Low SNR FreeDV Mode

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1 Introduction

After 10 years development and on air experience with various FreeDV waveforms, we would like to develop a new waveform that outperforms and replaces a variety of existing modes such as 700C/D/E and 1600. Requirements include [3]:

1. Better performance than SSB at 0dB SNR on MPP and MPD channels.
2. A single mode that can handle MPP, MPD, GEO (e.g. QO-100), and replace several existing FreeDV modes, simplifying the end user experience.
3. For compliance with Export Control regulations, the minimum speech codec bit rate is 700 bit/s.

It is acceptable for performance to gradually decrease as the multipath channel quality declines, but we would like the decline to be gradual, e.g. a few dB more power for operation on MPD versus MPP.

This document explores ways we can improve the existing OFDM modem waveforms in order to meet these requirements.

1.1 Glossary

Acronym	Explanation
AWGN	Additive White Gaussian Noise - a communications channel with flat frequency response and additive noise
CP	Cyclic Prefix
FEC	Forward Error Correction
ISI	Inter Symbol Interference
LEO	Low earth orbit satellite channel, AWGN with large freq offset and Doppler shift (high rate of change of freq offset)
GEO	Geosynchronous satellite channel, AWGN but high phase noise and large freq offset
PTT	Push To Talk - voice communications where only one person is transmitting at any one time. Common in two way radio but not mobile telephones
MPP	Multipath Poor channel, 1 Hz Doppler spread, 2ms delay spread, typical for US and Australian interstate propogation
MPD	Multipath Disturbed channel, 2 Hz Doppler spread, 4ms delay spread, typical for UK Winter NVIS propogation

Table 1: Glossary of Acronyms

[1]Can be expressed as a linear ratio E_b/N_0 or $10log_{10}(E_b/N_0)$ dB

Symbol	Explanation	Units
B	Noise bandwidth	Hz
B_d	Doppler spreading bandwidth for HF channel model	Hz
E_b/N_0	Energy per bit on spectral noise density	dimensionless, dB[1]
N_p	Pilot insertion rate	dimensionless
R_b	Bit rate	Bits/second
R_s	Symbol rate	symbols/second
T_s	Symbol period	seconds
SNR	Signal to Noise Ratio	dB
S	Signal Power	Watts
N	Noise Power	Watts

Table 2: Glossary of Symbols

2 Modem and Channel Models

In this section we will develop theoretical models to help us explore performance limits.

For practical PTT voice systems algorithmic delay is limited to a few 100ms, which limits the FEC codeword size and hence the performance of the code. For PSK channels a threshold $E_b/N_0 = 2\,\text{dB}$ and a code rate $R = 0.5$ is typical, where E_b/N_0 is the energy per payload data bit (coded E_b/N_0). The lowest (threshold) SNR for a viable voice link is given by:

$$\begin{aligned} \frac{S}{N} &= \frac{E_b R_b}{N_0 B} \\ SNR &= 10log_{10}\left(\frac{E_b}{N_0}\right) + 10log_{10}\left(\frac{R_b}{B}\right) \quad \text{[dB]} \end{aligned} \tag{1}$$

where R_b is the payload data bit rate, and B is the bandwidth in which we measure SNR. Given $Rb = 700$ and $B = 3000$ we have:

$$\begin{aligned} SNR &= 2 + 10log_{10}(700/3000) \\ &= -4.3\,\text{dB} \end{aligned} \tag{2}$$

This is ideal performance for an AWGN channel. In practice we must allocate some power to symbols used for synchronisation, such as pilot symbols used for frequency and phase estimation, or unique word bits used for frame synchronisation. Synchronisation algorithms often struggle at low SNRs, introducing additional "implementation" losses.

Performance on multipath channels is significantly worse, in our use cases typically 5 dB. On these channels, we may allocate some carrier power to deal with intersymbol interference (for example a cyclic prefix in OFDM modems).

