

$\partial_t \psi + \frac{M}{\epsilon} \int_{\Omega} \frac{|u(x,t)|^2}{2} \psi \Delta \psi + \int_{\Omega} p = 0, \quad \nabla \psi = 0, \quad \psi(x,0) = \psi_0(x), \quad \psi(x,t) = \psi_0(x)$

# Exploiting Imprecise Information Sources in Sequential Decision Making Problems under Uncertainty

**N.Drougard**

under D.Dubois, J-L.Farges and F.Teichteil-Königsbuch supervision

doctoral school: EDSYS    institution: ISAE-SUPAERO

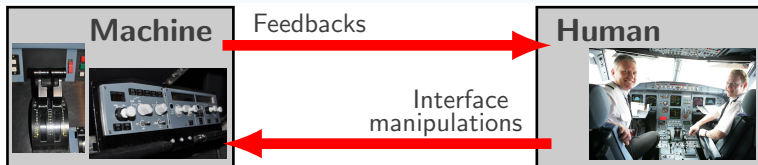
laboratory: ONERA-The French Aerospace Lab



retour sur innovation

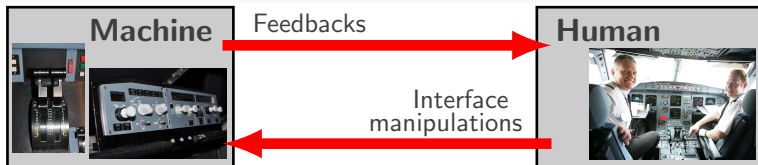
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joint work with **Sergio Pizziol** – Context



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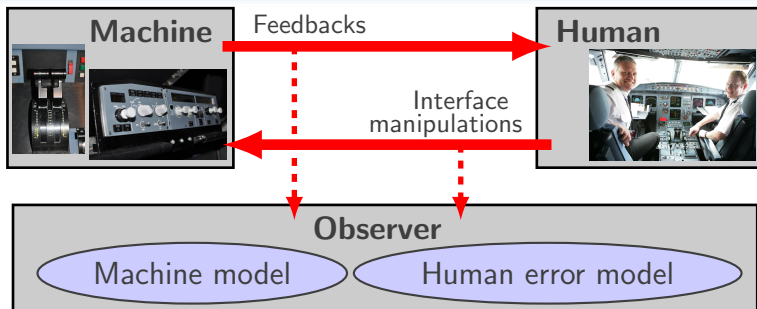
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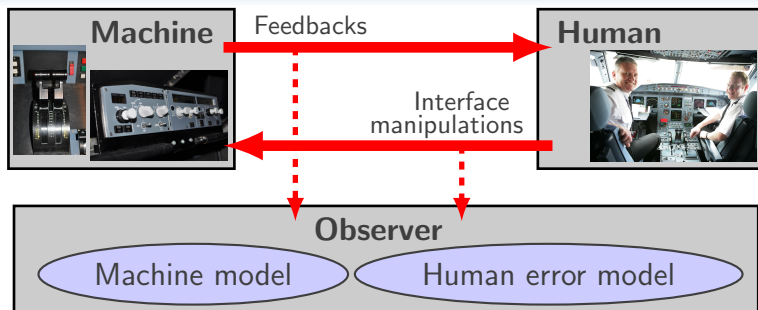
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**Issue:** incorrect human assessment of the machine state  
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$\pi$ -POMDP without actions:  $\pi$ -Hidden Markov Process

- **system space**  $\mathcal{S}$ : set of human assessments → **hidden**
- **observation space**  $\mathcal{O}$ : feedbacks/human manipulations

# Example: Human-Machine Interaction (HMI)

Human error model from expert knowledge

Machine with states  $A, B, C, \dots$

state  $s_A \in \mathcal{S}$ : “human thinks machine state is  $A$ ”

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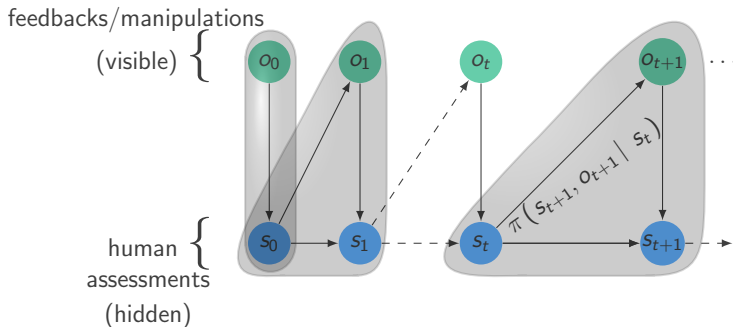
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■ impossible cases: possibility degree 0

