

INTRODUCTION TO CAUSAL INFERENCE

Michael Kühhirt

November 27, 2017

v.2017-11-27-1:24pm

Estimation of total effects II

LAST WEEK: COVARIATE ADJUSTMENT USING REGRESSION

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- Q: Hmmm, that's exactly what we want if we want to learn about the ATE, right?
- A: Yup! Important caveat though: adjustment can only equalize the distribution of measured covariates (unlike randomization, remember?).

TODAY: COVARIATE ADJUSTMENT USING IPT WEIGHTING

- 1. Nonparametric IPT weighting
- 2. Parametric estimation of IPT weights
- 3. Comparing the results of different estimators
- 4. Checks for model misspecification and positivity violations



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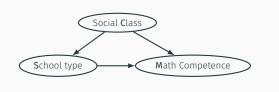
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This doesn't make much of a difference for estimating total effects, but it can be important for estimating direct effects and cumulative effects later on.

TOY DATA FOR DEMONSTRATION OF NONPARAMETRIC ADJUSTMENT



Data generating process for **07estte1a.dta** (N = 100,000):

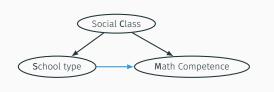
$$C = \varepsilon_C$$

$$S = 0.2C + \varepsilon_S$$

$$M = 20C - 5S + 5SC + \varepsilon_M$$

C is binary (1=higher class; 0=lower class); S is binary (1=private; 0=public); M is continuous

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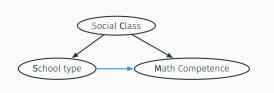
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Causal parameter of interest (unobservable):

$$ATE_{S\rightarrow M} = E(M^{S=private}) - E(M^{S=public}) = 45 - 49 = -4$$

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$$ATE_{S\rightarrow M} = E(M^{S=private}) - E(M^{S=public}) = 45 - 49 = -4$$

Statistical parameter, δ , that identifies ATE_{S $\rightarrow M$}:

$$\begin{split} \delta &= \sum_{c} E(M|S = \text{private}, C = c) P(C = c) - \sum_{c} E(M|S = \text{public}, C = c) P(C = c) \\ &= E\Big(M \frac{U_{S = \text{private}}}{P(S = \text{private}|C = c)}\Big) - E\Big(M \frac{U_{S = \text{public}}}{P(S = \text{public}|C = c)}\Big) \end{split}$$

(Hernán and Robins, 2018, Part I, p. 24)

$$\begin{split} \hat{\delta} &= \hat{E} \Big(M \frac{U_{S=private}}{\hat{P}(S=private | C=c)} \Big) - \hat{E} \Big(M \frac{U_{S=public}}{\hat{P}(S=public | C=c)} \Big) \\ &= \hat{E}_{w}(M | S=private) - \hat{E}_{w}(M | S=public) \end{split}$$

with $\hat{E}_W(.)$ being the average after weighting each unit u with $\widehat{\mathit{IPTW}} = \frac{1}{\widehat{P}(S=s \mid C=c)}$

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- 5. Result: estimate of $ATE_{S \to M}$

CALCULATE $\hat{P}(S = s | C = c)$

. tab pschool hiclass, col

Key
frequency column percentage

	hicl	.ass	
pschool	0	1	Total
0	71,683	14,163	85,846
	89.85	70.05	85.85
1	8,099	6,055	14,154
	10.15	29.95	14.15
Total	79,782	20,218	100,000
	100.00	100.00	100.00

Source: 08estte2.do#1A

CALCULATE $\widehat{IPTW} = 1/\hat{P}(S = s | C = c)$

```
. gen iptw=.
(100,000 missing values generated)
. replace iptw=1/.1015 if hiclass==0 & pschool==1
(8,099 real changes made)
. replace iptw=1/.8985 if hiclass==0 & pschool==0
>
(71,683 real changes made)
. replace iptw=1/.2995 if hiclass==1 & pschool==1
(6,055 real changes made)
. replace iptw=1/.7005 if hiclass==1 & pschool==0
(14,163 real changes made)
```

Source: 08estte2.do#1B

APPLY IPTW AND CALCULATE $\hat{E}_{w}(M|S = private) - \hat{E}_{w}(M|S = public)$

pschool	mean
0	49.00899
1	45.10707

. reg mathcomp pschool [pw=iptw], nohead nopvalues (sum of wgt is 2.0001e+05)

mathcomp	Coef.	Robust Std. Err.	[95% Conf.	Interval]
pschool	-3.90	0.12	-4.14	-3.67
_cons	49.01	0.04	48.92	49.10

Source 08estte2.do#10

$$\hat{\delta} = -3.9$$

After weighting, conditioning on C is no longer necessary!

