Turing’s Thesis

Turing's thesis (1930):

Any computation carried out
by mechanical means
can be performed by a Turing Machine

Algorithm:

An algorithm for a problem is a Turing Machine which solves the problem

The algorithm describes the steps of the mechanical means

This is easily translated to computation steps of a Turing machine

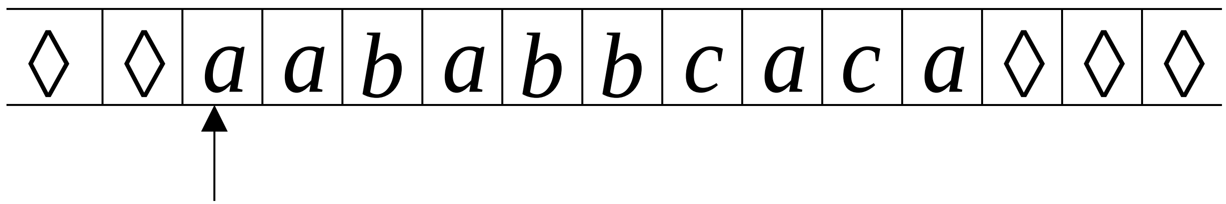
When we say: There exists an algorithm

We mean: There exists a Turing Machine that executes the algorithm

Variations
of the
Turing Machine

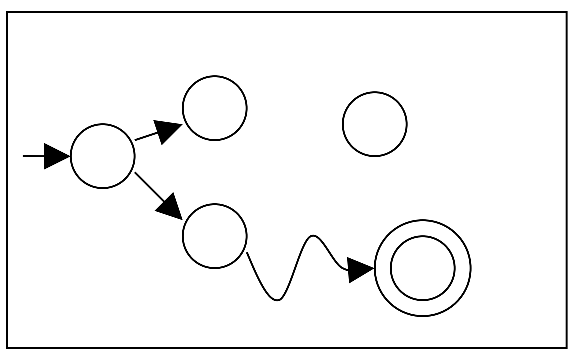
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

Variations of the Standard Model

Turing machines with:

- Stay-Option
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

Different Turing Machine **Classes**

Same Power of two machine classes:

both classes accept the
same set of languages

We will prove:

each new class has the same power
with Standard Turing Machine

(accept Turing-Recognizable Languages)

Same Power of two classes means:

for any machine M_1 of first class

there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

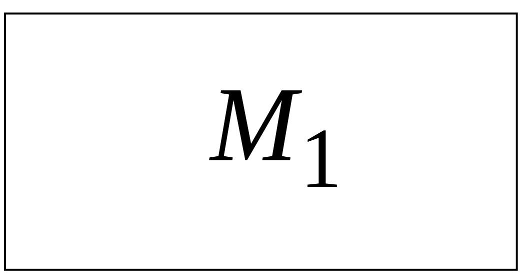
and vice-versa

Simulation:

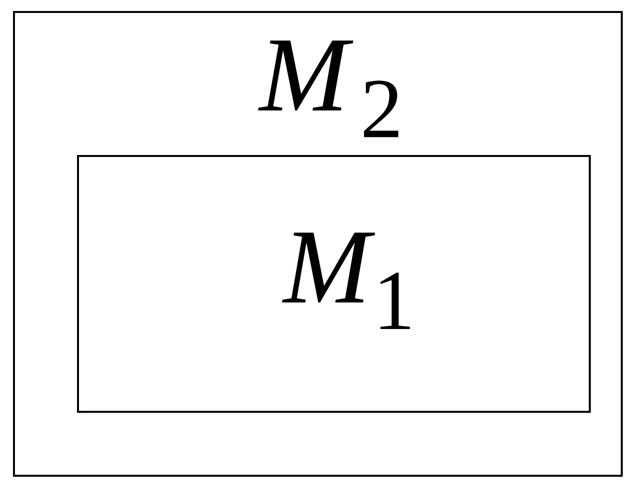
A technique to prove same power.

Simulate the machine of one class with a machine of the other class

First Class
Original Machine



Second Class
Simulation Machine



simulates M_1

Configurations in the Original Machine M_1
have corresponding configurations
in the Simulation Machine M_2

$$\begin{array}{lccccccc}
 & & & & M_1 & & & \\
\text{Original Machine:} & d_0 & \succ & d_1 & \succ & \cdots & \succ & d_n \\
 & \updownarrow & & \updownarrow & & & & \updownarrow \\
\text{Simulation Machine:} & d'_0 & \overset{*}{\succ} & d'_1 & \overset{*}{\succ} & \cdots & \overset{*}{\succ} & d'_n \\
 & & & & M_2 & & &
\end{array}$$

