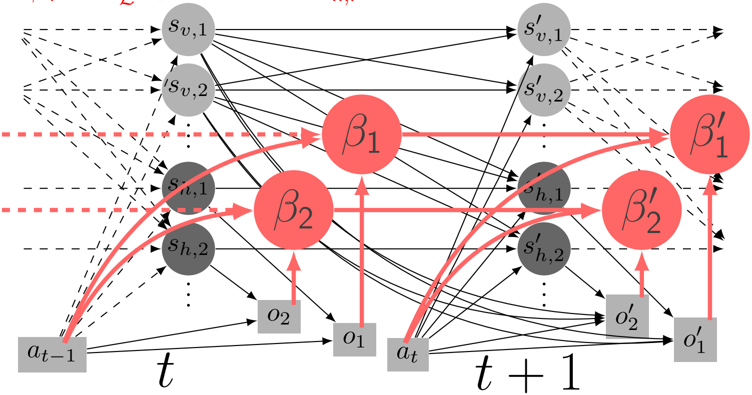
d-Separation $\Rightarrow$ $(s_v, \beta) = (s_{v,1}, \dots, s_{v,m}, \beta_1, \dots, \beta_l)$

$\beta_i \in \Pi_{\mathcal{L}}^{\mathcal{S}_{h,i}}$, belief over $s_{h,i}$



Simplifying computations with π-MOMDPs

Natural factorization: belief independence

$\perp\!\!\!\perp$ assumptions on state & observation variables
$\rightarrow$ belief variable factorization
$\rightarrow$ **additional** computation savings

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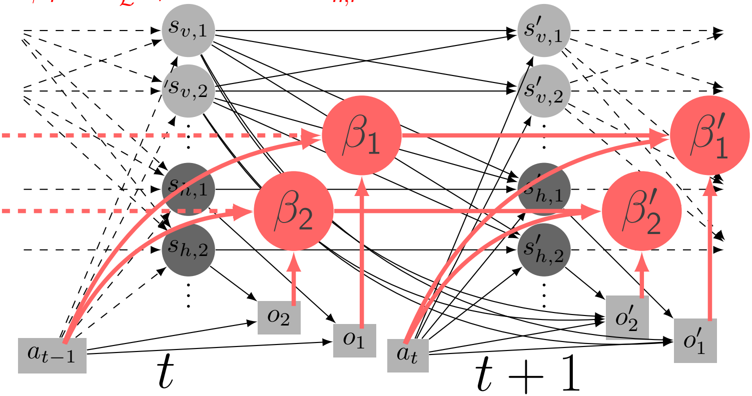

Simplify computations with π-MOMDPs

Experiments – perfect sensing: Navigation problem

PPUDD vs SPUDD (*Hoey et al.*, 1999)

Navigation benchmark: reach a goal – spots with accident risk

M1 (resp. M2) optimistic (resp. pessimistic) criterion

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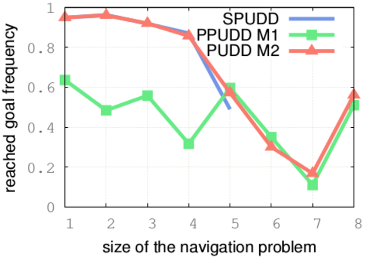
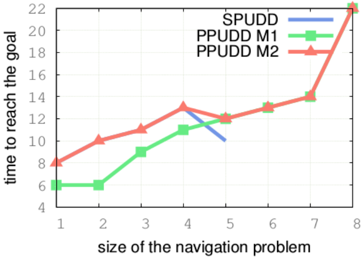
Performance, function of the problem index

reached goal frequency



higher is better

steps to reach the goal

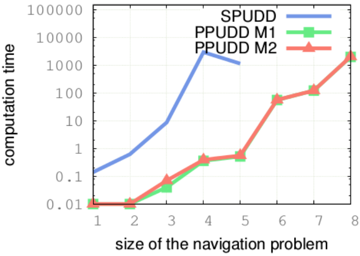


lower is better

Simplify computations with π-MOMDPs

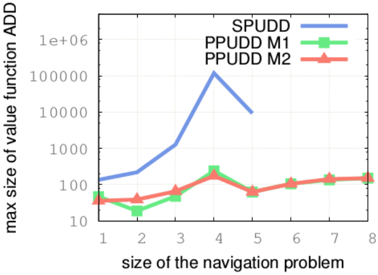
Experiments – perfect sensing: Navigation problem

computation time



lower is better

max size of ADDs



lower is better

- PPUDD + M2 (pessimistic criterion)
  faster with same performance as SPUDD
- SPUDD only solves the first 5 instances
- verified intuition: ADDs are smaller

Simplify computations with π-MOMDPs

Experiments – imperfect sensing: RockSample problem

PPUDD vs APPL (*Kurniawati et al.*, 2008, solver MOMDP)
symbolic HSVI (*Sim et al.*, 2008, solver POMDP)
RockSample benchmark: recognize and sample "good" rocks

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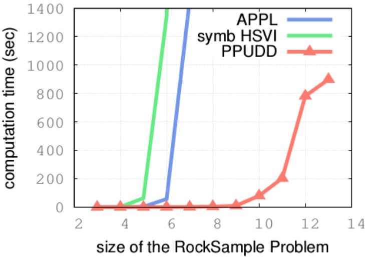
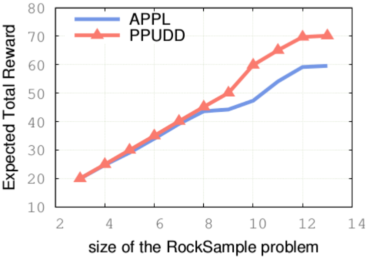


lower is better

average of rewards

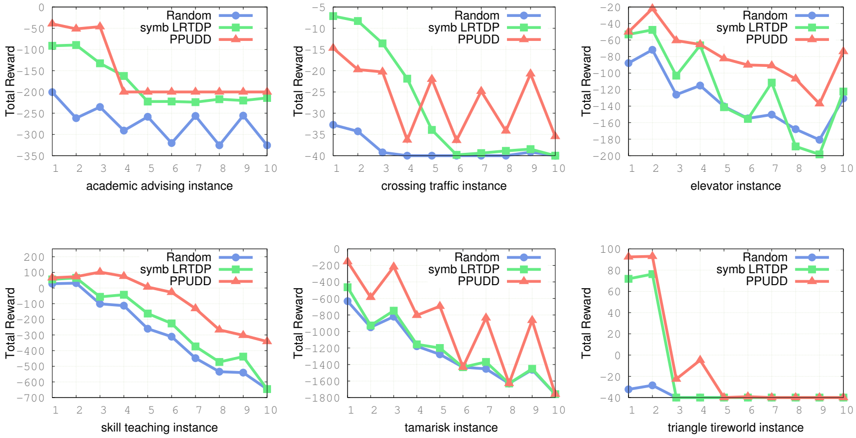


higher is better

- approximate model + exact resolution solver can be **better than** exact model + approximate resolution solver