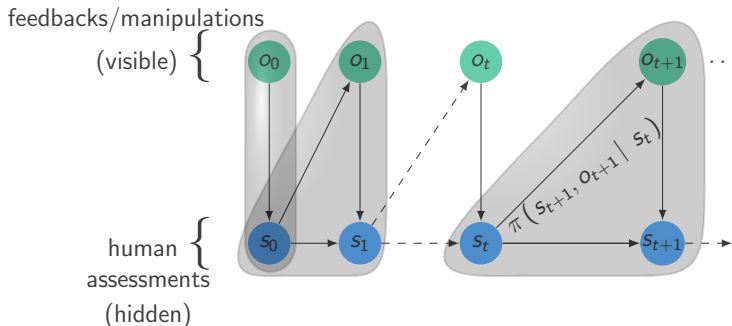
# Qualitative Possibilistic Hidden Markov Process:

$\pi$ -HMP, detection & diagnosis tool for HMI (with Sergio Pizziol)



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- **estimation** of the human assessment  
 $\Leftrightarrow$  **possibilistic belief state**
- **detection** of human assessment errors + **diagnosis**
- validated with pilots on flight simulator missions

# Applicability of the $\pi$ -POMDPs

## three advancements

- lack of proof of optimality in indefinite horizon settings
- criterion/algorithm/proof
- curse of dimensionality:
  - belief space size of a  $\pi$ -POMDP: exponential in  $\#\mathcal{S}$
- in practice, part of  $s \in \mathcal{S}$  is visible
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- lack of possibilistic strategy evaluation
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**Indefinite Horizon, Mixed-Observability, Simulations**  
*contribution UAI 2013*

# Indefinite Horizon

criterion, DP scheme, optimal strategy

## **indefinite horizon criterion:**

maximizing

$$\min_{t=0}^{\# \delta} \min \left\{ \pi(s' | s, \delta_t(s)), \Psi(s) \right\}$$

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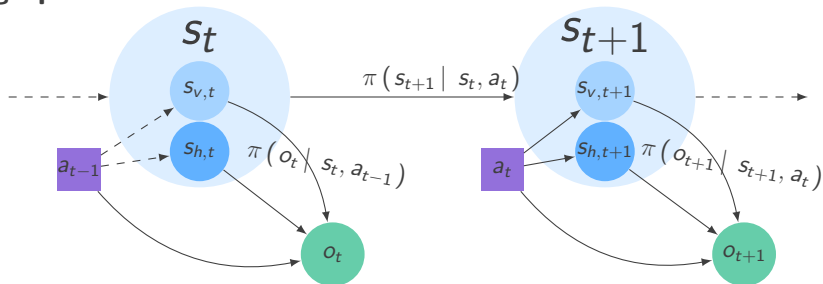
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- criterion **non decreasing** with iterations
- action update for states increasing the criterion
- **proof of optimality** of the resulting **stationary** strategy

## Mixed-Observability (MOMDP, *Ong et al., 2005*)

## $\pi$ -Mixed-Observable Markov Decision Process ( $\pi$ -MOMDP)

**graphical model** of a  $\pi$ -MOMDP:



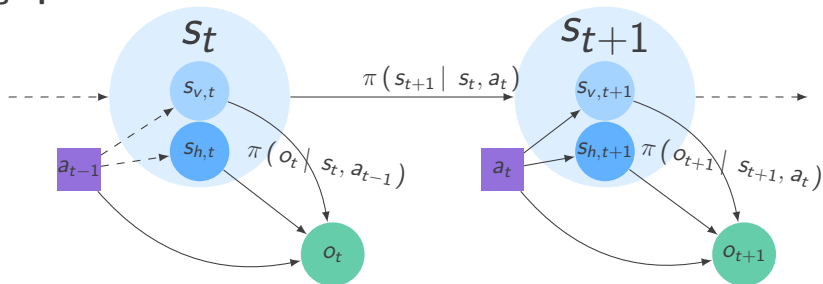
**Mixed-Observability:** system state  $s \in \mathcal{S} = \mathcal{S}_y \times \mathcal{S}_h$