Accepting Configuration

Original Machine: d_f

$$\updownarrow$$

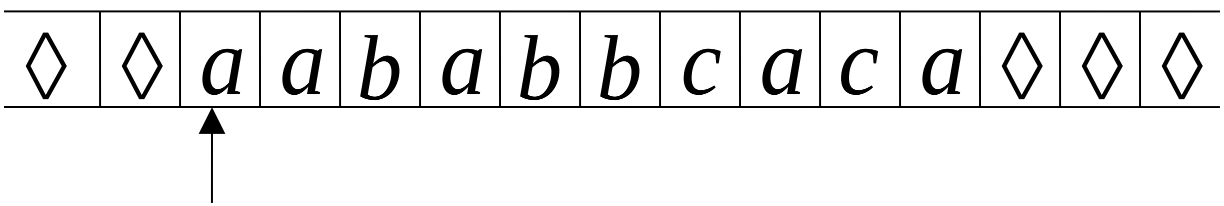
Simulation Machine: d'_f

the Simulation Machine
and the Original Machine
accept the same strings

$$L(M_1) = L(M_2)$$

Turing Machines with Stay-Option

The head can stay in the same position

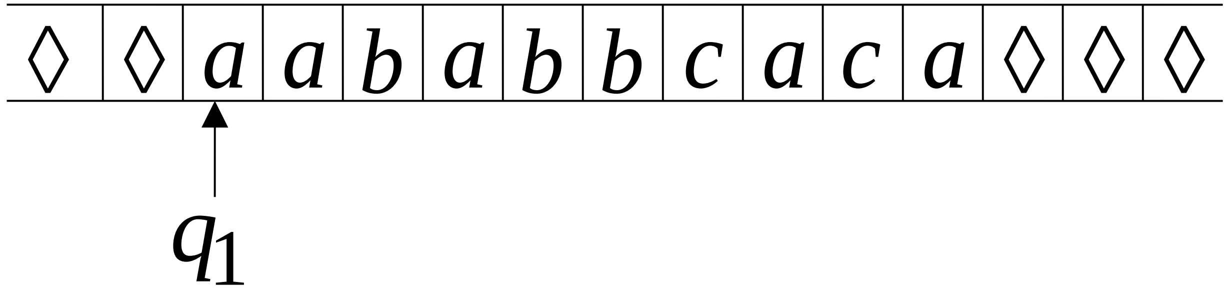


Left, Right, Stay

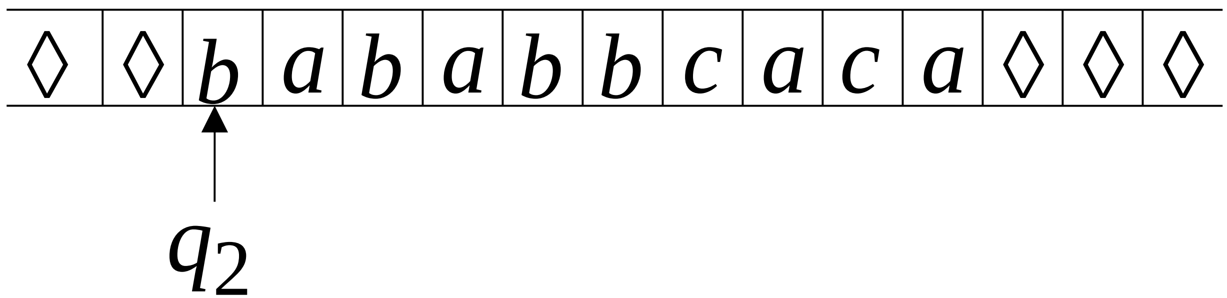
L,R,S: possible head moves

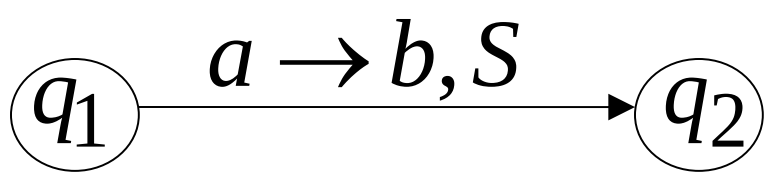
Example:

Time 1



Time 2





Theorem: Stay-Option machines have the same power with Standard Turing machines

Proof:

1. Stay-Option Machines simulate Standard Turing machines

2. Standard Turing machines simulate Stay-Option machines

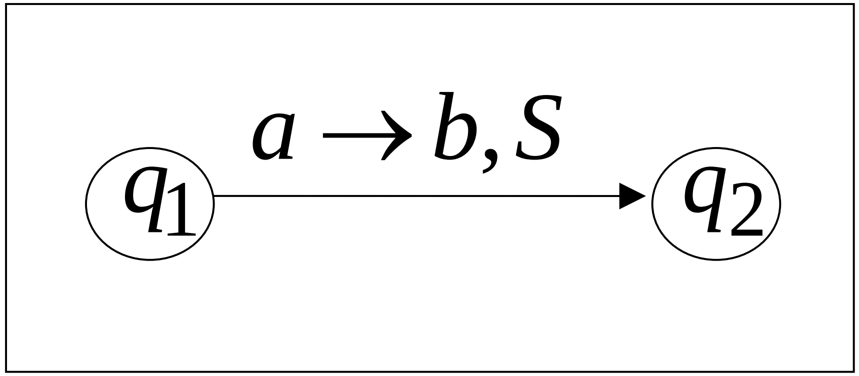
1. Stay-Option Machines
simulate Standard Turing machines

Trivial: any standard Turing machine
is also a Stay-Option machine

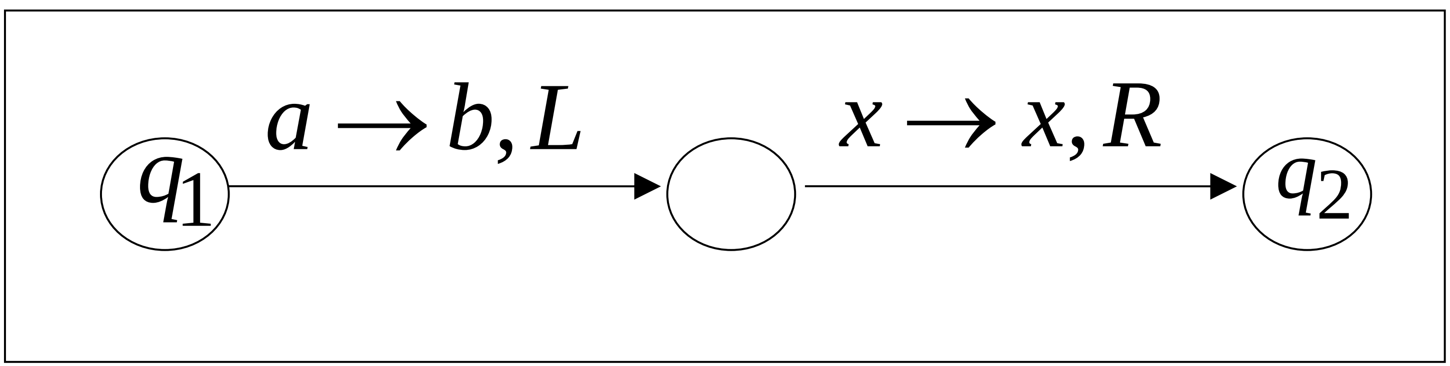
2. Standard Turing machines simulate Stay-Option machines

We need to simulate the stay head option with two head moves, one left and one right

