FM Demodulator SNR

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My take on an expression for FM demodulator SNR above threshold, based mainly on Carlson [1] but also the noise triangle/phasor model discussed in various online and text sources. The theoretical derivation is compared to the companion Octave simulation and measurements for a commercial land mobile radio.

Consider a FM signal $r(t)$ modulated by a sinusoidal message signal $x(t)$:

$$\begin{aligned} r(t) &= A_c\cos(2\pi f_c t + \phi(t)) \\ \phi(t) &= 2\pi f_d \int_0^t x(t) \\ x(t) &= A\cos(2\pi f_m t) \end{aligned} \tag{1}$$

where A_c is the carrier level, f_c the carrier frequency, $x(t)$ the message signal of peak amplitude $A \leq 1$, and the peak deviation is f_d. We assume we can extract the phase term $\phi(t)$ from the carrier phase $2\pi f_c t$. An ideal FM demodulator can be modelled as:

$$y(t) = \frac{d\phi}{dt} \tag{2}$$

Giving the demodulated message signal:

$$y_s(t) = 2\pi f_d A\cos(2\pi f_m t) \tag{3}$$

The average power at the demodulator output is then:

$$S = (2\pi f_d)^2 \frac{A^2}{2} \tag{4}$$

Consider the FM carrier plus noise signal in Figure 1. As a thought experiment consider the signal has been frequency shifted such that the carrier frequency is now 0 and the carrier phasor lies stationary along the real axis. The sum of the carrier and noise vector has variations in magnitude and phase, however our demodulator is only sensitive to changes in phase.

Let A_n be the magnitude of the signal from a small slice of the input noise spectrum, effectively a small interfering sine wave summed with the carrier. As the noise vector rotates, we can observe ϕ_n will be changing or modulated by the noise vector. Consider the component of the noise vector at right angles

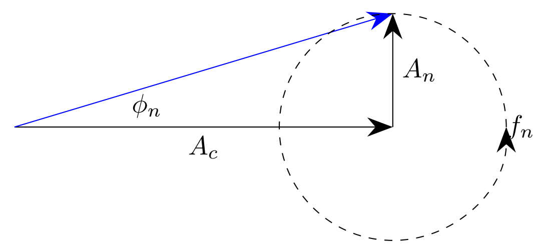

Figure 1: Phasor diagram showing the sum (blue) of the carrier signal with magnitude A_c and a small noise vector of magnitude A_n. Consider the noise vector to be a small slice of the input noise spectrum at frequency f_n from the carrier. The noise vector rotates around the tip of the carrier vector at frequency f_n, creating sinusoidal modulation in ϕ_n.

to the carrier (our demodulator ignores the AM component in phase with the carrier). The peak of the angle (deviation of the noise signal) occurs when $\tan(\phi_n) = A_n/A_c$. We therefore express ϕ_n as:

$$\phi_n = \mathrm{atan}(A_n/A_c)\mathrm{sin}(2\pi f_n t) \tag{5}$$

Passing this through our ideal demodulator (2):

$$y_n(t) = \frac{d\phi_n}{dt} = 2\pi f_n (A_n/A_c)\mathrm{cos}(2\pi f_n t) \tag{6}$$

where for $A_c >> A_n$ we can use the approximation $\mathrm{atan}(x) = x$. This is an interesting result:

1. The slice of noise at $f_c + f_n$ is demodulated as a baseband sine wave at f_n.
2. The amplitude of the demodulated noise is a function of f_n.

This leads to the "noise triangle" visualisation of FM where the noise is increasing as a function of frequency, unlike linear modulation schemes like SSB that have constant noise across frequency. The demodulator output power from the slice of noise at the demodulator input at frequency $f_c + f_n$ is:

$$N(f_n) = \frac{1}{2}(2\pi f_n (A_n/A_c))^2 \tag{7}$$

The total noise power in the demodulator output is the noise power per unit bandwidth summed over the interval $[-f_m, f_m]$, where f_m is the maximum frequency of the message signal $x(t)$. Using the 1:1 frequency relationship between a noise slice at the demodulator input and output, we can integrate (7) over the interval $[-f_m, f_m]$ to find the total noise power:

$$N = (2\pi)^2 \frac{N_0}{C} \int_{-f_m}^{f_m} f_n^2 df_n = (2\pi)^2 \frac{N_0}{3C} f_m^3 \tag{8}$$

where $N_0 = A_n^2$ is the demodulator input power per unit bandwidth, and $C = A_c^2$ is the carrier power. The signal to noise ratio at the FM demodulator output is then [1]:

$$\frac{S}{N} = 3\beta^2 \frac{A^2}{2} \frac{C}{N_0 f_m} \tag{9}$$

where the deviation $\beta = f_d/f_m$. Discussion:

1. The term $\frac{C}{N_0 f_m}$ is equivalent to a SSB product detector SNR with modulation bandwidth f_m, which makes (9) useful for comparing FM performance to SSB for a range of β.

2. Only noise from the interval $f_c \pm f_m$ contributes to the demodulator noise power and hence SNR. This gives an intuitive explanation of why FM SNR increases with the deviation β: as f_d increases and f_m remains fixed, the signal power S increases as a function of f_d^2 while noise power remains the same.

3. Note (9) is not dependant on the total noise power at the input of the FM demodulator (which is set by the IF filter), just the noise density N_0. This can lead to some surprising results, e.g. the SNR of a small deviation test tone is quite independent of IF filter bandwidth.

4. The A term is useful when testing FM radios, for example it is common to use a test tone of 60% full deviation or $A = 0.6$.

For narrow band FM on land mobile radio (LMR), a common choice is $\beta = f_d/f_m = 2500/3000$. Figure 2 is a plot of the measured SNR from an Octave simulation. Expressing (9) in dB:

$$SNR_{dB} = 10\log_{10}\left(3\beta^2 \frac{A^2}{2}\right) + 10\log_{10}(C/N_0 f_m) SNR_{dB} = G_{FM} + CNR_{dB} \tag{10}$$

For this example $G_{FM} = 0.18$ dB, which produces SNR performance almost identical to SSB. Figure 3 is a plot of measured SNR against Rx input level for a commercial land mobile radio, with the same $\beta = f_d/f_m = 2500/3000$ and test tone level $A = 0.6$. The results agree with theory to within about 1dB. To compute the theoretical SNR the Rx level is mapped to CNR_{dB} using:

$$CNR_{dB} = \mathrm{Rx}_{dBm} - (-174 + NF_{dB} + 10\log_{10}(f_m)) \tag{11}$$

where $NF_{dB} = 5$ dB is the estimated radio noise figure.

References

[1] Paul B Crilly and A Bruce Carlson. *Communication Systems.* McGraw-Hill Education, 2009.

Figure 2: Theoretical (9) and simulated SNR versus CNR, both measured in a noise bandwidth of $f_m = 3000$ Hz, $\beta = 0.83$, $A = 1$.

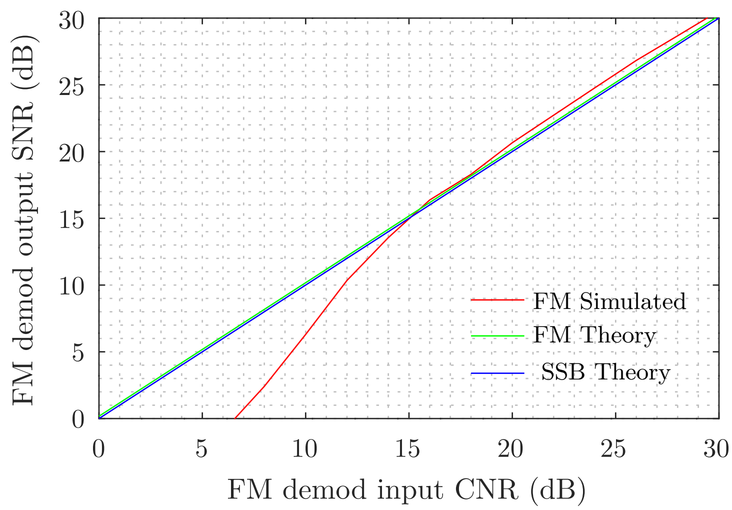

Figure 3: Measured SNR from a commercial LMR against Rx input level for $f_m = 3000$ Hz, $\beta = 0.83$, $A = 0.6$, $NF = 5$ dB.