Strategy Computation for Robotic Missions using Possibilistic Approaches

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retour sur innovation

Outline

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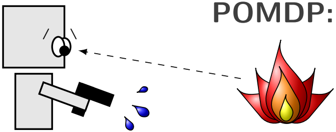

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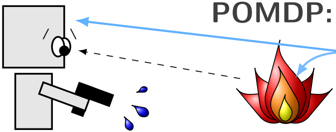

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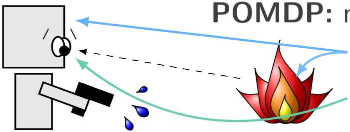

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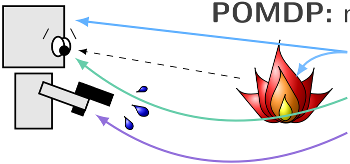

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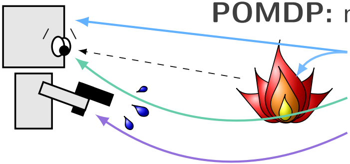

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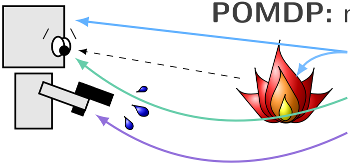

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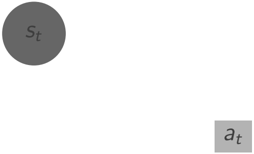

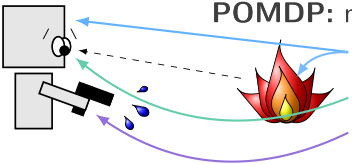

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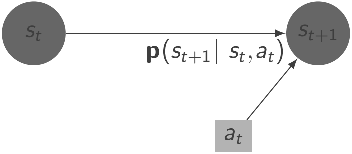

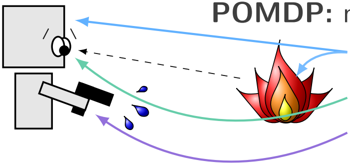

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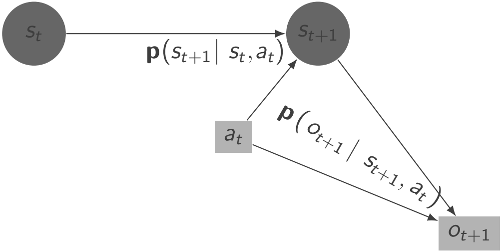

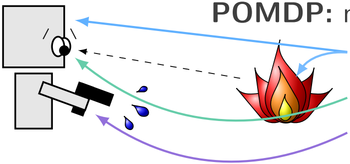

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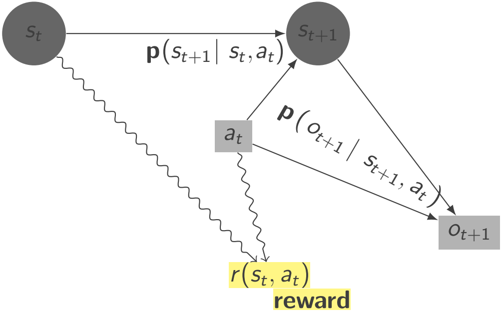

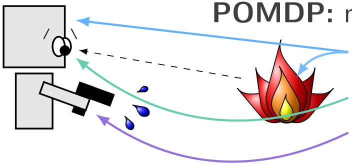

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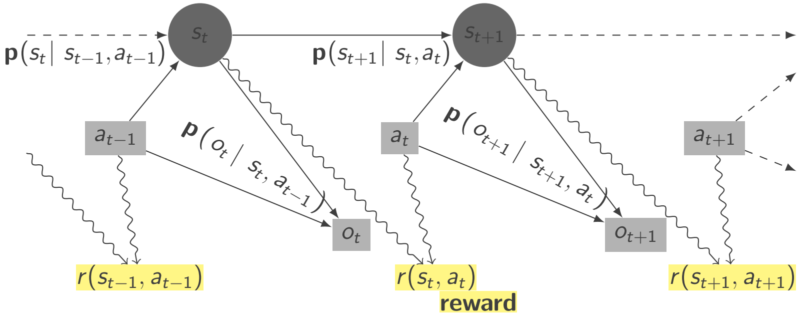

