Exploiting Imprecise Information Sources in Sequential Decision Making Problems under Uncertainty

N.Drougard

under **D.Dubois**, **J-L.Farges** and **F.Teichteil-Königsbuch** supervision

doctoral school: EDSYS **institution:** ISAE–SUPAERO

laboratory: ONERA–The French Aerospace Lab

retour sur innovation

Context

Autonomous robotics

Onera, Flight Dynamics & System control

Control Engineering, Artificial intelligence, Cognitive Sciences

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among many other works:

- autonomy and human factors
- decision making, planning
- experimental/industrial applications: UAVs, human-machine interaction, exploration robots

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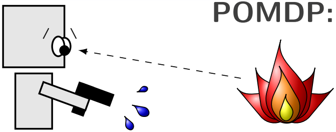
Partially Observable Markov Decision Processes (POMDPs)



POMDP: model for autonomous robotics

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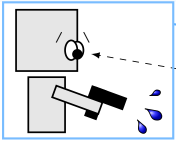

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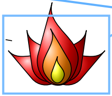

$s \in \mathcal{S}$: **system states**

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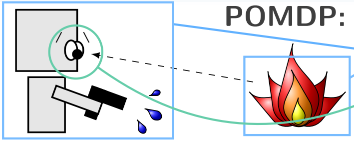

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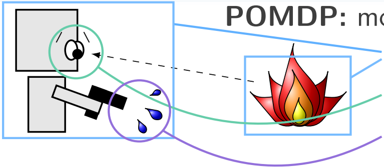

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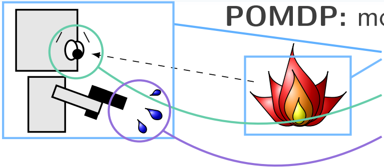

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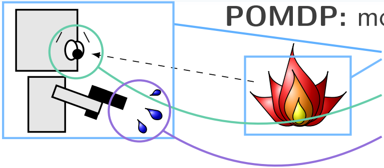

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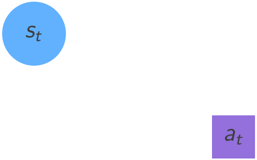

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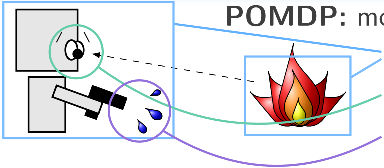

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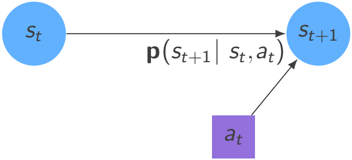

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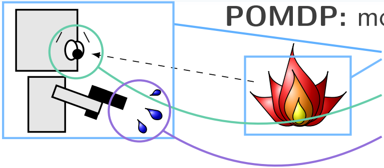

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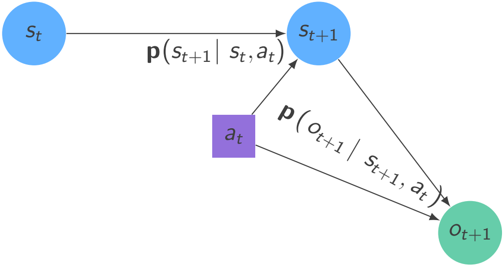

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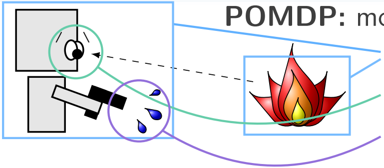

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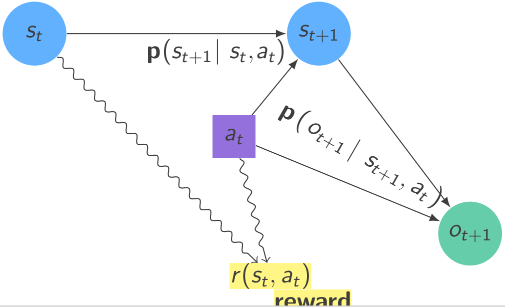

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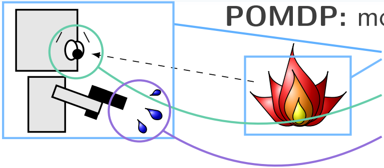

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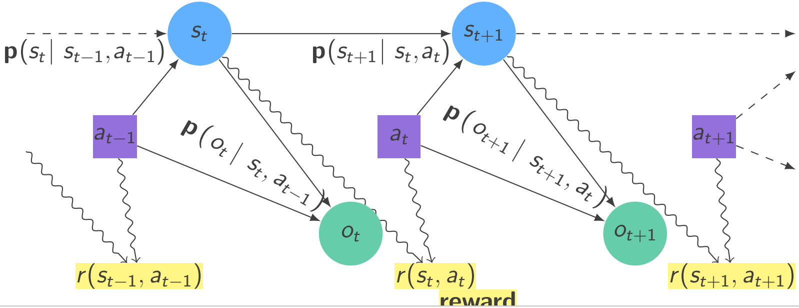