A more complete model is:

$$SNR = 10log_{10}\left(\frac{E_b}{N_0}\right) + 10log_{10}\left(\frac{R_b}{B}\right) + L_p + L_{il} + L_{cp} \tag{3}$$

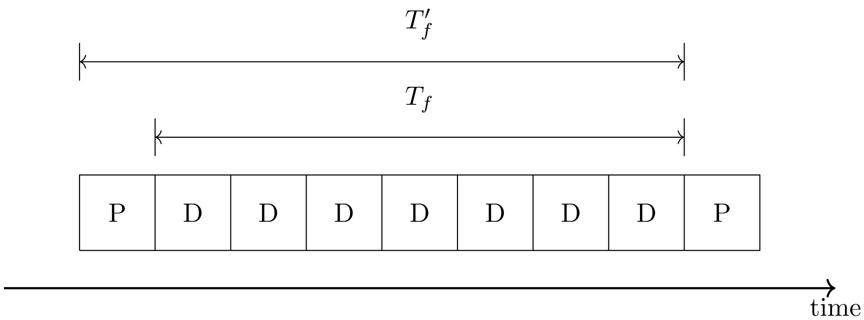
where L_p is the loss from power allocated to pilot symbols, L_{il} is the real world implementation loss, and L_{cp} is the loss in SNR due to the power allocated to the cyclic prefix.

TODO: Discuss Available bandwidth. 2000 Hz, using QPSK we can get 4000 bits/s. Less overheads (which can be expressed as SNR or bandwidth reduction?). We have 700 bits/s source code or 1400 bits/s with rate 0.5 FEC.

2.1 Pilot symbol overhead

In this section we explore the effect of inserting pilot symbols on the threshold SNR (1). Consider a sequence of $N_p - 1$ PSK data symbols that carry the modulated FEC codeword bits (e.g. data and parity bits) over the channel. We denote this sequence a modem *modem frame*. The frame of $N_p - 1$ symbols has

Figure 1: Modem Frame with $N_p = 8$, the pilot of the next modem frame is also shown.



a period of $T_f = (N_p - 1)T_s$ seconds, where T_s is the period of each symbol. We wish to insert a single pilot symbol after the data symbols, creating a new frame N_p symbols long, with period $T_f' = N_p T_s$. To maintain the same payload data rate:

$$\begin{aligned} T_f &= T_f' \\ (N_p - 1)T_s &= N_p T_s \\ R_s' &= R_s \frac{N_p}{N_p - 1} \end{aligned} \tag{4}$$

where the symbol rate $R_s = 1/T_s$. Expressing S/N (1) in terms of E_s and R_s:

$$\begin{aligned} \frac{S}{N} &= \frac{E_s R_s}{N_0 B} \\ \frac{S\,'}{N} &= \frac{E_s R_s'}{N_0 B} \\ &= \frac{E_s R_s N_s}{N_0 B (N_p - 1)} \\ \frac{S'/N}{S/N} &= \frac{N_p}{N_p - 1} \end{aligned} \tag{5}$$

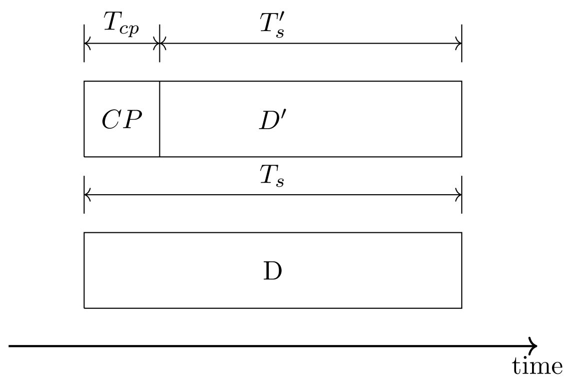
Thus when we insert pilots, the threshold S/N increases by a factor of $N_p/(N_p - 1)$. Expressed in dB:

$$\begin{aligned} 10log_{10}\left(\frac{S'}{N}\right) &= 10log_{10}\left(\frac{S}{N}\right) + 10log_{10}\left(\frac{N_s}{N_s - 1}\right) \\ SNR' &= SNR + 10log_{10}\left(\frac{N_p}{N_p - 1}\right) \\ SNR' &= SNR + L_p \quad \text{[dB]} \end{aligned} \tag{6}$$

where L_p can be considered the pilot symbol *loss* - the SNR degradation from the ideal performance (1) due to the insertion of pilot symbols. For example FreeDV 700D uses a pilot insertion rate of $N_s = 8$ results in $L_p = 10log_{10}(8/7) =$ 0.58 dB, thus we need 0.58 dB more SNR to acheive the threshold SNR for the voice link. To maintain $Rs = 700$ data symbols/second over the channel, we require $R'_s = (700)8/7 = 800$ symbols/second which introduces a 100 Hz bandwidth overhead.

