Low SNR FreeDV Mode

David Rowe VK5DGR

July 26, 2023

1 Glossary

Acronym	Explanation
AWGN	Additive White Gaussian Noise - a communications channel with flat frequency response and additive noise
CP	Cyclic Prefix
FEC	Forward Error Correction
PTT	Push To Talk - voice communications where only one person is transmitting at any one time. Common in two way radio but not mobile telephones
MPP	Multipath Poor channel, 1 Hz Doppler, 2ms delay spread, typical for US and Australian interstate propogation
MPD	Multipath Disturbed channel, 2 Hz Doppler, 4ms delay spread, typical for UK Winter NVIS propogation

Table 1: Glossary of Acronyms

Symbol	Explanation	Units
B	Noise bandwidth	Hz
E_b/N_0	Energy per bit on spectral noise density	dimensionless, dB[1]
N_s	Spacing between pilots (pilot insertion rate)	-
R_b	Bit rate	Bits/second
R_s	Symbol rate	symbols/second
T_s	Symbol period	seconds
SNR	Signal to Noise Ratio	dB
S	Signal Power	Watts
N	Noise Power	Watts

Table 2: Glossary of Symbols

[1] Can be expressed as a linear ratio E_b/N_0 or $10log_{10}(E_b/N_0)$ dB

2 Introduction

After 10 years development and on air experience with various FreeDV waveforms, we would like to develop a new waveform that outperforms and replaces a variety of existing modes such as 700C/D/E and 1600. Requirements include [1]:

1. Better performance than SSB at 0dB SNR on MPP and MPD channels.
2. A single mode that can handle MPP and MPD, and replace several existing FreeDV modes, simplifying the end user experience.
3. For compliance with Export Control regulations, the minimum speech codec bit rate is 700 bit/s.

3 Theory

In this section we will develop a theoretical model to help us explore performance limits. For practical PTT voice systems algorithmic delay is limited to a few 100ms, which limits the FEC codeword size and hence the performance of the code. For PSK channels a threshold $E_b/N_0 = 2\,\text{dB}$ and a code rate $R = 0.5$ is typical, where E_b/N_0 is the energy per payload data bit (coded E_b/N_0). The lowest (threshold) SNR for a viable voice link is given by:

$$\begin{aligned} \frac{S}{N} &= \frac{E_b R_b}{N_0 B} \\ SNR &= 10log_{10}\left(\frac{E_b}{N_0}\right) + 10log_{10}\left(\frac{R_b}{B}\right) \quad [\text{dB}] \end{aligned} \tag{1}$$

where R_b is the payload data bit rate, and B is the bandwidth in which we measure SNR. Given $Rb = 700$ and $B = 3000$ we have:

$$\begin{aligned} SNR &= 2 + 10log_{10}(700/3000) \\ &= -4.3\,\text{dB} \end{aligned} \tag{2}$$

This is ideal performance for an AWGN channel. In practice we must allocate some power to symbols used for synchronisation, such as pilot symbols used for frequency and phase estimation, or unique word bits used for frame synchronisation. Synchronisation algorithms often struggle at low SNRs, introducing additional ”implementation” losses.

Performance on multipath channels is significantly worse, in our use cases typically 5 dB. On these channels, we may allocate some carrier power to deal with intersymbol interference (for example a cyclic prefix in OFDM modems).

A more complete model is:

$$SNR = 10log_{10}\left(\frac{E_b}{N_0}\right) + 10log_{10}\left(\frac{R_b}{B}\right) + L_p + L_{il} + L_{cp} \tag{3}$$

where L_p is the loss from power allocated to pilot symbols, L_{il} is the real world implementation loss, and L_{cp} is the loss in SNR due to the power allocated to the cyclic prefix.

3.1 Pilot symbol overhead

In this section we explore the effect of inserting pilot symbols on the threshold SNR (1). Consider a sequence of $N_s - 1$ PSK data symbols that carry the modulated FEC codeword bits (e.g. data and parity bits) over the channel. We denote this sequence a *frame*. The frame of $N_s - 1$ symbols has a period of $T_f = (N_p - 1)T_s$ seconds, where T_s is the period of each symbol. We wish to insert a single pilot symbol after the data symbols, creating a new frame N_s symbols long, with period $T_f' = N_pT_s$. To maintain the same payload data rate:

$$\begin{aligned} T_f &= T_f' \\ (N_s - 1)T_s &= N_sT_s \\ R_s' &= R_s\frac{N_s}{N_s - 1} \end{aligned} \tag{4}$$

where the symbol rate $R_s = 1/T_s$. Expressing S/N (1) in terms of E_s and R_s:

$$\begin{aligned} \frac{S}{N} &= \frac{E_sR_s}{N_0B} \\ \frac{S\,'}{N} &= \frac{E_sR_s'}{N_0B} \\ &= \frac{E_bR_sN_s}{N_0B(N_s - 1)} \\ \frac{S'/N}{S/N} &= \frac{N_s}{N_s - 1} \end{aligned} \tag{5}$$