Stay-Option Machine



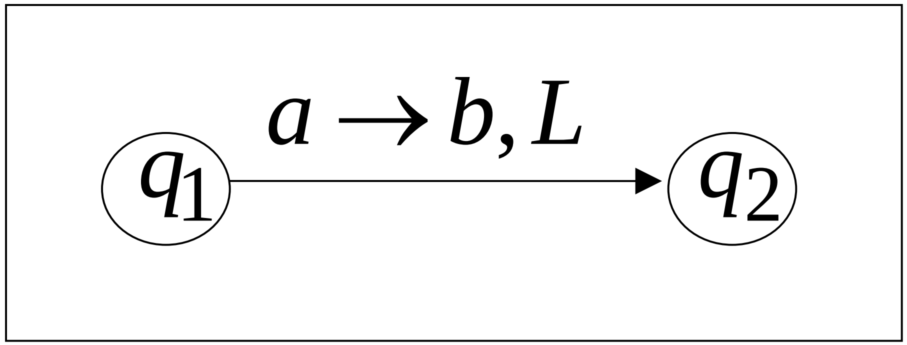
Simulation in Standard Machine



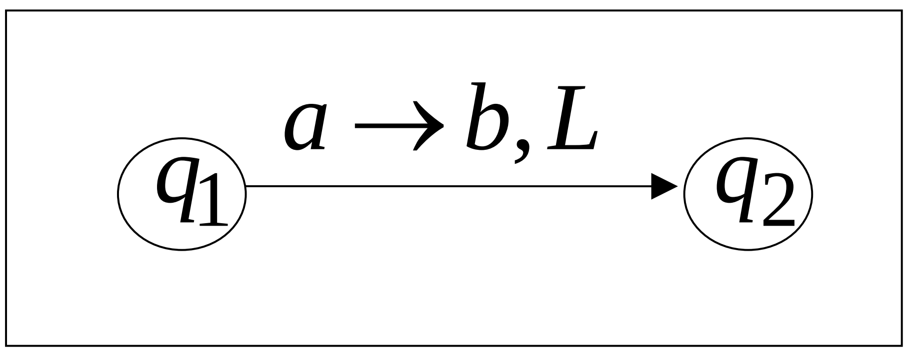
For every possible tape symbol x

For other transitions nothing changes

Stay-Option Machine

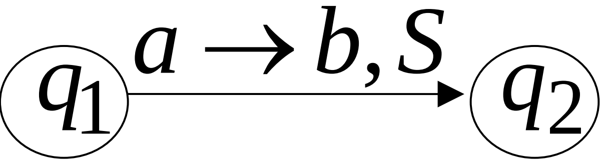


Simulation in Standard Machine

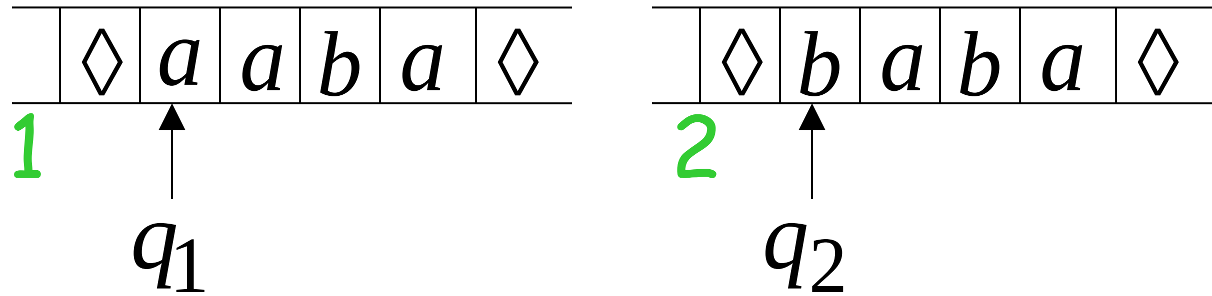


Similar for Right moves

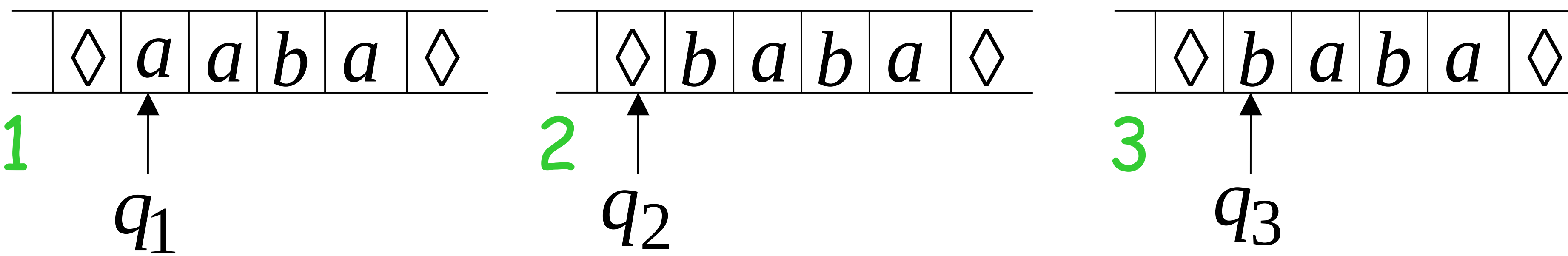
example of simulation



Stay-Option Machine:



Simulation in Standard Machine:



END OF PROOF

Multiple Track Tape

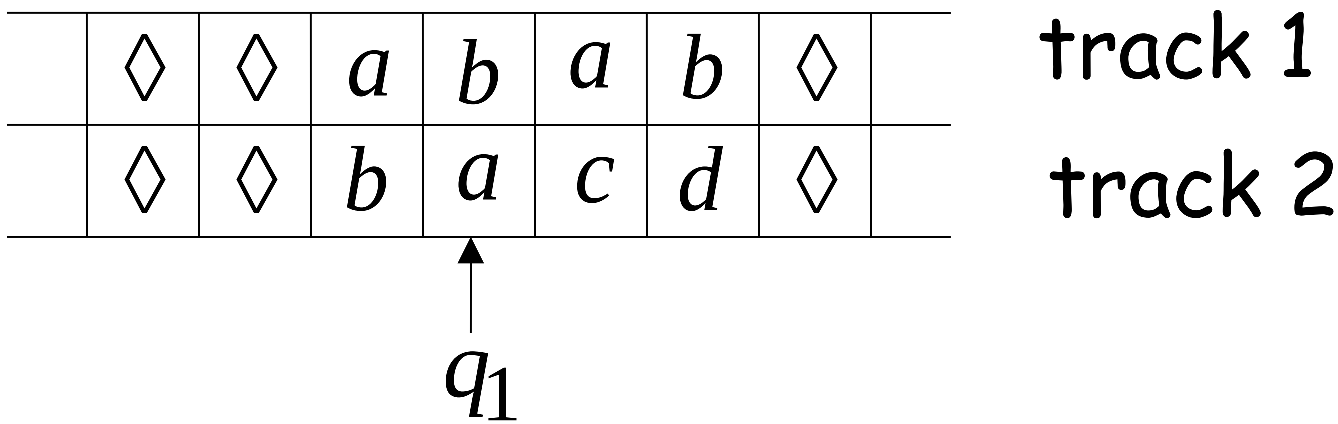
A useful trick to perform more complicated simulations

