

# INTRODUCTION TO CAUSAL INFERENCE

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v.2017-12-03-7:15pm

## Direct and indirect effects I

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A: Specific causal relations between variables (causal effect, common causes, conditioning on a common effect) that can be notated using directed acyclic graphs.

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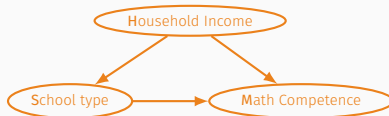
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A: With different nonparametric and parametric methods that calculate the (covariate-adjusted) group contrasts that identify the effect of interest.

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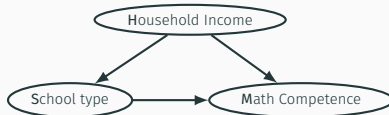
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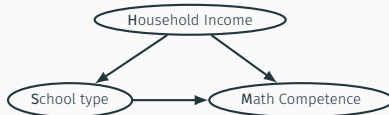
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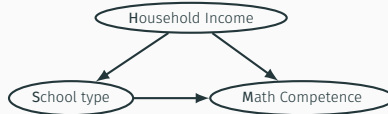
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*All covariates measured in the data; no conditioning on additional variables.*

4. Assess whether the causal parameter of interest can be identified with the available data and define the respective statistical parameter,  $\delta$ :

*The  $ATE_{S \rightarrow M}$  can be identified by the difference in mean math competence between private and public school students adjusted for  $Z = \{H\}$ .*

$$\delta = \sum_h E(M|S = \text{private}, H = h)P(H = h) - \sum_h E(M|S = \text{public}, H = h)P(H = h)$$



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## 7. Interpret the results and discuss assumptions:

*Statistical interpretation: On average, private school students have a 4.12 [95% CI: -4.31,-3.93] points lower math competence than public school students after adjusting for differences in household income using the regression model. The respective estimate after adjustment by IPT weighting is -3.78 [95% CI: -4.07,-3.48].*

*Counterfactual interpretation: Attending private school instead of public school leads to an average decrease in math competence by 4.12 [95% CI: -4.31,-3.93] points using regression adjustment. The respective estimate after adjustment by IPT weighting is -3.78 [95% CI: -4.07,-3.48].*

*The statistical interpretation rests on the assumption of no misspecification of the regression model or the IPT weight model, respectively. The counterfactual interpretation rests on the assumptions of positivity and that adjusting for household income d-separates every noncausal path, didn't d-separate any causal path, and didn't d-connect any noncausal path between school type and math competence.*

*The distribution of the SIPTW and overlap in the covariate distributions didn't provide any evidence for severe model misspecification or positivity violations.*

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TODAY:

## DEFINITION AND IDENTIFICATION OF DIRECT & INDIRECT EFFECTS

1. Terminology and goals
2. Definition of direct and indirect effects
3. Identification of direct and indirect effects

# TERMINOLOGY AND GOALS

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The total effect of X on Y can then potentially be decomposed into a direct effect and an indirect effect through M. Note that the direct effect is always defined relative to M, such that there may always be other mechanisms that bring it about.

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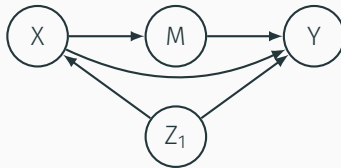
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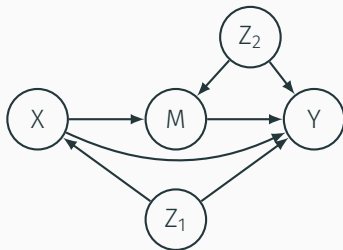
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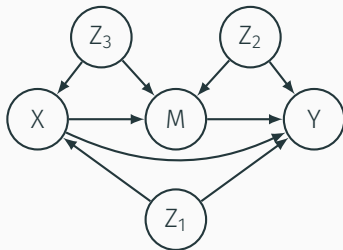
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# WHY ARE WE INTERESTED IN MEDIATION?

- Theory**
- explaining and understanding total effects (opening the “black box”)
  - test of alternative theories regarding mechanisms underlying total effect

- Practice**
- improvement of specific component(s) of interventions/programs
  - intervention on X not feasible but on M

- Methodology**
- alternative identification strategy for total effect when some noncausal paths between X and Y cannot be d-separated by simple covariate adjustment  
(“frontdoor identification”, see Pearl et al., 2016, Ch. 3.4)
  - but stronger (and also untestable) assumptions needed than for simple covariate adjustment

(VanderWeele, 2015, Ch. 1.3.1.)