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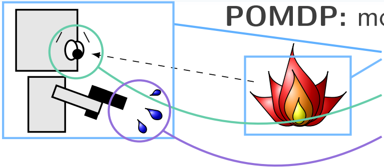

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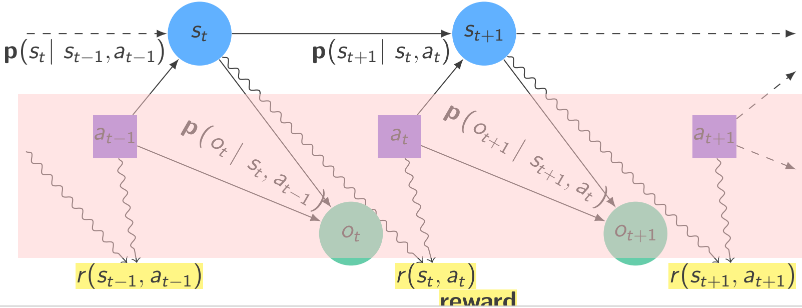

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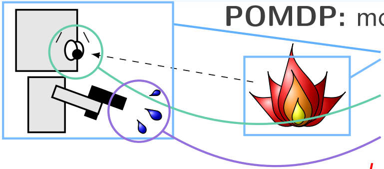

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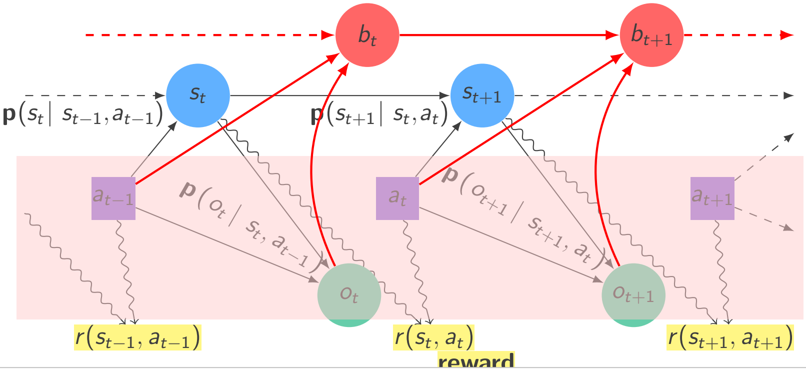
- b **belief**: **probability distribution**



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belief state, strategy, criterion

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probabilistic belief update – a selected, o' received

$$b_{t+1}(s') \propto \mathbf{p}\left(o' \mid s',a\right) \cdot \sum_{s \in \mathcal{S}} \mathbf{p}\left(s' \mid s,a\right) \cdot b_t(s)$$

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action choices: strategy $\delta(b_t) = a_t \in \mathcal{A}$

maximizing $\mathbb{E}_{s_0 \sim b_0} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot r\Big(s_t, \delta(b_t) \Big) \right]$, $0 < \gamma < 1$

Flaws of the POMDP model

POMDPs in practice

- optimal strategy computation **PSPACE-hard**

  (*Papadimitriou et al. 1987*)

- probabilities are **imprecisely known** in practice

- **prior ignorance** semantic/management?

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practical issues: Complexity, Vision and Initial Belief

- **POMDP optimal strategy computation undecidable** in infinite horizon (*Madani et al. 1999*)

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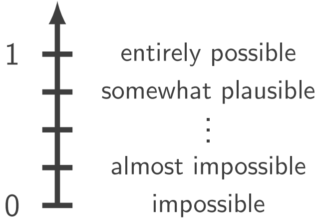
→ uniform probability distribution $\neq$ ignorance!