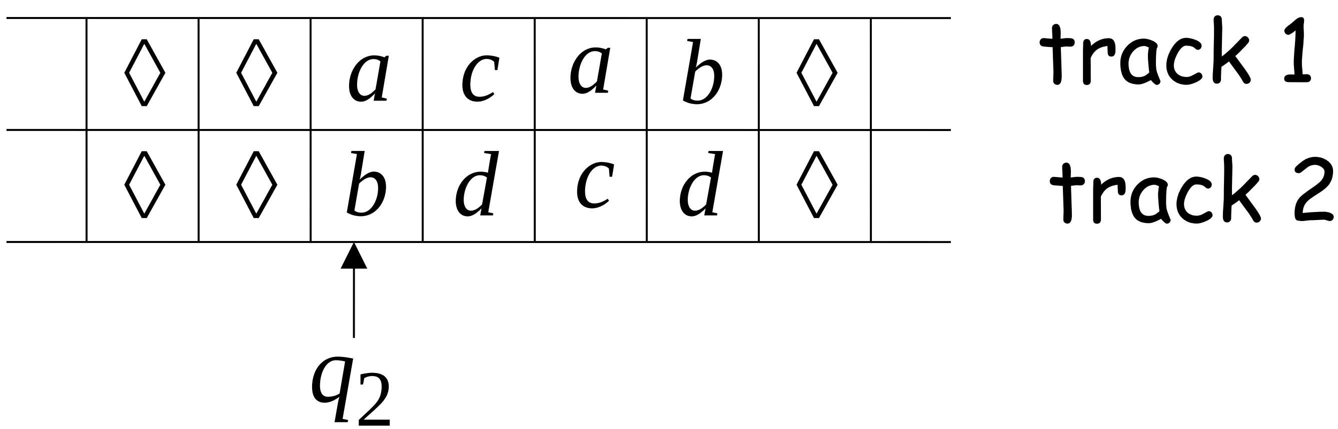
One Tape

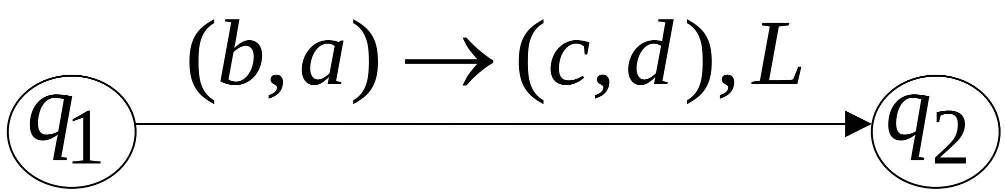
	$\Diamond$	$\Diamond$	a	b	a	b	$\Diamond$	track 1
	$\Diamond$	$\Diamond$	b	a	c	d	$\Diamond$	track 2

One head

One symbol (a,b)

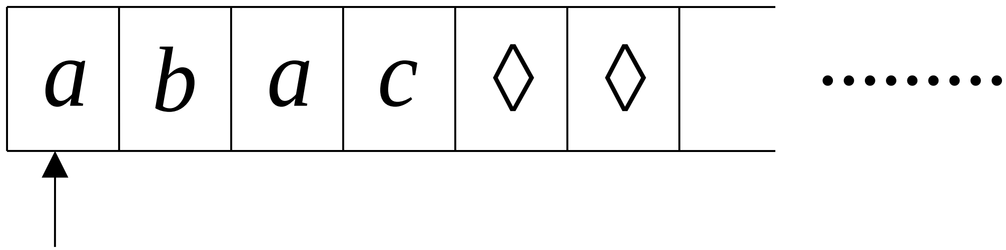

◊ ◊ a b a b ◊
◊ ◊ b a c d ◊
track 1
track 2
q_1


◊ ◊ a c a b ◊
◊ ◊ b d c d ◊
track 1
track 2
q_2


q_1
$(b,a) \rightarrow (c,d), L$
q_2

Semi-Infinite Tape

The head extends infinitely only to the right



- Initial position is the leftmost cell

- When the head moves left from the border, it returns to the same position

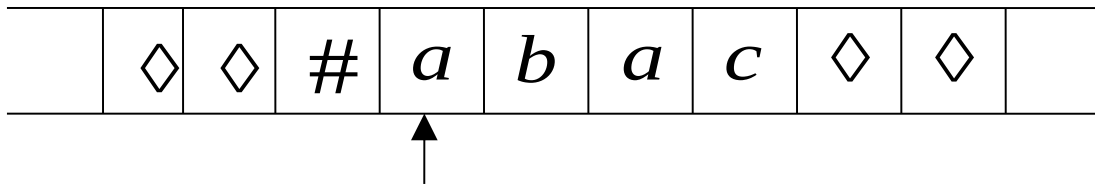
Theorem: Semi-Infinite machines have the same power with Standard Turing machines

Proof:

1. Standard Turing machines simulate Semi-Infinite machines

2. Semi-Infinite Machines simulate Standard Turing machines

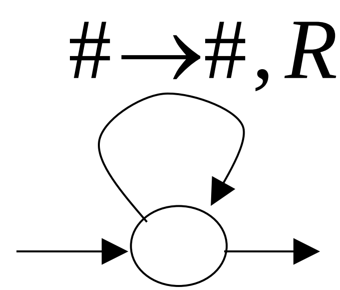
1. Standard Turing machines simulate Semi-Infinite machines:



Standard Turing Machine

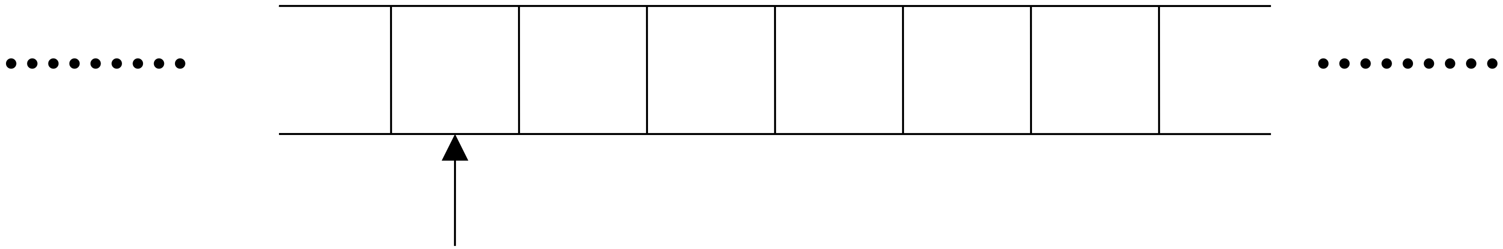
a. insert special symbol # at left of input string

b. Add a self-loop to every state (except states with no outgoing transitions)