## DEFINITION OF DIRECT AND INDIRECT EFFECTS

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# JOINT INTERVENTIONS AND NESTED COUNTERFACTUALS

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In contrast, direct and indirect effects correspond to specific *joint* interventions on  $X$  and  $M$ .

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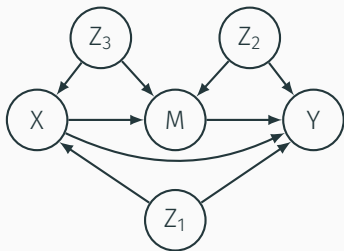
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The natural direct and the natural indirect effect thus create *nested counterfactuals* for  $Y$  as we use a (natural) counterfactual for  $M$  (under specific  $x$ ) which is then used to create a counterfactual for  $Y$ .



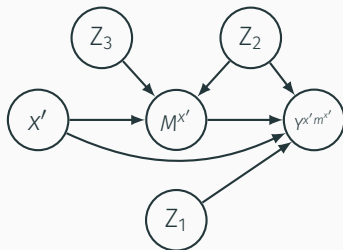
## FROM INTERVENTION TO EFFECT



# FROM INTERVENTION TO EFFECT

Intervention:

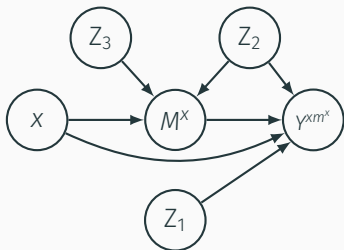
Set  $X$  to  $x'$



# FROM INTERVENTION TO EFFECT

Intervention:

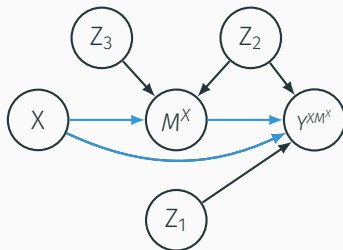
Set  $X$  to  $x$



# FROM INTERVENTION TO EFFECT

Intervention:

Change X



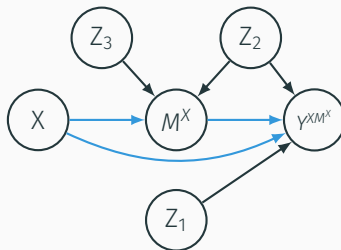
Average total effect

$$ATE = E(Y^{XM^X}) - E(Y^{X'M^{X'}}) = E(Y^X) - E(Y^{X'})$$

# FROM INTERVENTION TO EFFECT

Intervention:

Change X



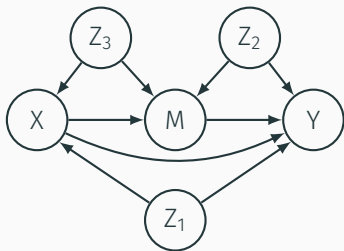
Example:  
NH poverty — X  
School resources — M  
Achievement — Y  
(Wodtke and Parbst, 2017)

Average total effect

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$$ATE = E(\text{Achievement}^{\text{poor}}) - E(\text{Achievement}^{\text{nonpoor}})$$

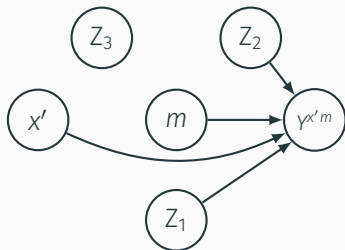
## FROM INTERVENTION TO EFFECT



# FROM INTERVENTION TO EFFECT

Joint intervention:

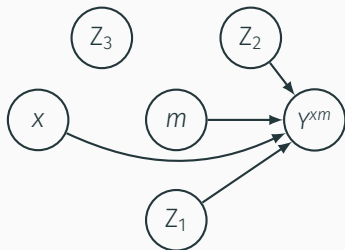
Set  $X$  to  $x'$  and  $M$  to  $m$



# FROM INTERVENTION TO EFFECT

Joint intervention:

Set  $X$  to  $x$  and  $M$  to  $m$

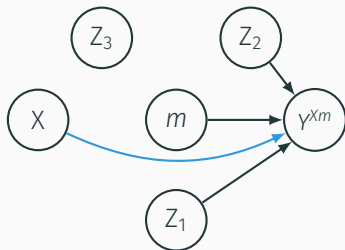




# FROM INTERVENTION TO EFFECT

Joint intervention:

Change X and fix M at m



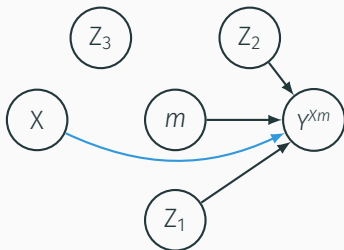
Average controlled direct effects

$$\text{ACDE}(m) = E(\gamma^{xm}) - E(\gamma^{x'm})$$

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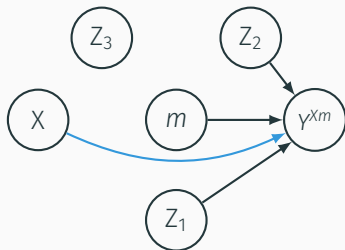
$$\text{ACDE}(m) = E(Y^{xm}) - E(Y^{x'm})$$

$$\text{ACDE}(\text{low}) = E(\text{Achievement}^{\text{poor,low}}) - E(\text{Achievement}^{\text{nonpoor,low}})$$

# FROM INTERVENTION TO EFFECT

Joint intervention:

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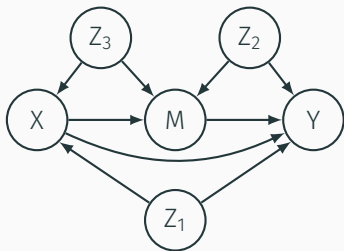
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Achievement — Y  
(Wodtke and Parbst, 2017)

Average controlled direct effects

$$\text{ACDE}(m) = E(Y^{xm}) - E(Y^{x'm})$$

$$\text{ACDE}(\text{high}) = E(\text{Achievement}^{\text{poor,high}}) - E(\text{Achievement}^{\text{nonpoor,high}})$$

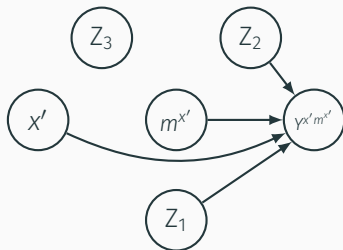
## FROM INTERVENTION TO EFFECT



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Joint intervention:

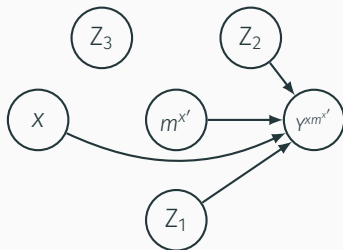
Set  $X$  to  $x'$  and  $M$  to  $m^{x'}$



# FROM INTERVENTION TO EFFECT

Joint intervention:

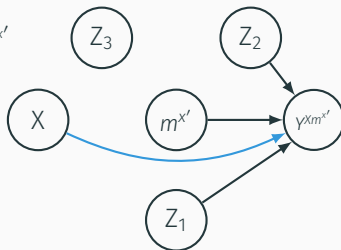
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Joint intervention:

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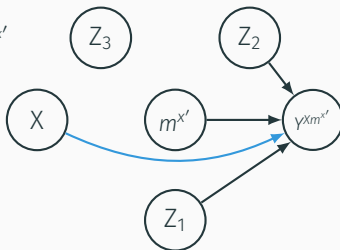
Average natural direct effect

$$\text{ANDE} = E(y^{xm^{x'}}) - E(y^{x'm^{x'}})$$

# FROM INTERVENTION TO EFFECT

Joint intervention:

Change X and fix M at  $m^{x'}$



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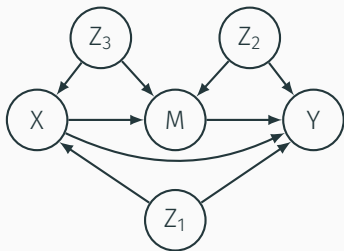
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$$\text{ANDE} = E(\text{Achievement}^{\text{poor}, \text{Resources}^{\text{nonpoor}}}) - E(\text{Achievement}^{\text{nonpoor}, \text{Resources}^{\text{nonpoor}}})$$



