Planning in Partially Observable Domains with Fuzzy Epistemic States and Probabilistic Dynamics.

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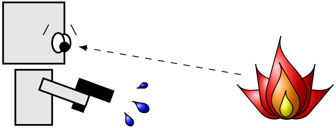
retour sur innovation

Plan

Plan

POMDP: model for sequential decision making under uncertainty



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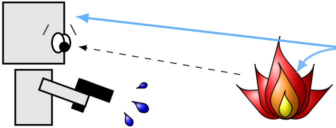

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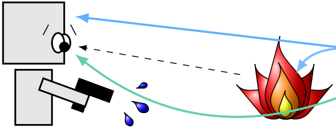

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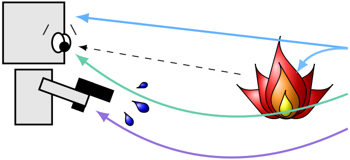

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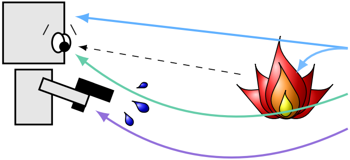

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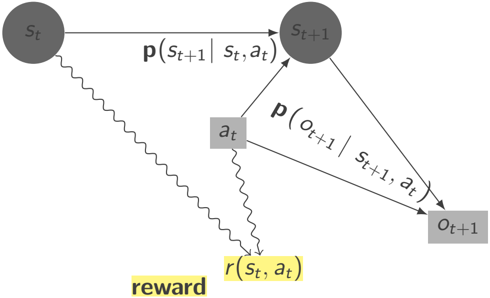
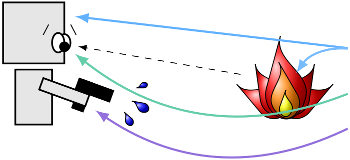


Context

Partially Observable Markov Decision Process (POMDP)

POMDP: model for sequential decision making under uncertainty



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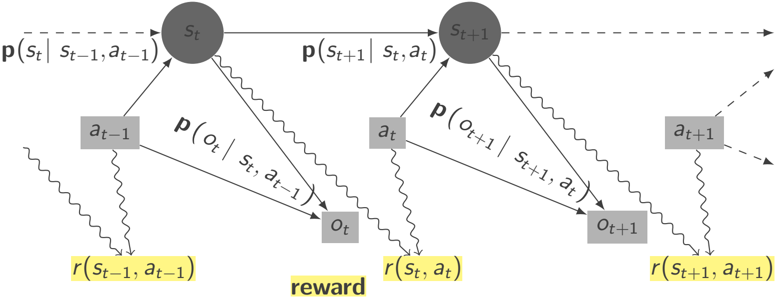

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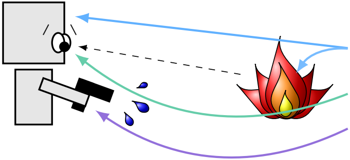

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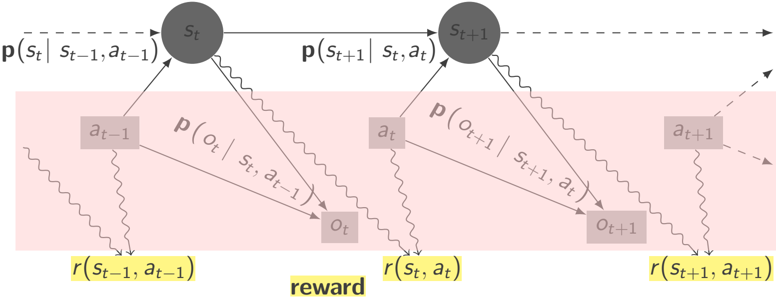

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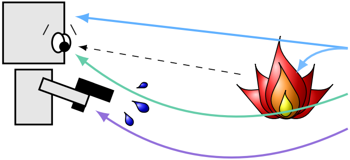

- $s \in \mathcal{S}$: **system states**;
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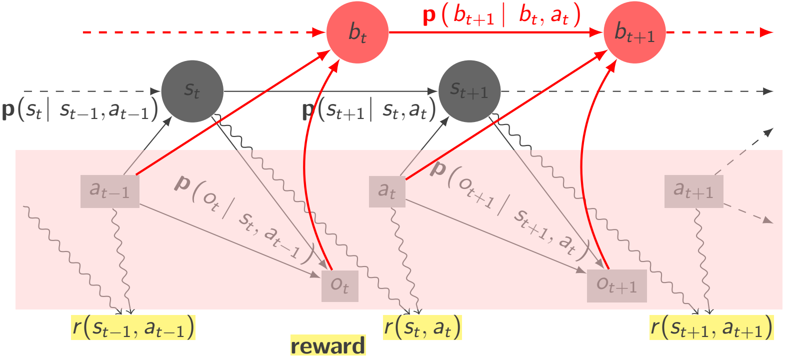
- $a \in \mathcal{A}$: **agent's actions**;
- b: **belief state**.



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- **transition** function $T(s, a, s') = \mathbf{p}\left(s' \mid s, a \right)$;
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$$b_{t+1}(s') \propto \mathbf{p}\left(o' \mid s', a \right) \cdot \sum_{s \in \mathcal{S}} \mathbf{p}\left(s' \mid s, a \right) \cdot b_t(s)$$

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action choices: strategy $\delta(b_t) = a_t \in \mathcal{A}$

$$\text{maximizing } \mathbb{E}_{s_0 \sim b_0} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot r\Big(s_t, \delta(b_t) \Big) \right], \; 0 < \gamma < 1.$$

- optimal strategy computation $\geqslant$ **PSPACE**;
- probabilities are **imprecisely known** in practice;
- agent's **ignorance** not taken into account.