Qualitative Possibility Theory

presentation – (max,min) "tropical" algebra

finite scale $\mathcal{L}$

usually $\{0, \frac{1}{k}, \frac{2}{k}, \ldots, 1\}$



events $e \subset \Omega$ (universe)

sorted using possibility **degrees** $\pi(e) \in \mathcal{L}$

$$\neq$$

quantified with **frequencies** $\mathbf{p}(e) \in [0, 1]$ (probabilities)

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1 — entirely possible

somewhat plausible

$\vdots$

almost impossible

0 — impossible

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$e_1 \neq e_2$, 2 events $\subset \Omega$

- $\pi(e_1) < \pi(e_2) \Leftrightarrow$ "e_1 is less plausible than e_2"

Qualitative Possibility Theory

Criteria from Sugeno integral

Probability / Possibility:

$+$	max
$\times$	min
$X \in \mathbb{R}$	$X \in \mathcal{L}$
$\mathbb{E}[X] = \sum_{x \in X} x \cdot \mathbf{p}(x)$	**optimistic:** $\mathbb{S}_{\Pi}[X] = \max_{x \in X} \min \{ x, \pi(x) \}$ **pessimistic:** $\mathbb{S}_{\mathcal{N}}[X] = \min_{x \in X} \max \{ x, 1 - \pi(x) \}$

Qualitative Possibility Theory

qualitative possibilistic POMDP (π-POMDP)

Sabbadin (UAI-98) introduces

the qualitative possibilistic POMDP

π-**POMDP:** $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T^{\pi}, O^{\pi}, \rho \rangle$

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- belief space trick: POMDP $\rightarrow$ MDP with **infinite** space
  π-POMDP $\rightarrow$ π-MDP with **finite** space
- problem becomes **decidable**
- $\forall s \in \mathcal{S}, \ \pi(s) = 1 \Leftrightarrow$ total ignorance about s

A possibilistic belief state

finite belief space

$$\Pi_{\mathcal{L}}^{\mathcal{S}} = \Big\{ \text{ possibility distributions } \Big\} : \; \#\Pi_{\mathcal{L}}^{\mathcal{S}} \sim \#\mathcal{L}^{\#\mathcal{S}} < +\infty$$

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joint distribution on $\mathcal{S} \times \mathcal{O}$ from b_t^{π}: $\pi\left(o', s' \,\middle|\, b_t^{\pi}, a \right)$

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- **Makovian update:** only depends on o', a and b_t^{π}

Overview

Qualitative Possibility Theory:
$\rightarrow$ simplification, imprecision/prior ignorance modeling

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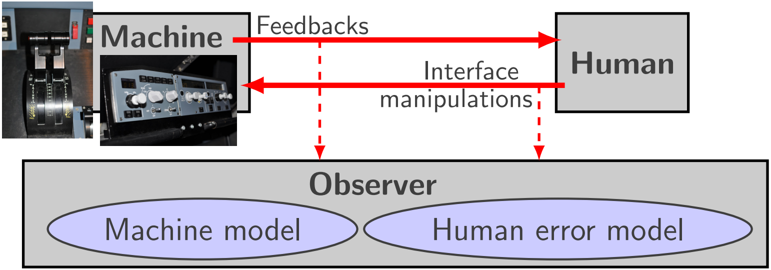
1. example of a qualitative possibilistic model
2. advancements and first use of the π-POMDP model
3. simplify computation: ADDs and factorization
4. probabilistic-possibilistic (hybrid) approach

Example: Human-Machine Interaction

joint work with Sergio Pizziol – Context



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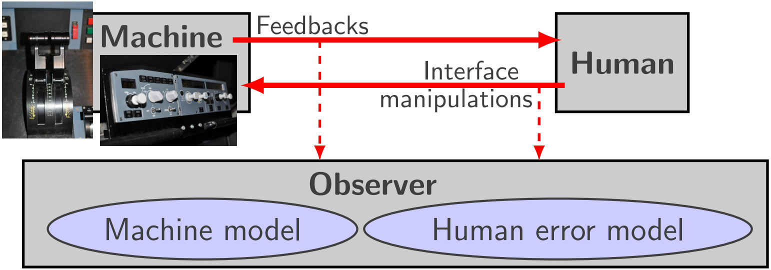

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$\rightarrow$ **accident risk**

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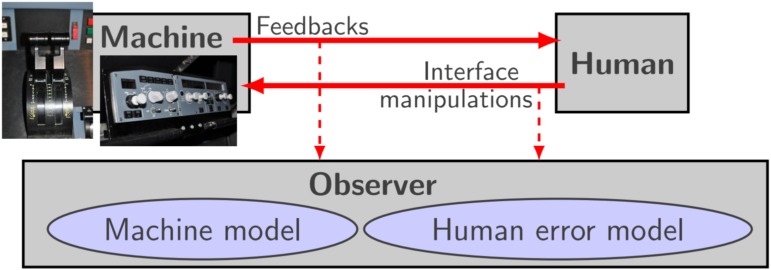

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π-POMDP without actions: π-Hidden Markov Process

- **system space** $\mathcal{S}$: set of human assessments $\rightarrow$ **hidden**
- **observation space** $\mathcal{O}$: feedbacks/human manipulations

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Human error model from expert knowledge

Machine with states A, B, C, ...

state $s_A \in \mathcal{S}$: "human thinks machine state is A"

