Exploiting Imprecise Information Sources in Sequential Decision Making Problems under Uncertainty

N.Drougard

under **D.Dubois**, **J-L.Farges** and **F.Teichteil-Königsbuch** supervision

doctoral school: EDSYS **institution:** ISAE–SUPAERO

laboratory: ONERA–The French Aerospace Lab

retour sur innovation

Overview

Overview

Onera, DCSD

Automatics, AI, Flight Mechanics, Cognitive Sciences

Onera, DCSD

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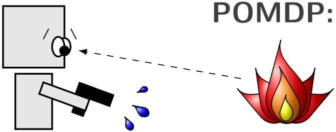
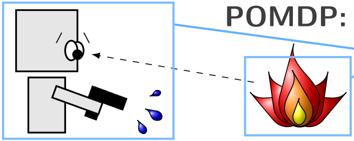
among many other works:

- autonomy, steering architectures and human factors
- decision making, planning
- experimental/industrial applications: UAVs,
  orbital systems,
  exploration robots



POMDP: model for autonomous robotics



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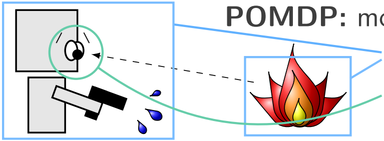
$s \in \mathcal{S}$: **system states**



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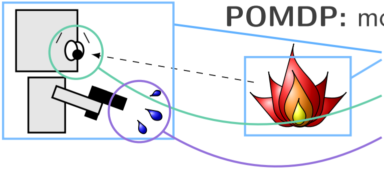

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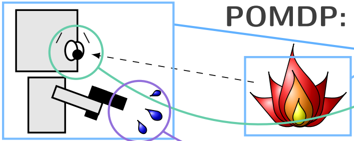

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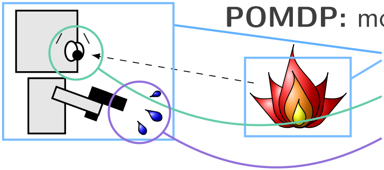
Context
Partially Observable Markov Decision Processes (POMDPs)



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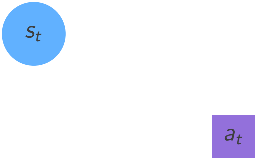

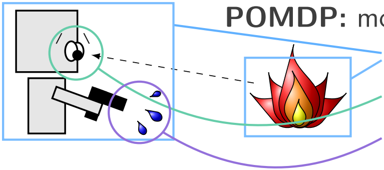

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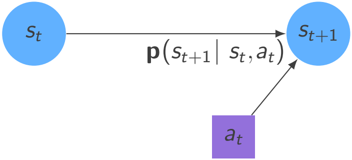

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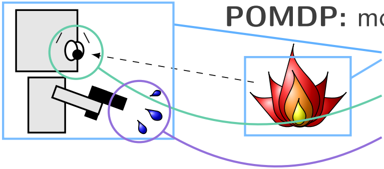

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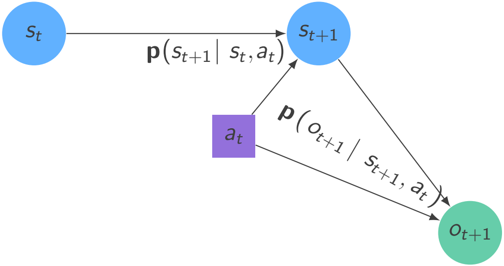

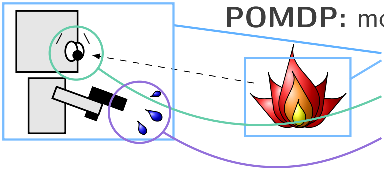

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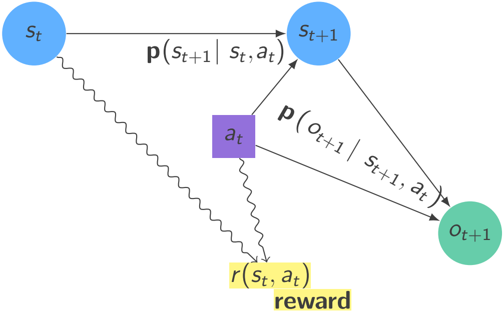

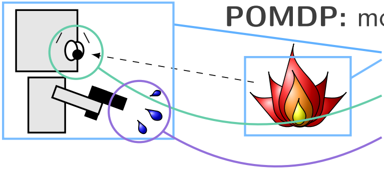

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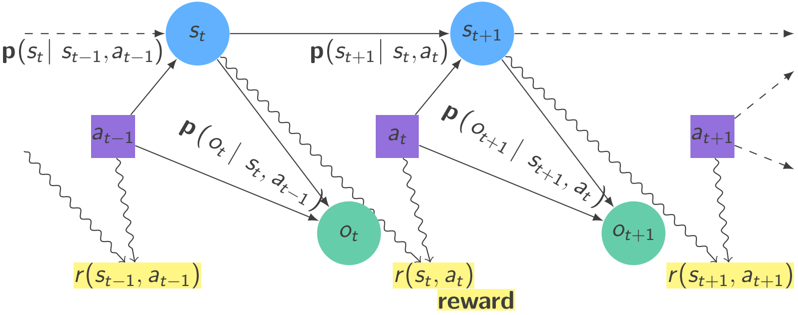

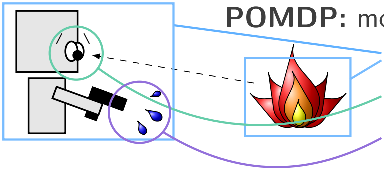

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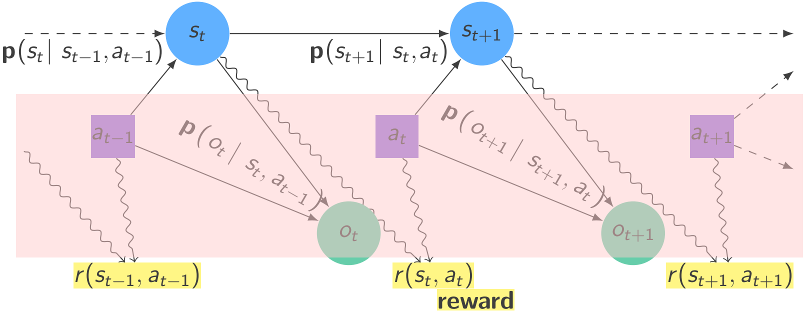

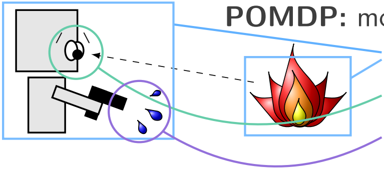

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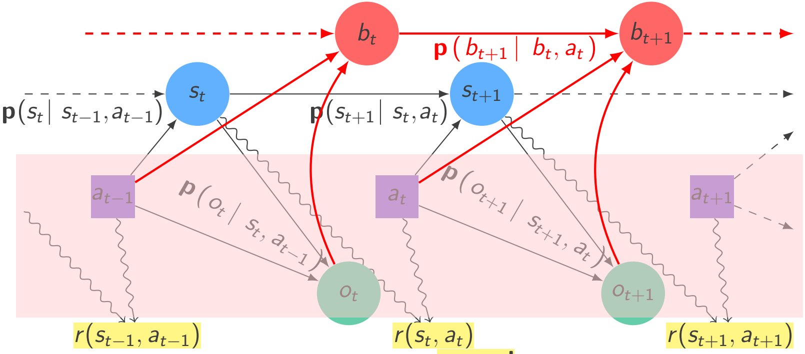
b **belief**



POMDP: $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$ *(Smallwood et al. 1973)*

- **transition** function $T(s,a,s') = \mathbf{p}\left(s' \mid s, a \right)$
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probabilistic belief update – a selected, o' received

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action choices: strategy $\delta(b_t) = a_t \in \mathcal{A}$

$$\text{maximizing } \mathbb{E}_{s_0 \sim b_0}\left[\sum_{t=0}^{+\infty} \gamma^t \cdot r\Big(s_t, \delta(b_t)\Big)\right], \; 0 < \gamma < 1$$

- optimal strategy computation $\geqslant$ **PSPACE** (*Papadimitriou et al. 1987*)
- probabilities are **imprecisely known** in practice
- agent's **ignorance** not taken into account

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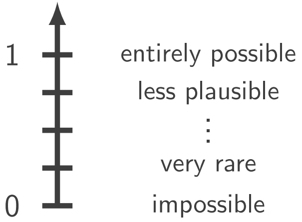
$\rightarrow$uniform probability distribution $\neq$ ignorance!