Why model ignorance?

knowledge is not always encouraged with POMDPs

- initial belief deterministic $s_0 = s_A$.

s_A

$b_0(s_A) = 1$

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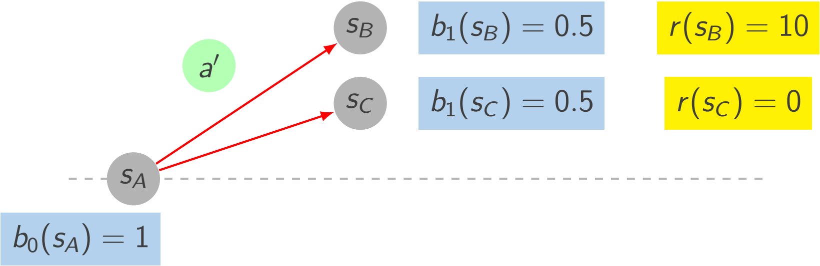

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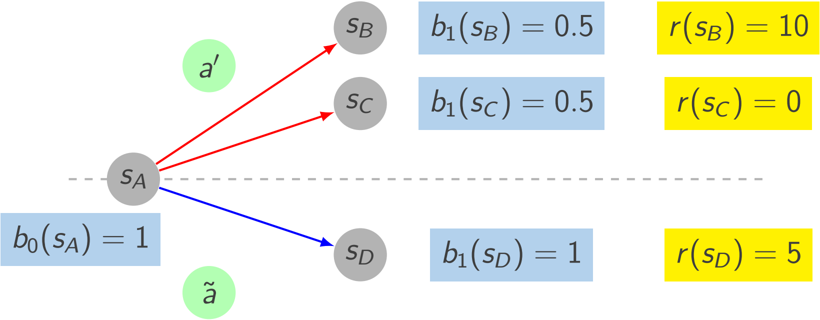

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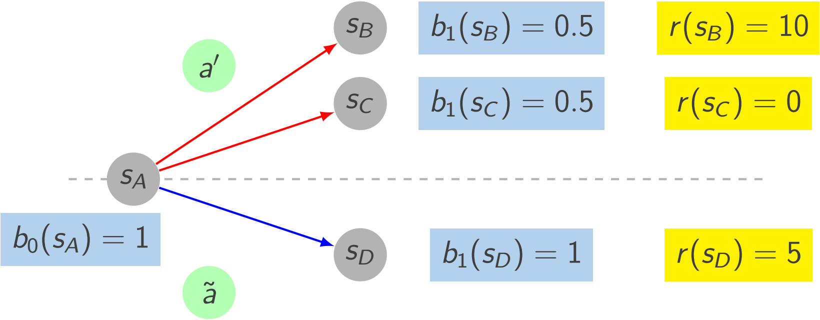

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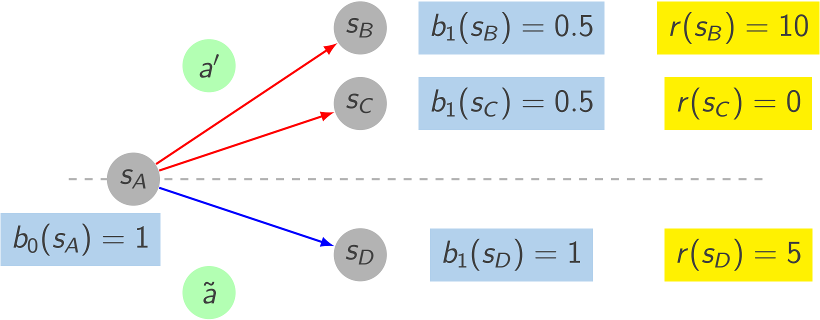

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$$\mathbb{E}_{s_0 \sim b_0}\left[\sum_{t=0}^{+\infty} \gamma^t \cdot r(s_t) \,\middle|\, a_0 = \tilde{a} \text{ ou } a' \right] = r(s_0) + 5\gamma.$$

the safe action is not prefered.

Qualitative Possibility Theory

- **simplification/imprecision** taken into account, BUT frequentist information lost;
- **ignorance** modeling;
- possibilistic belief states already studied: π-POMDP (*Sabbadin 98,Drougard 13,14*).

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- defined distributions π:
  $\mathbb{P} \rightarrow \pi$ transformations: pignistic, specific, ...

$1 = l_1 > l_2 > \ldots > l_{\#\mathcal{L}} = 0$ form the **finite scale** $\mathcal{L}$.

events $e \subset \Omega$ (universe)

sorted using possibility **degrees** $\pi(e) \in \mathcal{L}$,

$\neq$

quantified with **frequencies** $\mathbf{p}(e) \in [0, 1]$ (probabilities).

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Probability ($\mathbb{P}$) / Possibility (Π):

e_1 ou e_2	$\mathbf{p}(e_1) + \mathbf{p}(e_2 \cap \overline{e_1})$	$\max\left\{\pi(e_1), \pi(e_2)\right\}$
e_1 et e_2	$\mathbf{p}(e_1).\mathbf{p}\left(e_2 \mid e_1\right)$	$\min\left\{\pi(e_1), \pi(e_2 \mid e_1)\right\}$

Plan

$$\Pi_S = \left\{ \text{ possibility distributions } \right\} : \#\Pi_S < +\infty$$

$\rightarrow$ **belief space discretization**.

A possibilistic belief state

belief space discretization

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denoted by $b_{t+1}^\pi(s') \propto^\pi \pi\left(o', s' \,\middle|\, b_t^\pi, a\right)$

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- $\pi(o', s' | b_t^\pi, a) = \max_{s \in \mathcal{S}} \min \left\{ \pi(o' | s', a), \pi(s' | s, a), b_t^\pi(s) \right\}$;
- $\pi\left(o' \mid s', a\right) = \max_{s' \in \mathcal{S}} \pi\left(o', s' \mid b_t^\pi, a\right)$.