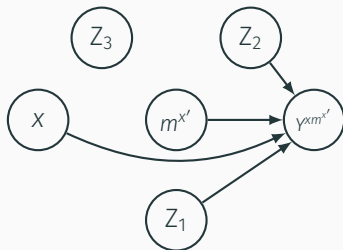
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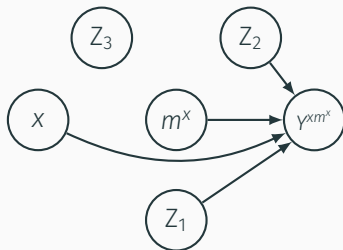
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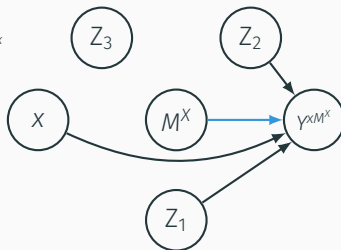
Set  $X$  to  $x$  and  $M$  to  $m^x$



# FROM INTERVENTION TO EFFECT

Joint intervention:

Fix  $X$  at  $x$  and  
change  $M$  from  $m^{x'}$  to  $m^x$



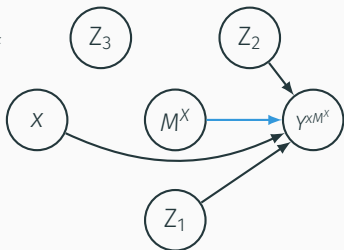
Average natural indirect effect

$$ANIE = E(Y^{xm^x}) - E(Y^{xm^{x'}})$$

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Fix  $X$  at  $x$  and  
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Example:  
NH poverty —  $X$   
School resources —  $M$   
Achievement —  $Y$   
(Wodtke and Parbst, 2017)

Average natural indirect effect

$$ANIE = E(Y^{xm^x}) - E(Y^{xm^{x'}})$$

$$ANIE = E(\text{Achievement}^{\text{poor}, \text{Resources}^{\text{poor}}}) - E(\text{Achievement}^{\text{poor}, \text{Resources}^{\text{nonpoor}}})$$

## Quiz 9.1. Definition of direct and indirect effects

Suppose you want to know to what degree the effect of living in a poor (vs. a nonpoor) neighborhood on children's academic achievement is mediated by school resources.

1. Provide the substantive definition of an average controlled direct effect!
2. Provide the substantive definition of the average natural direct effect!
3. Provide the substantive definition of the average natural indirect effect!

Please complete the quiz on Ilias.

# EFFECT DECOMPOSITION

$$ATE = ANIE + ANDE$$

- Intuition: The ATE captures the effect of X on Y while M runs its “natural course”.

(VanderWeele, 2015, p. 23)

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- Therefore:  $ATE \neq ANIE + ACDE(m)$  if there is an X-M interaction (i.e., it matters for the direct effect of X on Y which value M takes on).
- Without X-M interaction:  $ANDE = ACDE$

(VanderWeele, 2015, p. 23)

## EFFECT DECOMPOSITION: EXAMPLE

X and M are binary here:

$u$	$M^0$	$M^1$	$Y^{00}$	$Y^{01}$	$Y^{10}$	$Y^{11}$
1	0	1	18	18	20	25
2	1	0	45	43	40	48
3	1	0	34	40	41	38
4	0	0	30	20	23	25
...	...	...	...	...	...	...

For unit 2:

$$TE = Y^1 - Y^0 = Y^{1m^1} - Y^{0m^0} = Y^{10} - Y^{01} = 40 - 43 = -3$$

$$CDE(1) = Y^{11} - Y^{01} = 48 - 43 = 5$$

$$CDE(0) = Y^{10} - Y^{00} = 40 - 45 = -5$$

$$NDE = Y^{1m^0} - Y^{0m^0} = Y^{11} - Y^{01} = 48 - 43 = 5$$

$$NIE = Y^{1m^1} - Y^{1m^0} = Y^{10} - Y^{11} = 40 - 48 = -8$$

Calculate the effects for unit 1!

# INTERPRETATION AND RELEVANCE

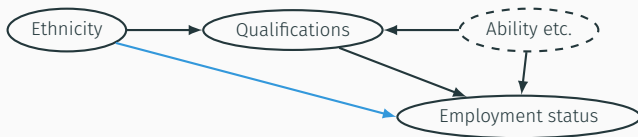
- ACDE(m)**
- effect of intervention that changes  $X$  and sets  $M$  to the same value for all  $u$
  - intervention (theoretically) feasible
  - particularly relevant for practice/policy

- ANDE/ANIE**
- effect of interventions that set  $X$  to the same value for all  $u$  and  $M$  to unit-specific counterfactual values
  - akin to disabling/switching off indirect causal path through  $M$  (for ANDE) or direct causal path not through  $M$  (for ANIE)
  - unit-specific intervention on  $M$  unfeasible (even theoretically), because each unit's  $M$  only observable under one  $x$ .
  - particularly relevant for theory, explanation, effect decomposition

(VanderWeele, 2015, p. 23)

# INTERPRETATION AND RELEVANCE: EXAMPLE

Labor market discrimination (e.g., Bertrand and Mullainathan, 2004):



Much empirical social research studies how much labor market inequality by ethnicity (or gender, or class, or parenthood status, ...) can be explained by employer discrimination (i.e., the effect of ethnicity that is not mediated by qualifications).