Qualitative Possibility Theory

presentation – (max,min) “tropical” algebra

finite scale $\mathcal{L}$

usually $\{0, \frac{1}{k}, \frac{2}{k}, \ldots, 1\}$



events $e \subset \Omega$ (universe)

sorted using possibility **degrees** $\pi(e) \in \mathcal{L}$

$$\neq$$

quantified with **frequencies** $\mathbf{p}(e) \in [0,1]$ (probabilities)

finite scale $\mathcal{L}$

usually $\{0, \frac{1}{k}, \frac{2}{k}, \dots, 1\}$

1 — entirely possible

less plausible

⋮

very rare

0 — impossible

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$\neq$

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$e_1 \neq e_2$, 2 events $\subset \Omega$

- $\pi(e_1) < \pi(e_2) \Leftrightarrow$ "e_1 is less plausible than e_2"

Probability / Possibility:

$+$	max
$\times$	min
$X \in \mathbb{R}$	$X \in \mathcal{L}$
$\mathbb{E}[X] = \sum_{x \in X} x \cdot \mathbf{p}(x)$	**optimistic:** $\mathbb{S}_{\Pi}[X] = \max_{x \in X} \min \{ x, \pi(x) \}$ **cautious:** $\mathbb{S}_{\mathcal{N}}[X] = \min_{x \in X} \max \{ x, 1 - \pi(x) \}$

Sabbadin (UAI-98) introduces

the qualitative possibilistic POMDP

π-**POMDP:** $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T^{\pi}, O^{\pi}, \rho \rangle$

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- **preference** function $\rho : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{L}$

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- belief space trick: POMDP $\rightarrow$ MDP with **infinite** $\mathcal{S}$
  π-POMDP $\rightarrow$ π-MDP with **finite** $\mathcal{S}$
- $\forall s \in \mathcal{S}$, $\pi(s) = 1 \Leftrightarrow$ total ignorance about s

A possibilistic belief state
finite belief space

$\Pi_{\mathcal{L}}^{\mathcal{S}} = \Big\{$ possibility distributions $\Big\}$: $\#\Pi_{\mathcal{L}}^{\mathcal{S}} \sim \#\mathcal{L}^{\#\mathcal{S}} < +\infty$

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joint distribution on $\mathcal{S} \times \mathcal{O}$ from b^{π}_t: $\pi \left(o', s' \mid b^{\pi}_t, a \right)$

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- the update **only depends on** o'**,** a **and** b^{π}_t

ONERA THE FRENCH AEROSPACE LAB

Qualitative Possibility Theory:

$\rightarrow$ simplification, ignorance and imprecision modeling

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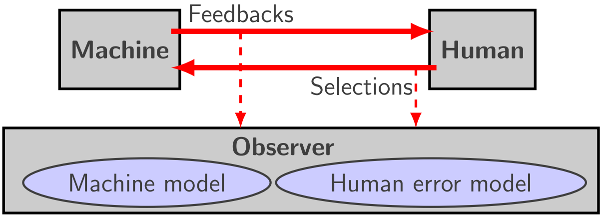
1. introduction
2. natural use of a qualitative possibilistic model
3. updates and first use of the π-POMDP model
4. simplify computation: ADDs and factorization
5. probabilistic-possibilistic (hybrid) approach
6. conclusion

Overview

Example: Human-Machine Interaction
joint work with Sergio Pizziol – Context



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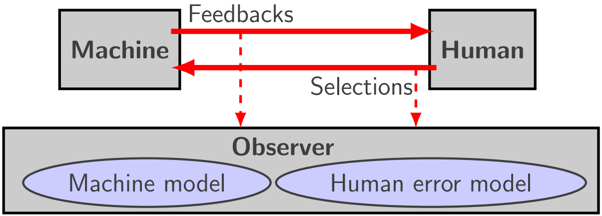

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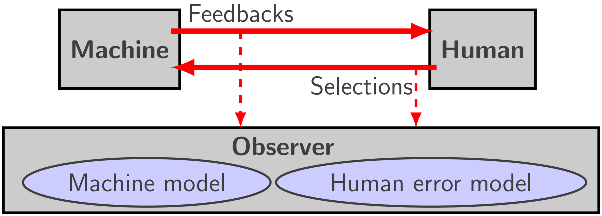

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π-POMDP without actions: π-Hidden Markov Process

- **system space** $\mathcal{S}$: set of human assessments $\rightarrow$ **hidden**
- **observation space** $\mathcal{O}$: feedbacks/human selections

Machine with states A, B, C, ...

state $s_A \in \mathcal{S}$: "human thinks machine state is A"

Example: Human-Machine Interaction
Human error model from expert knowledge

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Machine state $A \to B$

- **machine feedback** observation $o_f \in \mathcal{O}$:

human usually aware of feedbacks $\to \pi\left(s'_B, o'_f \mid s_A\right) = 1$
but may lost a feedback $\to \pi\left(s'_A, o'_f \mid s_A\right) = \frac{2}{3}$

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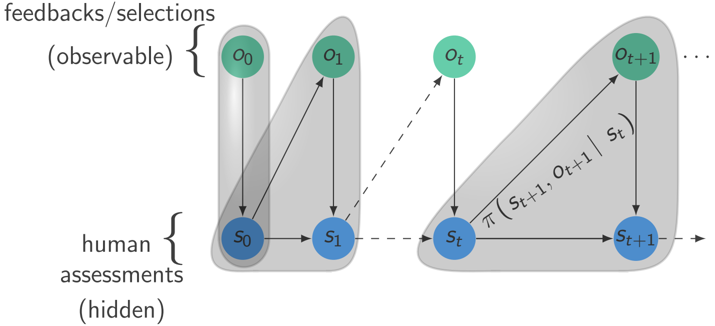
- impossible cases: possibility degree 0

Qualitative Possibilistic Hidden Markov Process:
diagnosis tool for Human-Machine interaction (with Sergio Pizziol)



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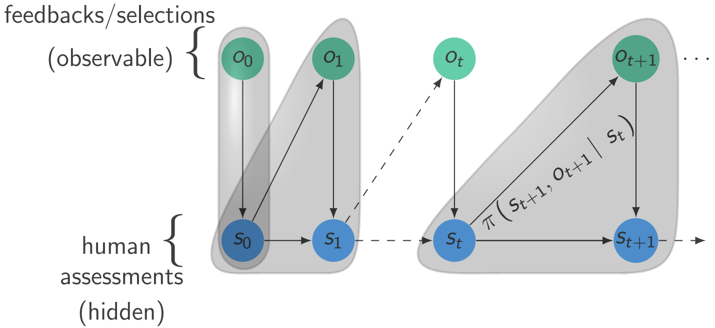
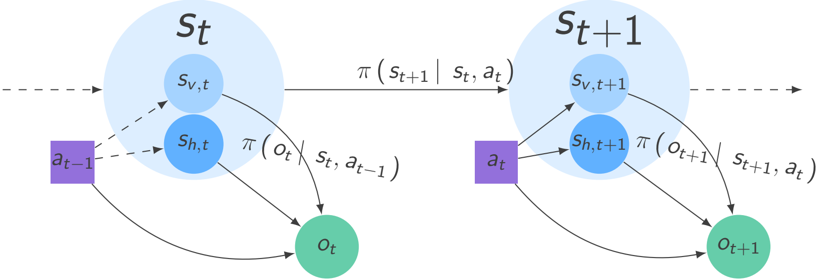


- **estimation** of the human assessment
  $\Leftrightarrow$ **possibilistic belief state**
- **detection** of human assessment errors
- **diagnosis** using *leximin* operator
- results on flight simulator missions with pilots

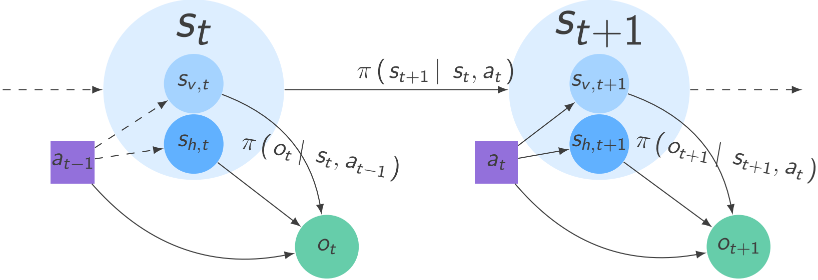
Overview

contribution (UAI-13):



Mixed-Observability: system state $s \in \mathcal{S} = \mathcal{S}_v \times \mathcal{S}_h$
i.e. state s = visible component s_v & hidden component s_h

contribution (UAI-13):



Mixed-Observability: system state $s \in \mathcal{S} = \mathcal{S}_v \times \mathcal{S}_h$
i.e. state s = visible component s_v & hidden component s_h

- belief states only over $\mathcal{S}_h$ (component s_v observed)

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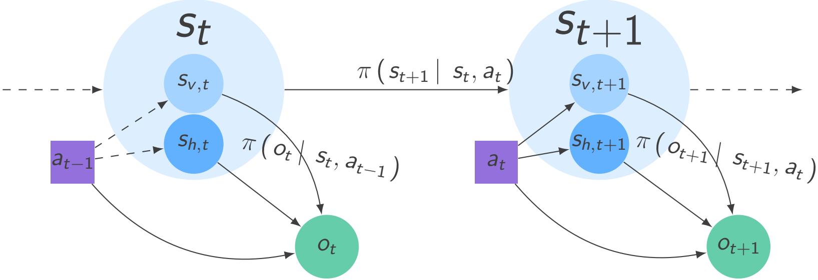

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- $\rightarrow$ π-POMDP: belief space $\Pi_{\mathcal{L}}^{\mathcal{S}}$ $\quad \#\Pi_{\mathcal{L}}^{\mathcal{S}} \sim \#\mathcal{L}^{\#\mathcal{S}}$

  $\rightarrow$ π-MOMDP: computations on $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$

  $\#\mathcal{X} \sim \#\mathcal{S}_v \cdot \#\mathcal{L}^{\#\mathcal{S}_h} \ll \#\Pi_{\mathcal{L}}^{\mathcal{S}}$

contribution (UAI-13): Undefined Horizon

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Dynamic Programming scheme: $\#$ iterations $< \#\mathcal{X}$

- assumption: $\exists$ artificial "stay" action
  as in classical planning/ γ counterpart
- value function $=$ criterion: non decreasing with horizon