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- the update **only depends on** o' **and** a.

numbering of the $n = \#\mathcal{S}$ system states:
$1 = b^{\pi}(s_1) \geqslant \ldots \geqslant b^{\pi}(s_n) \geqslant b^{\pi}(s_{n+1}) = 0.$

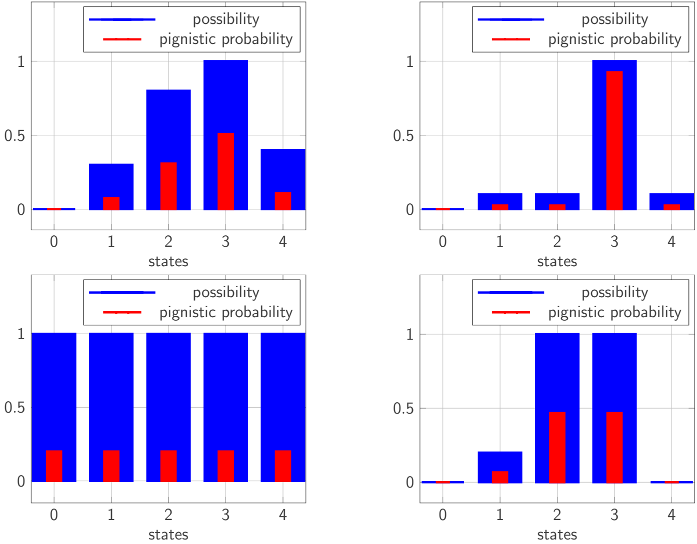
pignistic transformation – $P : \Pi_{\mathcal{S}} \rightarrow \mathbb{P}_{\mathcal{S}}$

$$\overline{b^{\pi}}(s_i) = \sum_{j=i}^{\#\mathcal{S}} \frac{b^{\pi}(s_j) - b^{\pi}(s_{j+1})}{j}.$$

- $\overline{b^{\pi}} \in \mathcal{B}_{\mathcal{S}}$ gravity center of the represented probabilistic distributions;
- Laplace principle: ignorance $\rightarrow$ uniform probability.

Pignistic transformation

Examples of pignistic transformations (red) of possibility distributions (blue)



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Approximation of the probabilities over the observations:

- $\mathbf{p}\left(o' \mid s, a\right) = \sum_{s' \in \mathcal{S}} O(s', a, o') \cdot T(s, a, s')$;
- $\mathbf{p}\left(o' \mid b^{\pi}, a\right) := \sum_{s \in \mathcal{S}} \mathbf{p}\left(o' \mid s, a\right) \cdot \overline{b^{\pi}}(s)$.

$$\Rightarrow \mathbf{p}\left((b^{\pi})' \middle| b^{\pi}, a\right) = \sum_{\substack{o' \text{ t.q.} \\ u(b^{\pi}, a, o') = (b^{\pi})'}} \mathbf{p}\left(o' \mid b^{\pi}, a\right).$$

notation: if $a \in \mathcal{A}$ selected, $o' \in \mathcal{O}$ received,

$$b^{\pi}_{t+1} = u\left(o', a, b^{\pi}_t\right) = \text{ update of } b^{\pi}_t.$$

imprecision of b^π = agent ignorance + discretization:
pessimistic reward about these imprecisions.

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Dual measure of $\Pi : 2^{\mathcal{S}} \rightarrow \mathcal{L}$

necessity $\mathcal{N}$ such that $\forall A \subseteq \mathcal{S}$, $\mathcal{N}(A) = 1 - \Pi(\overline{A})$.

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Choquet integral of r with respect to $\mathcal{N}$

$$Ch(r, \mathcal{N}) = \sum_{i=1}^{k} (r_i - r_{i+1}) \cdot \mathcal{N}\left(\left\{ r(s) \geqslant r_i \right\}\right) \quad (1)$$

$$\quad (2)$$

Choquet integral and rewards

pessimistic evaluation of the rewards – necessity measure

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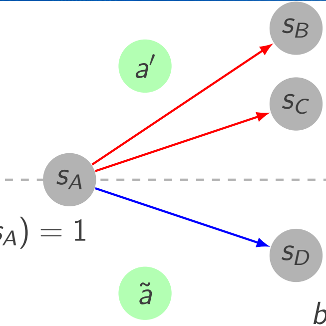
notation $\mathcal{L} = \left\{ l_1 = 1, l_2, l_3, \ldots, 0 \right\}$.



$b_0^\pi(s_A) = 1$

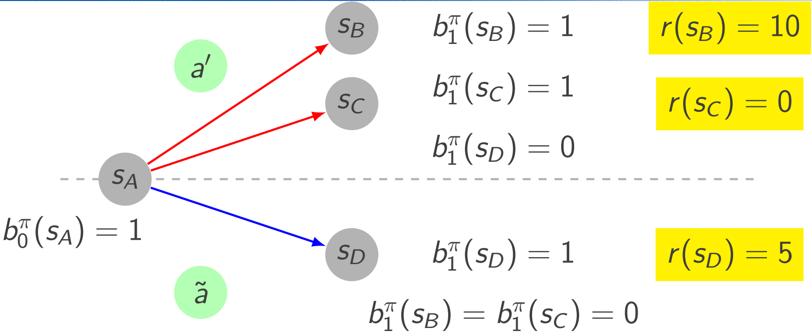
$b_1^\pi(s_B) = 1$ $\quad$ $\boxed{r(s_B) = 10}$