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

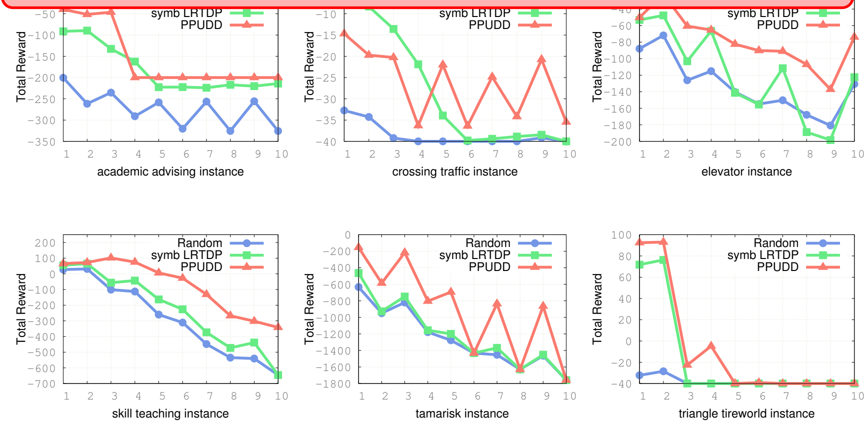
PPUDD + BDD mask over reachable states.



average of rewards over simulations – higher is better

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

PPUDD outperforms its probabilistic counterpart



average of rewards over simulations – higher is better

Qualitative possibilistic approach: benefits/drawbacks

towards a hybrid POMDP

- **granulated** belief space (discrete)
- efficient problem **simplification** (PPUDD 2× better than LRTDP with ADDs)
- **ignorance and imprecision** modeling

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- **granulated** belief space (discrete)
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- **ignorance and imprecision** modeling

- ADD methods $\prec$ state space search methods
  $\rightarrow$ winners of IPPC 2014: 2$\times$ better than PPUDD
- choice of the qualitative criterion (optimistic/pessimistic)
- preference $\rightarrow$ non additive degrees
  $\rightarrow$ same scale as possibility degrees (commensurability)
- coarse approximation of probabilistic model
  $\rightarrow$ no frequentist information

A hybrid model

a probabilistic POMDP with possibilistic belief states

hybrid approach

- agent knowledge = possibilistic belief states
- probabilistic dynamics & quantitative rewards

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Usefulness

→ **heuristic** for solving POMDPs:
results in a standard (finite state space) MDP
→ problem with **qualitative** & **quantitative** uncertainty

Transitions and rewards

belief-based transition and reward functions

- possibility distribution $\beta \rightarrow$ probability distribution $\overline{\beta}$ using poss-prob tranformations *(Dubois & Prade)*

Transition function on belief states

$$\Rightarrow \mathbf{p}\left(\beta' \middle| \overline{\beta}, a\right) = \sum_{\substack{o' \text{ t.q.} \\ update(\beta,a,o')=\beta'}} \mathbf{p}\left(o' \mid \overline{\beta}, a\right)$$

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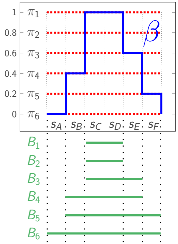
- reward pessimistic according to β



Pessimistic Choquet Integral

$$r(\beta, a) = \sum_{i=1}^{\#\mathcal{L}-1} (\pi_i - \pi_{i+1}) \cdot \min_{B_i} r(s, a)$$

$$B_i = \{ s \in \mathcal{S} \text{ s.t. } \beta(s) \geqslant \pi_i \}$$

Resulting MDP

translation from hybrid POMDP to MDP – **contribution (SUM-15):**

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$
output: the MDP $\langle \tilde{\mathcal{S}}, \mathcal{A}, \tilde{T}, \tilde{r}, \gamma \rangle$

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- **reward** $\tilde{r}(a, \beta) = \underline{Ch}\Big(r(a, .)\Big)$

$$\textbf{criterion:}\ \mathbb{E}_{\beta_t \sim \tilde{T}} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot \tilde{r}\left(\beta_t, d_t\right) \right]$$

Belief variable factorization

3 classes of state variables – **contribution (SUM-15)**

variable: **visible** $s'_v \in \mathbb{S}_v$

s'_v

---

inferred hidden $s'_h \in \mathbb{S}_h$

s'_h

---

fully hidden $s'_f \in \mathbb{S}_f$

s'_f

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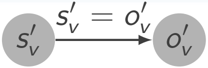

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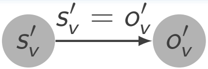

$$\beta_{t+1}(s'_v) = \mathbb{1}_{\{s'_v = o'_v\}}(s'_v)$$

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$\Leftrightarrow$ deterministic belief variable

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$$s'_v \xrightarrow{s'_v = o'_v} o'_v$$

---

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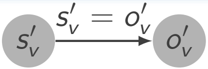

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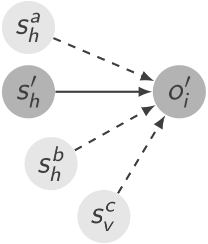

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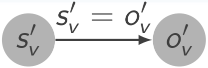

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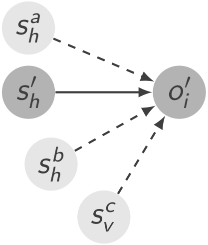

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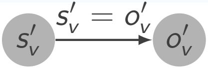

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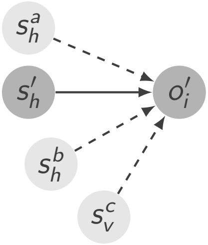

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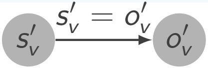

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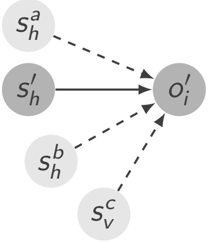

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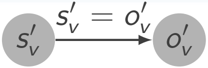

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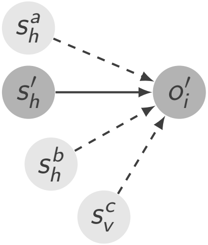

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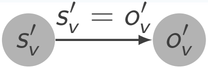

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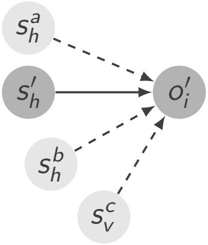

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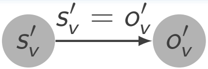

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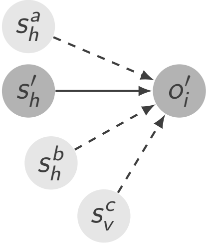

fully hidden $s'_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f.

$$\beta_{t+1}(s'_f) = \pi\left(s'_f \mid \beta_t, a\right)$$

Belief variable factorization

global belief state from marginal belief variables

bound over the global belief state

$$\beta_{t+1}(s'_1, \dots, s'_n) = \pi\left(s'_1, \dots, s'_n \mid a_0, o_1, \dots, a_t, o_{t+1}\right)$$

$$\leqslant \min\left\{ \min_{s'_j \in \mathbb{S}_v} \left[\mathbb{1}_{\left\{ s'_j = o'_j \right\}} \right], \min_{s'_j \in \mathbb{S}_f} \left[\beta_{t+1}(s'_j) \right], \min_{o'_i \in \mathbb{O}_h} \left[\beta_{t+1}\left(parents(o'_i) \right) \right] \right\}$$

