July 15 (Monday) Explanation of Gasbeam Theory

THE FIRST THING WE NEED FOR MY THEORY IS A WAY OF DEALING WITH COLLISIONS. IN DEALING WITH THIS, WE NOTE THAT IN THE LAB FRAME, EVEN HARD SPHERE ~~GO~~ COLLISIONS ~~RESULT~~ HAVE IN A FORWARD PEAKED RESULT (AS OPPOSED TO IN THE C.M. FRAME, WHERE YOU ~~HAVE~~ RANDOMIZING COLLISIONS). AS A RESULT, WE CANNOT SUCCESSFULLY TREAT THE COLLISIONS AS RANDOMIZING. INSTEAD, WE WILL USE THE CENTRAL LIMIT THEOREM TO AVOID REQUIRING KNOWLEDGE OF THE EXACT DIFFERENTIAL CROSS SECTION. WE DEFINE A CROSS SECTION

$$\Sigma = \int d\Omega \; \theta^2_{LAB} \frac{d\sigma}{d\Omega}$$

~~BUT THIS IS ACTUALLY WE~~ THIS COMPLICATES LIFE!

THUS THE CHANGE IN ANGLE AFTER TRAVELING Δz THROUGH A NUMBER DENSITY n (THIS IS A RANDOM WALK PROBLEM) IS

$$\Delta\theta = \sqrt{\Sigma n \Delta z}$$

SO THIS IS OUR FORMULA. WE MUST SIMPLY KEEP Δz SMALL ENOUGH ~~#~~ SO THAT $\Delta\theta$ IS SMALL. BUT WAIT! WHAT IS Σ? WELL, TO ANSWER THAT, I DECIDED TO USE A HARD SPHERE MODEL, SINCE H.S. CROSS SECTIONS ARE READILY AVAILABLE. SO I SIMPLY RAN A MONTE CARLO MODEL (BECAUSE THIS SEEMED THE EASIEST WAY) TO FIND $\langle \theta^2_{LAB} \rangle$ FOR HARD SPHERE COLLISIONS. I FOUND $\sqrt{\langle \theta^2_{LAB} \rangle} = 1.354$. SO $\Sigma = \sigma_{HS} (1.354)^2$. :)

THE SECOND THING TO MY THEORY IS TO CALCULATE PROFILES, GIVEN A DENSITY DISTRIBUTION. (NOTE: ALL DENSITY DISTRIBUTIONS USED ARE ASSUMED TO BE RANDOM OR ISOTROPIC DENSITY DISTRIBUTION. I.E. THAT PORTION OF THE GAS WHICH IS COLLIMATED DOESN'T COUNT. ~~THUS THE S~~ WH THUS THE STATEMENTS I AM ABOUT TO MAKE COULD BE SEEN AS DEFINITIONS OF THE DENSITY DISTRIBUTION n, RATHER THAN DERIVATIONS FROM IT.). MY METHOD IS SIMPLE A STRAIGHTFORWARD EXTENSION OF CLAUSING AND EVERYONE ELSE'S METHOD, ~~ETC~~ WITH THE DIFFERENCE BEING THAT MY GAS-GAS COLLISIONS ARE NOT RANDOMIZING. SO I ONLY HAVE TWO SOURCES OF GAS:

1. THAT COMING IN THE BACK OF THE TUBE.
2. THAT WHICH IS BOUNCING OFF THE WALLS.

THE FLUX COMING FROM EACH OF THESE SOURCES IS (ASSUMING LINEAR DENSITY DEPENDANCE, FOR THE INTEGRATED VALUES)

① $$\text{FLUX} = \int \frac{n v_0}{4}\, dA = \frac{n_{entrance}\pi r^2 v_0}{4}$$

② $$\text{FLUX} = \int \frac{n\, 2\pi r\, v_0}{4}\, dz = \left(\frac{n_{entrance}^2 - n_{exit}^2}{4\,\frac{dn}{dz}}\right) 2\pi r v_0$$

HOWEVER, HALF OF THE FLUX FROM SOURCE ② WILL BE GOING BACKWARDS, AND THUS WILL PROBABLY NOT LEAVE THE FRONT OF THE TUBE BEFORE HITTING A WALL, SO FOR OUR PURPOSES WE IGNORE IT, GIVING A REVISED VALUE FOR THE FLUX

② $$\text{FLUX} = \left(\frac{n_{entrance}^2 - n_{exit}^2}{4\,\frac{dn}{dz}}\right) \pi r v_0$$