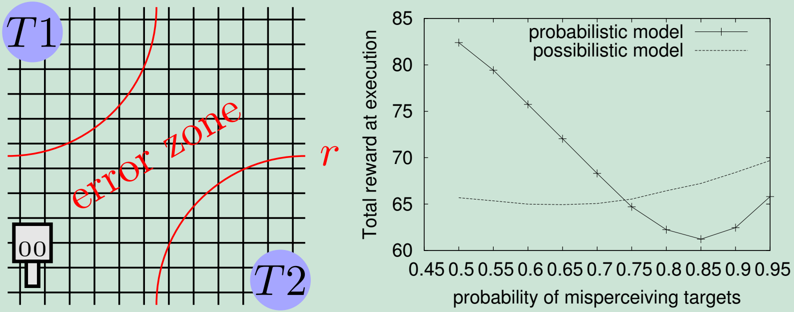
contribution (UAI-13): Undefined Horizon

Dynamic Programming scheme: $\#$ iterations $< \#\mathcal{X}$

- assumption: $\exists$ artificial "stay" action
  as in classical planning/ γ counterpart
- value function = criterion: non decreasing with horizon
- action update for states increasing the value function
- proof of optimality

- **goal:** reach the object $A = T1$ or $T2$
- noisy observations of the location of the object A

Recognition mission: robot on a grid, targets $T1$ & $T2$



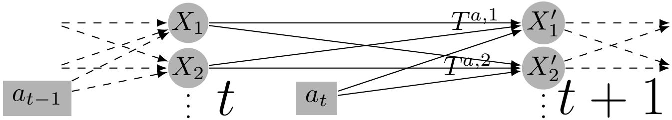
in reality, misperception probability in the error zone: $P_{bad} > \frac{1}{2}$

Overview

Factored π-MOMDP and computations with ADDs

qualitative possibilistic models to reduce complexity

contribution (AAAI-14): factored π-MOMDP
$\Leftrightarrow$ state space $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$ = Boolean variables $(X_1, \ldots, X_n)$
+ independence assumptions $\Leftarrow$ graphical model



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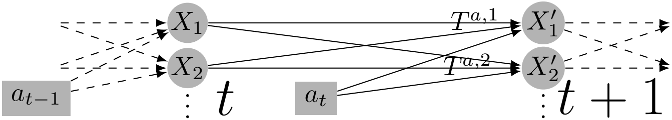
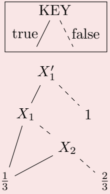


- **factorization:** transition functions $T_i^a = \pi\left(X'_i \mid parents(X'_i), a\right)$ stored as **Algebraic Decision Diagrams (ADD)**

  probabilistic case:
  SPUDD, *Hoey et al., UAI-99*



example of ADD

- probabilistic model: $+$ and $\times \Rightarrow$ new values created
  $\Rightarrow$ number of ADDs leaves **potentially huge**
- possibilistic model: min and max $\Rightarrow$ values $\in \mathcal{L}$ finite
  $\Rightarrow$ number of leaves bounded, **ADDs smaller**.

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PPUDD: Possibilistic Planning Using Decision Diagrams

- factorization $\Rightarrow$ DP steps divided into n stages
  $\rightarrow$ used ADDs smaller $\Rightarrow$ **faster computations**

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PPUDD: Possibilistic Planning Using Decision Diagrams

- factorization $\Rightarrow$ DP steps divided into n stages
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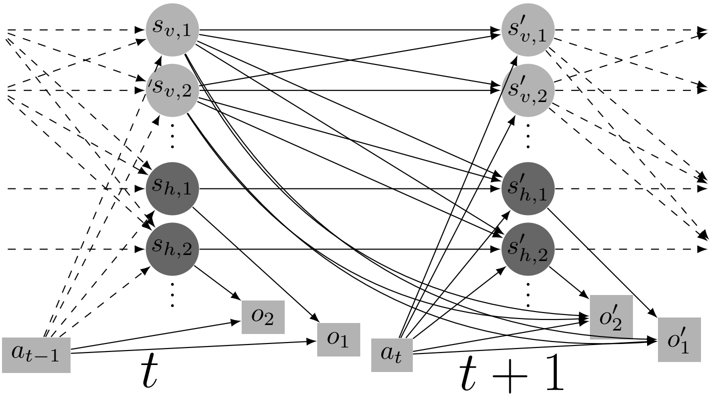
- computations on trees: *CU Decision Diagram Package*.

Simplify computations with π-MOMDPs

Natural factorization: belief independence

contribution (AAAI-14):

independent sensors, hidden states, ... $\Rightarrow$ graphical model



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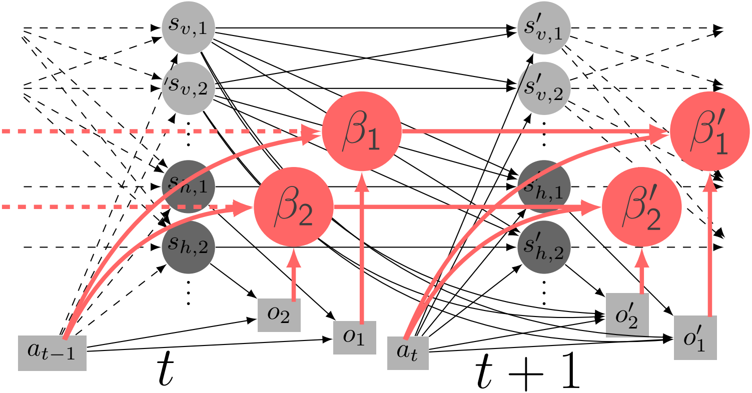
contribution (AAAI-14):

independent sensors, hidden states, ... $\Rightarrow$ graphical model

d-Separation $\Rightarrow$ $(s_v, \beta) = (s_{v,1}, \ldots, s_{v,m}, \beta_1, \ldots, \beta_l)$

$\beta_i \in \Pi_{\mathcal{L}}^{\mathcal{S}_{h,i}}$, belief over $s_{h,i}$



PPUDD vs SPUDD *Hoey et al.*

Navigation benchmark: reach a goal – spots with accident risk

M1 (resp. M2) optimistic (resp. cautious) criterion

PPUDD vs SPUDD *Hoey et al.*

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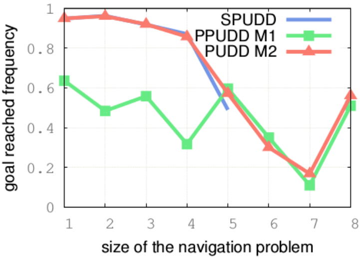
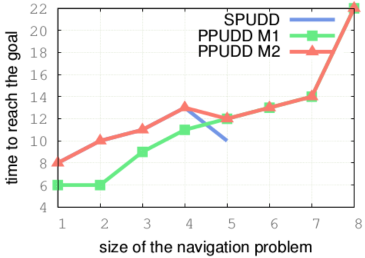
Performances, function of the instance size

reached goal frequency

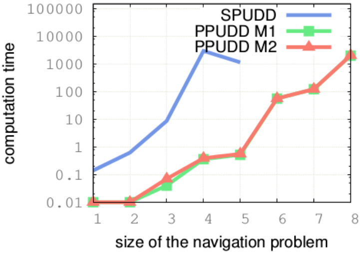


\# steps to reach the goal

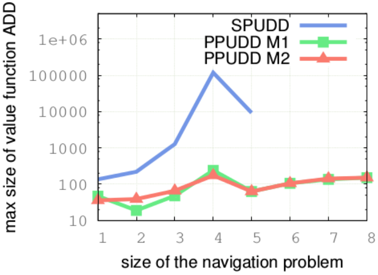


Simplify computations with π-MOMDPs
Experiments – perfect sensing: Navigation problem

computation time



max size of ADDs



- PPUDD + M2 (pessimistic criterion)
  faster with same performances as SPUDD
- SPUDD only solves the first 5 instances
- verified intuition: ADDs are smaller

PPUDD vs APPL *Kurniawati et al.*, solver MOMDP
symbolic HSVI *Sim et al.*, solver POMDP
RockSample benchmark: recognize and sample “good” rocks

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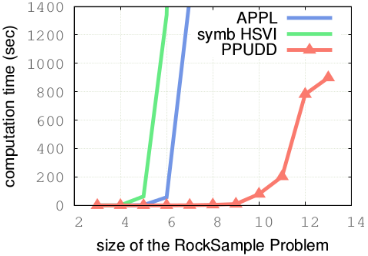
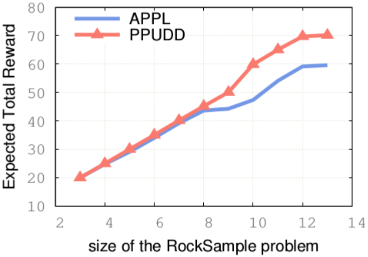
computation time:
probabilistic solvers, prec. 1
PPUDD, exact resolution



average of rewards
APPL stopped when
PPUDD end



- **approximate model + exact resolution solver**
  $\rightarrow$ improvement of computation time and performances

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

PPUDD + BDD mask over reachable states.