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human usually aware of feedbacks $\to \pi\left(s'_B, o'_f \mid s_A\right) = 1$

but may lose a feedback $\to \pi\left(s'_A, o'_f \mid s_A\right) = \frac{2}{3}$

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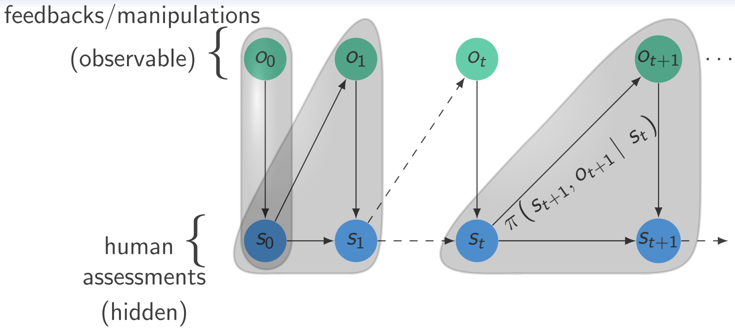
- impossible cases: possibility degree 0

Qualitative Possibilistic Hidden Markov Process:

diagnosis tool for Human-Machine interaction (with Sergio Pizziol)



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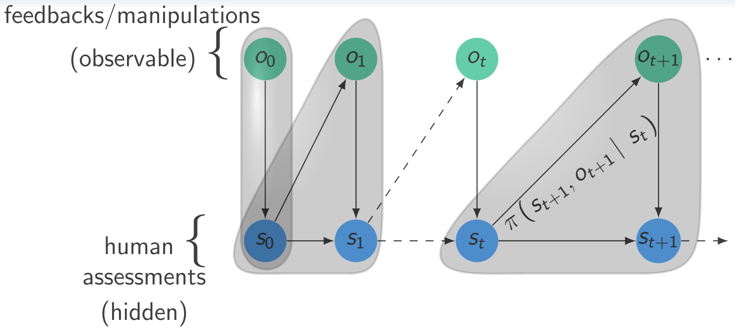
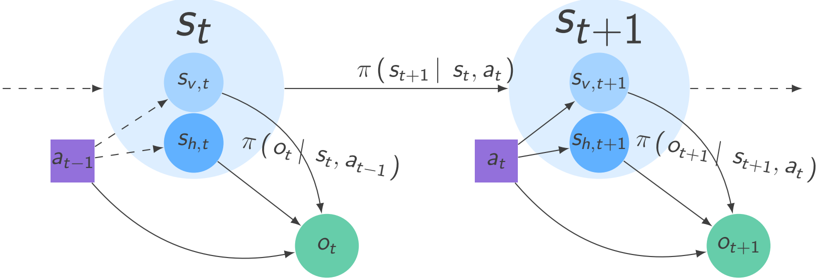


- **estimation** of the human assessment
  $\Leftrightarrow$ **possibilistic belief state**
- **detection** of human assessment errors + **diagnosis**
- validated with pilots on a flight simulator missions

Mixed-Observability (MOMDP) – *Ong et al. (RSS-05)*

π-Mixed-Observable Markov Decision Process (π-MOMDP)

contribution (UAI-13):



Mixed-Observability: system state $s \in \mathcal{S} = \mathcal{S}_v \times \mathcal{S}_h$

i.e. state s = visible component s_v & hidden component s_h

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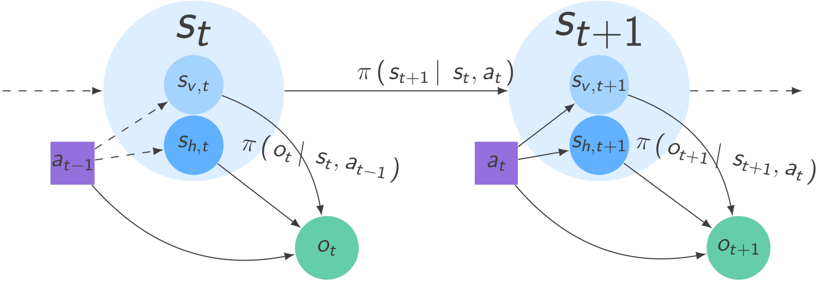

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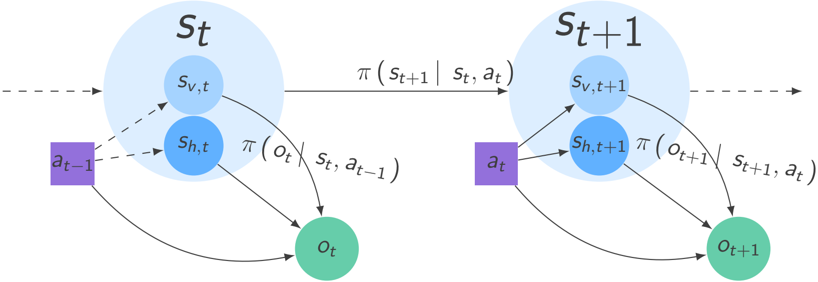