S AND C INDEPENDENT IN THE WEIGHTED PSEUDO-POPULATION

	tab	hiclass	pschool	[aw=iptw],	nofreq	col
--	-----	---------	---------	------------	--------	-----

hiclass	psch 0	ool 1	Total
0 1	79.78 20.22	79.79 20.21	79.78 20.22
Total	100.00	100.00	100.00

Source: 08estte2.do#1D

Weighting balances the distribution of covariates over values of the treatment by downweighting (upweighting) units with a large (small) probability to receive the treatment value with which they're observed.

S AND C DEPENDENT IN THE POPULATION

. tab hiclass pschool, nofreq col			
hiclass	pscho 0	ool 1	Total
0 1	83.50 16.50	57.22 42.78	79.78 20.22
Total	100.00	100.00	100.00

Source: 08estte2.do#1E

So in these data, units from higher (lower) social class are upweighted (downweighted) in the group of public school students and units from lower (higher) social class are upweighted (downweighted) in the group of private school students.

INTERPRETATION OF ESTIMATES

statistical On average, private school students have a 3.9 [95% CI:-4.14, -3.67] points lower math competence than public school students after adjusting for differences in social class (in the population from which the sample was drawn).

counterfactual Had every student attended private school instead of public school, average math competence would have been 3.9 [95% CI:—4.14, —3.67] points lower (in the population from which the sample was drawn).

Or:

Attending private school instead of public school leads to an average decrease in math competence by 3.9 points (in the population from which the sample was drawn).

INTERPRETATION OF ESTIMATES

Assumptions for interpretation (after nonparametric adjustment):

- statistical No random bias
 - No measurement bias (see Hernán and Robins, 2018, Ch.9)

counterfactual the previous plus

- No positivity violation There are private school students and public school students in each social class
- No confounding bias, no overcontrol bias, no endogenous selection bias We d-separated every noncausal path, we didn't d-separate any

causal path, we didn't d-connect any noncausal path from school type to math competence.

IPT WEIGHTING OF MARGINAL STRUCTURAL MODELS

Recall that we can use a marginal structural (mean) model (MSM) to define counterfactual means of Y and their dependence on the values of X:

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For a binary X:

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$$= (\theta_0 + \theta_1) - \theta_0$$
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Given the identification conditions for covariate adjustment, IPT weighting allows us to express the MSM in terms of an observable regression model in the pseudo-population, which can be estimated from (sample) data,

$$E(Y^{X=X}) = E_w(Y|X=X) = \beta_0 + \beta_1 X$$

with $ATE_{X\to Y} = \beta_1$.

Like the number of means for standardization, the number of IPTWs to be estimated increases with the number of covariates and the number of their values, e.g.,

with binary covariate and binary treatment:

$$2 \times 2 = 4$$
 IPTWs

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 - $2 \times 2 = 4$ IPTWs
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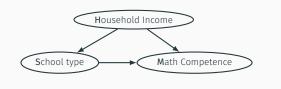
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This time, however, we don't escape it by modelling E(Y|X,Z) (i.e., the outcome) but by modelling P(X=x|Z) (i.e., the treatment).