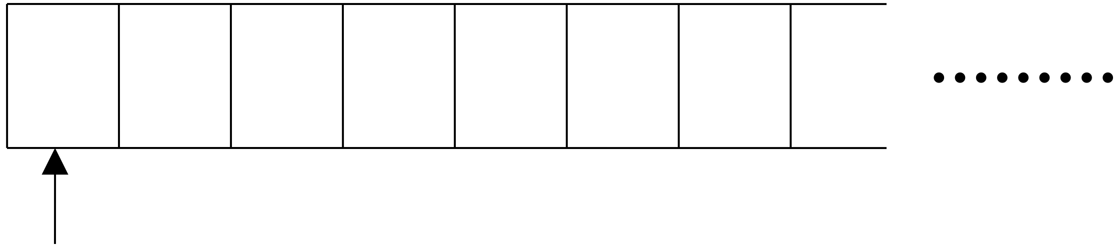


2. Semi-Infinite tape machines simulate Standard Turing machines:

Standard machine

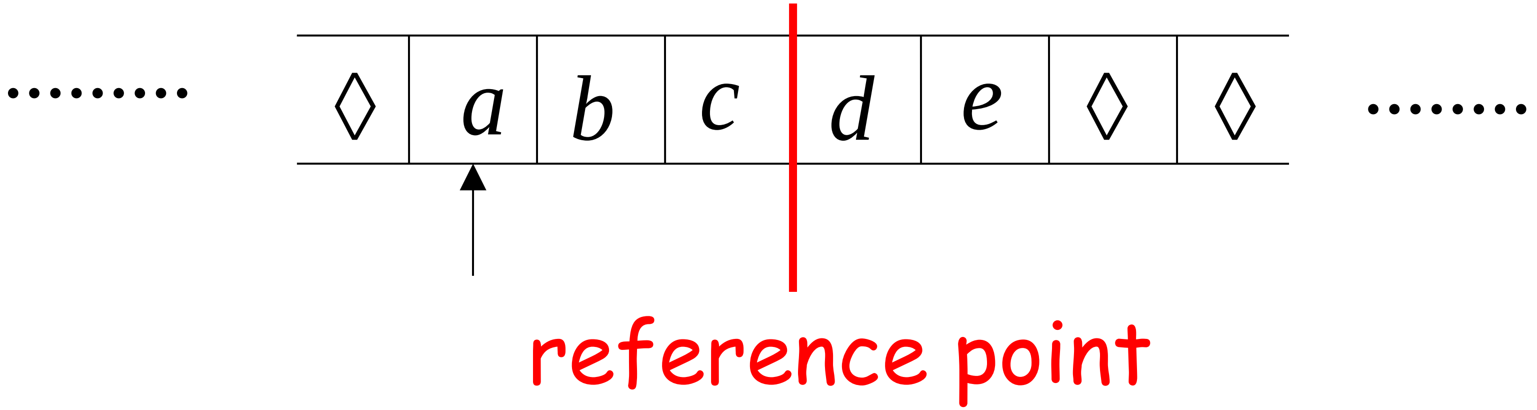


Semi-Infinite tape machine

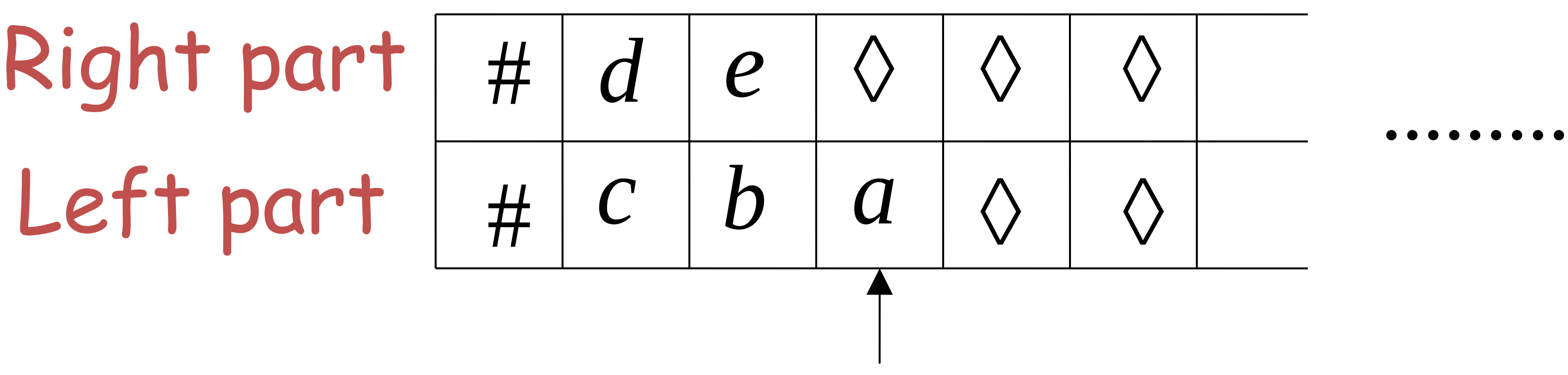


Squeeze infinity of both directions in one direction

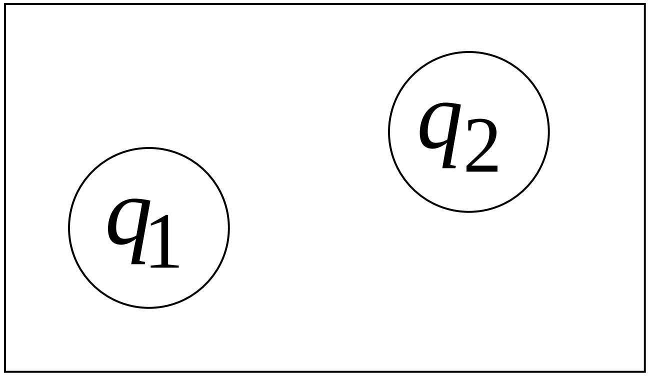
Standard machine



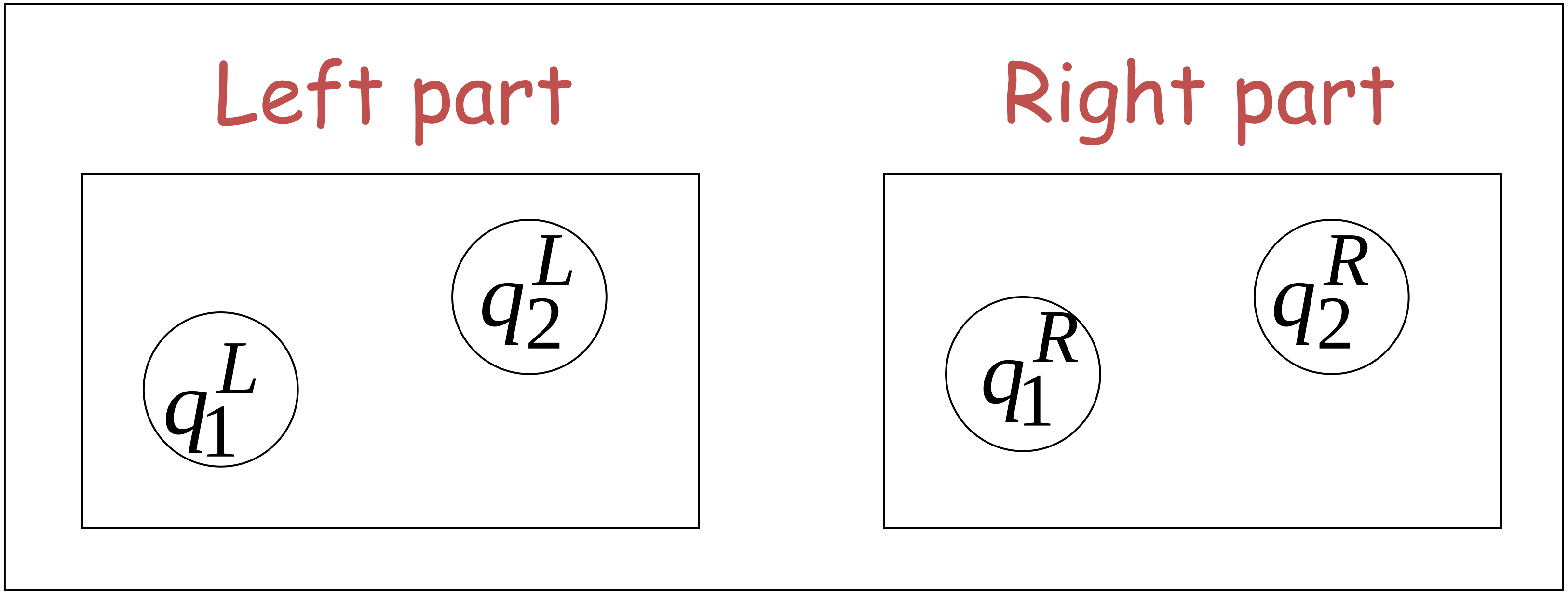
Semi-Infinite tape machine with two tracks