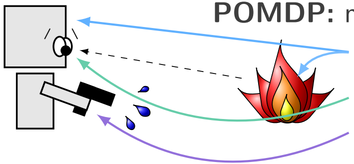

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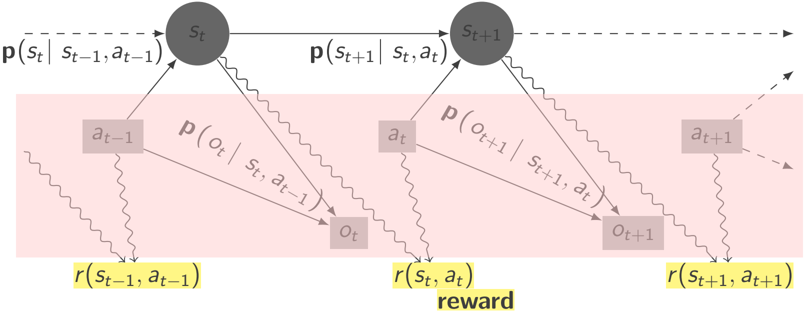

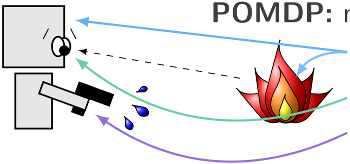

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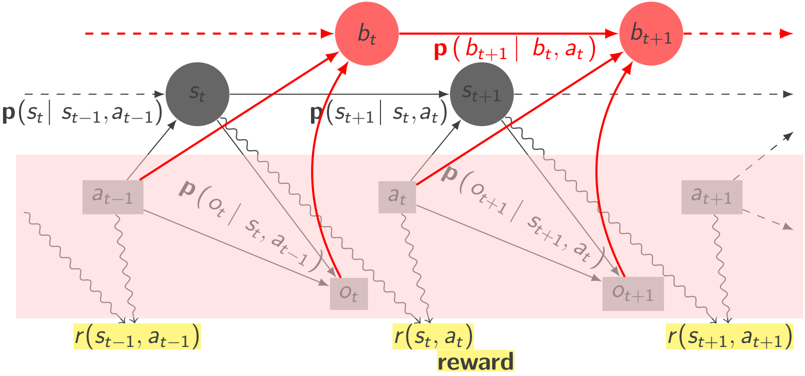
- b **belief.**



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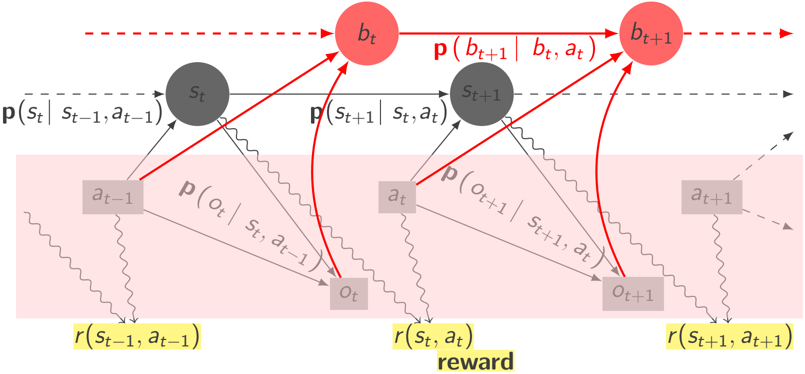
Context and Background

Bayes rule, Strategy, Criterion.

$$b_{t+1}(s') = nextBelief\left(b_t, a, \tilde{o}\right) = \frac{\mathbf{p}(\tilde{o}|s',a).\sum_s \mathbf{p}(s'|s,a)b_t(s)}{\sum_{\underline{s},\bar{s}} \mathbf{p}(\tilde{o}|\bar{s},a).\mathbf{p}(\bar{s}|\underline{s},a)b_t(\underline{s})}$$