PARAMETRIC ESTIMATION OF IPT

WEIGHTS

TOY DATA FOR DEMONSTRATION OF PARAMETRIC ADJUSTMENT

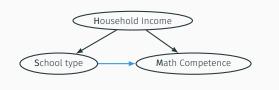


Data generating process
for
$$07estte1b.dta$$

 $(N = 100,000)$:
 $H = \varepsilon_H$
 $S = 0.05H + \varepsilon_S$
 $M = 7H - 0.3H^2 - 5S + .3SH + \varepsilon_M$

H is continuous; S is binary (1=private; 0=public); M is continuous

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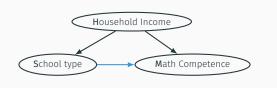
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(Hernán and Robins, 2018, Part I, p. 24)

ESTIMATE $\hat{P}(S = private | H = h)$ USING LOGISTIC REGRESSION

. logit pschool hhinc, nohead nopval

Iteration 0: log likelihood = -37035.778

Iteration 1: log likelihood = -32045.528

Iteration 2: log likelihood = -30931.622

Iteration 4: log likelihood = -30923.612

Iteration 5: log likelihood = -30923.618

pschool	Coef.	Std. Err.	[95% Conf.	<pre>Interval]</pre>
hhinc	0.60	0.01	0.59	0.61
_cons	-4.14	0.03	-4.19	-4.09

Source 08estte2.do#2A

These model parameters are used to estimate each unit's probability to attend private school, conditional on household income:

$$\widehat{P}(S = \text{private}|H = h) = \frac{exp(\widehat{\gamma}_0 + \widehat{\gamma}_1 h)}{1 + exp(\widehat{\gamma}_0 + \widehat{\gamma}_1 h)}$$
$$= \frac{exp(-4.14 + 0.6h)}{1 + exp(-4.14 + 0.6h)}$$

CALCULATE $1/\hat{P}(S = private | H = h)$ AND $1/\hat{P}(S = public | H = h)$

```
. * predict P(X=1|Z) from model
. predict pden, pr
.
.
. * generate IPT weight
. gen iptw = .
(100,000 missing values generated)
. replace iptw = 1/pden if pschool == 1 // for obs. with x=1
(12,173 real changes made)
. replace iptw = 1/(1-pden) if pschool == 0 // for obs. with x=0
(87,827 real changes made)
```

Source: 08estte2 .do#2B

$$\widehat{IPTW} = \frac{1}{\widehat{P}(S = s|H = h)}$$
 with $\widehat{IPTW} = \frac{1}{\widehat{P}(S = \text{private}|H = h)}$ for private school students and $\widehat{IPTW} = \frac{1}{\widehat{P}(S = \text{public}|H = h)} = \frac{1}{1 - \widehat{P}(S = \text{private}|H = h)}$ for public school students.

IPT WEIGHTING OF MARGINAL STRUCTURAL MODEL

. reg mathcomp pschool [pw = iptw], nohead nopval
(sum of wgt is 1.9777e+05)

mathcomp	Coef.	Robust Std. Err.	[95% Conf.	Interval]
pschool	-3.78	0.15	-4.07	-3.48
_cons	47.89	0.05	47.80	47.99

Source: 08estte2.do#2C

$$\widehat{E}_w(M|S=s) = \widehat{\beta}_0 + \widehat{\beta}_1 s$$

The estimates of the regression parameters can be used to estimate the ATE:

$$\hat{\delta} = (\hat{\beta}_0 + \hat{\beta}_1) - \hat{\beta}_0 = \hat{\beta}_1$$
= (47.89 - 3.78) - 47.89
= -3.78

INTERPRETATION OF ESTIMATES

statistical On average, private school students have a 3.78 [95% CI: -4.07,-3.48] points lower math competence than public school students after adjusting for differences in household income (in the population from which the sample was drawn).

counterfactual Had every student attended private school instead of public school, average math competence would have been 3.78 [95% CI: -4.07,-3.48] points lower (in the population from which the sample was drawn).

Or:

Attending private school instead of public school leads to an average decrease in math competence by 3.78 [95% CI: -4.07,-3.48] points (in the population from which the sample was drawn).

INTERPRETATION OF ESTIMATES

Assumptions for interpretation (after parametric adjustment):

- **statistical** No random bias
 - No measurement bias (see Hernán and Robins, 2018, Ch.9)
 - No (treatment) model misspecification

counterfactual the previous plus

 No positivity violation There is (sufficient) overlap between private school students and public school students in the distribution of household income.

No confounding bias, no overcontrol bias, no

endogenous selection bias Adjusting for household income d-separates every noncausal path. doesn't d-separate any causal path, and doesn't d-connect any noncausal path from school type to math competence.