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- min of marginals = a **less informative** belief state
- computed using **marginal belief states**
  → factorization & smaller state space

Conclusion
contributions

- **modeling efforts**: $\rightarrow$ human-machine interaction

Conclusion

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- **advances**: $\rightarrow$ mixed-observability modeling
  $\rightarrow$ indefinite horizon + optimality proof

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  $\rightarrow$ robust recognition mission with possibilistic beliefs
  $\rightarrow$ validation of the computation time reduction
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- **hybrid POMDP**
  $\rightarrow$ formalization
  $\rightarrow$ factored POMDP $\xrightarrow{\text{translation}}$ factored **finite** MPD

Conclusion

perspectives

- refined criteria, intermediate preferences
  (Weng 2005, Dubois & Fortemps 2005)
  $\Rightarrow$ finer π-POMDP
- state space heuristic search for π-POMDPs
- combination with reinforcement learning (*Sabbadin 2001*)

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hybrid model

- IPPC problems (factored POMDPs);
- tests of this approach:
  1. **simplification:** π distributions definition?
  2. **imprecision:** robust in practice?

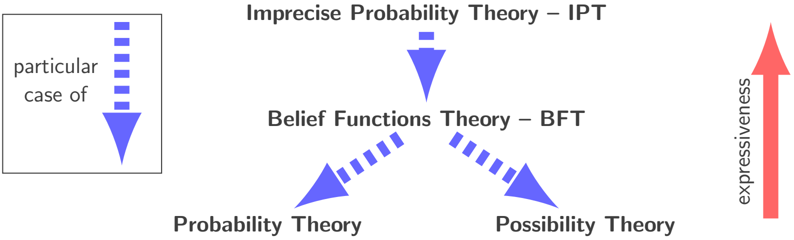
Thank you!

produced work:

- *Qualitative Possibilistic Mixed-Observable MDPs*, **UAI-2013**
- *Structured Possibilistic Planning Using Decision Diagrams*, **AAAI-2014**
- *Planning in Partially Observable Domains with Fuzzy Epistemic States and Probabilistic Dynamics*, **SUM-2015**
- *Processus Décisionnels de Markov Possibilistes à Observabilité Mixte*, Revue d'Intelligence Artificielle (**RIA french journal**)
- *A Possibilistic Estimation of Human Attentional Errors*, submitted to **IEEE-TFS journal**

Uncertainty theories

Most known uncertainty theories and their relations

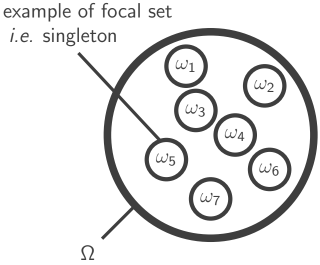


- IPT: most general uncertainty theory.
  Use of sets of probability measures over Ω.
- BFT: use of a mass function $m : 2^\Omega \to [0,1]$, with $\sum_{A \subset \Omega} m(A) = 1$.
  1. plausibility measure: $\forall A \subset \Omega$, $Pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$.
  2. belief function: $\forall A \subset \Omega$, $bel(A) = \sum_{B \subseteq A} m(B)$.

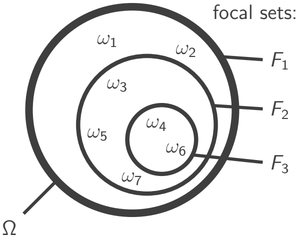
Focal sets of a mass function $m: 2^\Omega \to [0,1]$: subsets A of $\Omega = \{\omega_1, \dots, \omega_7\}$ such that $m(A) > 0$.

- if focal sets are all singletons
  $\to$ probability distribution ($bel = Pl = \mathbb{P}$)
- if focal sets are nested, e.g. $F_3 \subset F_2 \subset F_1 = \Omega$,
  $\to$ possibility distribution:
  bel=**necessity measure**, Pl=**possibility measure**.

probabilistic case



possibilistic case



Probabilistic belief update

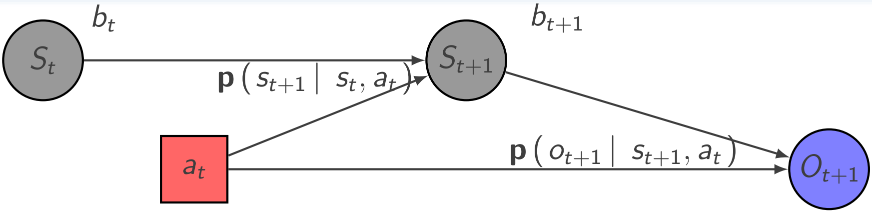


Figure : Bayesian Network illustrating the belief update

- the **system states** are the gray circular nodes,
- the **action** is the red square node ,
- and the **observation** is the blue circular node.

The belief state b_t (resp. b_{t+1}) is the probabilistic estimation of the current (resp. next) system state s_t (resp. s_{t+1})

probabilistic belief update

$$b_{t+1}(s') \propto \mathbf{p}\left(o' \mid s', a \right) \cdot \sum_{s \in S} \mathbf{p}\left(s' \mid s, a \right) \cdot b_t(s)$$

Rewritings of parameters

PROBABILISTIC parameters

- $T_j^a\left(\mathbb{S}, s_j'\right) = T_j^a\left(\mathcal{P}(s_j'), s_j'\right)$;
- $O_i^a\left(\mathbb{S}', o_i'\right) = O_i^a\left(\mathcal{P}(o_i'), o_i'\right)$.

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consequences on the joint distribution

$$\begin{aligned}\mathbf{p}\left(o_i', \mathcal{P}(o_i') \mid \mathbb{S}, a\right) &= O_i^a\left(\mathcal{P}(o_i'), o_i'\right) \cdot \prod_{s_j' \in \mathcal{P}(o_i')} T_i^a\left(\mathcal{P}(s_j'), s_j'\right) \\ &= \mathbf{p}\left(o_i', \mathcal{P}(o_i') \mid \mathcal{Q}(o_i'), a\right).\end{aligned}$$

Rewritings of parameters

PROBABILISTIC parameters

- $T^a_j\left(\mathbb{S}, s'_j\right) = T^a_j\left(\mathcal{P}(s'_j), s'_j\right)$;
- $O^a_i\left(\mathbb{S}', o'_i\right) = O^a_i\left(\mathcal{P}(o'_i), o'_i\right)$.

consequences on the joint distribution

$$\mathbf{p}\left(o'_i, \mathcal{P}(o'_i) \mid \mathbb{S}, a\right) = O^a_i\left(\mathcal{P}(o'_i), o'_i\right) \cdot \prod_{s'_j \in \mathcal{P}(o'_i)} T^a_i\left(\mathcal{P}(s'_j), s'_j\right)$$
$$= \mathbf{p}\left(o'_i, \mathcal{P}(o'_i) \mid \mathcal{Q}(o'_i), a\right).$$

observation probabilities

epistemic state

$$b^\pi(\mathbb{S}) \xrightarrow{\textbf{marginalization}} b^\pi(\mathcal{Q}(o'_i)) \xrightarrow{\textbf{pignistic transformation}} \overline{b^\pi}(\mathcal{Q}(o'_i))$$

$$\mathbf{p}\left(o'_i \mid b^\pi, a\right) = \sum_{2^{\mathcal{P}(o'_i)},\, 2^{\mathcal{Q}(o'_i)}} \mathbf{p}\left(o'_i, \mathcal{P}(o'_i) \mid \mathcal{Q}(o'_i), a\right) \cdot \overline{b^\pi}(\mathcal{Q}(o'_i))$$

Parameters rewritings

POSSIBILISTIC parameters

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marginal possibilistic belief states

$\forall o'_i \in \mathbb{O}$,

$$b^{\pi}_{t+1}\Big(\mathcal{P}(o'_i)\Big) \propto^{\pi} \pi\Big(o'_i, \mathcal{P}(o'_i) \Big| a_0, o_1, \ldots, a_{t-1}, o_t\Big)$$

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denoted by $\pi\Big(o'_i, \mathcal{P}(o'_i) \Big| b^{\pi}_t, a\Big)$.