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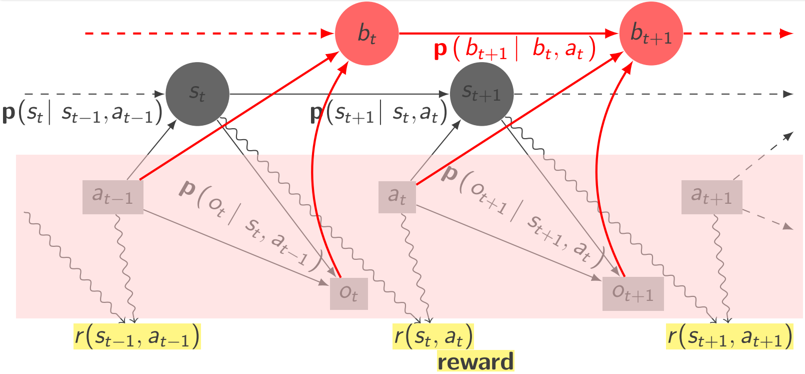
Actions choice: strategy $\delta(b_t) = a_t \in \mathcal{A}$

maximizing $\mathbb{E}[\sum_{t=0}^{+\infty} \gamma^t r(s_t, \delta(b_t))|b_0]$, $0 < \gamma < 1$.



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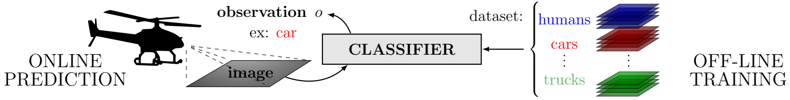

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Qualitative Possibility Theory:
$\rightarrow$ simplification, ignorance and imprecision modeling.

Qualitative Possibility Theory

$\mathcal{L}$ finite scale, ex: $\left\{0, \frac{1}{n}, \frac{2}{n}, \dots, 1\right\}$.

events $e \subset \Omega$ (sample space)

sorted with possibility **degrees** $\pi(e) \in \mathcal{L}$,

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Probability ($\mathbb{P}$) / Possibility (Π):

e_1 or e_2	$\mathbf{p}(e_1) + \mathbf{p}(e_2 \cap \overline{e_1})$	$\max\left\{\pi(e_1), \pi(e_2)\right\}$
e_1 and e_2	$\mathbf{p}(e_1).\mathbf{p}\left(e_2 \mid e_1\right)$	$\min\left\{\pi(e_1), \pi(e_2 \mid e_1)\right\}$

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Possibilistic models: π-MOMDPs

possibilistic POMDPs (π-POMDPs): *Sabbadin UAI-98*.

- finite belief space $\#\mathcal{B} = \#\mathcal{L}^{\#\mathcal{S}} - (\#\mathcal{L} - 1)^{\#\mathcal{S}}$

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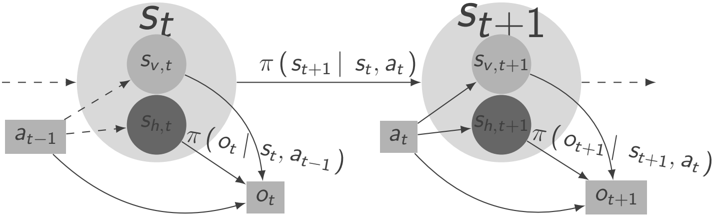

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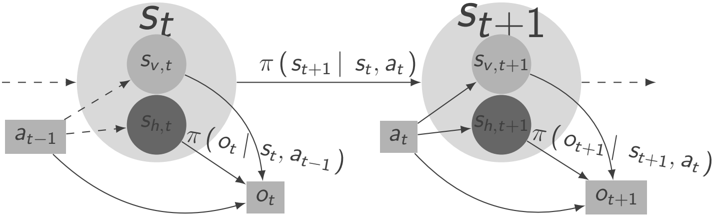

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- beliefs are only over $\mathcal{S}_h$ (component s_v observed),
- computations on $\mathcal{X} = \mathcal{S}_v \times \mathcal{B}_h$ whose size is

$$\#\mathcal{X} = \#\mathcal{S}_v \cdot \left(\#\mathcal{L}^{\#\mathcal{S}_h} - (\#\mathcal{L}-1)^{\#\mathcal{S}_h}\right) \ll \#\mathcal{B}.$$

π-MOMDPs in practice

contribution (UAI13): Infinite Horizon

Dynamic Programming scheme: $\#$ iterations $< \#\mathcal{X}$.