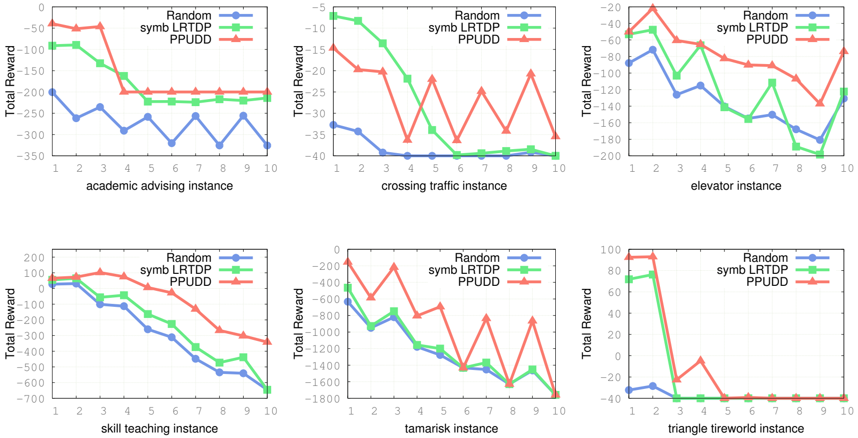


Figure : average of rewards over simulations

Overview

Qualitative possibilistic approach: benefits/drawbacks

towards a hybrid POMDP

Qualitative Possibilistic models:

- **granulated** belief space (discrete)
- efficient problem **simplification** (PPUDD $2\times$ better than LRTDP with ADDs)
- **ignorance and imprecision** modeling

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- ADD methods $\prec$ state space search methods: winners of IPPC 2014, 2$\times$ better than PPUDD
- choice of the qualitative criterion (optimistic/pessimistic)
- non additive utility degrees
  same scale as possibility degrees (commensurability)
- frequentist information lost

A hybrid model

a probabilistic POMDP with possibilistic belief states

hybrid approach

- agent knowledge = possibilistic belief states
- probabilistic dynamics & quantitative rewards

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$\rightarrow$ **heuristic** for solving POMDPs:

results in a standard (finite state space) MDP

$\rightarrow$ problem with **qualitative** & **quantitative** uncertainty

Transitions and rewards

belief-based transition and reward functions

- possibility distribution $\beta \rightarrow$ probability distribution $\overline{\beta}$ using poss-prob tranformations (Dubois et al., FSS-92)

$$\Rightarrow \mathbf{p}\left(\beta' \middle| \beta, a\right) = \sum_{\substack{o' \text{ t.q.} \\ update(\beta,a,o')=\beta'}} \mathbf{p}\left(o' \mid \beta, a\right)$$

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- reward cautious according to β

Pessimistic Choquet Integral

$$r(\beta, a) = \sum_{i=1}^{\#\mathcal{L}-1} (l_i - l_{i+1}) \cdot \min_{\substack{s \in \mathcal{S} \text{ s.t.} \\ \beta(s) \geqslant l_i}} r(s, a)$$

Resulting MDP

translation summary **contribution (SUM-15):**

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$;
output: the MDP $\langle \tilde{\mathcal{S}}, \mathcal{A}, \tilde{T}, \tilde{r}, \gamma \rangle$:

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$$\textbf{criterion: } \mathbb{E}_{\beta_t \sim \tilde{T}} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot \tilde{r}\left(\beta_t, d_t\right) \right].$$

General variable classification **contribution (SUM-15):**

3 classes of state variables – state space factorization

variable: visible $s'_v \in \mathbb{S}_v$

s'_v

---

inferred hidden $s'_h \in \mathbb{S}_h$

s'_h

---

fully hidden $s'_f \in \mathbb{S}_f$

s'_f

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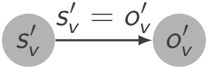

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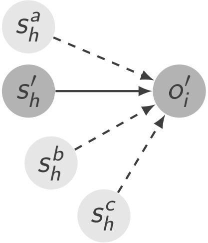

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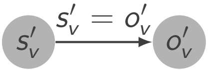

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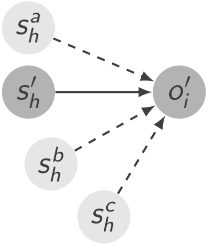

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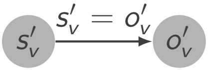

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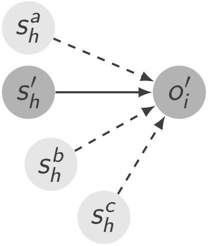

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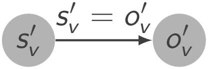

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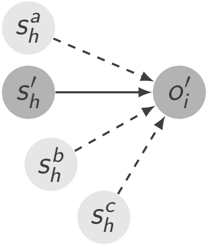

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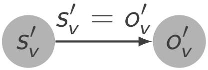

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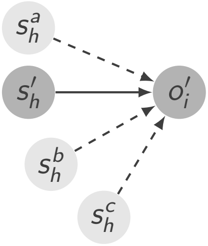

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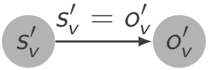

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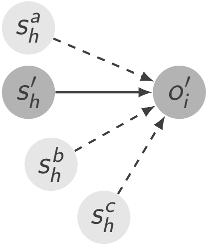

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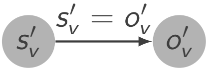

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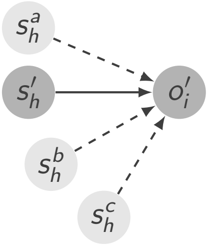
⚠ $\mathcal{P}(o'_i)$ may contain visible variables.



---

fully hidden $s'_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f.

$$\beta_{t+1}(s'_f) = \pi\left(s'_f \mid \beta_t, a\right)$$

Possibilistic belief variables
global belief state

bound over the global belief state

$$\beta_{t+1}(s'_1, \dots, s'_n) = \pi\left(s'_1, \dots, s'_n \mid a_0, o_1, \dots, a_t, o_{t+1}\right)$$

$$\leqslant \min\left\{ \min_{s'_j \in \mathbb{S}_v} \left[\mathbb{1}_{\left\{ s'_j = o'_j \right\}} \right], \min_{s'_j \in \mathbb{S}_f} \left[\beta_{t+1}(s'_j) \right], \min_{o'_i \in \mathbb{O}_h} \left[\beta_{t+1}\left(parents(o'_i) \right) \right] \right\}$$

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- min of marginals = **less informative** belief state
- computed using **marginal belief states**
  $\rightarrow$ factorization & smaller state space

Overview

Conclusion
contributions

- udates: $\rightarrow$ mixed-observability modeling
  $\rightarrow$ undefined horizon
- modeling: $\rightarrow$ human-machine interaction
  $\rightarrow$ robust recognition mission with possibilistic beliefs
- computations: factorization work & PPUDD algorithm (competitive solver, IPPC 2014)

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new refined criteria $\rightarrow$ finer π-POMDP

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new refined criteria $\rightarrow$ finer π-POMDP

quantitative information may be available: hybrid work

POMDP $\xrightarrow{\textbf{translation}}$ MDP with finite state space

- transition probabilities on the **possibilistic belief states**;

$\text{POMDP} \xrightarrow{\textbf{translation}} \text{MDP with finite state space}$

- transition probabilities on the **possibilistic belief states**;
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perspectives:

- IPPC problems (factored POMDPs);

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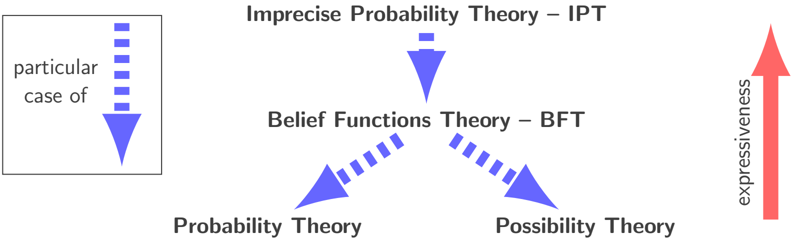
perspectives:

- IPPC problems (factored POMDPs);
- tests of this approach:
  1. **simplification:** π distributions definition?
  2. **imprecision:** robust in practice?

Thank you!

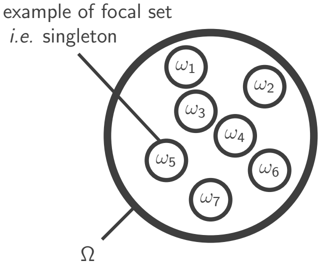


- IPT: most general uncertainty theory.
  Use of sets of probability measures over Ω.
- BFT: use of a mass function $m : 2^{\Omega} \to [0,1]$, with $\sum_{A \subset \Omega} m(A) = 1$.
  1. plausibility measure: $\forall A \subset \Omega$, $Pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$.
  2. belief function: $\forall A \subset \Omega$, $bel(A) = \sum_{B \subseteq A} m(B)$.

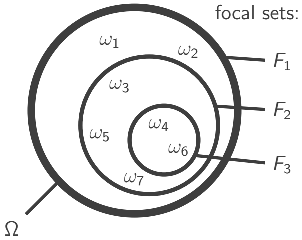
Focal sets of a mass function $m : 2^{\Omega} \to [0,1]$: subsets A of $\Omega = \{\omega_1, \ldots, \omega_7\}$ such that $m(A) > 0$.

- if focal sets are all singletons
  $\to$ probability distribution ($bel = Pl = \mathbb{P}$)
- if focal sets are nested, e.g. $F_3 \subset F_2 \subset F_1 = \Omega$,
  $\to$ possibility distribution:
  bel=**necessity measure**, Pl=**possibility measure**.

probabilistic case



possibilistic case



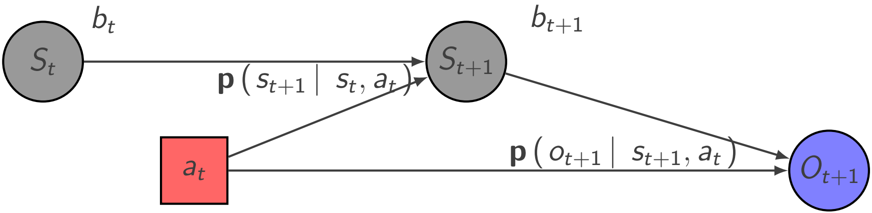


Figure : Bayesian Network illustrating the belief update

- the **system states** are the gray circular nodes,
- the **action** is the red square node ,
- and the **observation** is the blue circular node.