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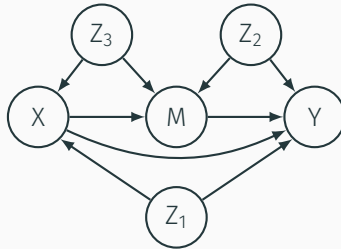
Is this research design able to capture the ANDE (if ethnic inequality in the labor market varies by qualification level)?



## IDENTIFICATION OF DIRECT AND INDIRECT EFFECTS

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# IDENTIFICATION CONDITIONS FOR DIRECT AND INDIRECT EFFECTS

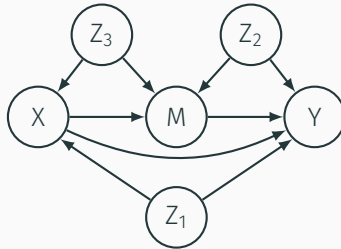


Example:  
NH poverty — X  
School resources — M  
Achievement — Y  
(Wodtke and Parbst, 2017)

# IDENTIFICATION CONDITIONS FOR DIRECT AND INDIRECT EFFECTS

## Assumptions:

- i.  $Z_1, Z_2, Z_3 \perp\!\!\!\perp X$
- ii.  $Z_1, Z_2, Z_3 \perp\!\!\!\perp Y$
- iii.  $Z_1, Z_2, Z_3 \perp\!\!\!\perp M$
- iv.  $Z_1, Z_2, Z_3 \perp\!\!\!\perp X \rightarrow M$



## Example:

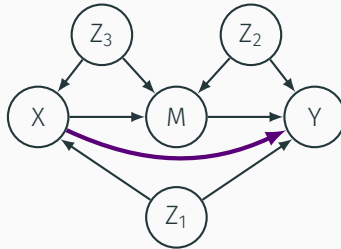
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ACDE(m), ANDE and ANIE can be identified with empirical data under specific substantive assumptions.

# IDENTIFICATION CONDITIONS FOR DIRECT AND INDIRECT EFFECTS

## Assumptions:

- i.  $X \rightarrow Y$  identifiable
- ii.  $X \rightarrow M$  identifiable
- iii.  $M \rightarrow Y$  identifiable
- iv.  $Z_1 \rightarrow X$  and  $Z_1 \rightarrow Y$



## Example:

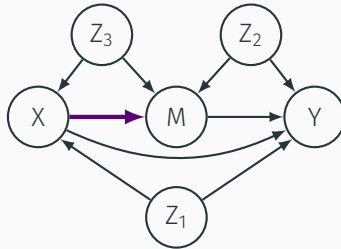
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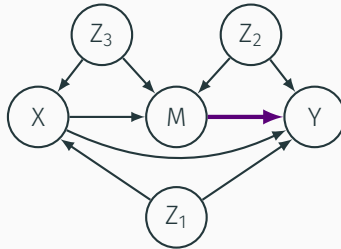
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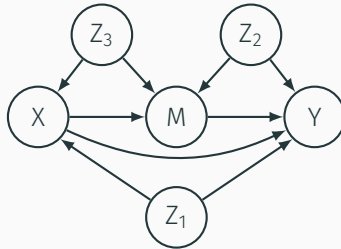
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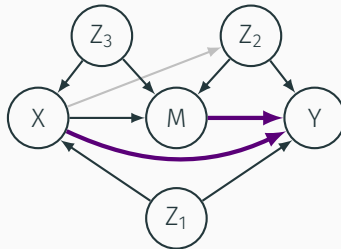
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Average controlled direct effect

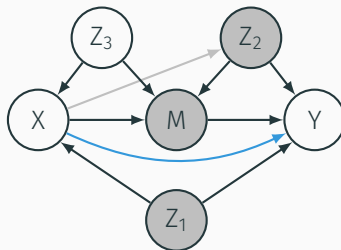
$$\text{ACDE}(m) = E(Y^{x,m}) - E(Y^{x',m})$$