2.2 Cyclic Prefix Overhead

Figure 2: Construction of composite symbol with a Cyclic Prefix CP pre-pended to a shortened data symbol D'.



Now we consider the SNR overhead for the Cycle Prefix (CP) used in OFDM modems to cope with delay spread on multipath channels. To achieve our payload data rate (e.g. 700 bits/s), we send symbols D across the channel at a constant symbol rate R_s, or one symbol every $T = T_s$ seconds. To cope with delay spread, we construct a composite symbol by pre-pending a Cyclic Prefix (CP) T_{cp} seconds in duration to a new symbol D' of T'_s seconds in duration. D and D' contain the same PSK symbol, and convey the same information over the channel. The new composite symbol is now $T' = T_{cp} + T'_s$ seconds long. The CP contains no additional information, it is just a cyclic extension of the single symbol D'. Thus we still send one symbol of data over the channel every T' seconds. To maintain the payload data rate over the channel, we must send the new composite symbol at the same rate as the original symbol:

$$\begin{aligned} T &= T' \\ T_s &= T_{cp} + T'_s \\ R'_s &= \frac{R_s}{1 - T_{cp}/T_s} \end{aligned} \tag{7}$$

It can be observed that $R'_s > R_s$, to account for the portion of the composite symbol allocated to the CP. For example with $R_s = 700$, $T_s = 0.02$, $T_{cp} = 0.002$, $R'_s = 700/(1 - 0.002/0.02) = 777.78$ symbols/second. Thus additional bandwidth is required to send the composite symbol including the cyclic prefix.

The increase in symbol rate does not directly affect BER performance if E_s/N_0 remains the same. For example if $Rs' = 2R_s$ we could send the symbol across the channel in $T_s/2$ seconds at power $2S$, followed by $T_s/2$ seconds of silence. The energy per symbol E_s and BER would remain the same.

For the composite symbol, the transmitter power S' is spread between the CP and D'. Given a constant power S', the energy for the symbol D' is given by:

$$\begin{aligned} E'_s &= \frac{S'}{R'_s} \\ &= \frac{S'}{R_s}(1 - T_{cp}/T_s) \end{aligned} \tag{8}$$

For the link BER to be maintained the energy per symbol must be unchanged, i.e. $E'_s = E_s$:

$$\begin{aligned} E_s &= \frac{S'}{R_s}(1 - T_{cp}/T_s) \\ \frac{S}{R_s} &= \frac{S'}{R_s}(1 - T_{cp}/T_s) \\ \frac{S}{N} &= \frac{S'}{N}(1 - T_{cp}/T_s) \\ \frac{S'}{N} &= \frac{S}{N}\left(\frac{1}{1 - T_{cp}/T_s}\right) \\ SNR' &= SNR - 10log_{10}(1 - T_{cp}/T_s) \\ &= SNR + L_{cp} \quad \text{[dB]} \\ L_{cp} &= -10log_{10}(1 - T_{cp}/T_s) \end{aligned} \tag{9}$$

Thus to close the link with the composite symbol the S/N must be increased by a factor of $1/(1 - T_s/T_{cp})$ compared to our ideal modem, to account for the energy allocated to the CP. For example FreeDV 700E has $T_s = 0.02$, $T_{cp} = 0.006$, giving $L_{cp} = -10log10(1 - 0.006/0.02) = 1.55$dB.

2.3 HF Channel Model

A common two path HF channel model [1] is given by:

$$y(t) = x(t)G_1(t) + x(t - d)G_2(t) \tag{10}$$

where G_1 and G_2 are two time varying, complex, Gaussian filtered random variables with *Doppler Spread* bandwidth B_d Hz, d is the path delay in seconds.