The belief state b_t (resp. b_{t+1}) is the probabilistic estimation of the current (resp. next) system state s_t (resp. s_{t+1})

probabilistic belief update

$$b_{t+1}(s') \propto \mathbf{p}\left(o' \mid s', a\right) \cdot \sum_{s \in \mathcal{S}} \mathbf{p}\left(s' \mid s, a\right) \cdot b_t(s)$$

PROBABILISTIC parameters

- $T_j^a\left(\mathbb{S}, s_j'\right) = T_j^a\left(\mathcal{P}(s_j'), s_j'\right)$;
- $O_i^a\left(\mathbb{S}', o_i'\right) = O_i^a\left(\mathcal{P}(o_i'), o_i'\right)$.

Rewritings of parameters
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consequences on the joint distribution

$$\begin{aligned}\mathbf{p}\left(o'_i, \mathcal{P}(o'_i) \mid \mathbb{S}, a\right) &= O_i^a\left(\mathcal{P}(o'_i), o'_i\right) \cdot \prod_{s'_j \in \mathcal{P}(o'_i)} T_i^a\left(\mathcal{P}(s'_j), s'_j\right) \\ &= \mathbf{p}\left(o'_i, \mathcal{P}(o'_i) \mid \mathcal{Q}(o'_i), a\right).\end{aligned}$$

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observation probabilities

epistemic state

$$b^\pi(\mathbb{S}) \xrightarrow{\textbf{marginalization}} b^\pi(\mathcal{Q}(o'_i)) \xrightarrow{\substack{\textbf{pignistic} \\ \textbf{transformation}}} \overline{b^\pi}(\mathcal{Q}(o'_i))$$

$$\mathbf{p}\left(o'_i \mid b^\pi, a\right) = \sum_{2^{\mathcal{P}(o'_i)}, 2^{\mathcal{Q}(o'_i)}} \mathbf{p}\left(o'_i, \mathcal{P}(o'_i) \mid \mathcal{Q}(o'_i), a\right) \cdot \overline{b^\pi}(\mathcal{Q}(o'_i))$$

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marginal possibilistic belief states

$\forall o'_i \in \mathbb{O}$,

$$b^{\pi}_{t+1}\Big(\mathcal{P}(o'_i)\Big) \propto^{\pi} \pi\Big(o'_i, \mathcal{P}(o'_i)\Big| a_0, o_1, \ldots, a_{t-1}, o_t\Big)$$

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denoted by $\pi\Big(o'_i, \mathcal{P}(o'_i) \Big| b^{\pi}_t, a\Big)$.

A possibilistic belief state

finite belief space

$$\Pi_{\mathcal{S}}^{\mathcal{L}} = \Big\{ \text{ possibility distributions } \Big\} : \ \#\Pi_{\mathcal{S}}^{\mathcal{L}} < +\infty$$

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$$b_{t+1}^\pi(s') = \begin{cases} 1 & \text{if } \pi\left(o', s' \mid b_t^\pi, a\right) = \pi\left(o' \mid b_t^\pi, a\right) \\ \pi\left(o', s' \mid b_t^\pi, a\right) & \text{otherwise.} \end{cases}$$

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- the update **only depends on** o' **and** a.

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if $V_{i+1}(x) > V_i(x)$, $\delta(x) = \operatorname{argmax}_{a \in \mathcal{A}} \max_{x' \in \mathcal{X}} \min\{\pi\left(x' \mid x, a \right), V_i(x')\}$.

Resulting π-MOMDP solver: PPUDD

- probabilistic model: $+$ and $\times \Rightarrow$ new values created, number of ADDs leaves **potentially huge**.
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PPUDD: Possibilistic Planning Using Decision Diagrams

$V^* \leftarrow 0$; $V^c \leftarrow \mu$; $\delta \leftarrow \overline{a}$;
while $V^* \neq V^c$ **do**
    $V^* \leftarrow V^c$;
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return (V^*, δ) ;

computations on trees: *CU Decision Diagram Package*.

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factorization

$\Rightarrow$ dynamic programming

divided into *n* stages

used ADDs smaller

$\rightarrow$ **faster computations.**

computations on trees: *CU Decision Diagram Package*.

numbering of the $n = \#\mathcal{S}$ system states:
$1 = b^\pi(s_1) \geqslant \ldots \geqslant b^\pi(s_n) \geqslant b^\pi(s_{n+1}) = 0.$

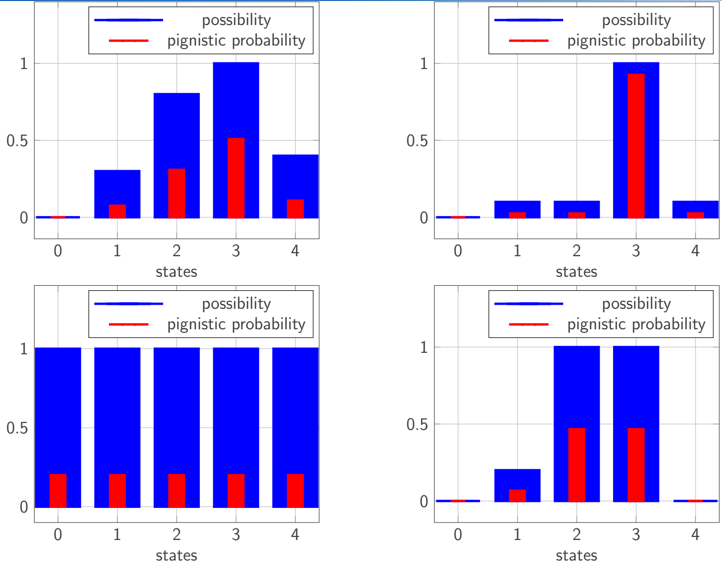
pignistic transformation – $P : \Pi_\mathcal{S} \to \mathbb{P}_\mathcal{S}$

$$\overline{b^\pi}(s_i) = \sum_{j=i}^{\#\mathcal{S}} \frac{b^\pi(s_j) - b^\pi(s_{j+1})}{j}.$$

- probability distribution $\overline{b^\pi}$ = **gravity center** of the represented probabilistic distributions;
- **Laplace principle**: ignorance $\to$ uniform probability.

Pignistic transformation

Examples of pignistic transformations (red) of possibility distributions (blue)



hybrid POMDP and π-POMDP

differences with possibilistic models

	hybrid POMDP	π-POMDP
transitions	probabilities	qualitative possibility
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situation	-some imprecisions -large POMDP	few quantitative
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hybrid model:

- only belief states are possibilistic:

$\rightarrow$ agent knowledge $=$ **possibility** distribution;

- probabilistic dynamics:

$\rightarrow$ **approximated** (prob.) transition between epistemic states.

factorized POMDP
definition

- $\mathcal{S}$ described by $\mathbb{S} = \{ s_1, \ldots, s_m \}$: $\mathcal{S} = s_1 \times \ldots \times s_m$.
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independences:

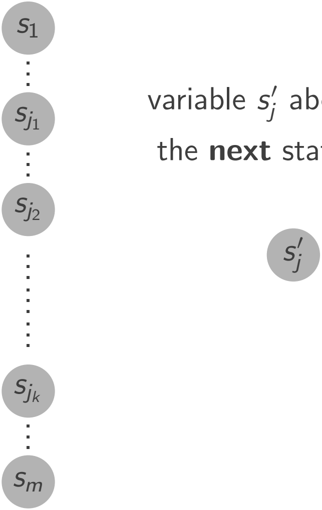
$\rightarrow \forall s'_i, s'_j \in \mathbb{S}', \qquad s'_i \perp\!\!\!\perp s'_j \mid \{\mathbb{S}, a \in \mathcal{A}\}$,

$\rightarrow \forall o'_i, o'_j \in \mathbb{O}', \qquad o'_i \perp\!\!\!\perp o'_j \mid \{\mathbb{S}', a \in \mathcal{A}\}$.

some variables does not interact with each other

variables about the **current** system state,

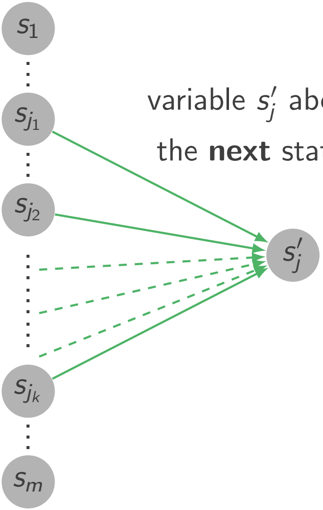
variable s'_j about the **next** state.



Notations

some variables does not interact with each other

variables about the **current** system state,



variable s'_j about the **next** state.

$$s_k \to s'_j$$
$$\Updownarrow$$
$\exists a \in \mathcal{A}$, such that

$T^a_j(\mathbb{S}, s'_j)$ depends on s_k.

variables about the **current** system state,

$s_k \rightarrow s'_j$

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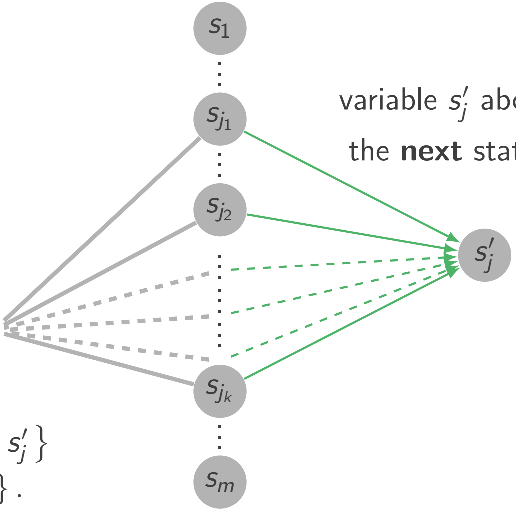
variable s'_j about the **next** state.