A possibilistic belief state

finite belief space

$$\Pi_{\mathcal{S}}^{\mathcal{L}} = \left\{ \text{ possibility distributions } \right\} \colon \ \#\Pi_{\mathcal{S}}^{\mathcal{L}} < +\infty$$

$\rightarrow$ **finite belief space**

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update – **possibilistic** belief state

$$b_{t+1}^\pi(s') = \begin{cases} 1 & \text{if } \pi\left(o', s' \mid b_t^\pi, a\right) = \pi\left(o' \mid b_t^\pi, a\right) \\ \pi\left(o', s' \mid b_t^\pi, a\right) & \text{otherwise.} \end{cases}$$

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- $\pi(o', s' | b_t^\pi, a) = \max_{s \in \mathcal{S}} \min \left\{ \pi(o' | s', a), \pi(s' | s, a), b_t^\pi(s) \right\}$;
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- the update **only depends on** o' **and** a.

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if $V_{i+1}(x) > V_i(x)$, $\delta(x) = \operatorname{argmax}\limits_{a \in \mathcal{A}} \max\limits_{x' \in \mathcal{X}} \min \{ \pi \left(x' \mid x, a \right), V_i(x') \}$.

Resulting π-MOMDP solver: PPUDD

- probabilistic model: $+$ and $\times$ $\Rightarrow$ new values created, number of ADDs leaves **potentially huge**.
- possibilistic model: min and max $\Rightarrow$ values $\in \mathcal{L}$ finite, number of leaves bounded, **ADDs smaller**.

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PPUDD: Possibilistic Planning Using Decision Diagrams

$V^* \leftarrow 0$; $V^c \leftarrow \mu$; $\delta \leftarrow \overline{a}$;
while $V^* \neq V^c$ **do**
    $V^* \leftarrow V^c$;
    for $a \in \mathcal{A}$ **do**
        $q^a \leftarrow$ swap each X_i variable in V^* with X_i' ;
        for $1 \leqslant i \leqslant n$ **do**
            $q^a \leftarrow \boxed{\min} \{ q^a, \pi(X_i' \mid parents(X_i'), a) \}$;
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        update δ to a where $q^a = V^c$ and $V^c > V^*$;
return (V^*, δ) ;

computations on trees: *CU Decision Diagram Package*.

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factorization

$\Rightarrow$ dynamic programming divided into *n* stages

used ADDs smaller

$\rightarrow$ **faster computations.**

computations on trees: *CU Decision Diagram Package*.

Pignistic transformation and transitions

Pignistic transformation

numbering of the $n = \#\mathcal{S}$ system states:
$1 = b^\pi(s_1) \geqslant \ldots \geqslant b^\pi(s_n) \geqslant b^\pi(s_{n+1}) = 0.$

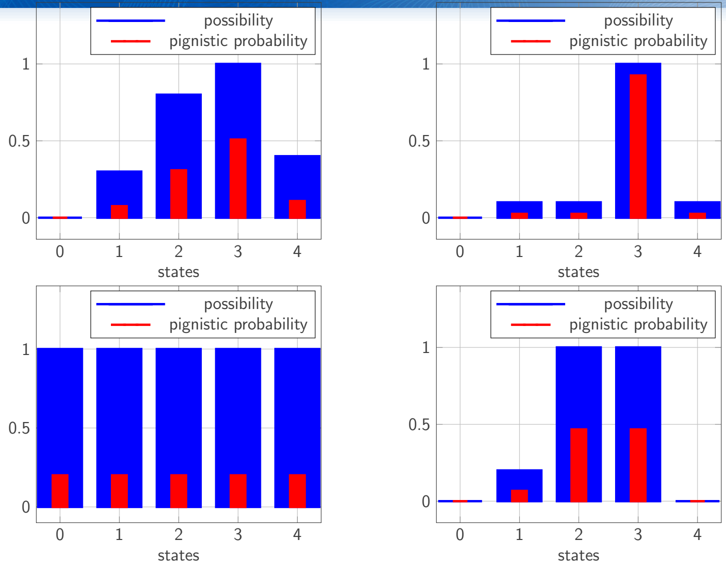
pignistic transformation – $P : \Pi_\mathcal{S} \rightarrow \mathbb{P}_\mathcal{S}$

$$\overline{b^\pi}(s_i) = \sum_{j=i}^{\#\mathcal{S}} \frac{b^\pi(s_j) - b^\pi(s_{j+1})}{j}.$$

- probability distribution $\overline{b^\pi}$ = **gravity center** of the represented probabilistic distributions;
- **Laplace principle**: ignorance → uniform probability.

Pignistic transformation

Examples of pignistic transformations (red) of possibility distributions (blue)



hybrid POMDP and π-POMDP

differences with possibilistic models

	hybrid POMDP	π-POMDP
transitions	probabilities	qualitative possibility
rewards	quantitative $\in \mathbb{R}$	qualitative $\in \mathcal{L}$
situation	-some imprecisions -large POMDP	few quantitative
issues	π definition	commensurability
in practice	MDP	π-MDP

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hybrid model:

- only belief states are possibilistic:

$\rightarrow$ agent knowledge $=$ **possibility** distribution;

- probabilistic dynamics:

$\rightarrow$ **approximated** (prob.) transition between epistemic states.

factorized POMDP
definition

- $\mathcal{S}$ described by $\mathbb{S} = \{ s_1, \ldots, s_m \}$: $\mathcal{S} = s_1 \times \ldots \times s_m$.
  Notation: $\mathbb{S}' = \{ s'_1, \ldots, s'_m \}$;

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- $\mathcal{O}$ described by $\mathbb{O} = \{ o_1, \ldots, o_n \}$: $\mathcal{O} = o_1 \times \ldots \times o_n$;
- **observation** function of o'_i,
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independences:

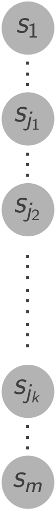
$\rightarrow \forall s'_i, s'_j \in \mathbb{S}', \qquad s'_i \perp\!\!\!\perp s'_j \mid \{\mathbb{S}, a \in \mathcal{A}\}$,

$\rightarrow \forall o'_i, o'_j \in \mathbb{O}', \qquad o'_i \perp\!\!\!\perp o'_j \mid \{\mathbb{S}', a \in \mathcal{A}\}$.