As $B_d << R_s$ we assume G_1 and G_2 are complex constants for the duration of a single symbol. Expressed in discrete time for the current symbol:

$$y(n) = x(n)G_1 + x(n - dF_s)G_2 \tag{11}$$

where F_s is the sample rate in Hz. Taking the z-transform:

$$\begin{aligned}
Y(z) &= X(z)G_1 + X(z)z^{-dF_s}G_2 \\
\frac{Y(z)}{X(z)} &= G_1 + z^{-dF_s}G_2 \\
H(z) &= G_1 + z^{-dF_s}G_2 \\
H(e^{j\omega}) &= G_1 + e^{-j\omega dF_s}G_2
\end{aligned} \tag{12}$$

For OFDM, the angular frequency of carrier c is given by $\omega = 2\pi cR_s/F_s$, which can be used to derive a single complex coefficient that describes the channel for carrier c:

$$H_c = G_1 + e^{-j2\pi cR_s d}G_2 \tag{13}$$

Lets examine the effects of intersymbol interference due to the delay spread. Consider the transmitter sample $x(0)$ where we transition from one PSK phase ϕ_1 to the next ϕ_2. Near the transition we can define $x(n)$ in terms of a step function $s(n)$:

$$\begin{aligned}
x(n) &= \begin{cases} e^{j(\omega n + \phi_1)}, & n < 0 \\ e^{j(\omega n + \phi_2)}, & n \geq 0 \end{cases} \\
&= s(n)e^{j(\omega n + \phi_2)} + (1 - s(n))e^{j(\omega n + \phi_1)} \\
s(n) &= \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}
\end{aligned} \tag{14}$$

where $\omega = 2\pi cR_s/F_s$ is the frequency of OFDM carrier c. Substituting into (11):

$$\begin{aligned}
y(n) = {} & G_1 s(n)e^{j(\omega n + \phi_2)} + G_1(1 - s(n - dF_s))e^{j(\omega n + \phi_1)} \\
& + G_2 e^{j(\omega(n - dF_s) + \phi_2)} + G_2(1 - s(n - dF_s))e^{j(\omega(n - dF_s) + \phi_1)}
\end{aligned} \tag{15}$$

At $n = 0$ we have a mixture of both symbols:

$$y(0) = G_1 s(n)e^{j(\omega n + \phi_2)} + G_2 e^{j(\omega(n - dF_s) + \phi_1)} \tag{16}$$

However by $y(dF_s)$ the ISI from ϕ_1 has gone:

$$y(dF_s) = G_1 s(n)e^{j(\omega dF_s + \phi_2)} + G_2 e^{j\phi_2} \tag{17}$$

Or more generally for $n \geq dF_s$:

$$\begin{aligned}
y(n) &= G_1 s(n)e^{j(\omega n + \phi_2)} + G_2 e^{j(\omega(n - dF_s) + \phi_2)} \\
&= e^{j(\omega n + \phi_2)}[G_1 + G_2 e^{-j\omega dF_s}] \\
&= e^{j(\omega n + \phi_2)}[G_1 + G_2 e^{-j2\pi cR_s d}] \\
&= e^{j(\omega n + \phi_2)}H_c
\end{aligned} \tag{18}$$

Thus if we start detecting $y(n)$ after the longest delay term we can recover the transmitted PSK symbol without ISI.

3 Waveform Improvements

In this section notes on (proposed) waveform improvements are presented.

The general strategy is to propose an innovation, and explore with analysis/maths and Octave simulation. Try low risk approaches to start with, then iterate. To simulate performance with voice codec, use a threshold of PER=0.1, BER=0.01 voice codec threshold for modem tests alone.

3.1 Equalisation

Pilot symbols are used to the estimate the channel, and equalise (correct) the phase of the received symbols. We require an equalisation system that works with $B_d = 2$ Hz Doppler Spread that has a low implementation loss L_{il} on noisy channels, and has a reasonable latency (we can't use symbols far in the future).

If we detect symbols after ISI has settled, the channel for each OFDM carrier resolves to a single complex constant H_c. This can be considered a complex random variable with bandwidth B_d.

