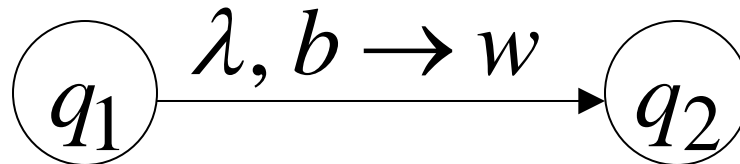
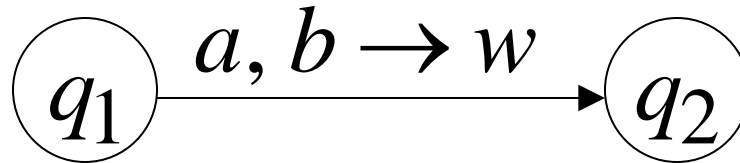


DPDA

Deterministic PDA

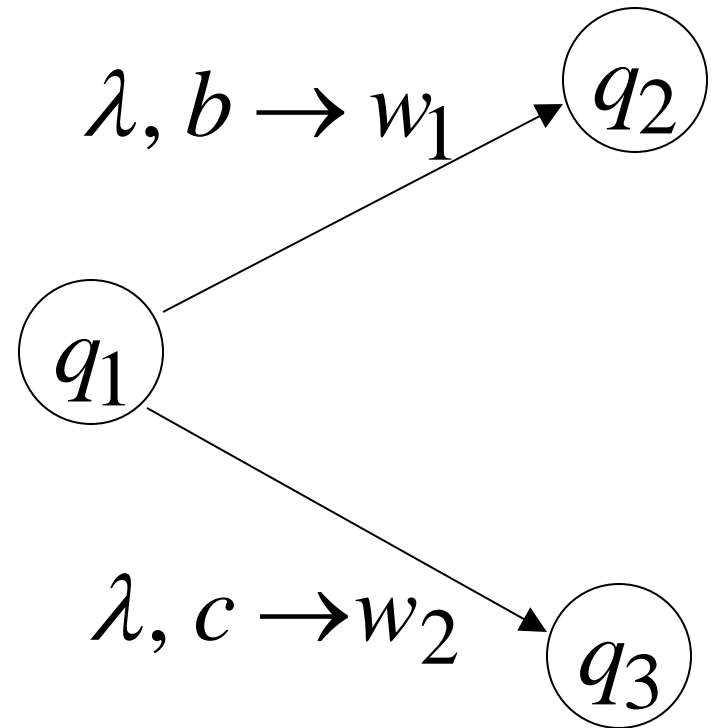
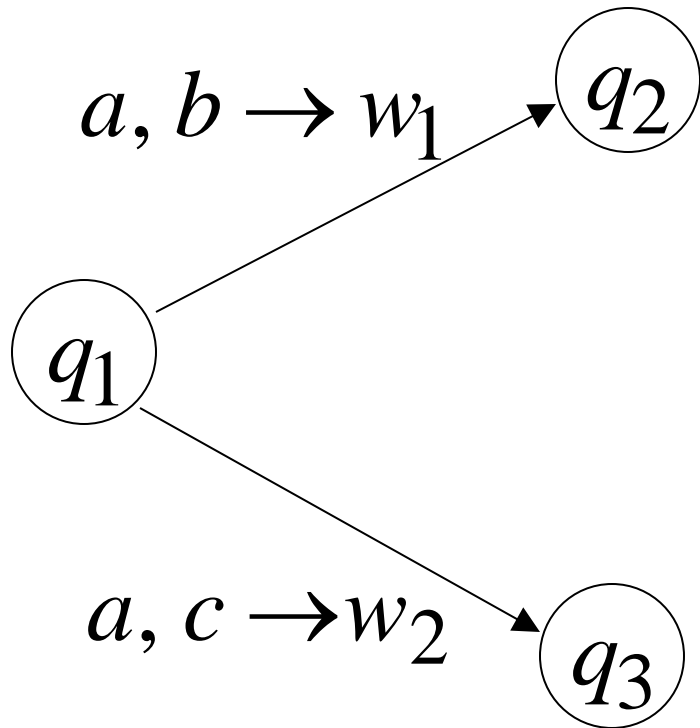
# Deterministic PDA: DPDA

Allowed transitions:



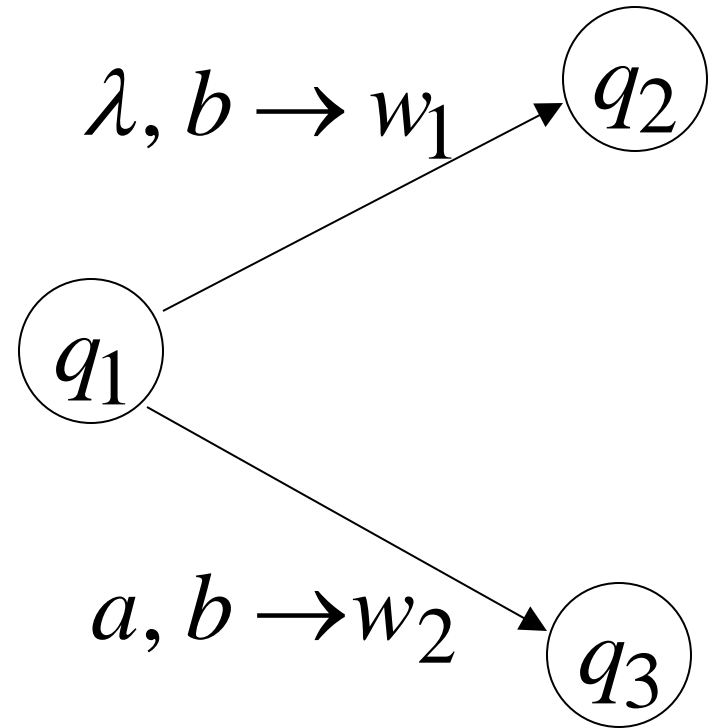
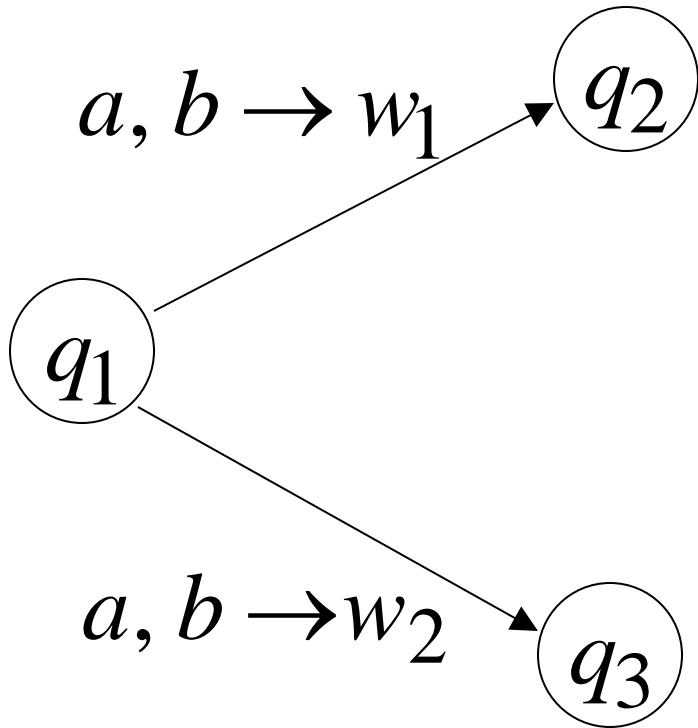
(deterministic choices)

## Allowed transitions:



(deterministic choices)

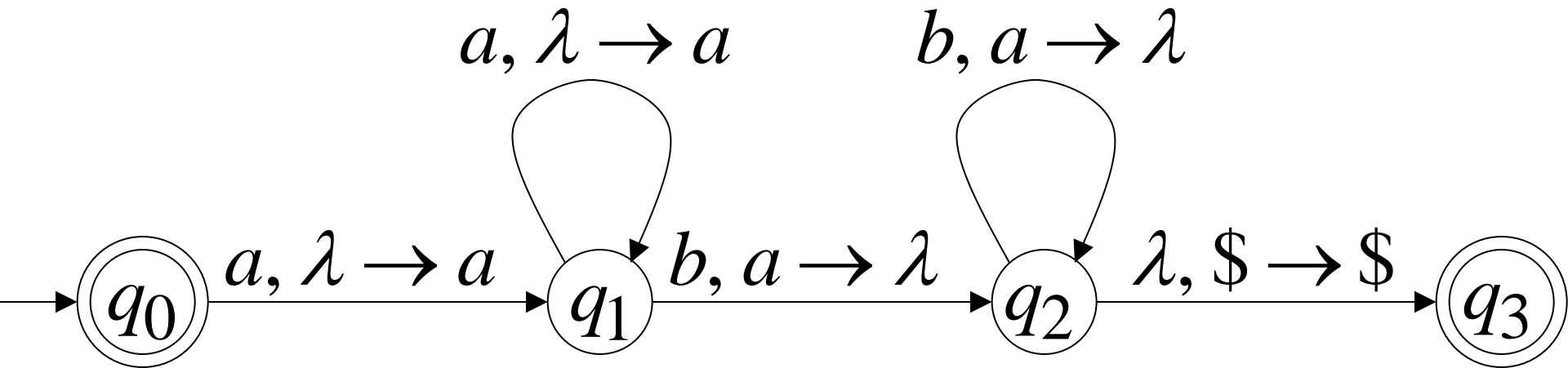
Not allowed:



(non deterministic choices)

# DPDA example

$$L(M) = \{a^n b^n : n \geq 0\}$$



$WC\ W^R, W \in \{a, b\}^*$

## Definition:

A language  $L$  is **deterministic context-free** if there exists some DPDA that accepts it

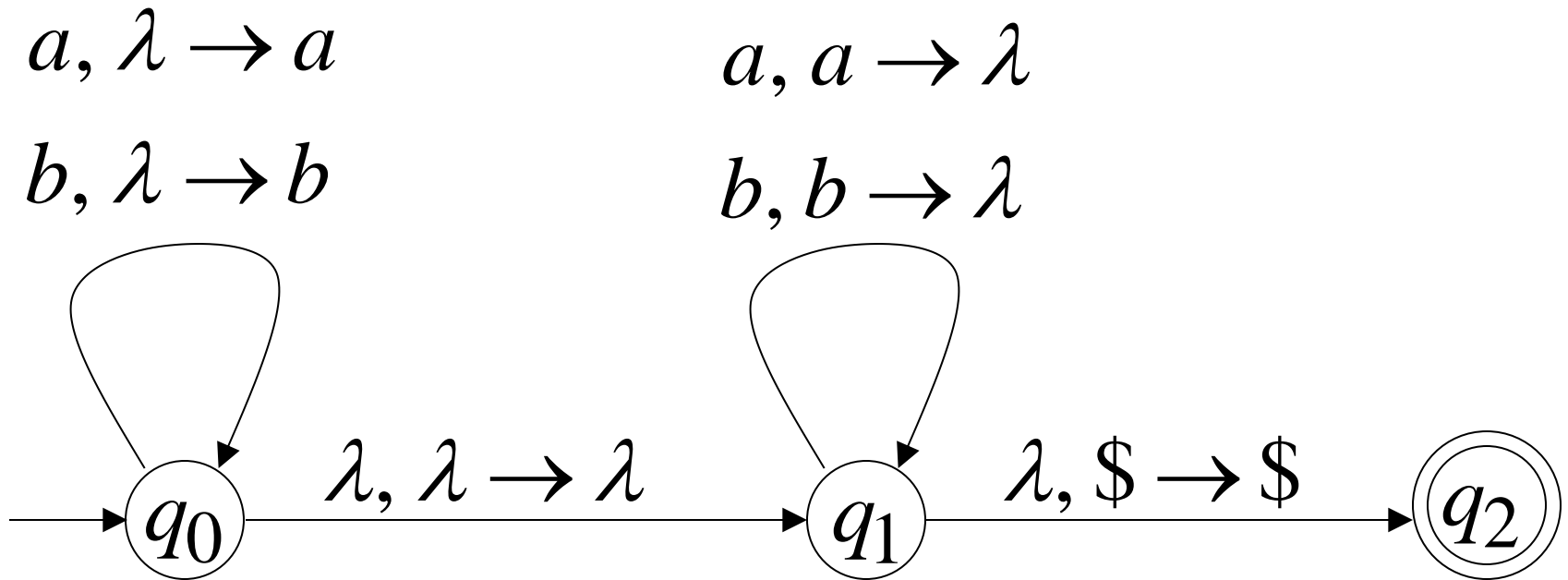
## Example:

The language  $L(M) = \{a^n b^n : n \geq 0\}$

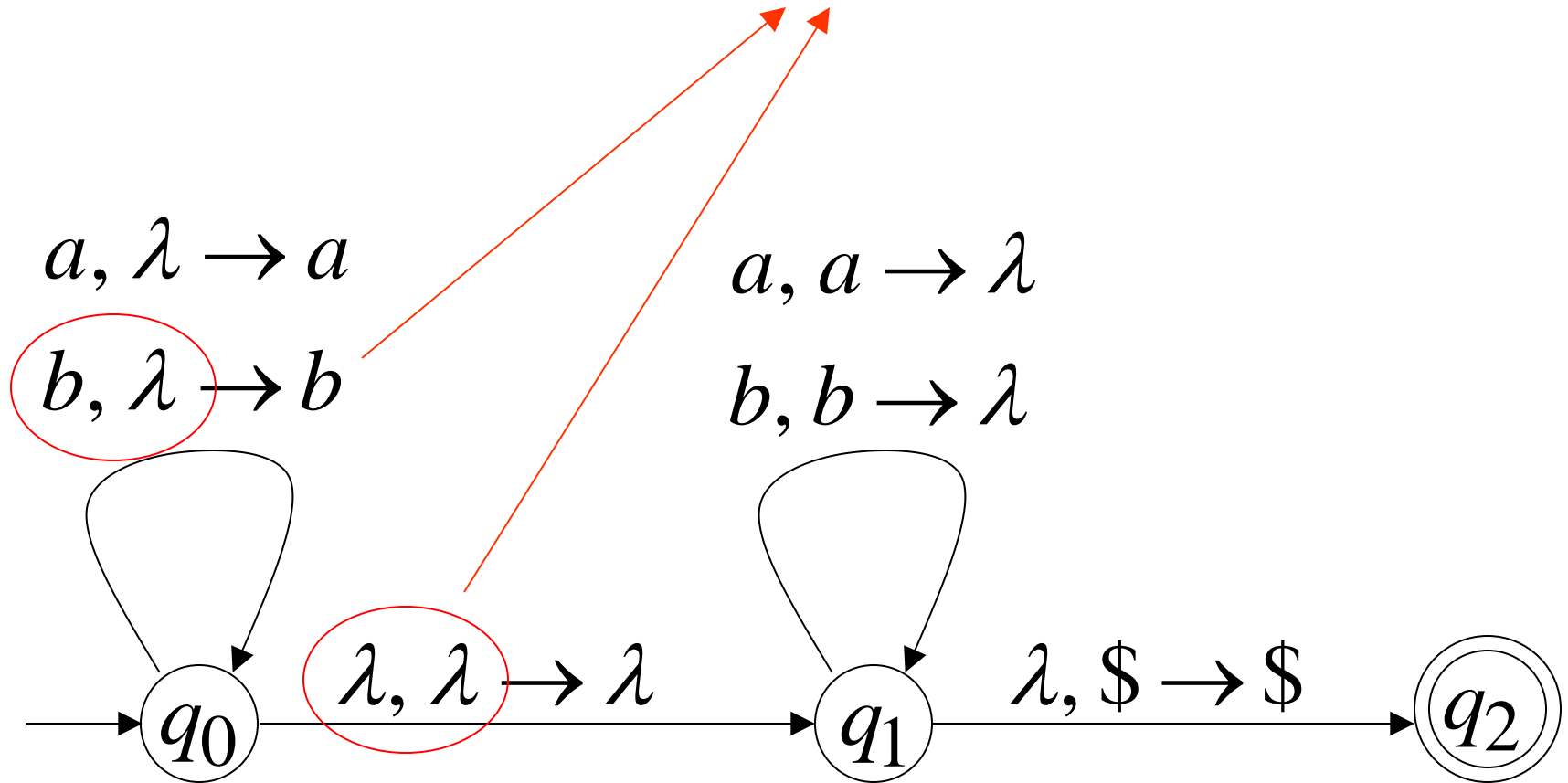
**is deterministic context-free**

# Example of Non-DPDA (PDA)

$$L(M) = \{vv^R : v \in \{a,b\}^*\}$$



Not allowed in DPDAs





PDA<sub>s</sub>

Have More Power than

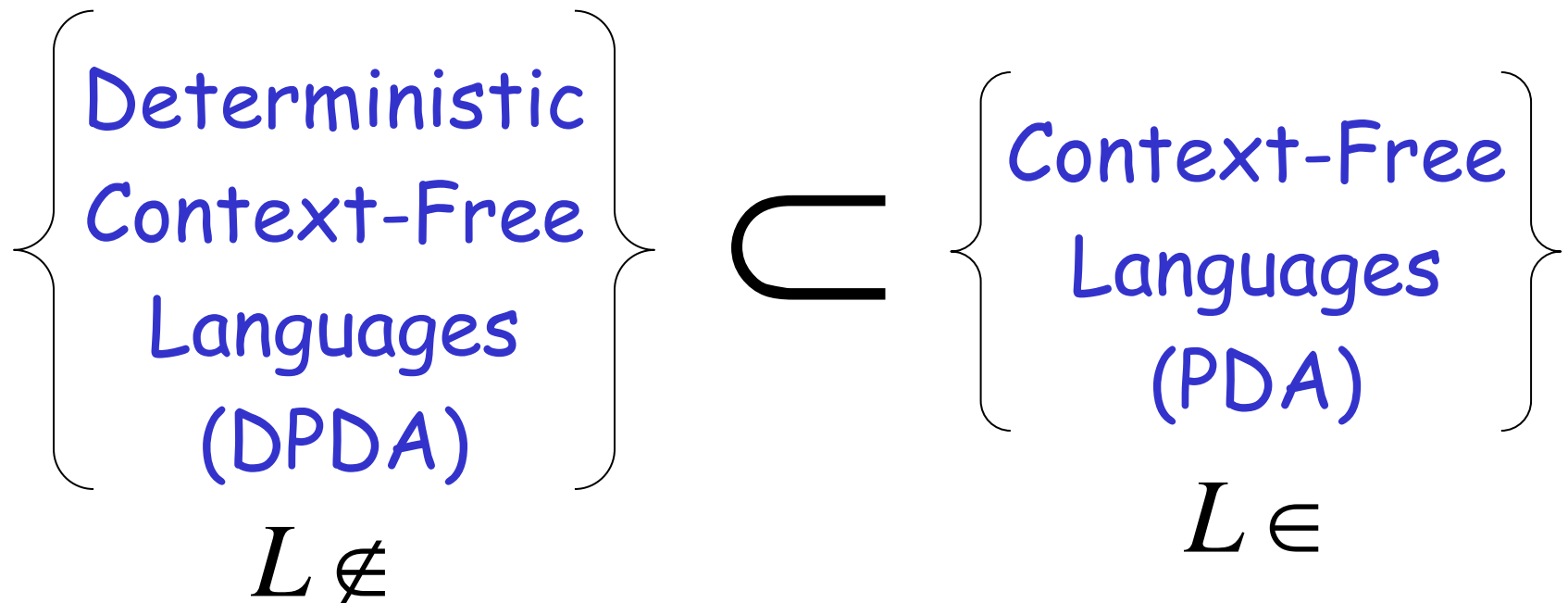
DPDA<sub>s</sub>

It holds that:

$$\left\{ \begin{array}{l} \text{Deterministic} \\ \text{Context-Free} \\ \text{Languages} \\ \text{(DPDA)} \end{array} \right\} \subseteq \left\{ \begin{array}{l} \text{Context-Free} \\ \text{Languages} \\ \text{PDAs} \end{array} \right\}$$

Since every DPDA is also a PDA

We will actually show:



We will show that there exists  
a context-free language  $L$  which is not  
accepted by any DPDA

The language is:

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\} \quad n \geq 0$$

We will show:

- $L$  is context-free
- $L$  is **not** deterministic context-free

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Language  $L$  is context-free

Context-free grammar for  $L$  :

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Language  $L$  is context-free

Context-free grammar for  $L$  :

$$S \rightarrow S_1 \mid S_2 \qquad \{a^n b^n\} \cup \{a^n b^{2n}\}$$

$$S_1 \rightarrow aS_1b \mid \lambda \qquad \{a^n b^n\}$$

$$S_2 \rightarrow aS_2bb \mid \lambda \qquad \{a^n b^{2n}\}$$

# Theorem:

The language  $L = \{a^n b^n\} \cup \{a^n b^{2n}\}$

is **not** deterministic context-free

(there is **no** DPDA that accepts  $L$  )

**Proof:** Assume for contradiction that

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

is deterministic context free

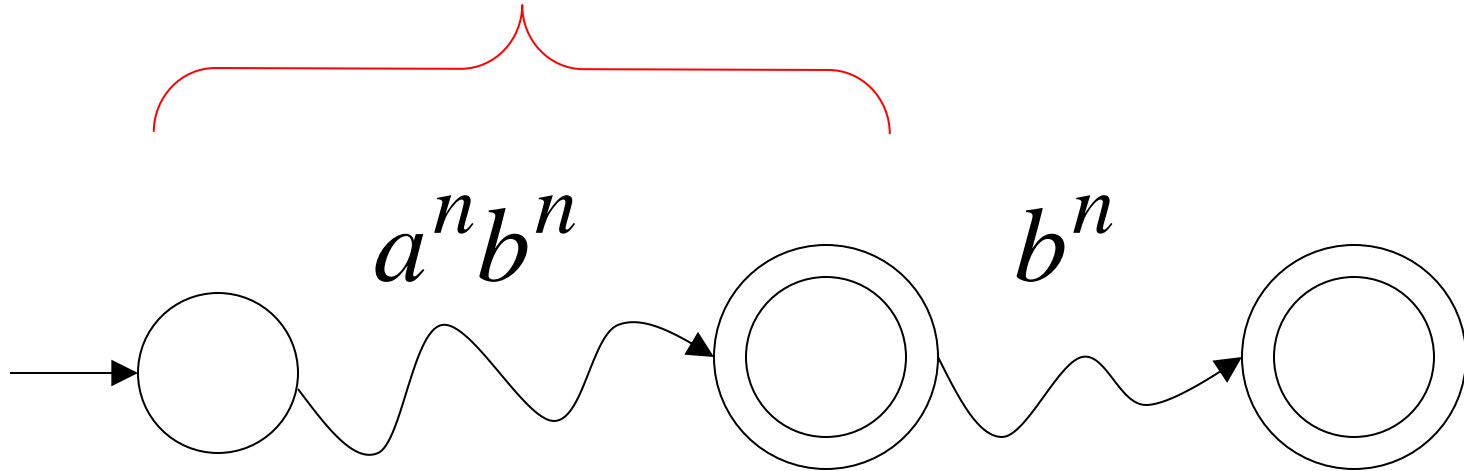
Therefore:

there is a DPDA  $M$  that accepts  $L$



DPDA  $M$  with  $L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$

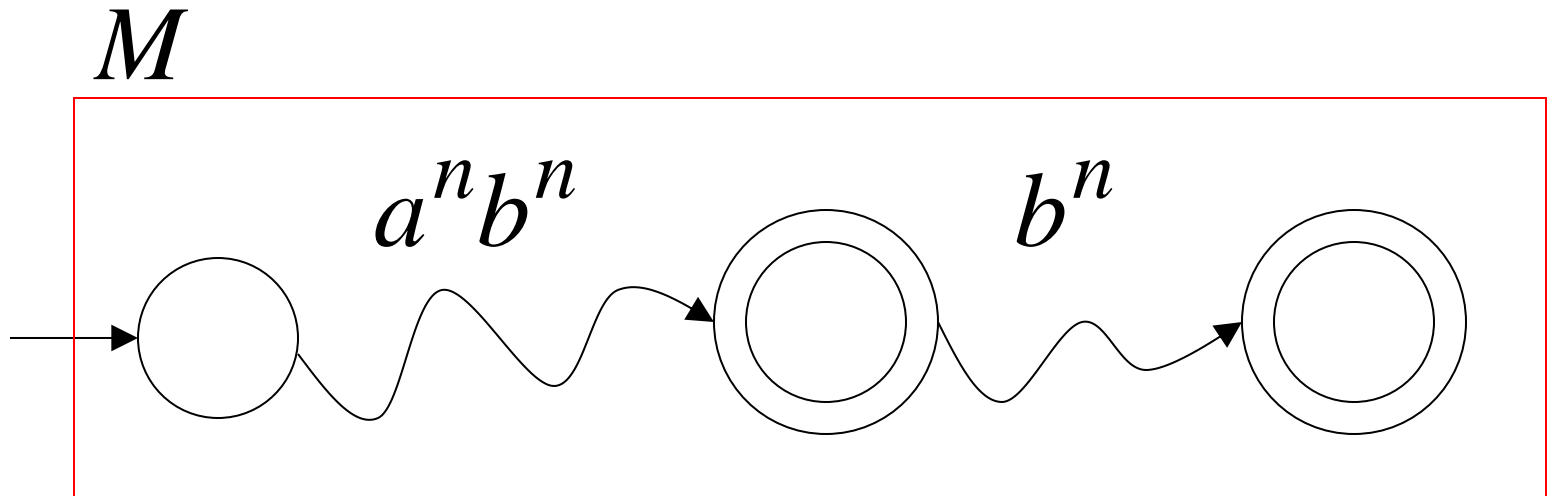
accepts  $a^n b^n$



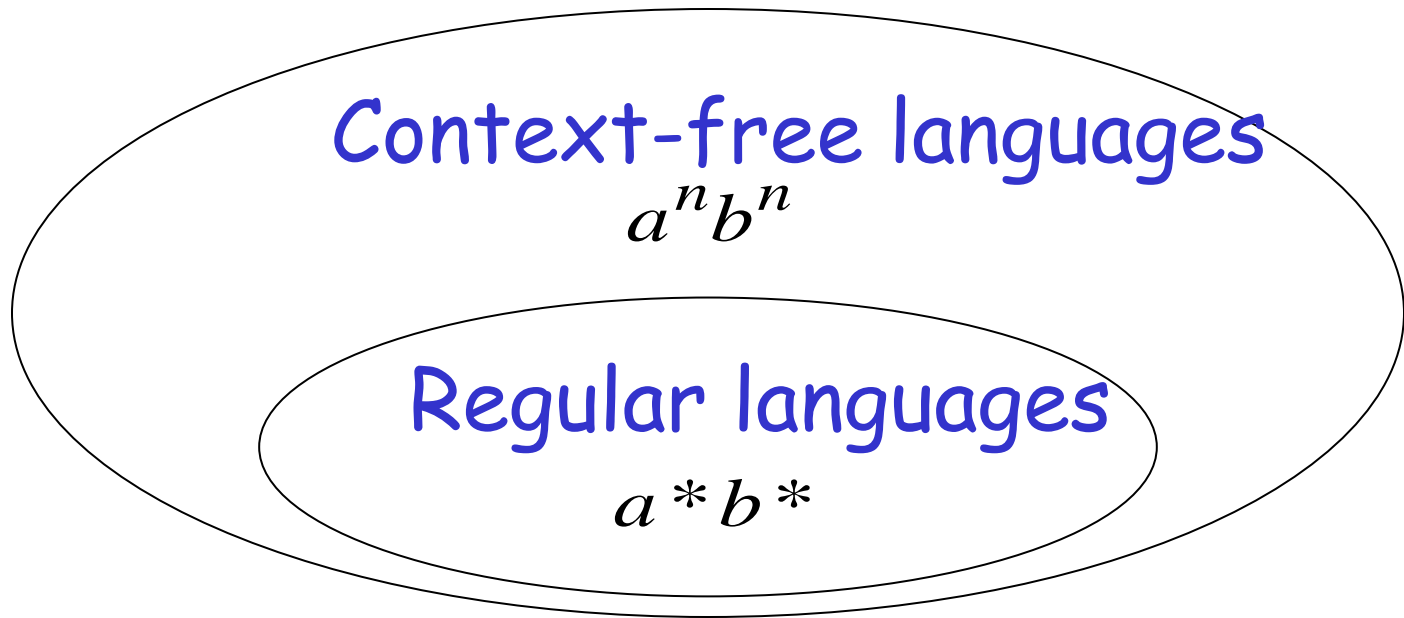
accepts  $a^n b^{2n}$

DPDA  $M$  with  $L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$

Such a path exists due to determinism



**Fact 1:** The language  $\{a^n b^n c^n\}$   
is **not** context-free



(we will prove this at a later class using  