$b_1^\pi(s_C) = 1$ $\quad$ $\boxed{r(s_C) = 0}$

$b_1^\pi(s_D) = 0$

$b_1^\pi(s_D) = 1$ $\quad$ $\boxed{r(s_D) = 5}$

$b_1^\pi(s_B) = b_1^\pi(s_C) = 0$



- $Ch\left(r, N_{b_1^\pi} \;\middle|\; a_0 = \tilde{a}\right) = r(s_D, a') = 5,$
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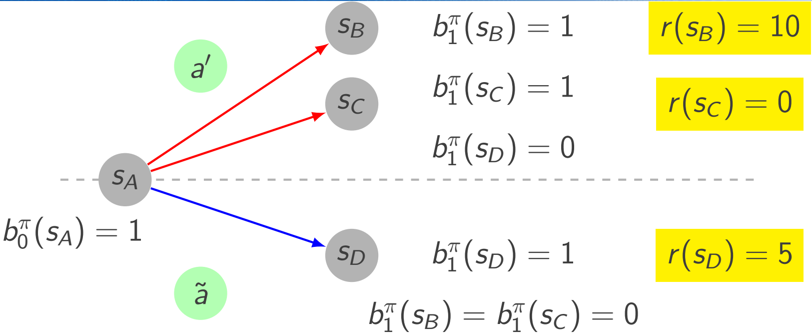
the safe action is prefered! **dispersion reduced**

Choquet integral and rewards

back to the example about ignorance



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if $\mathcal{N}_{b_1^\pi}$ replaced by $b_1 \Rightarrow Ch(r, b_1) = \mathbb{E}_{s \sim b_1}\left[r(s, a)\right].$

resulting MDP
translation summary

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$;
output: the MDP $\langle \tilde{\mathcal{S}}, \mathcal{A}, \tilde{T}, \tilde{r}, \gamma \rangle$:

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- **reward** $\tilde{r}(a, b^\pi) = Ch\Big(r(a, .), \mathcal{N}_{b^\pi}\Big)$,
  $\mathcal{N}_{b^\pi}$ necessity measure computed from b^π.

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$;
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  transitions $\tilde{T}\Big(b^\pi, a, (b^\pi)' \Big) = \mathbf{p}\Big((b^\pi)' \Big| b^\pi, a \Big)$;
- **reward** $\tilde{r}(a, b^\pi) = Ch\Big(r(a, .), \mathcal{N}_{b^\pi} \Big)$,
  $\mathcal{N}_{b^\pi}$ necessity measure computed from b^π.

$$\textbf{criterion: } \mathbb{E}_{(b_t^\pi) \sim \tilde{T}} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot \tilde{r}\left(b_t^\pi, d_t \right) \right].$$

hybrid POMDP and π-POMDP

differences with possibilistic models

	hybrid POMDP	π-POMDP
transitions	probabilities	qualitative possibility
rewards	quantitative $\in \mathbb{R}$	qualitative $\in \mathcal{L}$
situation	-some imprecisions -large POMDP	few quantitative
issues	π definition	commensurability
in practice	MDP	π-MDP

hybrid POMDP and π-POMDP
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in practice	MDP	π-MDP

hybrid model:

- only belief states are possibilistic:

$\rightarrow$ agent knowledge $=$ **possibility** distribution;

- probabilistic dynamics:

$\rightarrow$ **approximated** (prob.) transition between epistemic states.

Plan

factorized POMDP
definition

- $\mathcal{S}$ described by $\mathbb{S} = \{ s_1, \ldots, s_m \}$: $\mathcal{S} = s_1 \times \ldots \times s_m$. Notation: $\mathbb{S}' = \{ s'_1, \ldots, s'_m \}$;

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- **transition** function of s'_j, $T^a_j(\mathbb{S}, s'_j) = \mathbf{p}\left(s'_j \,\middle|\, \mathbb{S}, a \right)$, $\forall j \in \{1, \ldots, m\}$ et $\forall a \in \mathcal{A}$;

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- $\mathcal{O}$ described by $\mathbb{O} = \{ o_1, \ldots, o_n \}$: $\mathcal{O} = o_1 \times \ldots \times o_n$;

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- **observation** function of o'_i,
  $O^a_i(\mathbb{S}', o'_i) = \mathbf{p}\left(o'_i \mid \mathbb{S}', a \right)$, $\forall i \in \{1, \ldots, n\}$ et $\forall a \in \mathcal{A}$.

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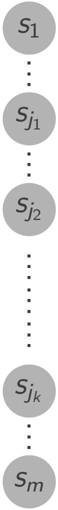
independences:

$\rightarrow \forall s'_i, s'_j \in \mathbb{S}', \qquad s'_i \perp\!\!\!\perp s'_j \mid \{\mathbb{S}, a \in \mathcal{A}\}$,

$\rightarrow \forall o'_i, o'_j \in \mathbb{O}', \qquad o'_i \perp\!\!\!\perp o'_j \mid \{\mathbb{S}', a \in \mathcal{A}\}$.

variables about the **current** system state,



variable s'_j about the **next** state.

Notations

some variables does not interact with each other

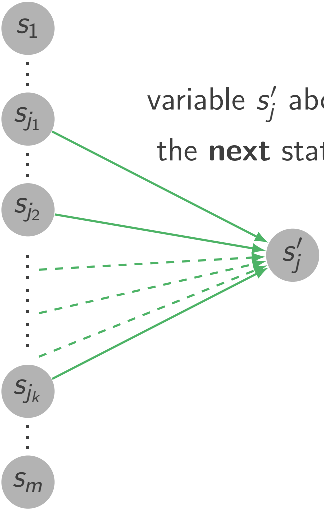
variables about the **current** system state,

$$s_k \rightarrow s'_j$$

$$\Updownarrow$$

$\exists a \in \mathcal{A}$, such that

$T^a_j(\mathbb{S}, s'_j)$ depends on s_k.