i.e. state  $s$  = visible component  $s_v$  & hidden component  $s_h$

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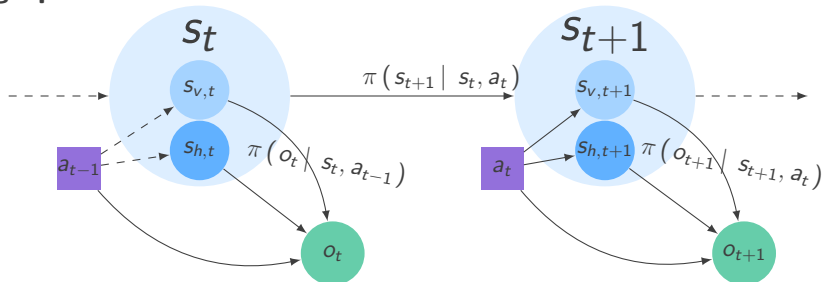
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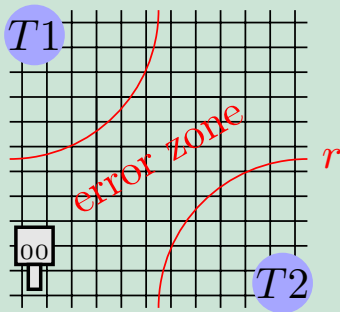
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- $\rightarrow \pi$ -MOMDP: computations on  $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$
- $\#\mathcal{X} \sim \#\mathcal{S}_v \cdot \#\mathcal{L}^{\#\mathcal{S}_h} \ll \#\Pi_{\mathcal{L}}^{\mathcal{S}}$

# $\pi$ -MOMDP for robotics with imprecise probabilities

simulations with machine vision behavior imprecisely known

- **goal:** reach the object  $A = T1$  or  $T2$
- noisy observations of the location of the object  $A$

Recognition mission: robot on a grid, targets  $T1$  &  $T2$

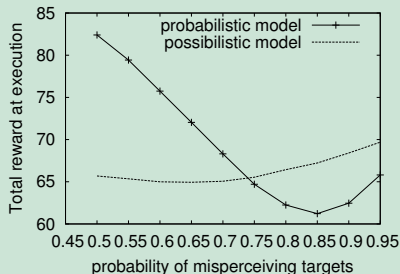
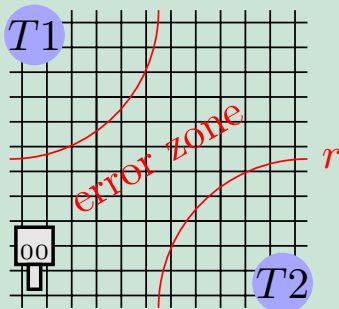


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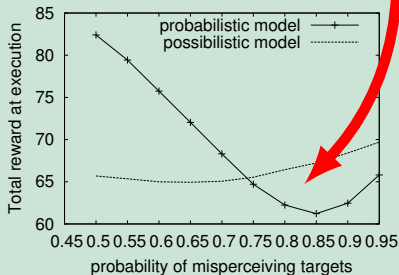
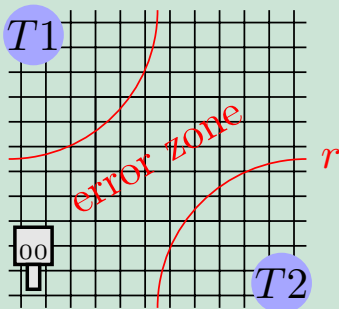
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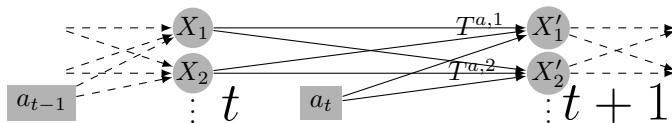


# Factored $\pi$ -MOMDP and computations with ADDs

qualitative possibilistic models to reduce complexity

**contribution (AAAI-14):** factored  $\pi$ -MOMDP

$\Leftrightarrow$  state space  $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$  = Boolean variables  $(X_1, \dots, X_n)$   
 + independence assumptions  $\Leftarrow$  graphical model

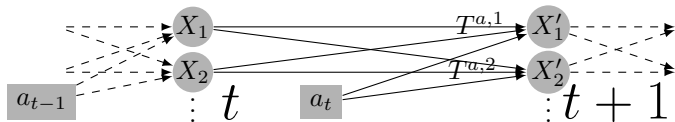


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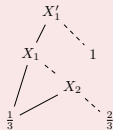
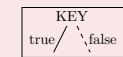
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- **factorization:** transition functions  $T_i^a = \pi(X'_i \mid \text{parents}(X'_i), a)$  stored as **Algebraic Decision Diagrams (ADD)**

probabilistic case:

SPUDD (Hoey et al., 1999)



example of ADD

# Simplify computations with $\pi$ -MOMDPs

Resulting  $\pi$ -MOMDP solver: PPUDD

- probabilistic model:  $+$  and  $\times \Rightarrow$  new values created  
 $\Rightarrow$  number of ADDs leaves **potentially huge**
- possibilistic model:  $\min$  and  $\max \Rightarrow$  values  $\in \mathcal{L}$  finite  
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## PPUDD: Possibilistic Planning Using Decision Diagrams

- factorization  $\Rightarrow$  each DP steps divided into  $n$  stages  
 $\rightarrow$  smaller ADDs  $\Rightarrow$  **faster computations**

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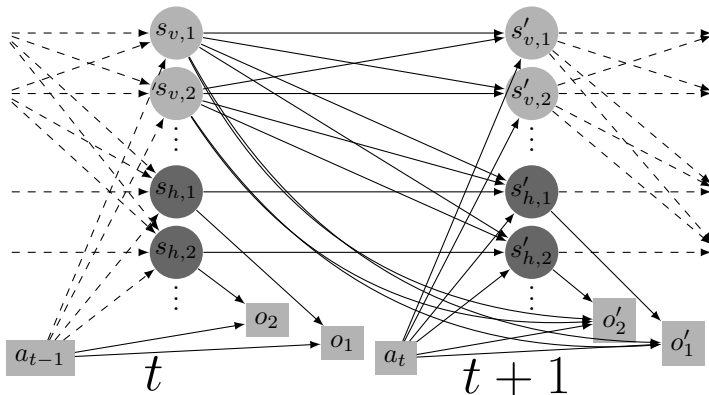
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- computations on trees: *CU Decision Diagram Package*.

# Simplifying computations with $\pi$ -MOMDPs

Natural factorization: belief independence

**contribution (AAAI-14):**

independent sensors, hidden states, ...  $\Rightarrow$  graphical model



# Simplifying computations with $\pi$ -MOMDPs

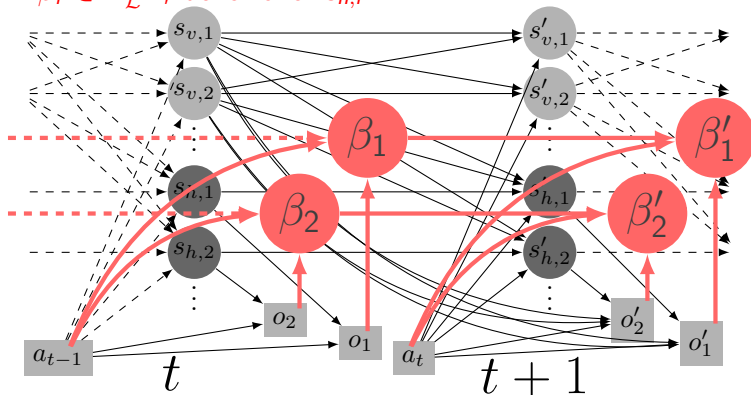
Natural factorization: belief independence

## contribution (AAAI-14):

independent sensors, hidden states, ...  $\Rightarrow$  graphical model

d-Separation  $\Rightarrow (s_v, \beta) = (s_{v,1}, \dots, s_{v,m}, \beta_1, \dots, \beta_l)$

$\beta_i \in \Pi_{\mathcal{L}}^{s_{h,i}}$ , belief over  $s_{h,i}$



# Simplifying computations with $\pi$ -MOMDPs

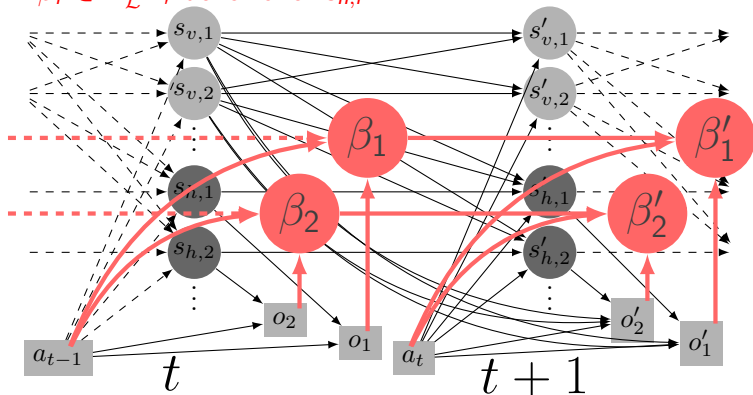
Natural factorization: belief independence

