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AI- Quiz -8

1.) To prove

$$\left|\max_a f(a) - \max_a g(a)\right| \leq \max_a |f(a) - g(a)|$$

$f, g : [0,1] \to [0, 1]$

Solution

let function f achieves global maxima at $a = a_x$ and function g achieves global maxima at $a = a_y$

Note: a_x and a_y may or may not be equal

Case:1 $f(a_x) > g(a_y)$

at $a = a_x$ $f(x)$ achieves maxima and $g(x)$ does not considering $a_x \neq a_y$ then it is obvious that $|f(a_x) - g(a_x)|$ will be greater than LHS since $|g(a_y)| \geq |g(a_x)|$ so new difference will be larger. If $a_x = a_y$ then LHS = RHS which holds true for given inequality.

Case - 2 $f(a_x) < g(a_y)$

Similar to previous argument new difference on RHS $|f(a_y) - g(a_y)|$ or any other arbitrary value of a for which diff $|f(x) - g(y)|$ becomes maximum becomes greater then LHS as $f(a_y) < f(a_x)$.

Using Case 1 and Case 2 arguments we have concluded that weather global maxima of f is greater, lesser or equal to global maxima of g in each case ~~differen~~ max difference will be greater than or equal to difference of maximas. i.e.

$$\left| \max_a f(a) - \max_a g(a) \right| \leq \max_a |f(a) - g(a)|$$

Hence proved.

Q-2 Prove that bellman operator is contraction

$$B(V(s)) = \max_a \left[\sum_{s',r} P(s',r|s,a)\,(r + \gamma V(s')) \right] \quad \text{— (1)}$$

$$B(U(s)) = \max_a \left[\sum_{s',r} P(s',r|s,a)\,(r + \gamma U(s')) \right] \quad \text{— (2)}$$

taking difference of (1) & (2)

$$|B(V(s)) - B(U(s))|$$

$$= \left| \max_a \left[\sum_{s',r} P(s',r|s,a)(r + \gamma V(s')) \right] - \max_a \left[\sum_{s',r} P(s',r|s,a)(r + \gamma U(s')) \right] \right|$$

$$= \left| \underbrace{\sum_{s',r} r P(s',r|s,a)} + \max_a \left[\sum_{s',r} \gamma P(s',r|s,a) V(s') \right] - \underbrace{\sum_{s',r} r P(s',r|s,a)} + \max_a \left[\sum_{s',r} \gamma P(s',r|s,a) U(s') \right] \right|$$

Same

$$= \left| \max_a \left[\sum_{s',r} \gamma P(s',r|s,a) V(s') \right] - \max_a \left[\sum_{s',r} \gamma P(s',r|s,a) U(s') \right] \right| \quad \text{— (3)}$$

③ is similar to $|\max_a f(x) - \max_a g(x)|$ so

$|\max_a f(x) - \max_a g(x)| \leq \max_a |f(x) - g(x)|$

③ $\Rightarrow$

$$\left|\max_a \sum_{s',r} P(s',r|s,a)\, v(s') - \max_a \sum P(s',r|s,a)\, U(s')\right|$$

$$\leq \max_a \left| \sum_{s',r} P(s',r|s,a)\, v(s') - \sum_{s',r} P(s',r|s,a)\, U(s')\right|$$

Using triangle inequality

$$\left|\sum x\right| \leq \sum |x| \quad \text{for } x \in R.$$

$$\left|\max_a \sum_{s',r} P(s',r|s,a)\, v(s') - \max_a \sum P(s',r|s,a)\, U(s')\right|$$

$$\leq \max_a \sum_{s',r} P(s',r|s,a)\, |v(s') - U(s')|$$

④

④ is equivalent

$$|B(v) - B(U)| \leq |B(v - U)|$$

⑤

⑤ is equation theit is enough to prove that

bellman is contraction

Hence proved.