Thus when we insert pilots, the threshold S/N increases by a factor of $N_s/(N_s - 1)$. Expressed in dB:

$$\begin{aligned} 10log_{10}\left(\frac{S'}{N}\right) &= 10log_{10}\left(\frac{S}{N}\right) + 10log_{10}\left(\frac{N_s}{N_s - 1}\right) \\ SNR' &= SNR + 10log_{10}\left(\frac{N_s}{N_s - 1}\right) \\ SNR' &= SNR + L_p \quad \text{[dB]} \end{aligned} \tag{6}$$

where L_p can be considered the pilot symbol *loss* - the SNR degradation from the ideal performance (1) due to the insertion of pilot symbols. For example FreeDV 700D uses a pilot insertion rate of $N_s = 8$ results in $L_p = 10log_{10}(8/7) = 0.58\,\text{dB}$, thus we need 0.58 dB more SNR to acheive the threshold SNR for the voice link.

3.2 Cyclic Prefix Overhead

Now we consider the SNR overhead for the Cycle Prefix (CP) used in OFDM modems to cope with delay spread on multipath channels. To achieve our payload data rate (e.g. 700 bits/s), we send symbols D across the channel at a constant symbol rate R_s, or one symbol every $T = T_s$ seconds. To cope with delay spread, we construct a composite symbol by pre-pending a Cyclic Prefix (CP) T_{cp} seconds in duration to a new symbol D' of T_s' seconds in duration. D and D' contain the same PSK symbol, and convey the same information over the channel. The new composite symbol is now $T' = T_{cp} + T_s'$ seconds long. The CP contains no additional information, it is just an extension of the single symbol D'. Thus we still send one symbol of data over the channel every T' seconds. To maintain the payload data rate over the channel, we must send the new composite symbol at the same rate as the original symbol:

$$\begin{aligned} T &= T' \\ T_s &= T_{cp} + T_s' \\ R_s' &= \frac{R_s}{1 - T_{cp}/T_s} \end{aligned} \tag{7}$$

It can be observed that $R_s' > R_s$, to account for the portion of the composite symbol allocated to the CP. For example with $R_s = 700$, $T_s = 0.02$, $T_{cp} = 0.002$, $R_s' = 700/(1 - 0.002/0.02) = 777.78$ symbols/second.

For the composite signal, the transmitter power S is spread between the CP and D':

$$S = \frac{T_{cp}}{T_s} S + \left(1 - \frac{T_{cp}}{T_s}\right) S \tag{8}$$

Only the RH term contributes to the demodulation of D':

$$\begin{aligned} \frac{S(1 - T_{cp}/T_s)}{N} &= \frac{E_s R_s'}{N_0 B} \\ \frac{S}{N} &= \frac{E_s R_s'}{N_0 B} \frac{1}{(1 - T_{cp}/T_s)} \\ &= \frac{E_s R_s}{N_0 B} \frac{1}{(1 - T_{cp}/T_s)^2} \\ SNR &= 10log10\left(\frac{E_s R_s}{N_0 B}\right) - 20log10(1 - T_{cp}/T_s) \\ &= 10log10\left(\frac{E_s R_s}{N_0 B}\right) + L_{cp} \quad \text{[dB]} \end{aligned} \tag{9}$$

Thus to close the link with the composite symbol the S/N must be increased by a factor of $1/(1 - T_s/T_{cp})^2$ compared to our ideal modem. This accounts for the energy allocated to the CP, and the shorter period T_s' of the symbol D'. The squared term suggest SNR is quite sensitive to increasing T_{cp}, which is necessary to handle fast fading, high delay spread channels such as MPD. For example FreeDV 700E has $Ts = 0.02$, $T_{cp} = 0.006$, giving $L_{cp} = -20log10(1 - 0.006/0.02) = 3.1$dB.

4 Unorganised Notes

TODO:

- Challenges, fast fading, ISI, PAPR
- simulation model to check results
- Can we include PAPR into model?
- Expression for Fading channels, block error rate, why 2020 is a lemon.
- Table of FreeDV waveforms and values, plugged into formula, effect of increasing pilot symbol rate.
- Check performance of a few codes against 2dB figure above.
- Where we can gain, diversity, PAPR reduction, reduced overheads for fast fading and ISI (discuss)
- single tone/equaliser/hybrid OFDM-EQ
- Joint low latency/high latency on two carriers diversity
- Tx & Rx diversity
- Wades techniques (ref)
- undersampled pilots

References

[1] David Rowe. FreeDV-020 WP4000 Low SNR Mode.