pumping lemma for context-free languages)

**Fact 2:** The language  $L \cup \{a^n b^n c^n\}$   
is **not** context-free

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

(we can prove this using pumping lemma  
for context-free languages)

We will construct a PDA that accepts:

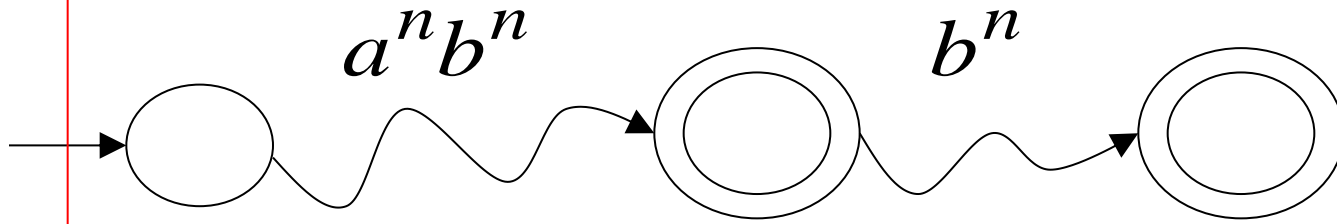
$$L \cup \{a^n b^n c^n\}$$

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

which is a contradiction!