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i.e. state s = visible component s_v & hidden component s_h

- belief states only over $\mathcal{S}_h$ (component s_v observed)
- $\rightarrow$ π-POMDP: belief space $\Pi_{\mathcal{L}}^{\mathcal{S}}$ $\quad \#\Pi_{\mathcal{L}}^{\mathcal{S}} \sim \#\mathcal{L}^{\#\mathcal{S}}$

  $\rightarrow$ π-MOMDP: computations on $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$

  $\#\mathcal{X} \sim \#\mathcal{S}_v \cdot \#\mathcal{L}^{\#\mathcal{S}_h} \ll \#\Pi_{\mathcal{L}}^{\mathcal{S}}$

Use of the π-MOMDP in practice

undeterminate horizon

contribution (UAI-13): undeterminate Horizon

Use of the π-MOMDP in practice

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Dynamic Programming scheme: $\#$ iterations $< \#\mathcal{X}$

- assumption: $\exists$ artificial **“stay” action**
  as in classical planning/ γ counterpart
- criterion **non decreasing** with iterations

Use of the π-MOMDP in practice

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Dynamic Programming scheme: $\#$ iterations $< \#\mathcal{X}$

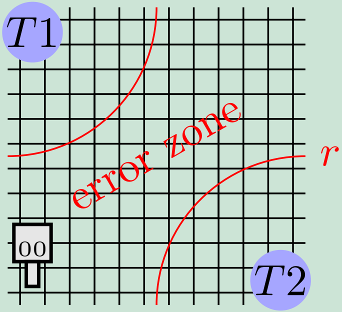
- assumption: $\exists$ artificial **"stay" action**
  as in classical planning/ γ counterpart
- criterion **non decreasing** with iterations
- action update for states increasing the criterion
- proof of optimality

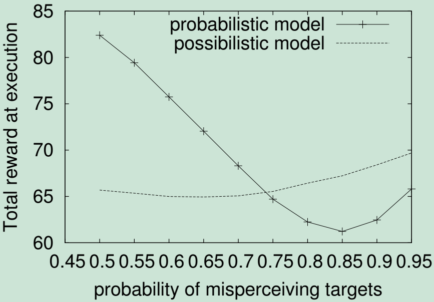
Use of the π-MOMDP in practice

simulations

- **goal:** reach the object $A = T1$ or $T2$
- noisy observations of the location of the object A

Recognition mission: robot on a grid, targets $T1$ & $T2$



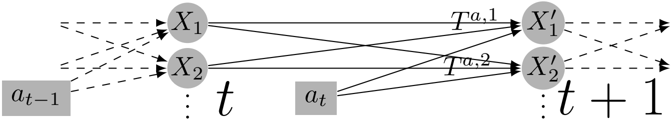


in reality, misperception probability in the error zone: $P_{bad} > \frac{1}{2}$

Factored π-MOMDP and computations with ADDs

qualitative possibilistic models to reduce complexity

contribution (AAAI-14): factored π-MOMDP
$\Leftrightarrow$ state space $\mathcal{X} = \mathcal{S}_v \times \Pi_{\mathcal{L}}^{\mathcal{S}_h}$ = Boolean variables $(X_1, \dots, X_n)$
+ independence assumptions $\Leftarrow$ graphical model



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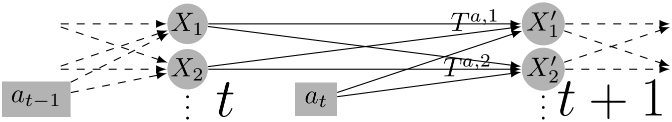
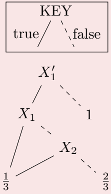


- **factorization:** transition functions $T_i^a = \pi\left(X'_i \mid parents(X'_i), a\right)$ stored as **Algebraic Decision Diagrams (ADD)**

probabilistic case:
SPUDD (*Hoey et al., 1999*)



example of ADD

Simplify computations with π-MOMDPs

Resulting π-MOMDP solver: PPUDD

- probabilistic model: $+$ and $\times$ $\Rightarrow$ new values created
  $\Rightarrow$ number of ADDs leaves **potentially huge**
- possibilistic model: min and max $\Rightarrow$ values $\in \mathcal{L}$ finite
  $\Rightarrow$ number of leaves bounded, **ADDs smaller**.