$\perp\!\!\!\perp$  assumptions on state & observation variables

→ belief variable factorization

→ **additional** computation savings

$\beta_i \in \Pi_{\mathcal{L}}^{S_{h,i}}$ , belief over  $s_{h,i}$





# Simplify computations with $\pi$ -MOMDPs

Experiments – perfect sensing: Navigation problem

PPUDD vs SPUDD (*Hoey et al.*, 1999)

**Navigation benchmark:** reach a goal – spots with accident risk  
M1 (resp. M2) optimistic (resp. pessimistic) criterion

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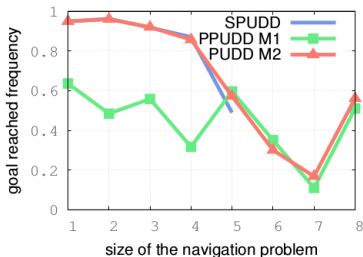
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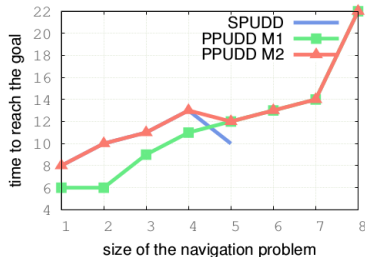
## Performances, function of the problem index

reached goal frequency



the higher the better

# steps to reach the goal

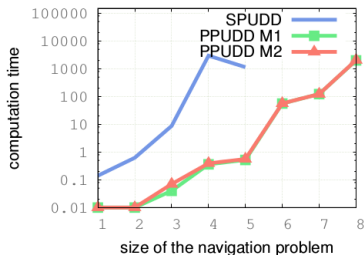


the lower the better

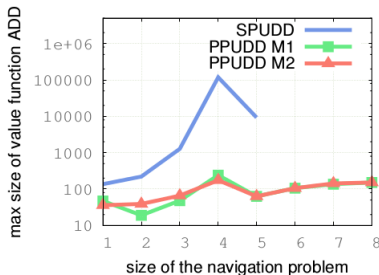
# Simplify computations with $\pi$ -MOMDPs

Experiments – perfect sensing: Navigation problem

computation time



max size of ADDs



- PPUDD + M2 (pessimistic criterion)  
**faster with same performances as SPUDD**
- SPUDD only solves the first 5 instances
- verified intuition: ADDs are smaller

# Simplify computations with $\pi$ -MOMDPs

Experiments – imperfect sensing: RockSample problem

PPUDD vs APPL (*Kurniawati et al.*, 2008, solver MOMDP)

symbolic HSVI ( *Sim et al.*, 2008, solver POMDP)

**RockSample benchmark:** recognize and sample “good” rocks

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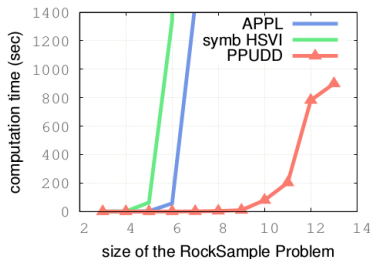
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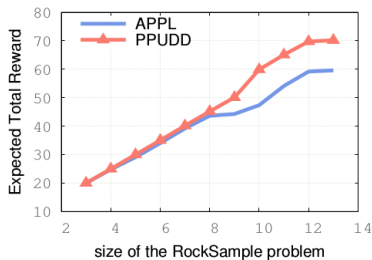
computation time:

probabilistic solvers, prec. 1  
PPUDD. exact resolution



average of rewards

APPL stopped when  
PPUDD end



- **approximate model + exact resolution solver**  
→ improvement of computation time and performances



# Thank you!

produced work:

- *Qualitative Possibilistic Mixed-Observable MDPs*, **UAI-2013**
- *Structured Possibilistic Planning Using Decision Diagrams*, **AAAI-2014**
- *Planning in Partially Observable Domains with Fuzzy Epistemic States and Probabilistic Dynamics*, **SUM-2015**
- *Processus Décisionnels de Markov Possibilistes à Observabilité Mixte*, *Revue d'Intelligence Artificielle (RIA journal)*
- *A Possibilistic Estimation of Human Attentional Errors*, submitted to **IEEE-TFS journal**