Standard machine



Semi-Infinite tape machine



Standard machine

$$q_1 \xrightarrow{a \rightarrow g, R} q_2$$

Semi-Infinite tape machine

Right part

$$q_1^R \xrightarrow{(a, x) \rightarrow (g, x), R} q_2^R$$

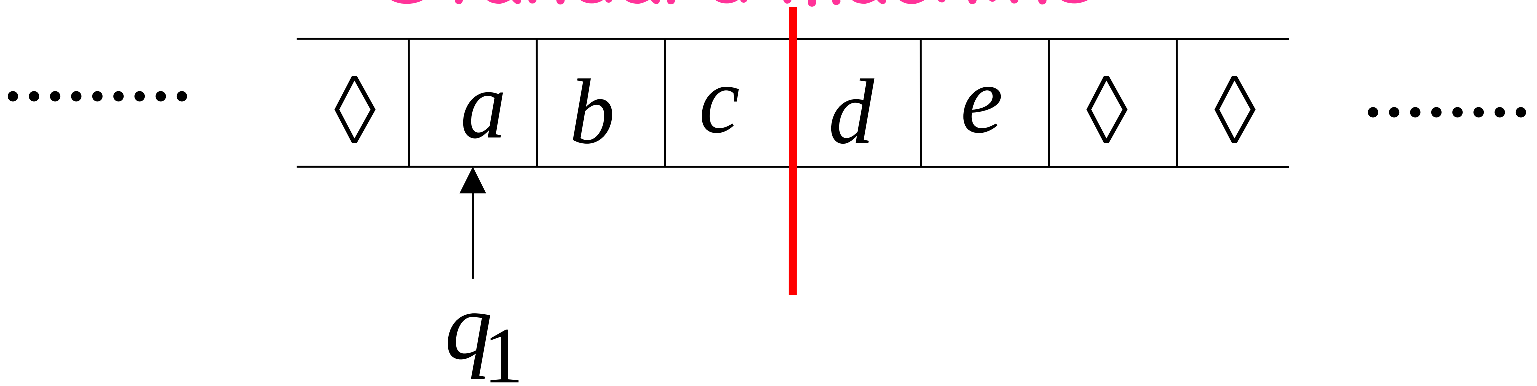
Left part

$$q_1^L \xrightarrow{(x, a) \rightarrow (x, g), L} q_2^L$$

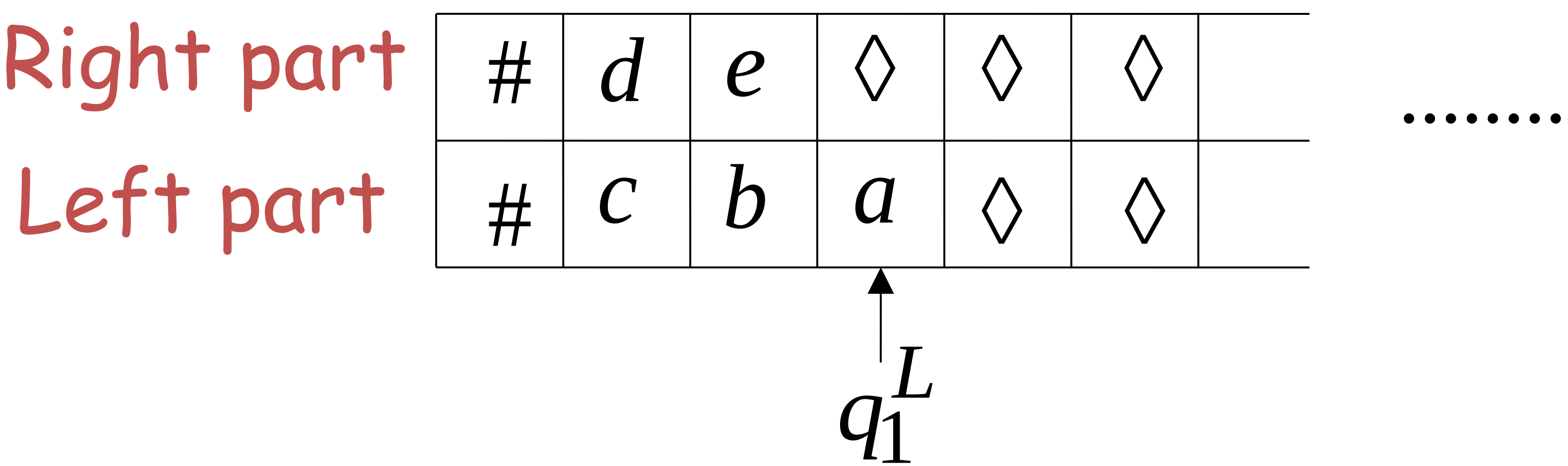
For all tape symbols x

Time 1

Standard machine



Semi-Infinite tape machine



Time 2

Standard machine

....	◊	g	b	c	d	e	◊	◊	....