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PPUDD: Possibilistic Planning Using Decision Diagrams

- factorization $\Rightarrow$ each DP steps divided into n stages
  $\rightarrow$ smaller ADDs $\Rightarrow$ **faster computations**

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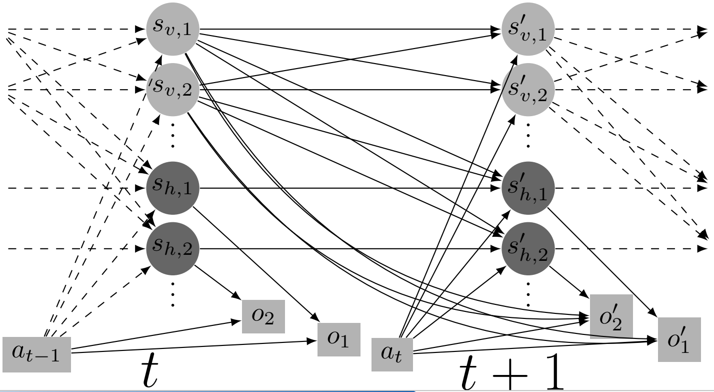
- computations on trees: *CU Decision Diagram Package*.

Simplifying computations with π-MOMDPs

Natural factorization: belief independence

contribution (AAAI-14):

independent sensors, hidden states, . . . $\Rightarrow$ graphical model



Simplifying computations with π-MOMDPs

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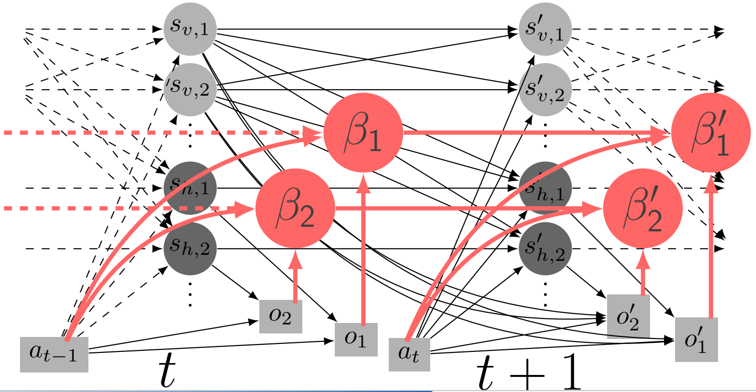
d-Separation $\Rightarrow$ $(s_v, \beta) = (s_{v,1}, \dots, s_{v,m}, \beta_1, \dots, \beta_l)$

$\beta_i \in \Pi_{\mathcal{L}}^{\mathcal{S}_{h,i}}$, belief over $s_{h,i}$



Simplify computations with π-MOMDPs

Experiments – perfect sensing: Navigation problem

PPUDD vs SPUDD (*Hoey et al.*)
Navigation benchmark: reach a goal – spots with accident risk
M1 (resp. M2) optimistic (resp. pessimistic) criterion

Simplify computations with π-MOMDPs

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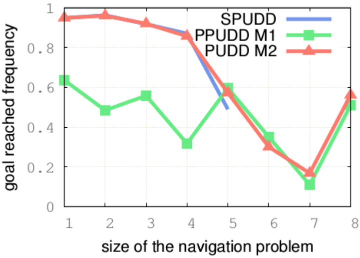
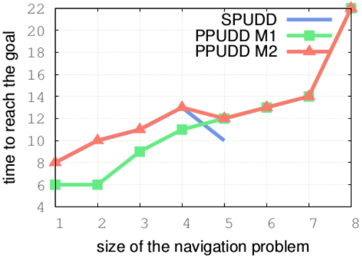
Performances, function of the instance size

reached goal frequency



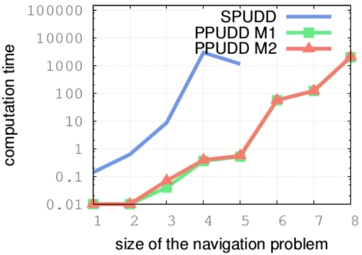
steps to reach the goal



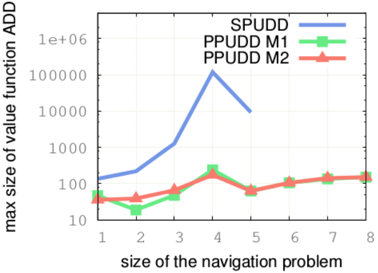
Simplify computations with π-MOMDPs

Experiments – perfect sensing: Navigation problem

computation time



max size of ADDs



- PPUDD + M2 (pessimistic criterion) **faster with same performances** as SPUDD
- SPUDD only solves the first 5 instances
- verified intuition: ADDs are smaller