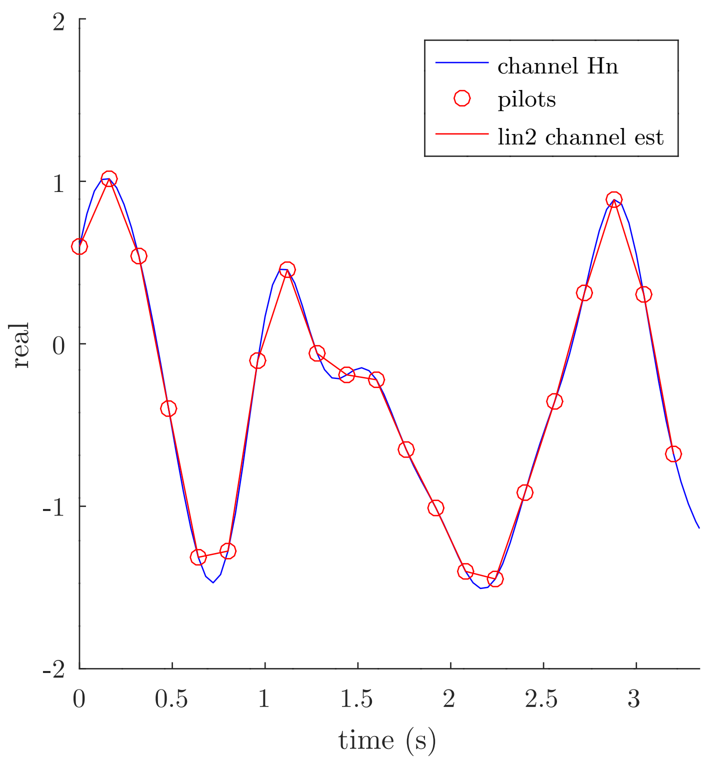
Consider the two path time domain channel model (11). Two additive terms of bandwidth B add linearly, so by linearity the result also has bandwidth B. The sum is a random modulation of bandwidth B_d about the symbol centre frequency. The Doppler bandwidth B_d therefore defines the bandwidth required for equalisation. One caveat - if B_d an appreciable fraction of R_s the DFT orthogonality may break down to some extent as energy falls into adjacent DFT bins.

FreeDV 700D samples pilots at $1/(N_p Ts) = 6.25$Hz, which should be an adequate sample rate for our target channels. 700D currently uses a block average over a 2D array (4 pilots in time, across 3 carriers in frequency) of 12 pilots, this can be interpreted as a filter with all coefficients $c_i = 1/12$. It is effective on MPP channels ($B_d = 1$ Hz), but breaks down on MPD channels ($B_d = 2$ Hz). This suggests a resampler with a wider bandwidth might enable a waveform similar to 700D to be used on MPD channels.

To explore pilot resampling algorithms, a simulation was written to test candidate algorithms, and plot BER versus E_b/N_0 curves. One interpretation of resampling 4 pilots in time is a 4 point FIR filter. This was explored, along with the existing 12 pilot window (700D), and 2 point linear resampler (as used in FreeDV 700E). Figure 3 shows a resampler in action.

Various resamplers were tried. Attempts were made to design 4 sample FIR filters (e.g. with a *sinc*() impulse response) however these performed poorly. The *mean12* algorithm used for 700D worked well on AWGN and MPP channels, but has a sluggish frequency response, and can't follow fading with $B_d > 1$ Hz. However on AWGN and slower fading channels it filters the channel noise well, resulting in good low SNR performance. This is consistent with on air reports

Figure 3: Equaliser *lin2* resampler in action for a MPD channel. The blue continuous line is the simulated channel H_c at each symbol, the red dots are the pilot symbols, and the red line the channel estimates from the linearly interpolated pilots. Just the real part is plotted.



of 700D. The *lin2* algorithm as used on 700E doesn't filter channel, noise very well, but was very hard to beat on fast fading channels, and works with $B_d > 2$ Hz.

Attempts were therefore made to improve the performance of *lin2* on AWGN and slower fading channels. Combining pilots from adjacent carriers is used in FreeDV 700D to reduce estimation noise. Examining (13) we can see some symmetry in the RH term in the phase between carrier $c-1$ and carrier $c+1$ which supports averaging over adjacent carriers to estimate H_c, especially when the term $2\pi R_s d$ is small. A more robust approach is to perform a least squares fit **??** of three equations with G_1 and G_2 as the two unknowns, and H_{-1}, H_0, H_1

as the three pilot samples centred around the current carrier:

$$
\begin{aligned}
G_1 + G_2 e^{j2\pi R_s d} &= H_{-1} \\
G_1 + G_2 &= H_0 \\
G_1 + G_2 e^{-j2\pi R_s d} &= H_1 \\
\begin{bmatrix} 1 & e^{j2\pi R_s d} \\ 1 & 1 \\ 1 & e^{-j2\pi R_s d} \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} &= \begin{bmatrix} H_{-1} \\ H_0 \\ H_1 \end{bmatrix} \\
Ag &= h \\
g &= (A^T A)^{-1} A^T h \\
\overline{H_0} &= G_1 + G_2
\end{aligned}
\tag{19}
$$

where $\overline{H_0}$ is the smoothed estimate of the current carriers pilot symbol. Note that the channel delay d is actually an unknown (and indeed the entire model is an approximation of the real channel). It was found that in simulation, choosing $d = 0.002$ of $d = 0.004$ produced reasonable results, possibly due to the symmetry of the The $(A^T A)^{-1} A^T$ term can then be precomputed as all the parameters of A are known.