DPDA  $M$

$$L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

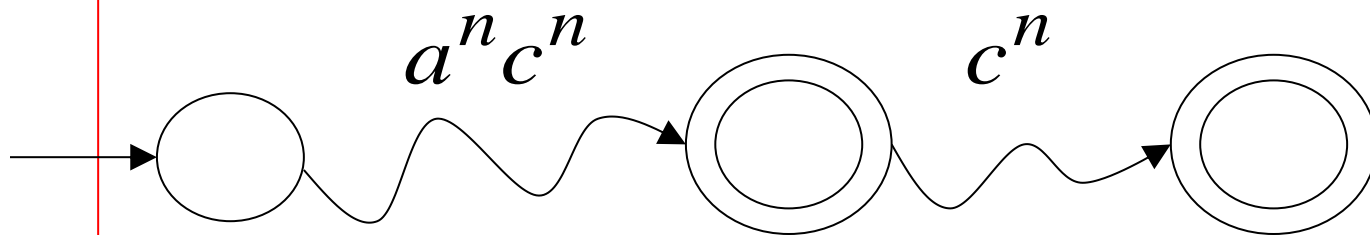


Modify  $M$

Replace  $b$   
with  $c$

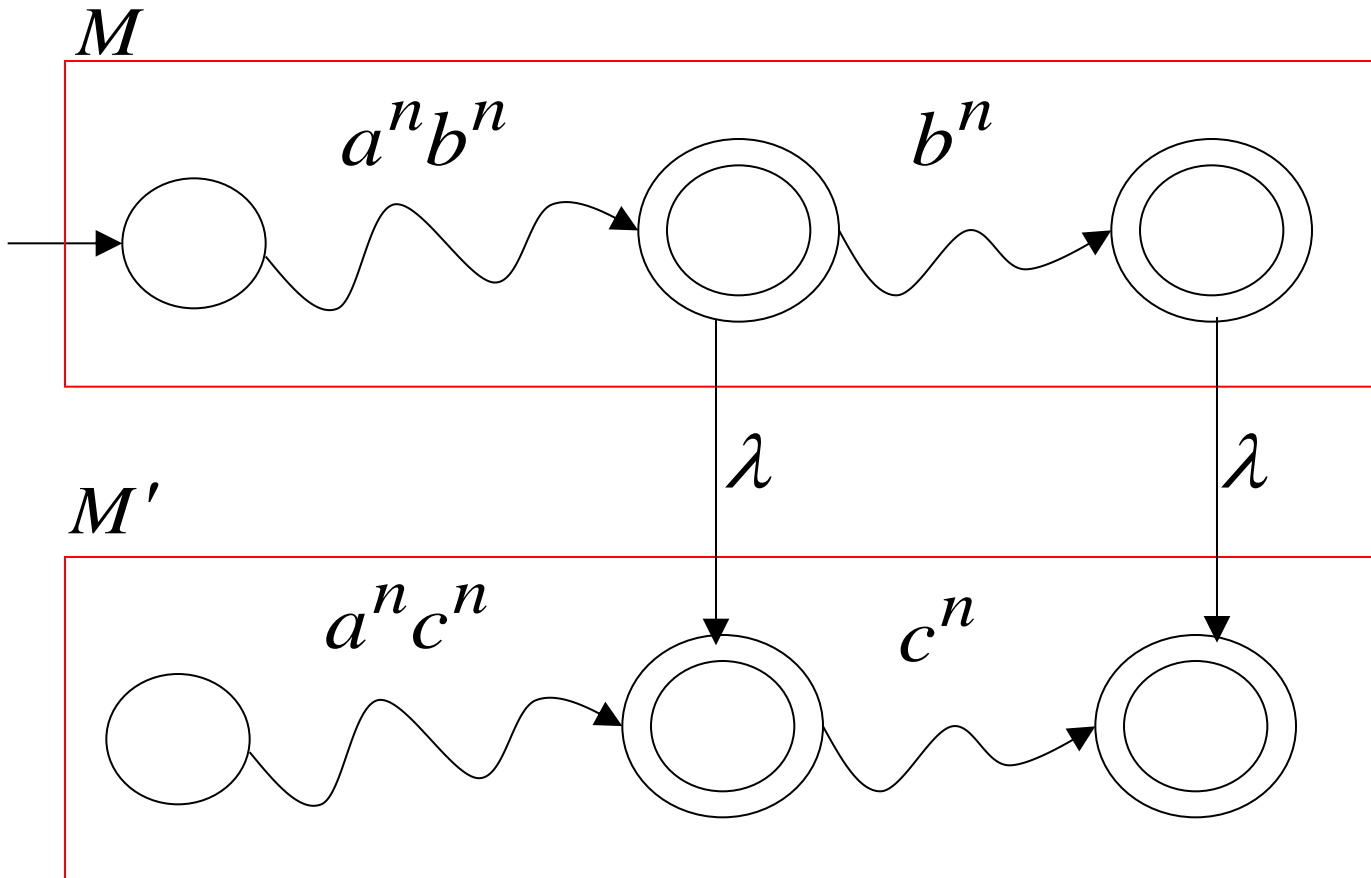
DPDA  $M'$

$$L(M') = \{a^n c^n\} \cup \{a^n c^{2n}\}$$



A PDA that accepts  $L \cup \{a^n b^n c^n\}$

Connect the final states of  $M$   
with the final states of  $M'$



Since  $L \cup \{a^n b^n c^n\}$  is accepted by a PDA  
it is context-free

**Contradiction!**

(since  $L \cup \{a^n b^n c^n\}$  is not context-free)



Therefore:

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Is not deterministic context free

There is no DPDA that accepts it

End of Proof