Simplify computations with π-MOMDPs

Experiments – imperfect sensing: RockSample problem

PPUDD vs APPL (*Kurniawati et al.*, solver MOMDP)
symbolic HSVI (*Sim et al.*, solver POMDP)
RockSample benchmark: recognize and sample "good" rocks

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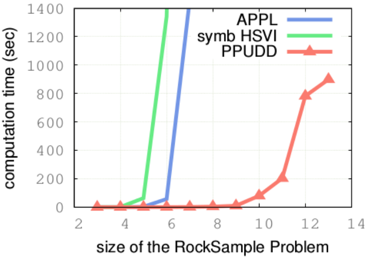
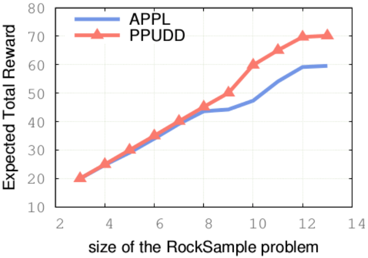
computation time:

probabilistic solvers, prec. 1
PPUDD, exact resolution



average of rewards

APPL stopped when
PPUDD end



- **approximate model + exact resolution solver**
  $\rightarrow$ improvement of computation time and performances

IPPC 2014 – ADD-based approaches: PPUDD vs symbolic LRTDP

PPUDD + BDD mask over reachable states.

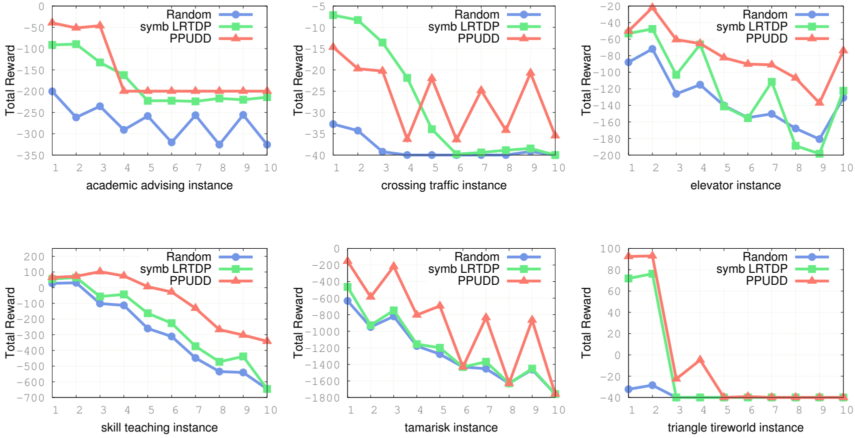


Figure : average of rewards over simulations

Qualitative possibilistic approach: benefits/drawbacks

towards a hybrid POMDP

- **granulated** belief space (discrete)
- efficient problem **simplification** (PPUDD 2$\times$ better than LRTDP with ADDs)
- **ignorance and imprecision** modeling

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- ADD methods $\prec$ state space search methods
  $\rightarrow$ winners of IPPC 2014: $2\times$ better than PPUDD
- choice of the qualitative criterion (optimistic/pessimistic)
- preference $\rightarrow$ non additive degrees
  $\rightarrow$ same scale as possibility degrees (commensurability)
- coarse approximation of probabilistic model
  $\rightarrow$ no frequentist information

A hybrid model

a probabilistic POMDP with possibilistic belief states

hybrid approach

- agent knowledge $=$ possibilistic belief states
- probabilistic dynamics & quantitative rewards

A hybrid model

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Usefullness

$\rightarrow$ **heuristic** for solving POMDPs:

results in a standard (finite state space) MDP

$\rightarrow$ problem with **qualitative** & **quantitative** uncertainty

Transitions and rewards

belief-based transition and reward functions

- possibility distribution $\beta \rightarrow$ probability distribution $\overline{\beta}$ using poss-prob tranformations *(Dubois et al., FSS-92)*

Transition function on belief states

$$\Rightarrow \mathbf{p}\left(\beta' \middle| \overline{\beta}, a\right) = \sum_{\substack{o' \text{ t.q.} \\ update(\beta,a,o')=\beta'}} \mathbf{p}\left(o' \mid \overline{\beta}, a\right)$$

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- reward cautious according to β

Pessimistic Choquet Integral

$$r(\beta, a) = \sum_{i=1}^{\#\mathcal{L}-1} (l_i - l_{i+1}) \cdot \min_{\substack{s \in \mathcal{S} \text{ s.t.} \\ \beta(s) \geqslant l_i}} r(s, a)$$