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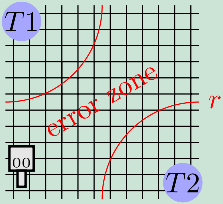
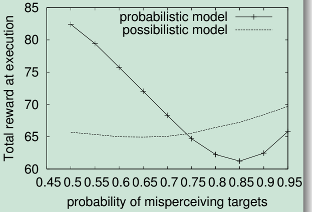
Recognition mission: robot on a grid $g \times g$, 2 targets $T1$, $T2$.

- **goal:** reach the object $A = T1$ or $T2$;
- noisy observations of the targets natures: $\mathbf{p}\left(o' \mid s', a\right)$.





Actually, misperception in the error zone is: $P_{bad} > \frac{1}{2}$.

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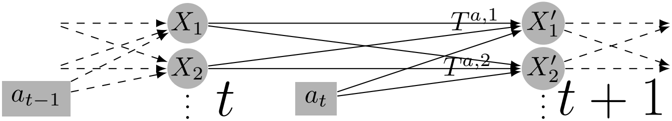
Factorization and symbolic solver

contribution (AAAI14): factored π-MOMDP
$\Leftrightarrow$ state space $\mathcal{X} = \mathcal{S}_v \times \mathcal{B}_h =$ Boolean variables $(X_1, \ldots, X_n)$
$+$ independence assumptions $\Leftarrow$ graphical model.



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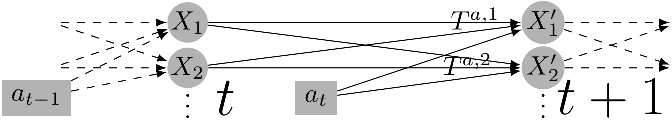
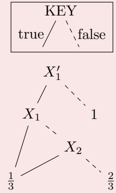


- transition functions
  $T_i^a = \pi\left(X_i' \mid \textit{parents}(X_i'), a \right)$
  represented by Algebraic Decision Diagrams (ADD).
  (SPUDD – *Hoey et al., UAI-99*).



example of ADD

Solver π-MOMDP résultant: PPUDD

- probabilistic model: $+$ and $\times$ $\Rightarrow$ new values created, number of ADDs leaves **potentially huge**.
- possibilistic model: min and max $\Rightarrow$ values $\in \mathcal{L}$ finite, number of leaves bounded, **ADDs smaller**.

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while $V^* \neq V^c$ **do**
    $V^* \leftarrow V^c$;
    for $a \in \mathcal{A}$ **do**
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        $V^c \leftarrow \boxed{\max}\{ q^a, V^c \}$;
        update δ to a where $q^a = V^c$ and $V^c > V^*$;
return (V^*, δ) ;

computations on trees: *CU Decision Diagram Package*.

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factorization

$\Rightarrow$ dynamic programming

divided into *n* stages

used ADDs smaller

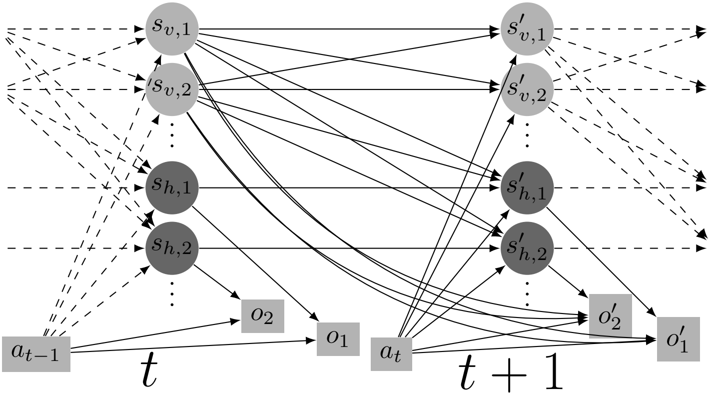
$\rightarrow$ **faster computations.**

computations on trees: *CU Decision Diagram Package*.

Natural factorisation: belief independence.

contribution (AAAI14): π-MOMDP following independence assumptions of the graphical model:

$\Rightarrow$ $(s_v, \beta) = (s_{v,1}, \ldots, s_{v,m}, \beta_1, \ldots, \beta_l)$, β_i belief over $s_{h,i}$.



Natural factorisation: belief independence.