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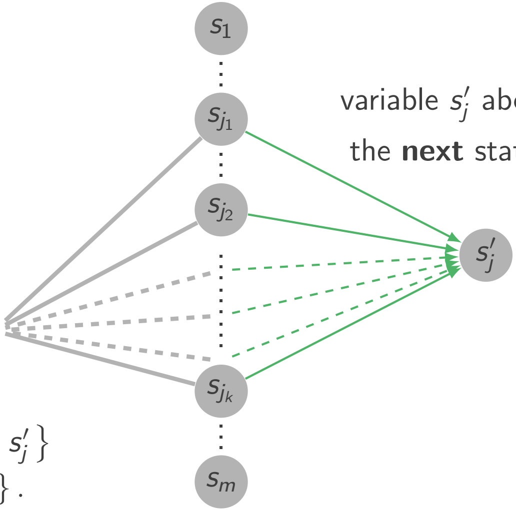
$\mathcal{P}(s'_j)$

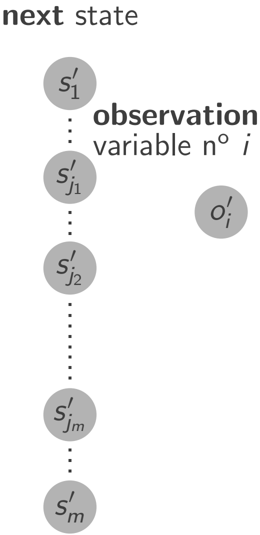


the parents of s'_j are:

$$\mathcal{P}(s'_j) = \{ s \in \mathbb{S} \mid s \rightarrow s'_j \}$$
$$= \{ s_{j_1}, s_{j_2}, \dots, s_{j_k} \} .$$

Notations

concerning observation variables



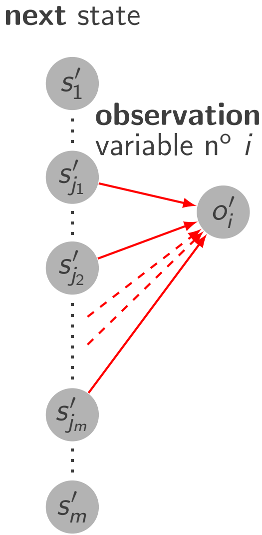
$$s'_j \to o'_i$$

$$\Updownarrow$$

$\exists a \in \mathcal{A}$, such that

$$O_i^a(\mathbb{S}', o'_i)$$

depends on s'_j.



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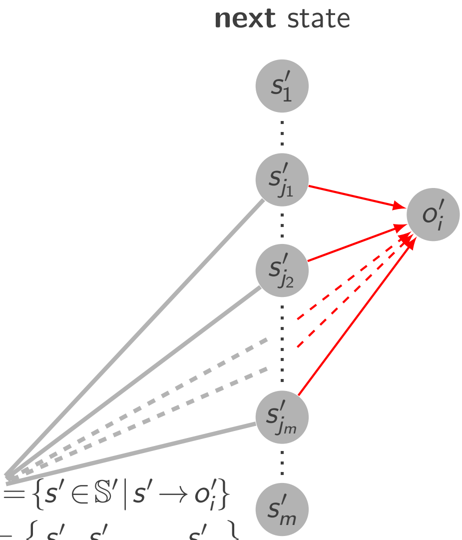

$$\mathcal{P}(o'_i) = \{s' \in \mathbb{S}' \,|\, s' \to o'_i\}$$
$$= \{ s'_{j1}, s'_{j2}, \ldots, s'_{jm} \}$$

Notations

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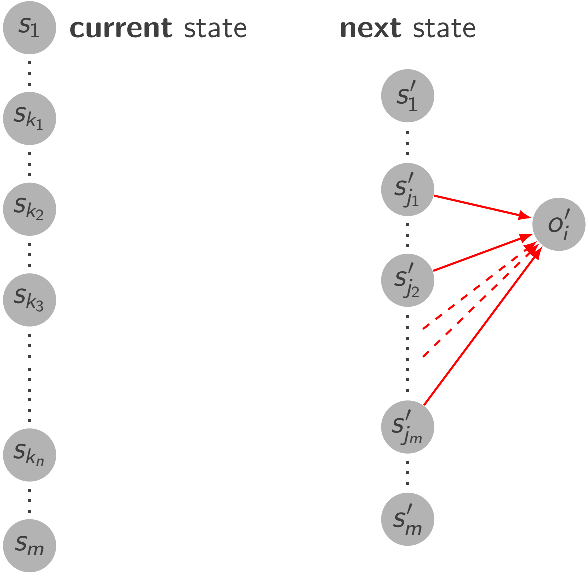

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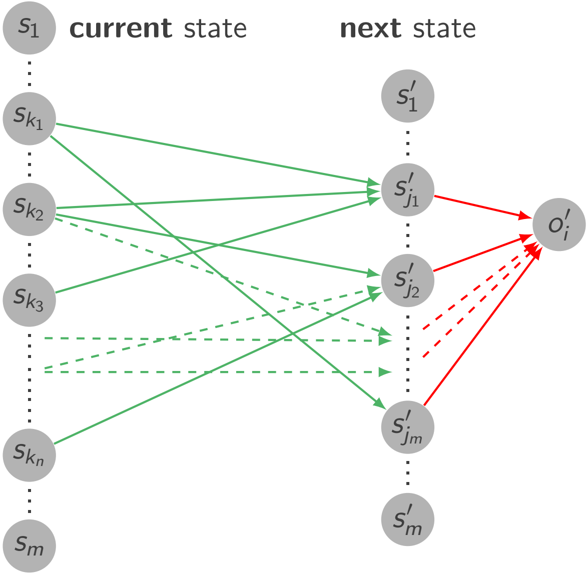