Notations

some variables does not interact with each other

variables about the **current** system state,



variable s'_j about the **next** state.

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$$s_k \rightarrow s'_j$$

$$\Updownarrow$$

$\exists a \in \mathcal{A}$, such that

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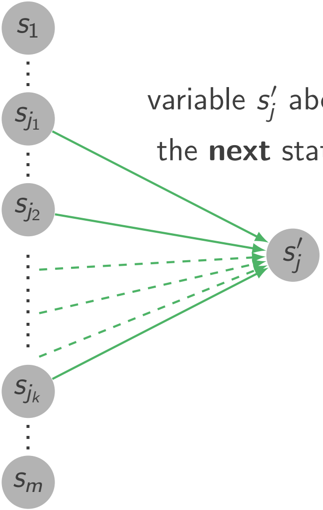

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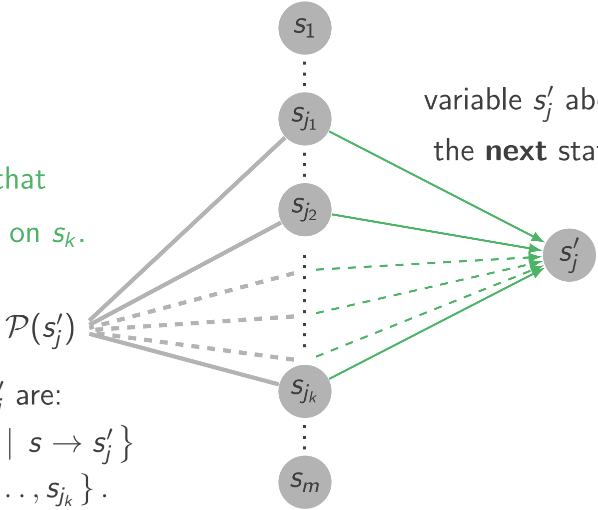


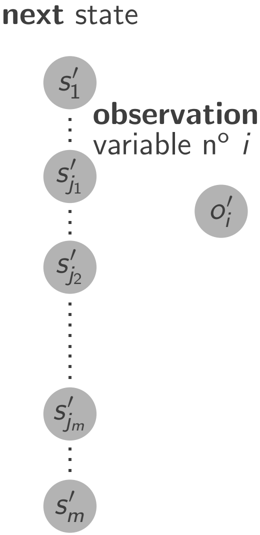
variable s'_j about the **next** state.

the parents of s'_j are:

$$\mathcal{P}(s'_j) = \left\{ s \in \mathbb{S} \mid s \to s'_j \right\} = \left\{ s_{j_1}, s_{j_2}, \dots, s_{j_k} \right\}.$$

Notations

concerning observation variables



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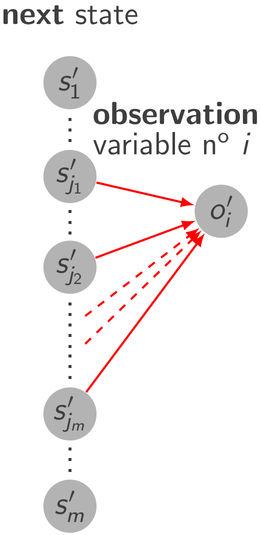

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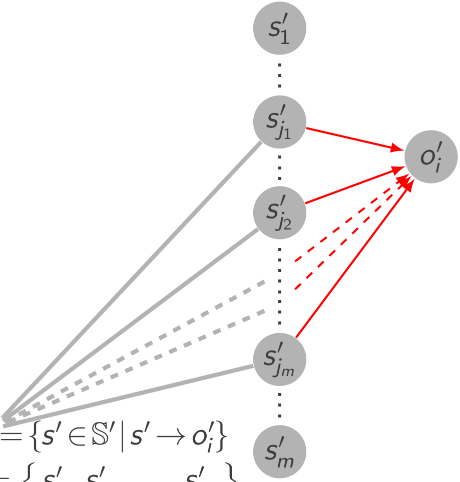

$$\mathcal{P}(o'_i) = \{s' \in \mathbb{S}' \mid s' \to o'_i\}$$
$$= \{ s'_{j_1}, s'_{j_2}, \dots, s'_{j_m} \}$$

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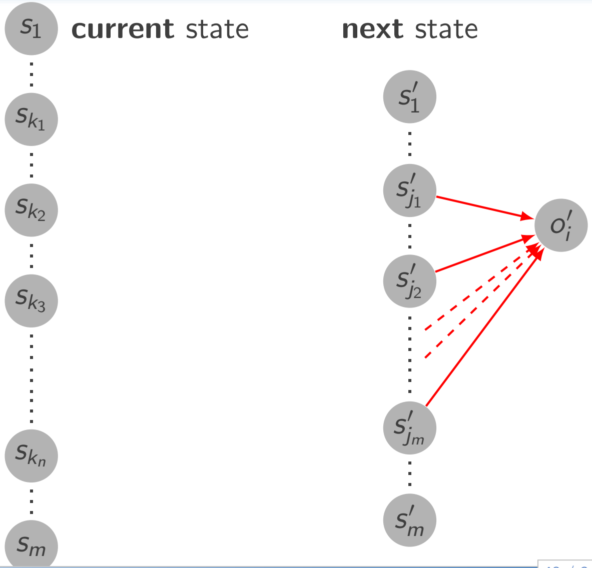

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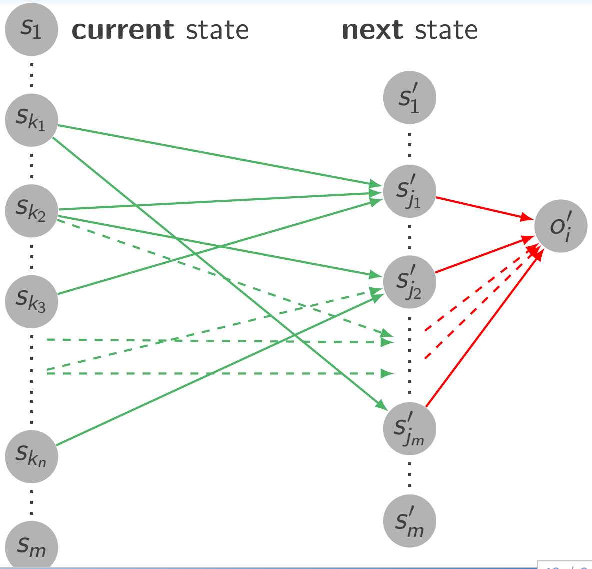

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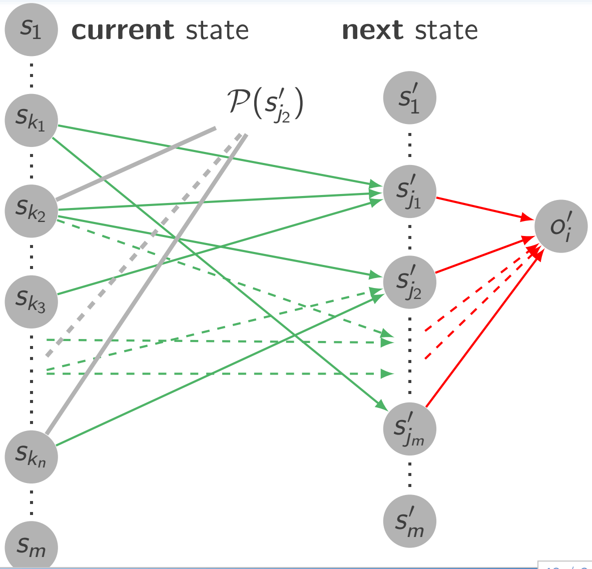