$\mathcal{P}(s'_j)$

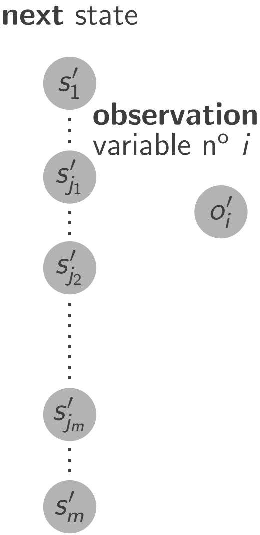


the parents of s'_j are:

$$\mathcal{P}(s'_j) = \{\, s \in \mathbb{S} \mid s \rightarrow s'_j \,\}$$
$$= \{\, s_{j_1}, s_{j_2}, \ldots, s_{j_k} \,\}.$$

Notations

concerning observation variables



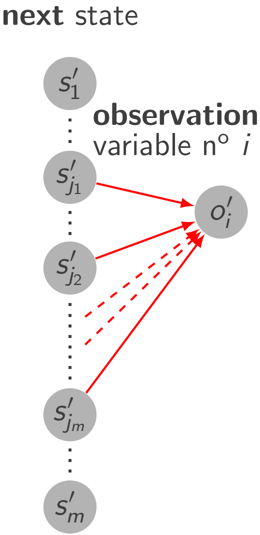
$$s'_j \to o'_i$$

$$\Updownarrow$$

$\exists a \in \mathcal{A}$, such that

$$O_i^a(\mathbb{S}', o'_i)$$

depends on s'_j.



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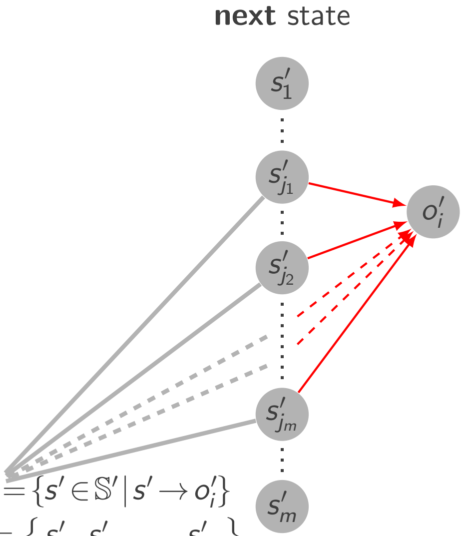

$$\mathcal{P}(o'_i) = \{s' \in \mathbb{S}' \mid s' \rightarrow o'_i\}$$
$$= \{\, s'_{j_1}, s'_{j_2}, \ldots, s'_{j_m} \,\}$$

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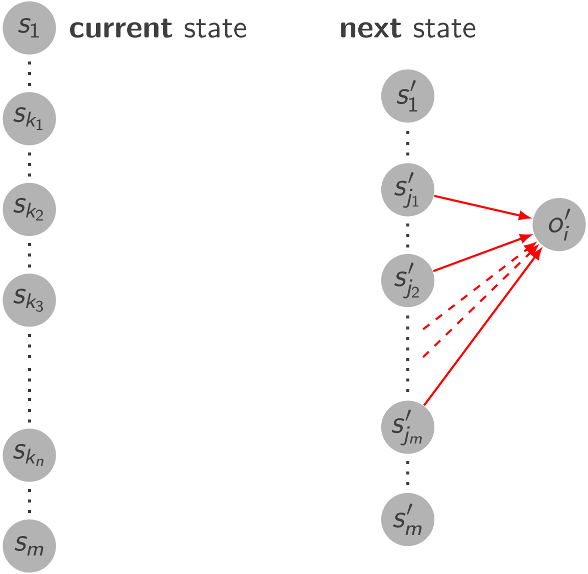

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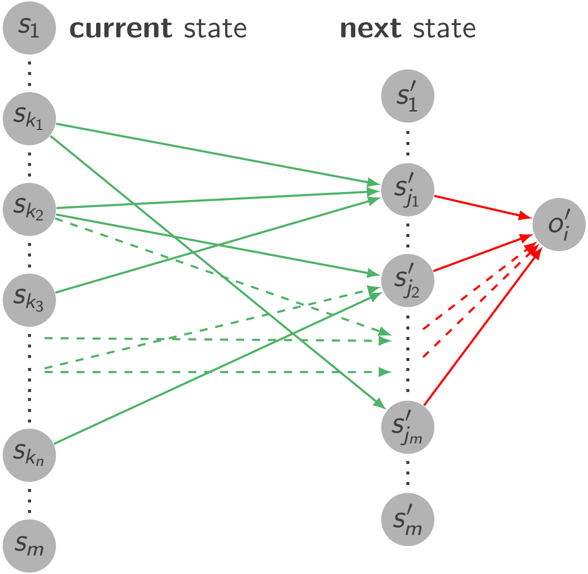

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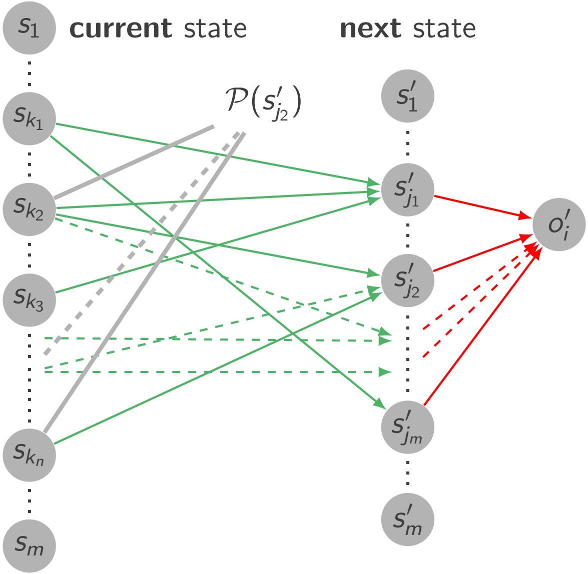

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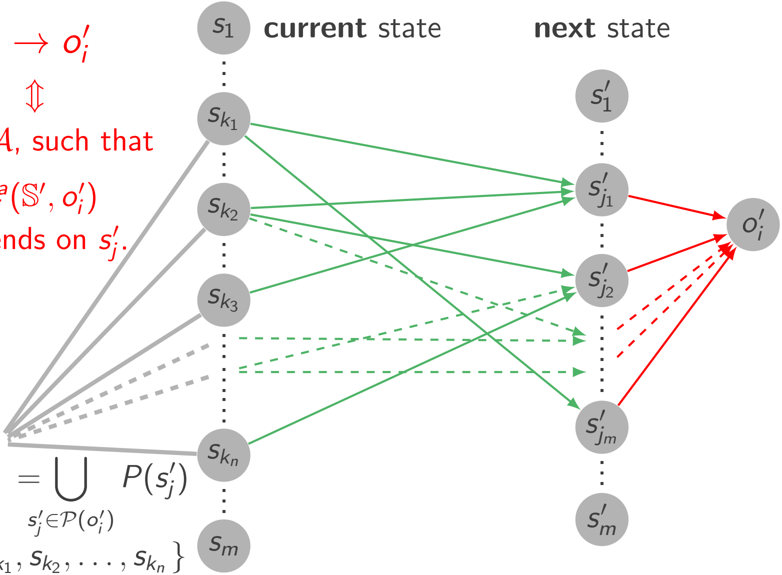

$$\mathcal{Q}(o'_i) = \bigcup_{s'_j \in \mathcal{P}(o'_i)} P(s'_j) = \{ s_{k_1}, s_{k_2}, \ldots, s_{k_n} \}$$

Variables de croyance

different according to the class of the variable

$$\lambda = \#\mathcal{L}$$

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  $\forall o_i \in \mathbb{O} \setminus \mathbb{S}_v$, $\lambda^{2^{p_i}} - (\lambda - 1)^{2^{p_i}}$ belief states,

  $\Rightarrow \lceil \log_2(\lambda^{2^{p_i}} - (\lambda - 1)^{2^{p_i}}) \rceil$ boolean variables β'_h .