Resulting MDP

translation from hybrid POMDP to MDP – **contribution (SUM-15):**

input: a POMDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, O, r, \gamma \rangle$
output: the MDP $\langle \tilde{\mathcal{S}}, \mathcal{A}, \tilde{T}, \tilde{r}, \gamma \rangle$:

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$$\textbf{criterion: } \mathbb{E}_{\beta_t \sim \tilde{T}} \left[\sum_{t=0}^{+\infty} \gamma^t \cdot \tilde{r}\left(\beta_t, d_t\right) \right].$$

General variable classification **contribution (SUM-15):**

3 classes of state variables – state space factorization

variable: **visible** $s'_v \in \mathbb{S}_v$

inferred hidden $s'_h \in \mathbb{S}_h$

fully hidden $s'_f \in \mathbb{S}_f$

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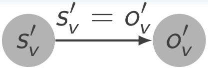

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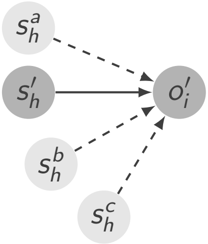

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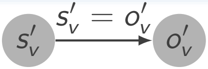

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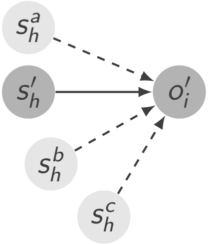

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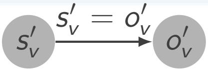

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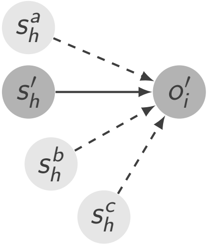

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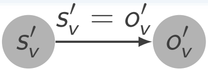

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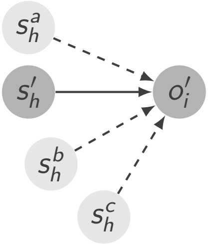
⚠ $\mathcal{P}(o'_i)$ may contain visible variables.



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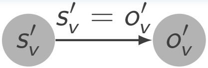

inferred hidden $s'_h \in \mathbb{S}_h$

$$\beta_{t+1}\Big(parents(o'_i)\Big) = \beta_{t+1}(s_h, s^a_h, s^b_h, s^c_h)$$

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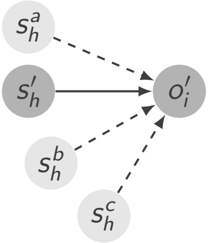

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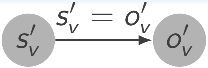
General variable classification **contribution (SUM-15):**

3 classes of state variables – state space factorization

variable: **visible** $s'_v \in \mathbb{S}_v$



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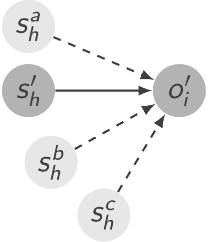

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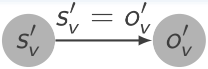

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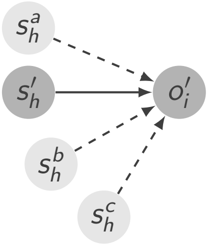
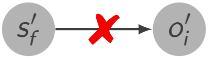


fully hidden $s'_f \in \mathbb{S}_f$

$\rightarrow$ observations don't inform belief state on s'_f.



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Possibilistic belief variables

global belief state

bound over the global belief state

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- min of marginals = a **less informative** belief state
- computed using **marginal belief states**
  $\rightarrow$ factorization & smaller state space

Conclusion

contributions

- **modeling efforts**: $\rightarrow$ human-machine interaction

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  $\rightarrow$ undeterminate horizon + optimality proof

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- **hybrid POMDP** $\xrightarrow{\text{translation}}$ MDP
  $\rightarrow$ probabilities on possibilistic belief states
  pessimistic rewards (Choquet integral)
  $\rightarrow$ factored POMDP $\xrightarrow{\text{translation}}$ factored MPD

Conclusion

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quantitative information may be available: hybrid work

- IPPC problems (factored POMDPs);
- tests of this approach:
  1. **simplification:** π distributions definition?
  2. **imprecision:** robust in practice?

Conclusion

Thank you!

produced work:

- *Qualitative Possibilistic Mixed-Observable MDPs*, **UAI-2013**
- *Structured Possibilistic Planning Using Decision Diagrams*, **AAAI-2014**
- *Planning in Partially Observable Domains with Fuzzy Epistemic States and Probabilistic Dynamics*, **SUM-2015**
- *Processus Décisionnels de Markov Possibilistes à Observabilité Mixte*, Revue d'Intelligence Artificielle (**RIA journal**)
- *A Possibilistic Estimation of Human Attentional Errors*, submitted to **IEEE-TFS journal**