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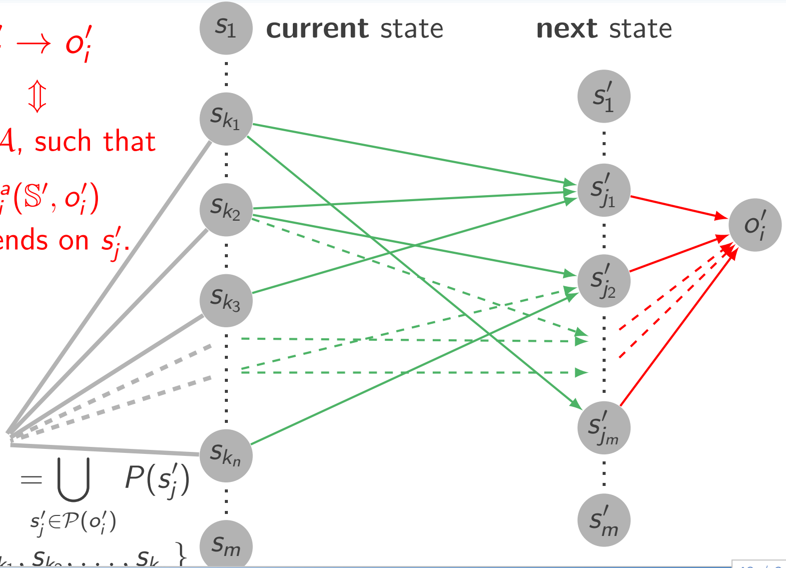

$$\mathcal{Q}(o'_i) = \bigcup_{s'_j \in \mathcal{P}(o'_i)} P(s'_j)$$

$$= \{ s_{k_1}, s_{k_2}, \ldots, s_{k_n} \}$$

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different according to the class of the variable

$$\lambda = \#\mathcal{L}$$

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  $\forall o_i \in \mathbb{O} \setminus \mathbb{S}_v$, $\lambda^{2^{p_i}} - (\lambda - 1)^{2^{p_i}}$ belief states,

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- $\forall s'_f \in \mathbb{S}_f$, $\lambda^2 - (\lambda - 1)^2 = 2\lambda - 1$ belief states,

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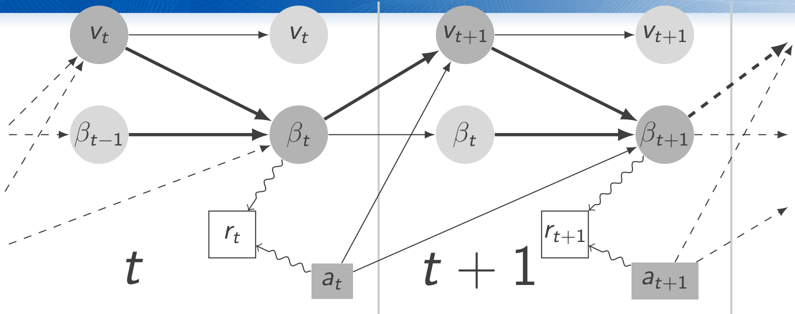
resulting MDP in practice

final structured MDP



\# **factorized model**'s variables: $\#\mathbb{O} + \#\mathbb{S}_v +$

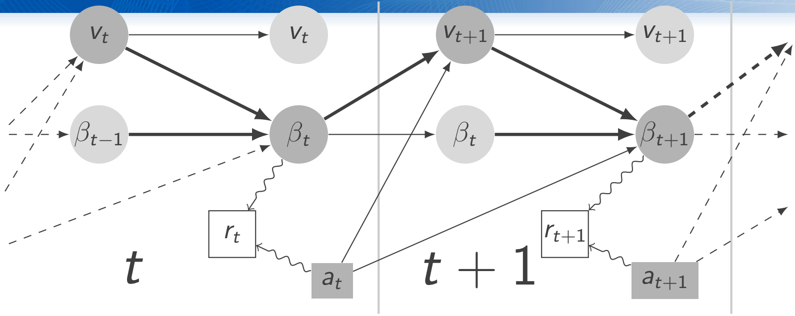
$$+\sum_{i=1}^{\#\mathbb{O}_h} \left\lceil \log_2 \left(\lambda^{2^{p_i}} - (\lambda-1)^{2^{p_i}} \right) \right\rceil + \#\mathbb{S}_f \cdot \left\lceil \log_2 \left(2\lambda - 1 \right) \right\rceil$$

$\ll$ \# **initial hybrid model**'s variables:

$$\left\lceil \log_2 \left(\lambda^{2^{\#\mathbb{S}}} - (\lambda-1)^{2^{\#\mathbb{S}}} \right) \right\rceil$$

resulting MDP in practice

final structured MDP



\# **factorized model**'s variables:

$$\leqslant \#\mathbb{O} + \#\mathbb{S}_v + \sum_{i=1}^{\#\mathbb{O}_h} \log_2(\lambda) \cdot 2^{p_i} + \#\mathbb{S}_f \cdot (1 + \log_2(\lambda))$$

$\ll$ \# **initial hybrid model**'s variables:

$$\geqslant \log_2(\lambda) \cdot (2^{\#\mathbb{S}} - 1).$$

Variable classification

3 classes of state variables – state space factorization

variable: visible $s'_v \in \mathbb{S}_v$

s'_v

inferred hidden $s'_h \in \mathbb{S}_h$

s'_h

fully hidden $s'_f \in \mathbb{S}_f$

s'_f

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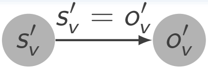

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$$s'_v \xrightarrow{s'_v = o'_v} o'_v$$

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---

inferred hidden $s'_h \in \mathbb{S}_h$

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---

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s'_f

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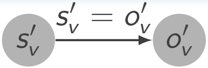

$\Leftrightarrow$ deterministic belief variable.