q_2

Semi-Infinite tape machine

Right part	#	d	e	◊	◊	◊		....
Left part	#	c	b	g	◊	◊		

q_2^L

At the border:

Semi-Infinite tape machine

Right part $\quad q_1^R \xrightarrow{(\#,\#)\rightarrow(\#,\#),R} q_1^L$

Left part $\quad q_1^L \xrightarrow{(\#,\#)\rightarrow(\#,\#),R} q_1^R$

Semi-Infinite tape machine

Time 1

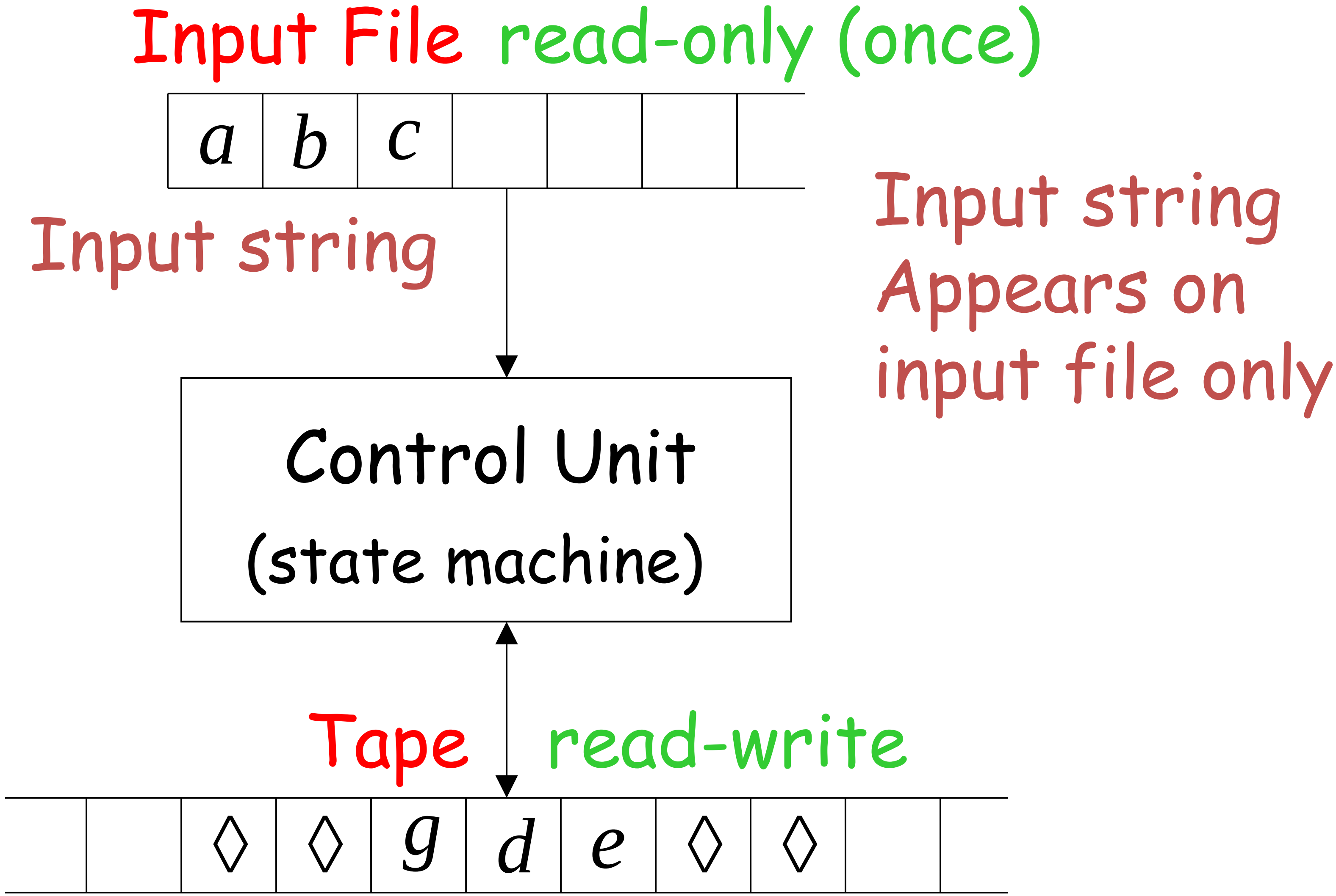
Right part	#	d	e	$\Diamond$	$\Diamond$	$\Diamond$		....
Left part	#	c	b	g	$\Diamond$	$\Diamond$		
	$\uparrow$ q_1^L							