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Average controlled direct effect

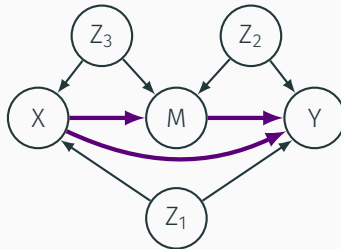
$$\begin{aligned} \text{ACDE}(m) &= E(Y^{x,m}) - E(Y^{x',m}) \\ &= \sum_z [E(Y|X = x, M = m, Z = z) - E(Y|X = x', M = m, Z = z)]P(Z = z) \end{aligned}$$

Adjustment of  $Z=\{Z_1, Z_2, M\}$  (or Randomization of X and Assignment of M)

# IDENTIFICATION CONDITIONS FOR DIRECT AND INDIRECT EFFECTS

Assumptions:

- i.  $X \rightarrow Y$  identifiable
- ii.  $X \rightarrow M$  identifiable
- iii.  $M \rightarrow Y$  identifiable
- iv. No effect  $X \rightarrow Z_2$



Example:  
NH poverty — X  
School resources — M  
Achievement — Y  
(Wodtke and Parbst, 2017)

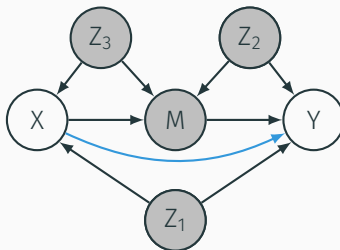
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$$\text{ANDE} = E(Y^{xm^{x'}}) - E(Y^{x'm^{x'}})$$

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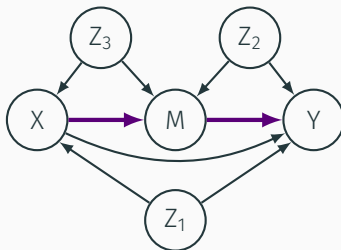
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Adjustment of  $Z=\{Z_1, Z_2, Z_3, M\}$  (or Randomization of X and Adjustment of  $Z=\{Z_2, M\}$ )

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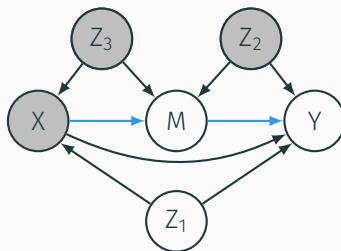
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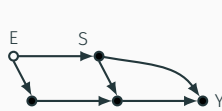
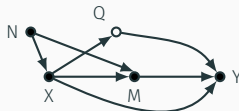
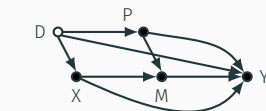
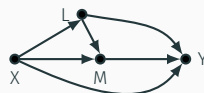
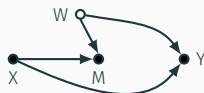
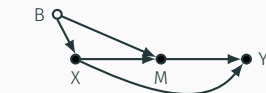
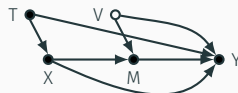
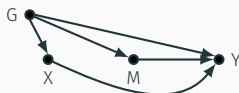
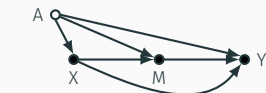
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## Quiz 6.2. Identification of direct and indirect effects

For which of each of the three scenarios is it possible to identify  $ACDE(m)$ ,  $ANDE$ , and  $ANIE$  by covariate adjustment (i.e., assumptions i.-iv. hold)?



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Unlike for identifying total effects, the causal model for mediation must also consider noncausal paths from M to Y.

If assumptions i.-iv. hold, direct and indirect effects can be identified by covariate adjustment (or a combination of randomization of X and covariate adjustment).

# ROADMAP FOR CAUSAL INFERENCE

1. Specify the causal model
2. Define the causal parameter of interest  
(along with the target population)
3. Link the causal model to the available empirical data
4. Assess whether the causal parameter of interest can be identified with the available data and define the respective statistical parameter
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NEXT WEEK:

## ESTIMATION OF DIRECT & INDIRECT EFFECTS USING REGRESSION

1. Nonparametric estimators
2. Flexible regression estimators
3. Classic regression estimators

THANK YOU FOR YOUR ATTENTION!

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