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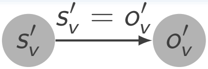

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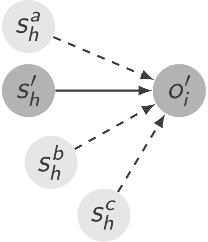

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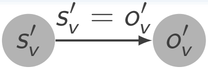

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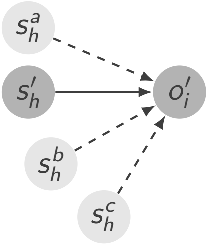

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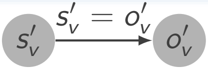

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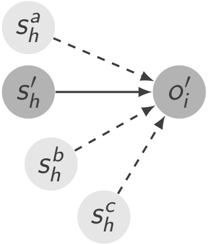

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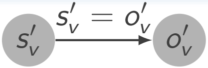

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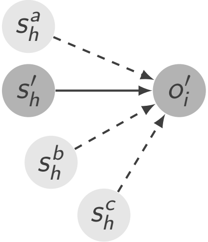

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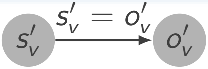

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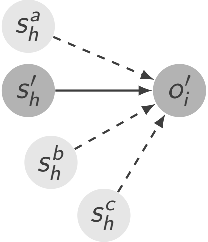

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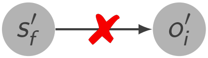

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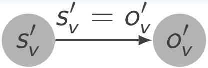

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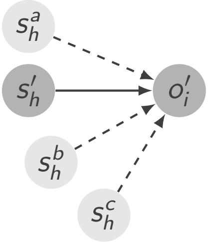

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$$b^\pi_{t+1}(s'_f) = \max_{\mathcal{P}(s'_f)} \min \left\{ \pi\left(s'_f \middle| \mathcal{P}(s'_f), a\right), b^\pi_t\left(\mathcal{P}(s'_f)\right) \right\}$$

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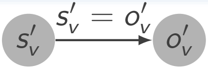

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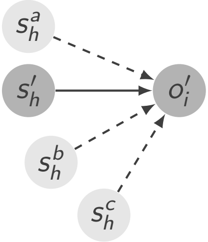
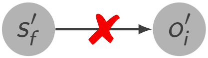


⚠ $\mathcal{P}(o'_i)$ may contain visible variables

fully hidden $s'_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f



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Toy example: 2 machine states, 3 occurrences

columns		1	2	3	4	5
SITUATION						
v'	v_A	1				1
	v_B		1			
	v_C	1			1	
h	s_A	1	1		1	
	s_B			1		1
BEHAVIOUR						
h'	s_A					1
	s_B		1		1	
EFFECT		$\overline{e}$	$\tilde{e}$	$\overline{e}$	$\widehat{e}$	$\underline{e}$
POSSIBILITY		1	ε	1	λ	δ

Probability / Possibility :

e_1 or e_2	$\mathbf{p}(e_1) + \mathbf{p}(e_2 \cap \overline{e_1})$	$\max\left\{ \pi(e_1), \pi(e_2) \right\}$
e_1 and e_2	$\mathbf{p}(e_1).\mathbf{p}\left(e_2 \mid e_1 \right)$	$\min\left\{ \pi(e_1), \pi\left(e_2 \mid e_1\right) \right\}$

Back to general POMDP: Partially Observable Criteria

Rewriting: belief dependent reward (belief trick)

- $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ reward function
- $\rho : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{L}$ preference function

Probability / Possibility:

Probability	Possibility
$R(b_t, d_t) = \sum_s r(s, d_t) \cdot b_t(s)$	**optimistic:** $\overline{\Psi}(\beta_t, \delta_t) = \max_s \min \left\{ \rho(s, \delta_t), \beta_t(s) \right\}$ **pessimistic:** $\underline{\Psi}(\beta_t, \delta_t) = \min_s \max \left\{ \rho(s, \delta_t), 1 - \beta_t(s) \right\}$
$\mathbb{E}[r(S_t, d_t)] = \mathbb{E}[R(b_t, d_t)]$	$\mathbb{S}_{\Pi}[\rho(S_t, d_t)] = \mathbb{S}_{\Pi}[\overline{\Psi}(\beta_t, d_t)]$ $\mathbb{S}_{\mathcal{N}}[\rho(S_t, d_t)] = \mathbb{S}_{\mathcal{N}}[\underline{\Psi}(\beta_t, d_t)]$

Note: $\mathbb{S}_{\Pi}[\underline{\Psi}(\beta_t, d_t)]$; $\mathbb{S}_{\mathcal{N}}[\overline{\Psi}(\beta_t, d_t)]$?

Why model ignorance?

knowledge is not always encouraged with POMDPs

- initial belief deterministic $s_0 = s_A$.

s_A

$b_0(s_A) = 1$

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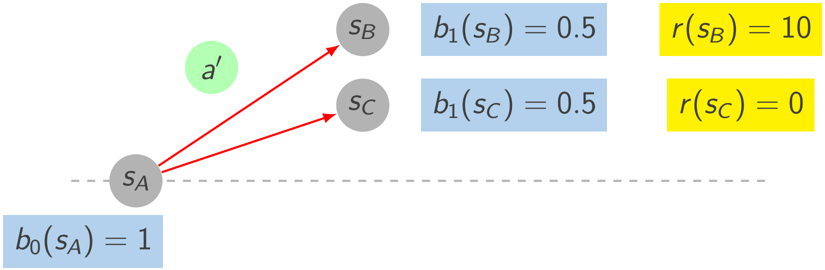

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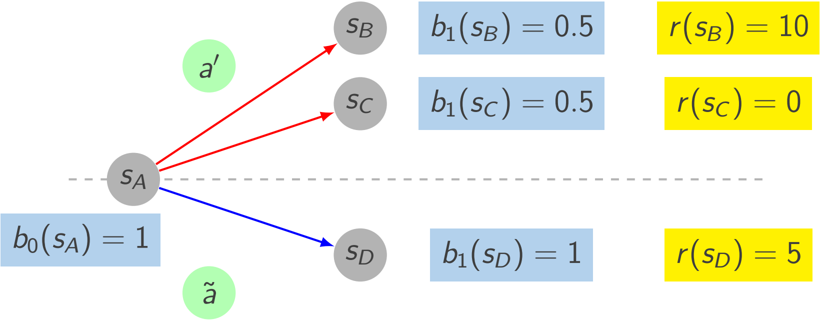

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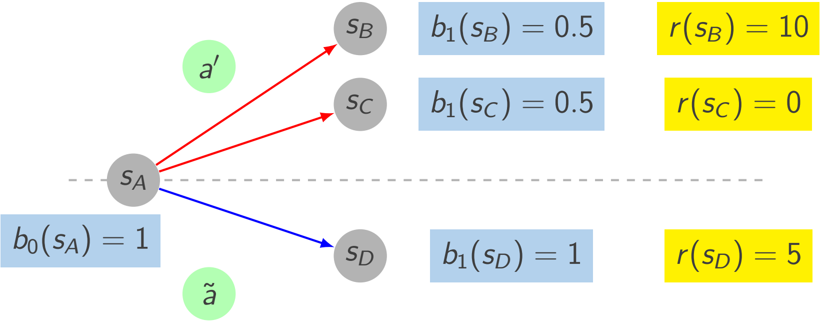

- $\{ s_B, s_C, s_D \} \xrightarrow{\text{deterministic}} s_E, \; r(s_E) = 0.$

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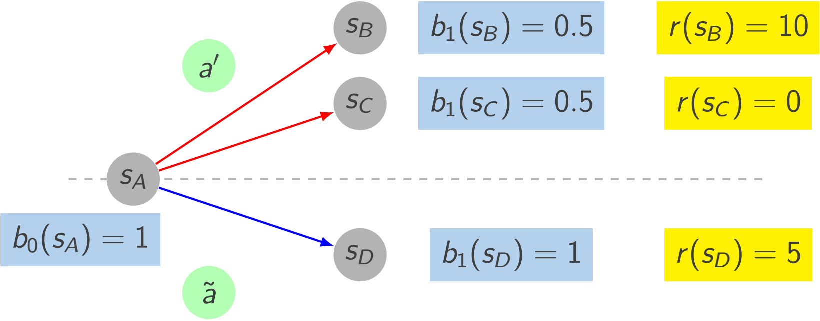