Algorithm	Mode	Colour	AWGN	MPP	MPD
mean12	700D	blue	0.0	0.5	unusable
lin2	700E	green	1.5	2.0	2.0
lin2ls	700X	magenta	0.5	0.5	0.5

Table 3: Comparison of equaliser algorithms for $N_p = 8$. The last three numbers are implementation loss in dB, smaller is better. 700X has the advantage of low implementation loss across channels with a single waveform, but is 0.5dB poorer than 700D on the (rare) AWGN channel. The 700E results are an approximation, as that waveform uses $N_p = 5$.

Benefit - wide Doppler bandwidth, small pilot overhead, low noise channel estimator, one waveform for AWGN through to MPD.

3.2 Multipath

n=2 diversity to handle multipath. Say 3dB gain on MPP/MPD. Research combining techniques. Risks are copies will have half power so estimates will have more noise.

3.3 FEC

If we use diversity, I am not sure how much FEC we can also use, as we may run out of bandwidth. Alternative is MAP techniques or some combination of MAP and FEC.

3.4 Delay Spread (ISI)

1. We need a CP long enough to handle MPD (4ms plus guard)

2. Try longer T_s which will mean less overhead. However this implies lower R_s which may be impacted by frequency spreading effect of Doppler. Caveat (as in equalisation section) is possible issues with Doppler spread and frequency offset tracking as R_s reduces.

3. Measure implementation loss or EVM against ISI, we might be able to get away with some ISI, it is acceptable to have performance drop off for MPD, but it needs to be gradual rather than breaking.

4. 700C had just 13ms symbols but dealt with ISI pretty well - this might be worth exploring.

3.5 Acquisition

TODO: also include frequency offset estimation and tracking. Sensitivity of Doppler spread, freq offset errors with reduced R_s. Can we use othogonality property to track freq offset, e.g. iterative adjustments of offset to peak OFDM carriers?

4 Further Work

This section presents topics useful to explore in future.

- Can we include PAPR into model? Can we have different trajectories for QPSK symbols around unit circle that reduce PAPR? No I don't think that helps when multiple carriers are added together. ECSSB techniques may be useful.

- Expression for Fading channels, block error rate, why 2020 is a lemon.

- Table of FreeDV waveforms and values, plugged into formula, effect of increasing pilot symbol rate.

- Where we can gain, diversity, PAPR reduction, reduced overheads for fast fading and ISI (discuss)

- Wades MAP techniques (ref). This has a lot of promise, need an effective way to simulate and establish benefit with a modest amount of work. Without a rate 0.5 code this would free up a lot of bandwidth to deal with ISI better, e.g. we could have large gaps between symbols. Or use parallel tone modem to reduce effects of ISI. A high rate code on top of this may be useful option (if it can converge). Can we combine MAP with extra bits? Index optimisation also a simple approach.

- Extending (19) to more carriers, it is essentially estimating the entire channel. Literature search, I'm sure this is nothing new. Determine if (19) is a reasonable approximation for real HF channels - we have derived it from a simulation model.

5 References

[1] ITU-R F.1487: Testing of HF modems with bandwidths of up to about 12 kHz using ionospheric channel simulators, 2000.

[2] P Bergadà, RM Alsina-Pagès, JL Pijoan, M Salvador, JR Regué, D Badia, and S Graells. Digital transmission techniques for a long haul hf link: Dsss versus ofdm. *Radio Science*, 49(7):518–530, 2014.

[3] David Rowe. FreeDV-020 WP4000 Low SNR Mode.

Figure 4: Performance curves for equaliser algorithms over various channels. *mean12* is the algorithm used for 700D, and *lin2* for 700E. Note *mean12* (blue) is very close to theory for AWGN but breaks down on MPP.