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Weighting with the SIPTW as defined above also creates a pseudo-population in which X is independent of measured covariates Z.

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But SIPTW have some desirable statistical properties compared to nonstabilized weights:

- $\boldsymbol{\cdot}$ The pseudo-population has the same size as the original population.
- The confidence intervals around estimates are typically narrower than for weighting with IPTW.

ESTIMATE $\hat{P}(S = \text{private}) \text{ AND } \hat{P}(S = \text{public})$

```
. logit pschool, nohead nopvalues
Iteration 0: log likelihood = -37035.778
Iteration 1: log likelihood = -37035.778
```

pschool	Coef.	Std. Err.	[95% Conf.	Interval]
_cons	-1.98	0.01	-2.00	-1.96

```
.
. * predict P(X=1) from model
. predict pnum, pr
.
. * calculate stabilized IPTW, with P(X=0) = 1 - P(X=1)
. gen siptw = .
(100,000 missing values generated)
. replace siptw = pnum*iptw if pschool == 1 // for obs. with x=1
(12,173 real changes made)
. replace siptw = (1-pnum)*iptw if pschool == 0 // for obs. with x=0
(87,827 real changes made)
```

Source: 08estte2.do#2D

$$\widehat{SIPTW} = \widehat{P}(S = s) \times \widehat{IPTW}$$

USING SIPTW LEADS TO THE SAME RESULT AS BEFORE

. reg mathcomp pschool [pw = siptw], robust nohead nopvalues (sum of wgt is 9.9994e+04)

mathcomp	Coef.	Robust Std. Err.	[95% Conf.	Interval]
pschool	-3.78	0.15	-4.07	-3.48
_cons	47.89	0.05	47.80	47.99

Source: 08estte2.do#2E

COMPARING THE RESULTS OF DIFFERENT ESTIMATORS

LET'S COMPARE (MOST OF) OUR ESTIMATES FROM 07estte1b.dta

Table 1: Estimates for ATE of school type on math competence based on 07estte1b.dta

	Unadj.	Simple RA	Flex. RA	SIPTW	SIPTW-RA
1.pschool	4.556 (0.122)	-4.123 (0.0963)	-4.166 (0.108)	017 7 0	-3.978 (0.117)
N	100000	100000	100000	100000	100000

Standard errors in parentheses

Source: 08estte2.do#3B

With the exception of the estimate based on the unadjusted difference in mean math competence between private and public school students all estimates are relatively similar (around -4). Our identification analysis based on the graphical causal model told us to adjust for household income. So we prefer the estimates after adjustment and consider the unadjusted estimate structurally biased (here: confounding bias).

The estimates in the column 'SIPTW-RA' come from a regression model of M on S and H (i.e., the 'simple' model) that was additionally weighted with the SIPTW. This estimator is thus based on an outcome model and a treatment model and is considered 'doubly robust', because it's unbiased if either model is correctly specified (Hernán and Robins, 2018, Part II, p. 29).

Comparing the results of estimators for the ATE based on outcome models (like regression) and those based on treatment models (like IPT weighting) is valuable to detect (severe) model misspecification and should therefore be done whenever feasible.

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If results are similar, however, severe model misspecification in either model is unlikely and thus provides some confidence that the estimates aren't biased in that way.

Importantly, similar results don't guarantee that we have adjusted for the right variables (something which can only be judged on the basis of a causal model). So similar results from different estimators don't somehow confirm that there is no structural bias.

CHECKS FOR MODEL MISSPECIFICATION AND

POSITIVITY VIOLATIONS

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To detect positivity violations we can check whether

- the distribution of Z (largely) overlaps for different values of X,
- the distribution of $\hat{P}(X = 1|Z)$ (largely) overlaps for different values of X,
- the mean of the SIPTW is close to 1 and its standard deviation is (way) below 1 (deviations from this may also indicate severe misspecification of the treatment model).