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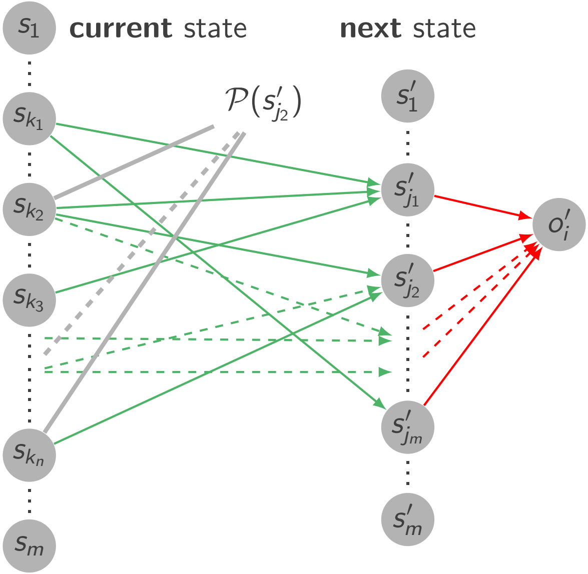

Notations
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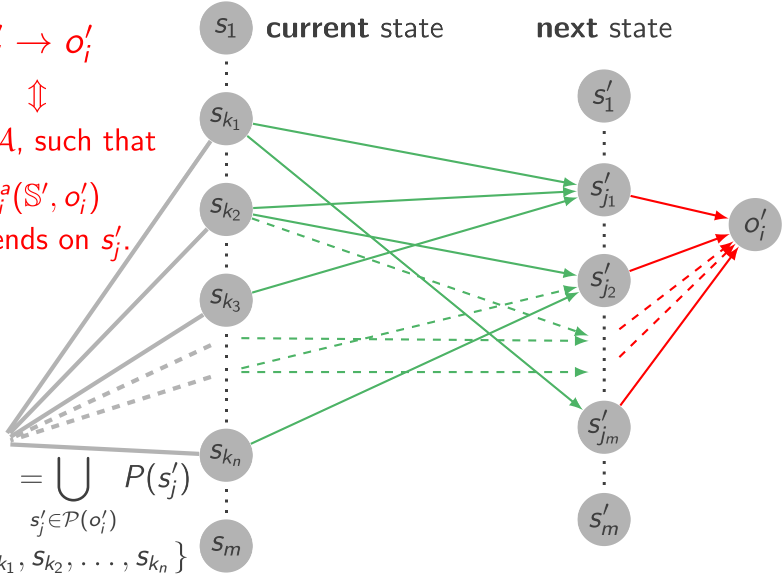

$$\mathcal{Q}(o'_i) = \bigcup_{s'_j \in \mathcal{P}(o'_i)} P(s'_j)$$

$$= \{ s_{k_1}, s_{k_2}, \dots, s_{k_n} \}$$

- $T_j^a\left(\mathbb{S}, s'_j\right) = T_j^a\left(\mathcal{P}(s'_j), s'_j\right)$;
- $O_i^a\left(\mathbb{S}', o'_i\right) = O_i^a\left(\mathcal{P}(o'_i), o'_i\right)$.

- $T_j^a\left(\mathbb{S}, s_j'\right) = T_j^a\left(\mathcal{P}(s_j'), s_j'\right)$;
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consequences on the joint distribution

$$\begin{aligned}\mathbf{p}\left(o_i', \mathcal{P}(o_i') \mid \mathbb{S}, a\right) &= O_i^a\left(\mathcal{P}(o_i'), o_i'\right) \cdot \prod_{s_j' \in \mathcal{P}(o_i')} T_i^a\left(\mathcal{P}(s_j'), s_j'\right) \\ &= \mathbf{p}\left(o_i', \mathcal{P}(o_i') \mid \mathcal{Q}(o_i'), a\right).\end{aligned}$$

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observation probabilities

epistemic state

$$b^\pi(\mathbb{S}) \xrightarrow{\textbf{marginalization}} b^\pi(\mathcal{Q}(o_i')) \xrightarrow[]{\substack{\textbf{pignistic}\\\textbf{transformation}}} \overline{b^\pi}(\mathcal{Q}(o_i'))$$

$$\mathbf{p}\left(o_i' \mid b^\pi, a\right) = \sum_{2^{\mathcal{P}(o_i')}, 2^{\mathcal{Q}(o_i')}} \mathbf{p}\left(o_i', \mathcal{P}(o_i') \mid \mathcal{Q}(o_i'), a\right) \cdot \overline{b^\pi}(\mathcal{Q}(o_i'))$$

Parameters rewritings
POSSIBILISTIC parameters

- $\pi\left(s'_j \mid \mathbb{S}, a\right) = \pi\left(s'_j \mid \mathcal{P}(s'_j), a\right)$;
- $\pi\left(o'_i \mid \mathbb{S}', a\right) = \pi\left(o'_i \mid \mathcal{P}(o'_i), a\right)$.

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marginal possibilistic belief states

$\forall o'_i \in \mathbb{O}$,

$$b^{\pi}_{t+1}\Big(\mathcal{P}(o'_i)\Big) \propto^{\pi} \pi\Big(o'_i, \mathcal{P}(o'_i)\Big| a_0, o_1, \ldots, a_{t-1}, o_t\Big)$$

- $\pi\left(s'_j \mid \mathbb{S}, a\right) = \pi\left(s'_j \mid \mathcal{P}(s'_j), a\right)$;
- $\pi\left(o'_i \mid \mathbb{S}', a\right) = \pi\left(o'_i \mid \mathcal{P}(o'_i), a\right)$.