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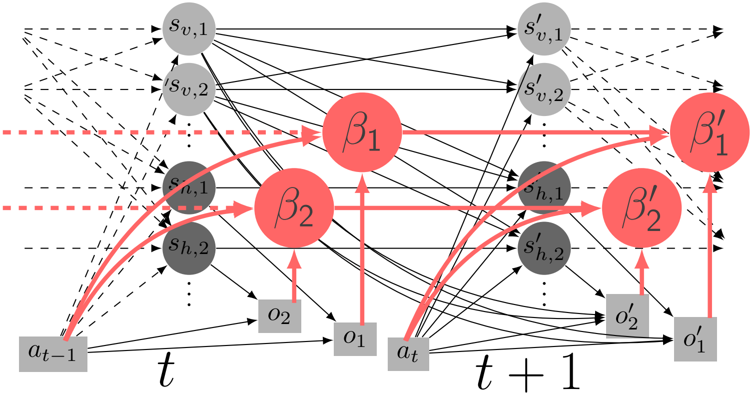

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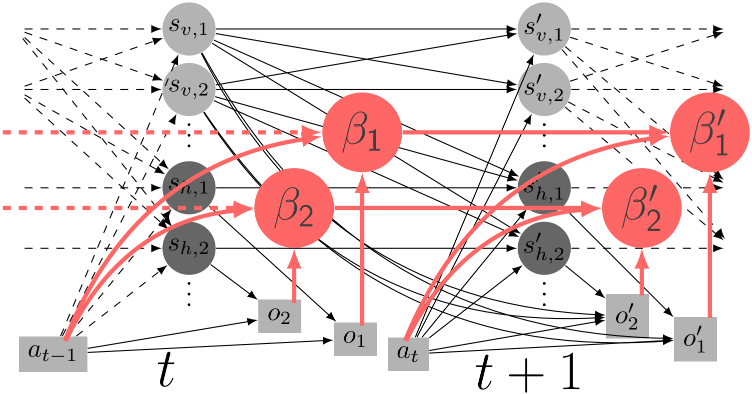
assumptions: independent captors, hidden states...



Experiments: Navigation problem – agent = robot.

PPUDD vs SPUDD (*Hoey et al.*)
Navigation benchmark: reach a goal; spots with accident risk.
2 possibilistic translations: M1 (optimistic) et M2 (cautious).

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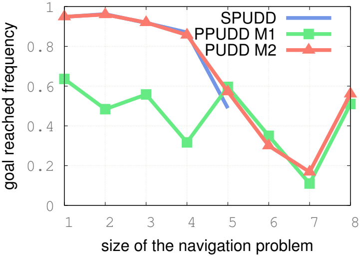
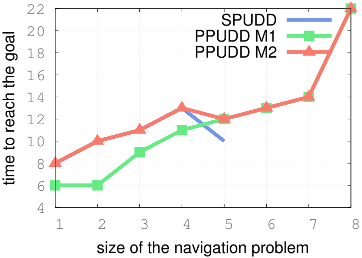
Performances, function of the instance size

reached goal frequency

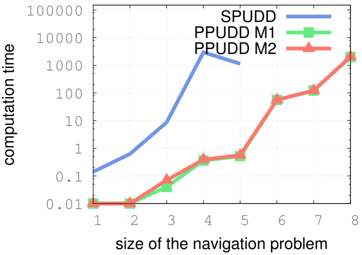


time to reach the goal

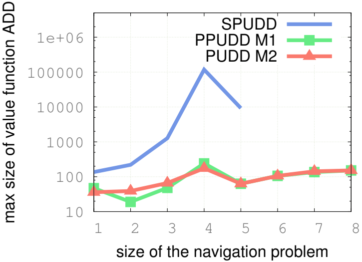


Experiments: Navigation problem – agent = robot.

computation time



max size of ADDs



- PPUDD + M2 (pessimistic translation) **faster and same performances** as SPUDD;
- SPUDD only solves the 5 first instances;
- verified intuition: ADDs are smaller.

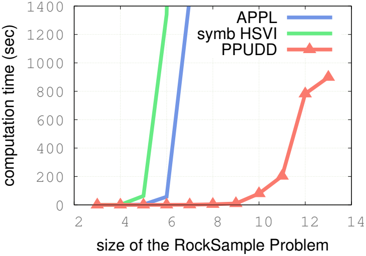
Experiments: RockSample problem – agent = robot.