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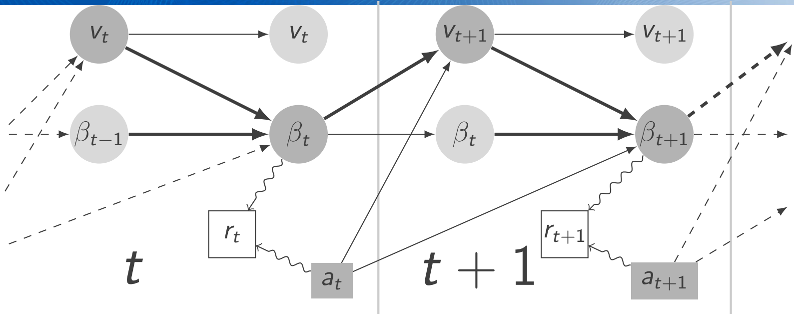
- $\forall s'_f \in \mathbb{S}_f$, $\lambda^2 - (\lambda - 1)^2 = 2\lambda - 1$ belief states,

  $\Rightarrow \lceil \log_2(2\lambda - 1) \rceil$ boolean variables β'_f.

resulting MDP in practice
final structured MDP

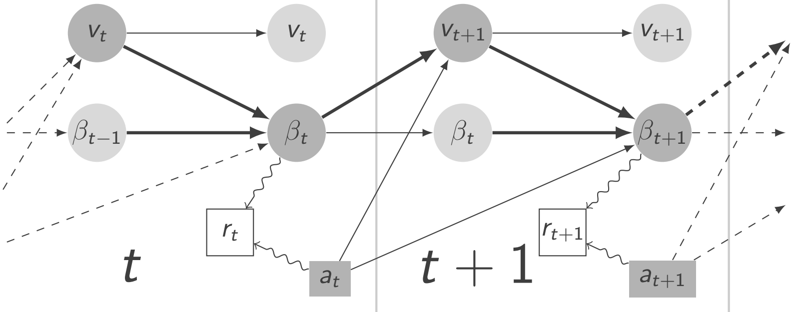


\# **factorized model**'s variables: $\#\mathbb{O} + \#\mathbb{S}_v +$

$$+\sum_{i=1}^{\#\mathbb{O}_h} \left\lceil \log_2\left(\lambda^{2^{p_i}} - (\lambda-1)^{2^{p_i}}\right) \right\rceil + \#\mathbb{S}_f \cdot \left\lceil \log_2\left(2\lambda - 1\right) \right\rceil$$

$\ll$ \# **initial hybrid model**'s variables:

$$\left\lceil \log_2\left(\lambda^{2^{\#\mathbb{S}}} - (\lambda-1)^{2^{\#\mathbb{S}}}\right) \right\rceil$$



factorized model's variables:

$$\leqslant \#\mathbb{O} + \#\mathbb{S}_v + \sum_{i=1}^{\#\mathbb{O}_h} \log_2(\lambda) \cdot 2^{p_i} + \#\mathbb{S}_f \cdot (1 + \log_2(\lambda))$$

$\ll$ # **initial hybrid model**'s variables:

$$\geqslant \log_2(\lambda) \cdot (2^{\#\mathbb{S}} - 1).$$

Variable classification

3 classes of state variables – state space factorization

variable: visible $s'_v \in \mathbb{S}_v$

inferred hidden $s'_h \in \mathbb{S}_h$

fully hidden $s'_f \in \mathbb{S}_f$

Variable classification

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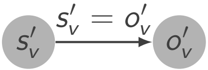

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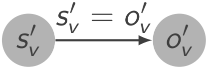

$$\mathbf{p}\left(s'_v \mid b^\pi_t, a\right) = \sum_{2^{\mathcal{P}(s'_v)}} T^a\left(\mathcal{P}(s'_v), s'_v\right) \cdot \overline{b^\pi_t}\Big(\mathcal{P}(s'_v)\Big)$$

---

inferred hidden $s'_h \in \mathbb{S}_h$

---

fully hidden $s'_f \in \mathbb{S}_f$

Variable classification

3 classes of state variables – state space factorization

$$s'_v \xrightarrow{s'_v = o'_v} o'_v$$

variable: visible $s'_v \in \mathbb{S}_v$

$\Leftrightarrow$ deterministic belief variable.

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s'_h

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s'_f

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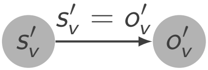

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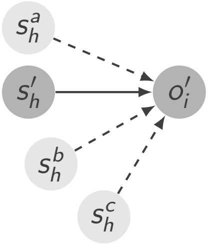

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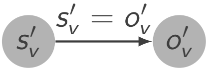

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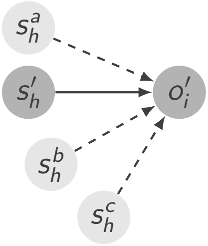
$$b^\pi_{t+1}(\mathcal{P}(o'_i)) = b^\pi_{t+1}(s_h, s^a_h, s^b_h, s^c_h)$$



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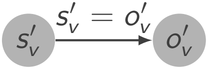

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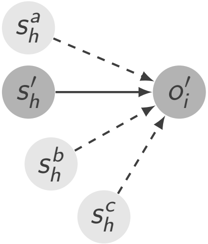

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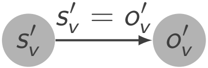

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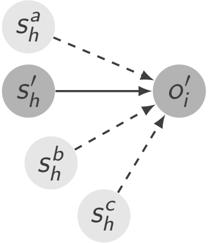
⚠ $\mathcal{P}(o'_i)$ may contain visible variables



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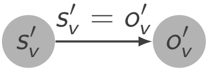

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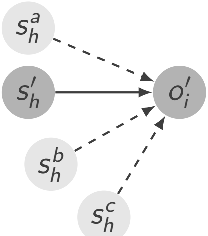

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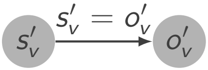

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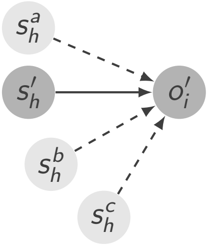

fully hidden $s'_f \in \mathbb{S}_f$

$$b^\pi_{t+1}(s'_f) = \max_{2^{\mathcal{P}(s'_f)}} \min\left\{\pi\left(s'_f \middle| \mathcal{P}(s'_f), a\right), b^\pi_t\left(\mathcal{P}(s'_f)\right)\right\}$$

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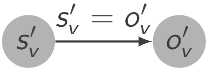

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---

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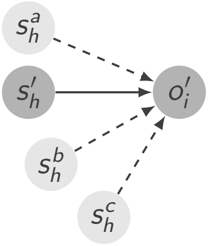

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fully hidden $s'_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f

$$b^\pi_{t+1}(s'_f) = \max_{2^{\mathcal{P}(s'_f)}} \min \left\{ \pi\left(s'_f \middle| \mathcal{P}(s'_f), a\right), b^\pi_t\left(\mathcal{P}(s'_f)\right) \right\}$$

Toy example: 2 machine states, 3 occurrences

columns		1	2	3	4	5
SITUATION						
v'	v_A	1				1
	v_B		1			
	v_C	1			1	
h	s_A	1	1		1	
	s_B			1		1
BEHAVIOUR						
h'	s_A					1
	s_B		1		1	
EFFECT		$\overline{e}$	$\tilde{e}$	$\overline{e}$	$\widehat{e}$	$\underline{e}$
POSSIBILITY		1	ε	1	λ	δ

Probability / Possibility :

e_1 or e_2	$\mathbf{p}(e_1) + \mathbf{p}(e_2 \cap \overline{e_1})$	$\max\{\pi(e_1), \pi(e_2)\}$
e_1 and e_2	$\mathbf{p}(e_1).\mathbf{p}(e_2 \mid e_1)$	$\min\{\pi(e_1), \pi(e_2 \mid e_1)\}$

Back to general POMDP: Partially Observable Criteria

Rewriting: belief dependent reward (belief trick)

- $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ reward function
- $\rho : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{L}$ preference function

Probability / Possibility:

Probability	Possibility
$R(b_t, d_t) = \sum_s r(s, d_t) \cdot b_t(s)$	**optimistic:** $\overline{\Psi}(\beta_t, \delta_t) = \max_s \min \{ \rho(s, \delta_t), \beta_t(s) \}$ **pessimistic:** $\underline{\Psi}(\beta_t, \delta_t) = \min_s \max \{ \rho(s, \delta_t), 1 - \beta_t(s) \}$
$\mathbb{E}[r(S_t, d_t)] = \mathbb{E}[R(b_t, d_t)]$	$\mathbb{S}_\Pi[\rho(S_t, d_t)] = \mathbb{S}_\Pi[\overline{\Psi}(\beta_t, d_t)]$ $\mathbb{S}_\mathcal{N}[\rho(S_t, d_t)] = \mathbb{S}_\mathcal{N}[\underline{\Psi}(\beta_t, d_t)]$

Note: $\mathbb{S}_\Pi[\underline{\Psi}(\beta_t, d_t)]$; $\mathbb{S}_\mathcal{N}[\overline{\Psi}(\beta_t, d_t)]$?

Why model ignorance?
knowledge is not always encouraged with POMDPs

- initial belief deterministic $s_0 = s_A$.

s_A

$b_0(s_A) = 1$

Why model ignorance?

knowledge is not always encouraged with POMDPs

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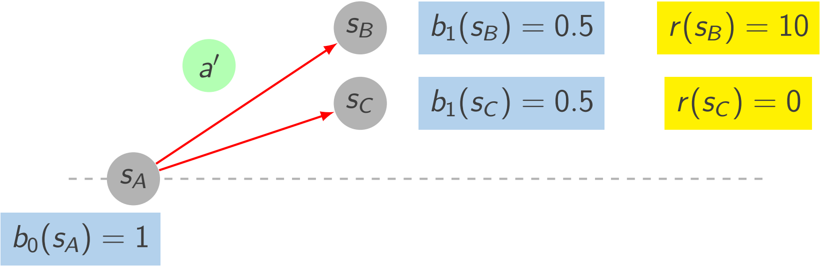

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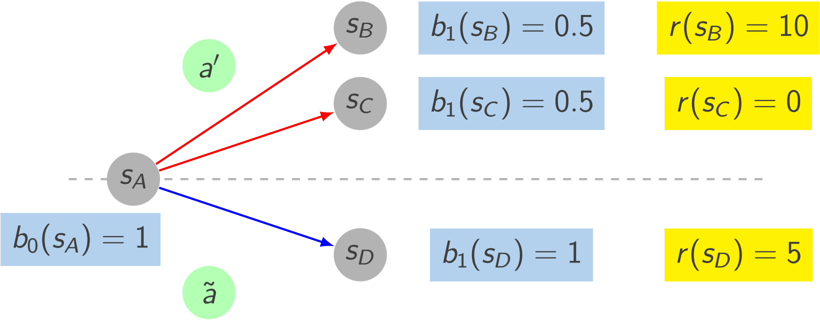