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$$= \max_{2^{\mathcal{Q}(o'_i)}} \min\left\{\pi\Big(o'_i, \mathcal{P}(o'_i)\Big|\mathcal{Q}(o'_i), a\Big), b^{\pi}_t\Big(\mathcal{Q}(o'_i)\Big)\right\}$$

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- $\pi\left(s'_j \mid \mathbb{S}, a\right) = \pi\left(s'_j \mid \mathcal{P}(s'_j), a\right)$;
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denoted by

$\pi\Big(o'_i, \mathcal{P}(o'_i)\Big| b^{\pi}_t, a\Big)$.

variable: visible $s_v \in \mathbb{S}_v$

s'_v

---

inferred hidden $s_h \in \mathbb{S}_h$

s'_h

---

fully hidden $s_f \in \mathbb{S}_f$

s'_f

Variable classification

3 classes of state variables

variable: visible $s_v \in \mathbb{S}_v$

$$s'_v \xrightarrow{s'_v = o'_v} o'_v$$

inferred hidden $s_h \in \mathbb{S}_h$

$$s'_h$$

fully hidden $s_f \in \mathbb{S}_f$

$$s'_f$$

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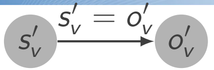

$$\mathbf{p}\left(s'_v \mid b^{\pi}_t, a\right) = \sum_{2^{\mathcal{P}(s'_v)}} T^a\left(\mathcal{P}(s'_v), s'_v\right) \cdot \overline{b^{\pi}_t}\left(\mathcal{P}(s'_v)\right).$$

inferred hidden $s_h \in \mathbb{S}_h$

fully hidden $s_f \in \mathbb{S}_f$

variable: visible $s_v \in \mathbb{S}_v$

$\Leftrightarrow$ deterministic belief variable.

$$s'_v \xrightarrow{s'_v = o'_v} o'_v$$

$$\mathbf{p}\left(s'_v \mid b_t^\pi, a\right) = \sum_{2^{\mathcal{P}(s'_v)}} T^a\left(\mathcal{P}(s'_v), s'_v\right) \cdot \overline{b_t^\pi}\Big(\mathcal{P}(s'_v)\Big).$$

---

inferred hidden $s_h \in \mathbb{S}_h$

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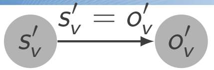
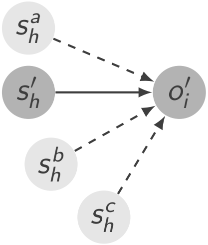


inferred hidden $s_h \in \mathbb{S}_h$



fully hidden $s_f \in \mathbb{S}_f$

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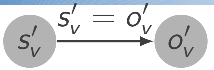
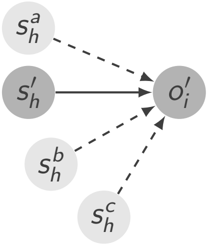


$$\mathbf{p}\left(s'_v \mid b_t^\pi, a\right) = \sum_{2^{\mathcal{P}(s'_v)}} T^a\left(\mathcal{P}(s'_v), s'_v\right) \cdot \overline{b_t^\pi}\Big(\mathcal{P}(s'_v)\Big).$$

inferred hidden $s_h \in \mathbb{S}_h$

$$b_{t+1}^\pi(\mathcal{P}(o'_i)) = b_{t+1}^\pi(s_h, s_h^a, s_h^b, s_h^c)$$



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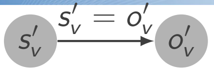

$$\mathbf{p}\left(s'_v \mid b^\pi_t, a\right) = \sum_{2^{\mathcal{P}(s'_v)}} T^a\left(\mathcal{P}(s'_v), s'_v\right) \cdot \overline{b^\pi_t}\Big(\mathcal{P}(s'_v)\Big).$$

---

inferred hidden $s_h \in \mathbb{S}_h$

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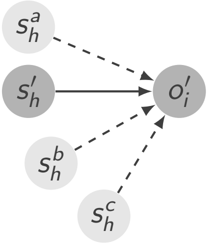

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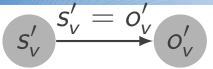

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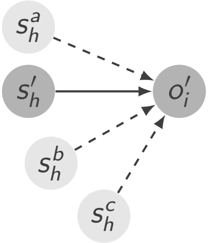

⚠ $\mathcal{P}(o'_i)$ may contain visible variables.

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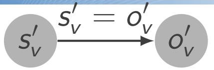

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$$b^\pi_{t+1}(\mathcal{P}(o'_i)) = b^\pi_{t+1}(s_h, s^a_h, s^b_h, s^c_h)$$

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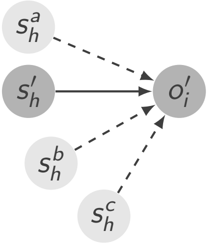

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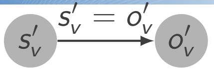

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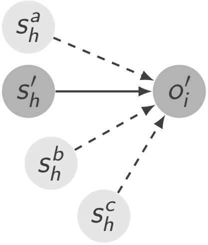


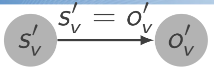
⚠ $\mathcal{P}(o'_i)$ may contain visible variables.

fully hidden $s_f \in \mathbb{S}_f$

$$b^\pi_{t+1}(s'_f) = \max_{2^{\mathcal{P}(s'_f)}} \min \left\{ \pi\left(s'_f \middle| \mathcal{P}(s'_f), a\right), b^\pi_t\left(\mathcal{P}(s'_f)\right) \right\}.$$

variable: visible $s_v \in \mathbb{S}_v$

$\Leftrightarrow$ deterministic belief variable.



$$\mathbf{p}\left(s'_v \mid b^\pi_t, a\right) = \sum_{2^{\mathcal{P}(s'_v)}} T^a\left(\mathcal{P}(s'_v), s'_v\right) \cdot \overline{b^\pi_t}\left(\mathcal{P}(s'_v)\right).$$

---

inferred hidden $s_h \in \mathbb{S}_h$

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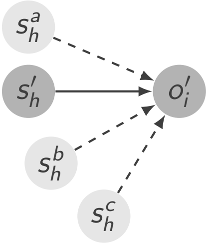

---

fully hidden $s_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f.