PPUDD vs APPL (*Kurniawati et al.*, solver MOMDP);
symbolic HSVI (*Sim et al., solver POMDP*).
RockSample benchmark: recognize and sample "good" rocks;

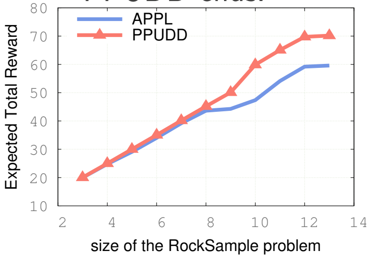
Experiments: RockSample problem – agent = robot.

PPUDD vs APPL (*Kurniawati et al.*, solver MOMDP);
symbolic HSVI (*Sim et al., solver POMDP*).
RockSample benchmark: recognize and sample "good" rocks;

computation time:
probabilistic solvers, prec. 1;
PPUDD, exact resolution.



average of rewards
APPL stopped when
PPUDD ends.



- **approximate model + exact resolution solver**
  → can improve of computation time and performances.

IPPC 2014 – MDP track. ADDs-based approaches: PPUDD vs symbolic LRTDP (*Bonet et al.*)

PPUDD + BDD mask over reachable states.

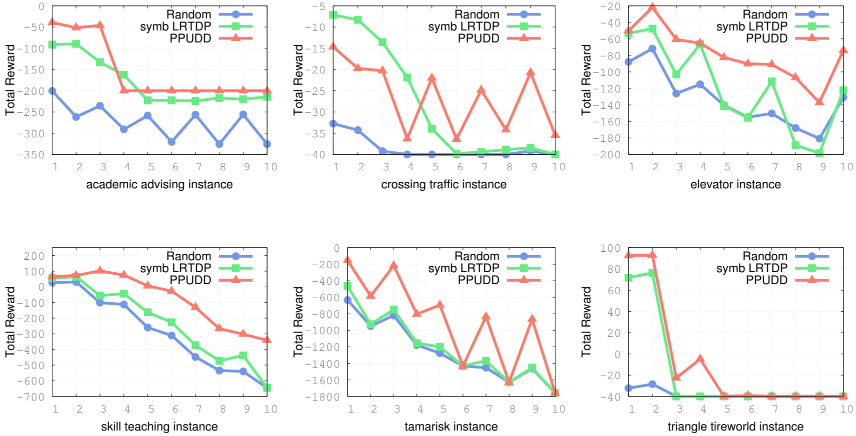


Figure : mean of rewards over simulations.

Outline

Possibility Theory:

- **granulated** belief space representation (discretization),
- efficient problem **simplification** (PPUDD $2\times$ better than LRTDP with ADDs);
- **ignorance and imprecision** modeling.

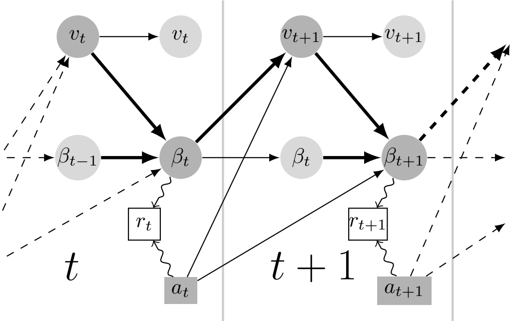
Conclusions/Perspectives

Possibility Theory:

- **granulated** belief space representation (discretization),
- efficient problem **simplification** (PPUDD $2\times$ better than LRTDP with ADDs);
- **ignorance and imprecision** modeling.

- ADD methods $\prec$ state space search methods: winners of IPPC 2014, (PROST & GOURMAND, $2\times$ better than PPUDD).
- choice of the qualitative criterion (optimistic/pessimistic);
- non additive utility degrees, from the same scale as possibility degrees.

Work in progress...
towards a hybrid POMDP



POMDP $\rightarrow$ POMDP with possibilistic beliefs

- transition probability distributions over possibilistic beliefs;
- reward aggregation: Choquet integral;
- factored POMDPs leads to factorized MPDs;
- resolution with any MDP solver.

Thank you.