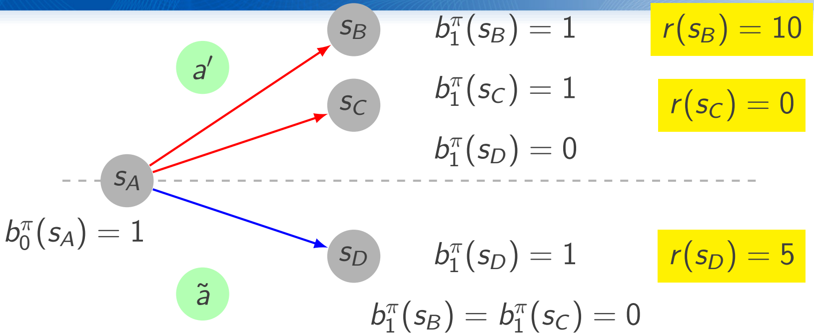
- $\{ s_B, s_C, s_D \} \xrightarrow{\text{deterministic}} s_E, \; r(s_E) = 0.$

$$\mathbb{E}_{s_0 \sim b_0}\left[\sum_{t=0}^{+\infty} \gamma^t \cdot r(s_t) \,\middle|\, a_0 = \tilde{a} \text{ or } a' \right] = r(s_0) + 5\gamma.$$

the safe action is not prefered.

Why model ignorance?

Choquet integral and rewards



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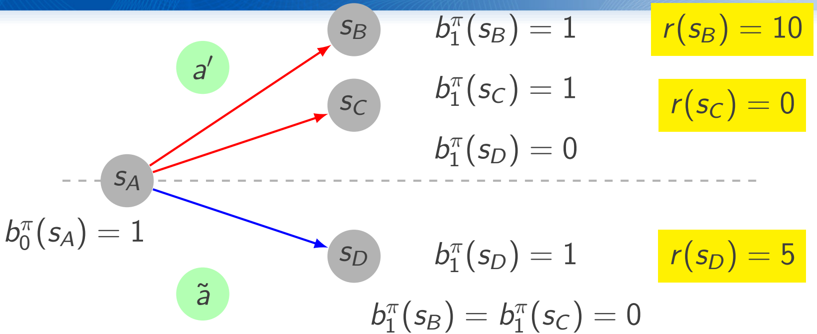

- $Ch\left(r, N_{b_1^\pi} \;\middle|\; a_0 = \tilde{a}\right) = r(s_D, \tilde{a}) = 5,$
- $Ch\left(r, N_{b_1^\pi} \;\middle|\; a_0 = a'\right) = \min_{s \in \mathcal{S}} r(s, a') = 0.$

the safe action is prefered! **dispersion reduced**

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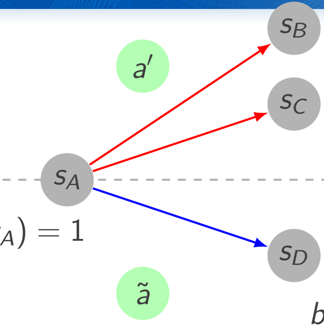

$b_0^\pi(s_A) = 1$

$b_1^\pi(s_B) = 1$ $r(s_B) = 10$

$b_1^\pi(s_C) = 1$ $r(s_C) = 0$

$b_1^\pi(s_D) = 0$

$b_1^\pi(s_D) = 1$ $r(s_D) = 5$

$b_1^\pi(s_B) = b_1^\pi(s_C) = 0$

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the safe action is prefered! **dispersion reduced**

if $\mathcal{N}_{b_1^\pi}$ replaced by $b_1 \Rightarrow Ch(r, b_1) = \mathbb{E}_{s \sim b_1}\left[r(s,a)\right].$

Choquet integral and rewards

pessimistic evaluation of the rewards – necessity measure

imprecision of b^{π} = agent ignorance + discretization:
pessimistic reward about these imprecisions.

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Dual measure of $\Pi : 2^{\mathcal{S}} \rightarrow \mathcal{L}$

necessity $\mathcal{N}$ such that $\forall A \subseteq \mathcal{S}$, $\mathcal{N}(A) = 1 - \Pi(\overline{A})$.

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$r_1 > r_2 > \ldots > r_{k+1} = 0$ represents elements of $\left\{ r(s,a) \middle| s \in \mathcal{S} \right\}$.

Choquet integral of r with respect to $\mathcal{N}$

$$Ch(r, \mathcal{N}) = \sum_{i=1}^{k} (r_i - r_{i+1}) \cdot \mathcal{N}\left(\left\{ r(s) \geqslant r_i \right\} \right) \qquad (1)$$

$$(2)$$

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$$= \sum_{i=1}^{\#\mathcal{L}-1} (l_i - l_{i+1}) \cdot \min_{\substack{s \in \mathcal{S} \text{ s.t.} \\ b^{\pi}(s) \geqslant l_i}} r(s) \quad (2)$$

notation $\mathcal{L} = \left\{ l_1 = 1, l_2, l_3, \ldots, 0 \right\}$.

resulting MDP in practice

trick: "flipflop" variable

boolean variable "*flipflop*" f changes state at each time step
$\rightarrow$ defines 2 phases:

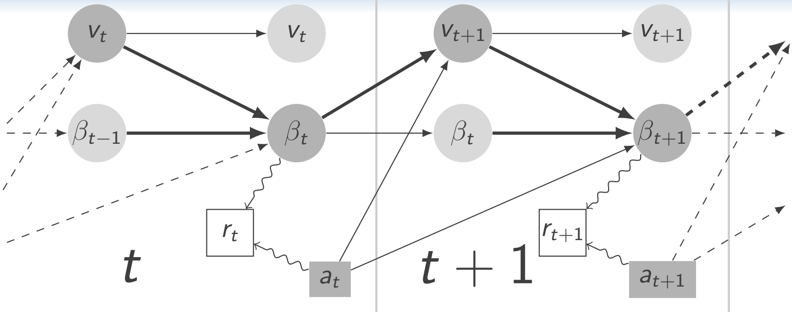
1. *observation generation*,
2. *belief update* (deterministic knowing the observation)

MDP variables:

$$\tilde{\mathbb{S}} = \textbf{beliefs}: \beta = \beta_v^1 \times \ldots \times \beta_v^{m_v} \times \beta_h^1 \times \ldots \times \beta_h^{m_h} \times \beta_f^1 \times \ldots \times \beta_f^{m_f} \times \textbf{visible variables}: v = f \times s_v^1 \times \ldots \times s_v^{m_v} \times o_1 \times \ldots \times o_k.$$

resulting MDP in practice

final structured MDP



$\tilde{\mathbb{S}} =$

beliefs: $\beta = \beta_v^1 \times \ldots \times \beta_v^{m_v} \times \beta_h^1 \times \ldots \times \beta_h^{m_h} \times \beta_f^1 \times \ldots \times \beta_f^{m_f}$

$\times$

visible variables : $v = f \times s_v^1 \times \ldots \times s_v^{m_v} \times o_1 \times \ldots \times o_k.$