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$$\mathbb{O}_h = \mathbb{O} \setminus \mathbb{S}_v.$$

bound over the global belief state

$$b_{t+1}(\mathbb{S}') = \pi\left(\mathbb{S}' \,\middle|\, a_0, o_1, \ldots, a_t, o_{t+1}\right)$$

$$\leqslant \beta_{t+1}(\mathbb{S}')$$
$$= \min\left\{ \min_{s'_j \in \mathbb{S}_v} \left[\mathbb{1}_{\left\{ s'_j = o'_j \right\}} \right], \min_{s'_j \in \mathbb{S}_f} \left[b^{\pi}_{t+1}(s'_j) \right], \min_{o'_i \in \mathbb{O}_h} \left[b^{\pi}_{t+1}\left(\mathcal{P}(o'_i) \right) \right] \right\}$$

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- β_t = **less informative** version of the belief state: $b^{\pi}_t \leqslant \beta_t$;
- computed using **marginal belief states** $\leftrightarrow$ factorization.

Variables de croyance

different according to the class of the variable

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boolean variable "*flipflop*" f changes state at each time step
$\rightarrow$ defines 2 phases:

1. *observation generation*,
2. *belief update* (deterministic knowing the observation).

MDP variables:

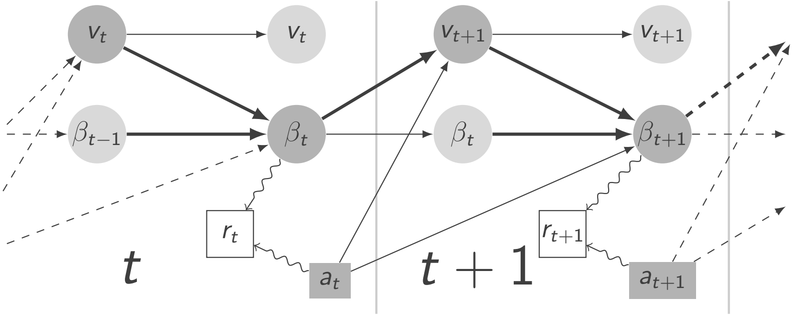
$$\tilde{\mathbb{S}} =$$

beliefs: $\beta = \beta_v^1 \times \ldots \times \beta_v^{m_v} \times \beta_h^1 \times \ldots \times \beta_h^{m_h} \times \beta_f^1 \times \ldots \times \beta_f^{m_f}$

$$\times$$

visible variables : $v = f \times s_v^1 \times \ldots \times s_v^{m_v} \times o_1 \times \ldots \times o_k$.

resulting MDP in practice
final structured MDP



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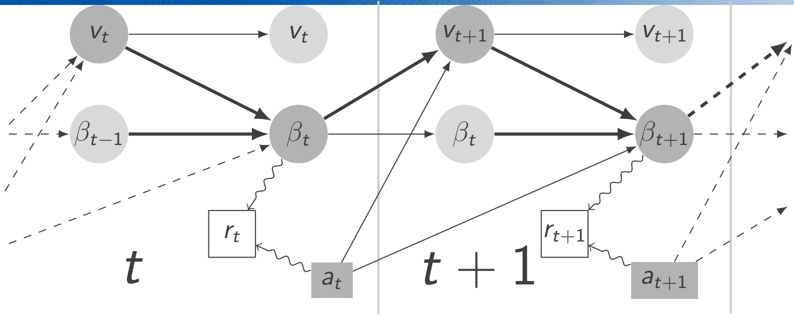
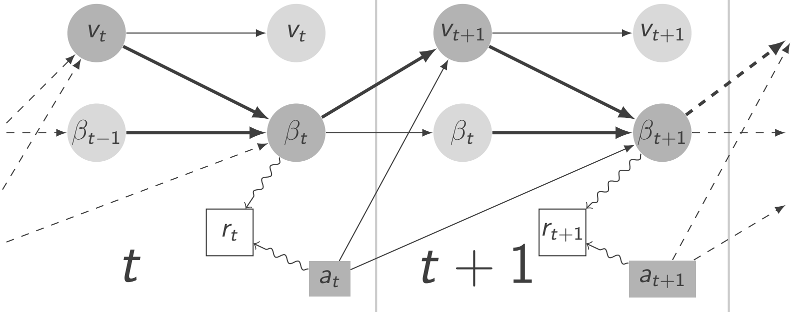


\# **factorized model**'s variables: $\#\mathbb{O} + \#\mathbb{S}_v +$

$$+\sum_{i=1}^{\#\mathbb{O}_h} \left\lceil \log_2\left(\lambda^{2^{p_i}} - (\lambda-1)^{2^{p_i}}\right)\right\rceil + \#\mathbb{S}_f \cdot \left\lceil \log_2\left(2\lambda - 1\right)\right\rceil$$

$\ll$ \# **initial hybrid model**'s variables:

$$\left\lceil \log_2\left(\lambda^{2^{\#\mathbb{S}}} - (\lambda-1)^{2^{\#\mathbb{S}}}\right)\right\rceil$$



\# **factorized model**'s variables:

$$\leqslant \#\mathbb{O} + \#\mathbb{S}_v + \sum_{i=1}^{\#\mathbb{O}_h} \log_2(\lambda) \cdot 2^{p_i} + \#\mathbb{S}_f \cdot (1 + \log_2(\lambda))$$

$\ll$ \# **initial hybrid model**'s variables:

$$\geqslant \log_2(\lambda) \cdot (2^{\#\mathbb{S}} - 1).$$

Plan

Conclusion

POMDP $\xrightarrow{\textbf{translation}}$ MDP with finite state space

- transition probabilities on the **possibilistic belief states**;

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Conclusion

POMDP $\xrightarrow{\textbf{translation}}$ MDP with finite state space

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perspectives:

- IPPC problems (factorized POMDPs);

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perspectives:

- IPPC problems (factorized POMDPs);
- test approaches:
  1. **simplification:** distributions π definition (π-normalization of $\mathbb{P}$, pignistic transformation, the more specific, . . .);
  2. **imprecision:** more robust?

Thank you!