GIVEN THESE FLUXES, IT IS A STRAIGHTFORWARD MONTE CARLO CALCULATION TO GET THE PROFILE AND FLOW RATE (USING OUR PREVIOUS METHOD FOR GAS-GAS COLLISIONS). OH. I ALMOST FORGOT TO MENTION. THE GAS FROM BOTH OF THESE SOURCES COMES OFF WITH A $\cos\theta$ DIST^N WHERE θ IS MEASURED FROM THE NORMAL OF

SO... WHAT IS n? OR RATHER n_{EXIT} AND $\frac{dn}{dz}$ (HENCEFORTH n', DEFINED TO BE POSITIVE). WELL THESE ARE TWO THINGS, WHICH IS A PAIN (SINCE YOU CAN GET LOST IN TWO DIMENSIONS). SO I DECIDED TO LINK THEM. TO DO THIS, I NOTED THAT THERE ARE TWO WAYS OF CALCULATING THE FLOW RATE ~~THROUGH~~ OF ~~A~~ ISOTROPIC GAS THROUGH A TUBE: ~~IT IS PROPORTION~~ THE FLOW RATE IS PROPORTIONAL TO THE EXIT DENSITY, ~~AS~~ AND THE FLOW RATE IS PROPORTIONAL TO THE DENSITY GRADIENT. I FIGURED THAT NEAR THE EXIT OF THE TUBE, THESE TWO FLOW RATES SHOULD BE EQUAL, SINCE ~~THE AMOUNT OF COLLIMATED~~ THE FLOW RATE DUE TO COLLIMATED GAS CAN'T BE CHANGING MUCH. SETTING THEM EQUAL, I FOUND THAT

$$n' = \frac{n_{exit}}{d} \quad d \leftarrow \text{DIAMETER OF TUBE.}$$

THIS WONT BE TRUE FAR AWAY FROM THE EXIT, BUT THE REGION NEAR THE EXIT HAS BY FAR THE MOST SIGNIFICANT EFFECT, SO ANY NONLINEAR EFFECTS FAR FROM THE EXIT AREN'T TOO IMPORTANT.

SO NOW WE HAVE ONLY ONE DIMENSION, SO WE CAN'T GET LOST! :) BUT WE STILL NEED TO KNOW WHAT n' IS. THE CORRECT ANSWER IS TO PICK AN n', USE OUR MONTE CARLO METHOD TO FIND THE FLOW RATE, AND IF IT IS LOWER THAN THE FLOW RATE WE WANT TO USE, INCREASE n', AND OTHERWISE DECREASE IT, UNTIL WE HAVE THE RIGHT FLOW RATE.

NOTE: WE DO NOT USE DRIVE PRESSURE (OR EQUIVALENTLY, THE ENTRANCE DENSITY) AS A PARAMETER, BECAUSE OF NONLINEAR EFFECTS. EXPERIMENTALLY THE FLOW RATE IS NOT PROPORTIONAL TO THE DRIVE PRESSURE, AND SINCE EXIT DENSITY IS SO MUCH MORE IMPORTANT THAN ENTRANCE DENSITY, WE FIND FLOW RATE TO BE A FAR PREFERABLE PARAMETER FOR CHARACTERIZING GAS BEAMS

Tuesday 7/16/96

REVISED EXPLANATION OF THE DERIVATION OF THE FORMULA

$$n' = \frac{n_{exit}}{d}$$

IN REALITY, I COULD FIND NO FORMULA IN DUSCHMAN FOR f.r. OF A SHORT TUBE AS A FUNCTION OF n'. I FOUND, HOWEVER AN EQUIVALENT ITEM, WHICH I WAS ABLE TO USE TO GET THE ABOVE FORMULA. I PREFER USING THE PHYSICAL EXPLANATION OF ~~SETTING~~ THE TWO FLOW RATES EQUAL, BUT MATHEMATICALLY, THIS IS A BIT EASIER.

DUSCHMAN GIVES (FOR SHORT TUBES) eq. (19a)

$$K = \frac{1}{1 + 0.5\frac{l}{r}} = \frac{1}{1 + \frac{l}{d}}$$ ← THIS IS AN EMPIRICAL FORMULA, ACCORDING TO DUSCHMAN

NOW

$$K \equiv \frac{n_{exit}}{n_{entrance}} = \frac{n_{exit}}{n_{exit} + n' l} = \frac{1}{1 + l\left(\frac{n'}{n_{exit}}\right)}$$

CLEARLY, THE TWO ~~we d~~ EQUATIONS ABOVE IMPLY THAT

$$\frac{n'}{n_{exit}} = \frac{1}{d} \Rightarrow n' = \frac{n_{exit}}{d}.$$

UNFORTUNATELY, I DON'T THINK THIS MATH IS VERY ENLIGHTENING. PERHAPS IT WOULD BE BETTER TO DERIVE FROM THIS K A f.r. AS A FN. OF n', THEN USE THAT IN OUR EXPLANATION.