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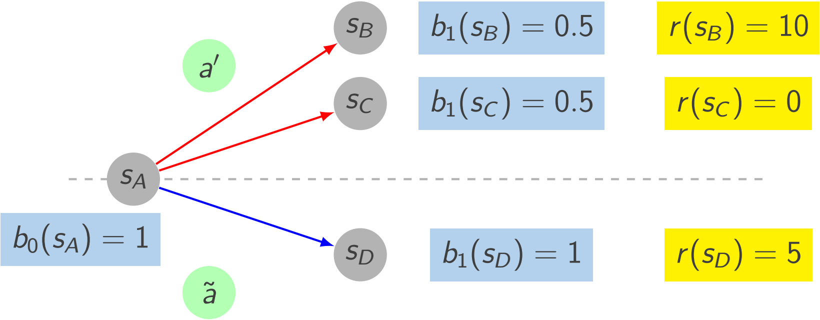

- $\{s_B, s_C, s_D\} \xrightarrow{\text{deterministic}} s_E,\ r(s_E) = 0.$

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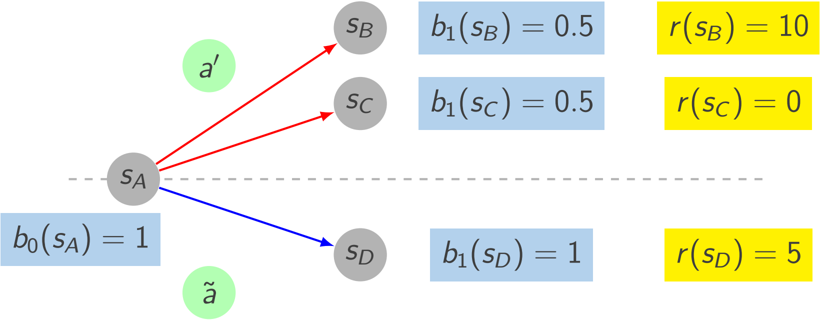


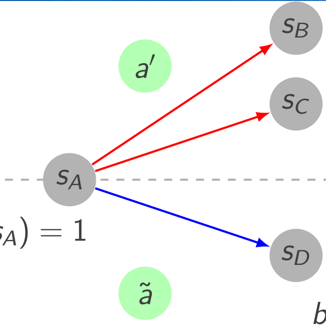
- $\{ s_B, s_C, s_D \} \xrightarrow{\text{deterministic}} s_E, \; r(s_E) = 0.$

$$\mathbb{E}_{s_0 \sim b_0} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot r(s_t) \,\middle|\, a_0 = \tilde{a} \text{ or } a' \right] = r(s_0) + 5\gamma.$$

the safe action is not prefered.

Why model ignorance?

Choquet integral and rewards



$b_0^\pi(s_A) = 1$

$b_1^\pi(s_B) = 1$ $\quad$ $r(s_B) = 10$

$b_1^\pi(s_C) = 1$ $\quad$ $r(s_C) = 0$

$b_1^\pi(s_D) = 0$

$b_1^\pi(s_D) = 1$ $\quad$ $r(s_D) = 5$

$b_1^\pi(s_B) = b_1^\pi(s_C) = 0$

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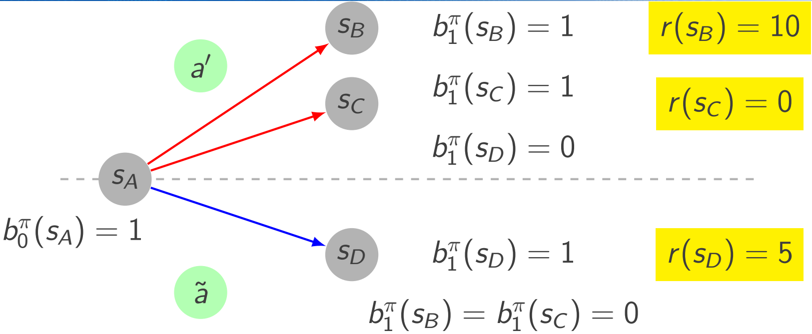
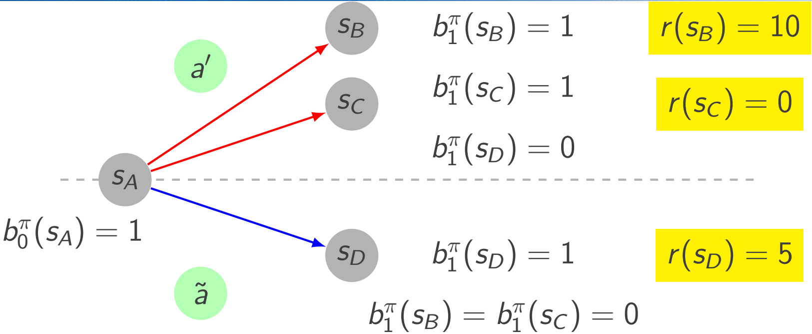


- $Ch\left(r, N_{b_1^\pi} \,\middle|\, a_0 = \tilde{a}\right) = r(s_D, \tilde{a}) = 5,$
- $Ch\left(r, N_{b_1^\pi} \,\middle|\, a_0 = a'\right) = \min_{s \in \mathcal{S}} r(s, a') = 0.$

the safe action is prefered! **dispersion reduced**

Why model ignorance?

Choquet integral and rewards



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the safe action is prefered! **dispersion reduced**

if $\mathcal{N}_{b_1^\pi}$ replaced by $b_1 \Rightarrow Ch(r, b_1) = \mathbb{E}_{s \sim b_1}\left[r(s, a)\right].$

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imprecision of b^π = agent ignorance + discretization:
pessimistic reward about these imprecisions.

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Dual measure of $\Pi : 2^{\mathcal{S}} \rightarrow \mathcal{L}$

necessity $\mathcal{N}$ such that $\forall A \subseteq \mathcal{S}$, $\mathcal{N}(A) = 1 - \Pi(\overline{A})$.

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$r_1 > r_2 > \ldots > r_{k+1} = 0$ represents elements of $\left\{ r(s,a) \middle| s \in \mathcal{S} \right\}$.

Choquet integral of r with respect to $\mathcal{N}$

$$Ch(r,\mathcal{N}) = \sum_{i=1}^{k} (r_i - r_{i+1}) \cdot \mathcal{N}\left(\left\{ r(s) \geqslant r_i \right\}\right) \qquad (1)$$

$$(2)$$

Choquet integral and rewards

pessimistic evaluation of the rewards – necessity measure

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pessimistic reward about these imprecisions.

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Choquet integral of r with respect to $\mathcal{N}$

$$
\begin{aligned}
Ch(r, \mathcal{N}) &= \sum_{i=1}^{k} (r_i - r_{i+1}) \cdot \mathcal{N}\left(\{ r(s) \geqslant r_i \} \right) && (1) \\
&= \sum_{i=1}^{\#\mathcal{L}-1} (l_i - l_{i+1}) \cdot \min_{\substack{s \in \mathcal{S} \text{ s.t.} \\ b^\pi(s) \geqslant l_i}} r(s) && (2)
\end{aligned}
$$

notation $\mathcal{L} = \{ l_1 = 1, l_2, l_3, \ldots, 0 \}$.

boolean variable "*flipflop*" f changes state at each time step
$\rightarrow$ defines 2 phases:

1. *observation generation*,
2. *belief update* (deterministic knowing the observation)

MDP variables:

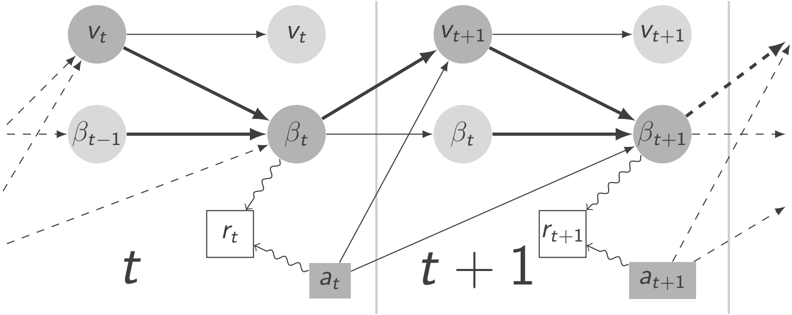
$$\tilde{\mathbb{S}} =$$

beliefs: $\beta = \beta_v^1 \times \ldots \times \beta_v^{m_v} \times \beta_h^1 \times \ldots \times \beta_h^{m_h} \times \beta_f^1 \times \ldots \times \beta_f^{m_f}$

$$\times$$

visible variables: $v = f \times s_v^1 \times \ldots \times s_v^{m_v} \times o_1 \times \ldots \times o_k$.

resulting MDP in practice
final structured MDP



$\tilde{\mathbb{S}} =$

beliefs: $\beta = \beta_v^1 \times \ldots \times \beta_v^{m_v} \times \beta_h^1 \times \ldots \times \beta_h^{m_h} \times \beta_f^1 \times \ldots \times \beta_f^{m_f}$

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visible variables : $v = f \times s_v^1 \times \ldots \times s_v^{m_v} \times o_1 \times \ldots \times o_k$.