CHECKING THE DISTRIBUTION OF THE SIPTW

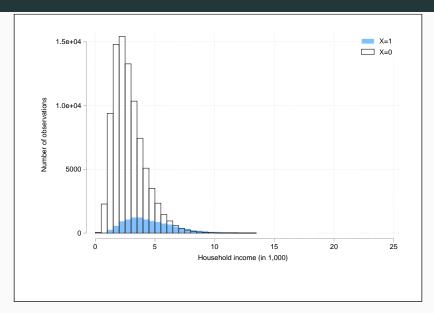
	um siptw, det			
		siptw		
	Percentiles	Smallest		
1%	.1757277	.1217373		
5%	.5339589	.1217443		
10%	.904268	.1217621	0bs	100,000
25%	.9173374	.1218755	Sum of Wgt.	100,000
50%	.9417001		Mean	.9999398
		Largest	Std. Dev.	.4615444
75%	.999531	20.916		
90%	1.159085	30.12489	Variance	.2130233
95%	1.430252	34.18182	Skewness	18.96017
99%	2.676924	35.33284	Kurtosis	946.447

Source: 08estte2.do#4A

Here, the mean is very close to 1 and the standard deviation is way below 1. Therefore, this test doesn't provide evidence for severe positivity violations (or misspecification of the treatment model).

If this wasn't the case, we'd need to limit the estimation to the covariate areas for which positivity holds and/or specify a more flexible treatment model (e.g., including nonlinearities and product terms between variables).

CHECKING OVERLAP OF COVARIATE DISTRIBUTIONS



Source: 08estte2.do#4B

CHECKING OVERLAP OF COVARIATE DISTRIBUTIONS

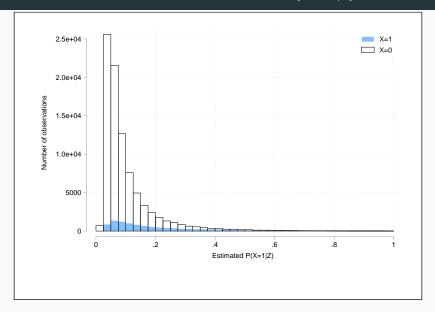
We can see that although the distribution of household income is shifted to the right for private school students, we find students from both school types across all values of income except the most extreme.

Generally, positivity is violated for areas of the covariate distribution for which there is no overlap.

Our estimates can only be generalized to areas of the covariate distribution for which there is overlap, because only these observations contribute to estimation.

In other words, for areas without overlap we can't compare the outcome for different values of X but similar values of Z.

CHECKING OVERLAP OF DISTRIBUTION OF $\hat{P}(X = 1|Z)$



Source: 08estte2.do#4C 27

CHECKING OVERLAP OF DISTRIBUTION OF $\hat{P}(X=1|Z)$

 $\hat{P}(X=1|Z)$ is a function of all covariates Z (here: only of houseold income) and can therefore be used as a summary measure of Z.

Overlap in the distribution of $\hat{P}(X=1|Z)$ between values of X can thus also be used to assess positivity. It is particularly helpful if there are many covariates and it is tedious to check overlap for all of them separately.

Unsurprisingly, we again see that there is overlap across the distribution except for the extremes.

SUMMARY: STRENGTHS & WEAKNESSES OF REGRESSION AND IPT WEIGHTING

Regression	IPT weighting

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Can handle continuous treatments	Can only handle categorical treatments well
Can't handle more complex settings	Can handle more complex settings
e.g., time-varying treatments	e.g., time-varying treatments

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On the other hand, criticism of estimates merely stating that assumptions *may* be violated is insufficient and uninformative.

"HOW SERIOUSLY DO WE TAKE OUR ESTIMATES?"

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Scientific discussion should systematically assess the plausibility of each assumption.

On the other hand, criticism of estimates merely stating that assumptions *may* be violated is insufficient and uninformative.

Any criticism, therefore, is required to outline a concrete reason for a (severe) violation (e.g., mention of a specific potential confounder not adjusted for in the analysis).

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Flexible modelling and comparison of estimators can be helpful to assess misspecification of estimation models.

NEXT WEEK: DEFINITION AND IDENTIFICATION OF DIRECT & INDIRECT EFFECTS

- 1. Terminology for mediation analysis
- 2. Counterfactual definition of direct and indirect effects
- 3. Graphical identification of direct and indirect effects
- 4. Treatment-induced mediator-outcome confounding

THANK YOU FOR YOUR ATTENTION!

REFERENCES

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