Time 2

Right part	#	d	e	$\Diamond$	$\Diamond$	$\Diamond$		....
Left part	#	c	b	g	$\Diamond$	$\Diamond$		
		$\uparrow$ q_1^R						

END OF PROOF

The Off-Line Machine



Theorem: Off-Line machines
have the same power with
Standard Turing machines

Proof:
1. Off-Line machines
simulate Standard Turing
machines

2. Standard Turing machines
simulate Off-Line machines

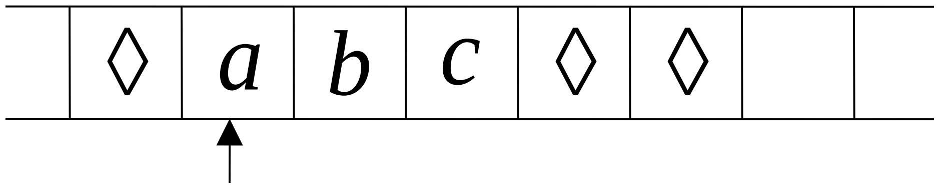
1. Off-line machines simulate Standard Turing Machines

Off-line machine:

1. Copy input file to tape

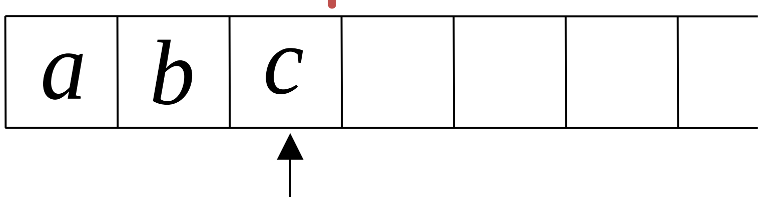
2. Continue computation as in Standard Turing machine

Standard machine

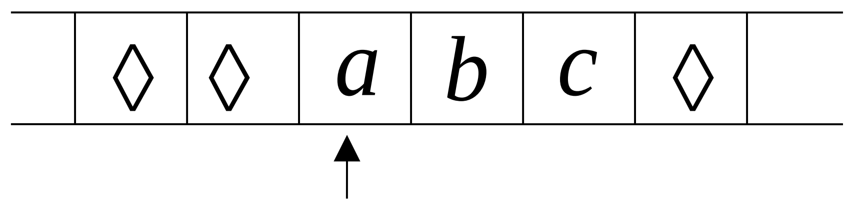


Off-line machine

Input File

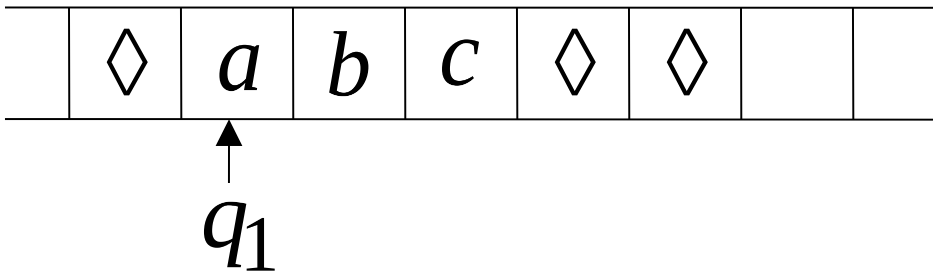


Tape



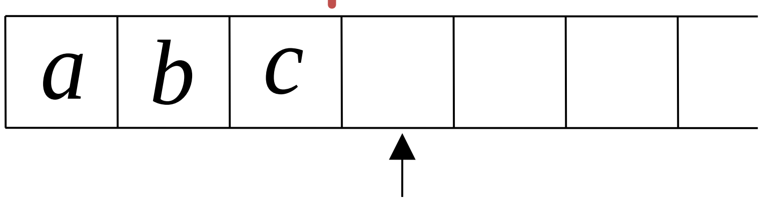
1. Copy input file to tape

Standard machine

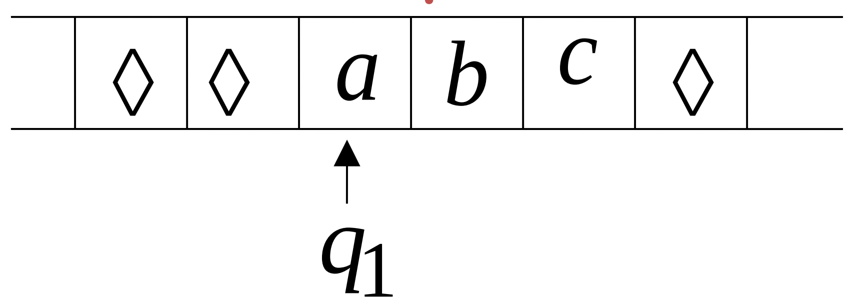


Off-line machine

Input File



Tape



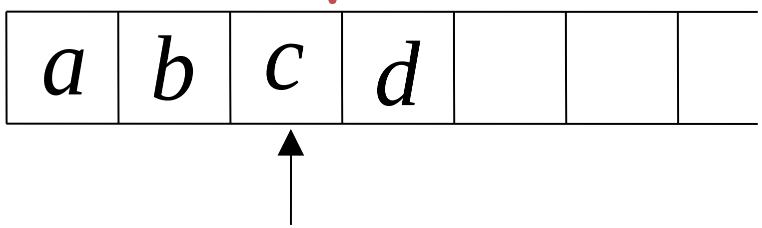
2. Do computations as in Turing machine

2. Standard Turing machines simulate Off-Line machines:

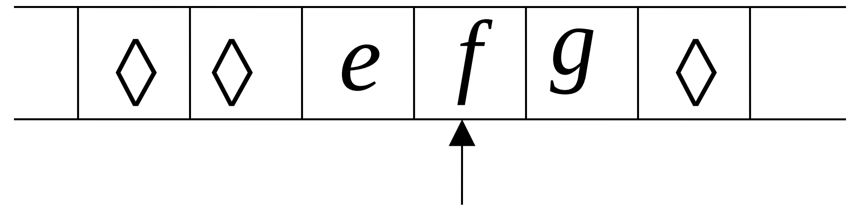
Use a Standard machine with
a four-track tape to keep track of
the Off-line input file and tape contents

Off-line Machine

Input File



Tape



Standard Machine -- Four track tape

	#	a	b	c	d			Input File
	#	0	0	1	0			head position
		e	f	g				Tape
		0	1	0				head position

Reference point (uses special symbol #)

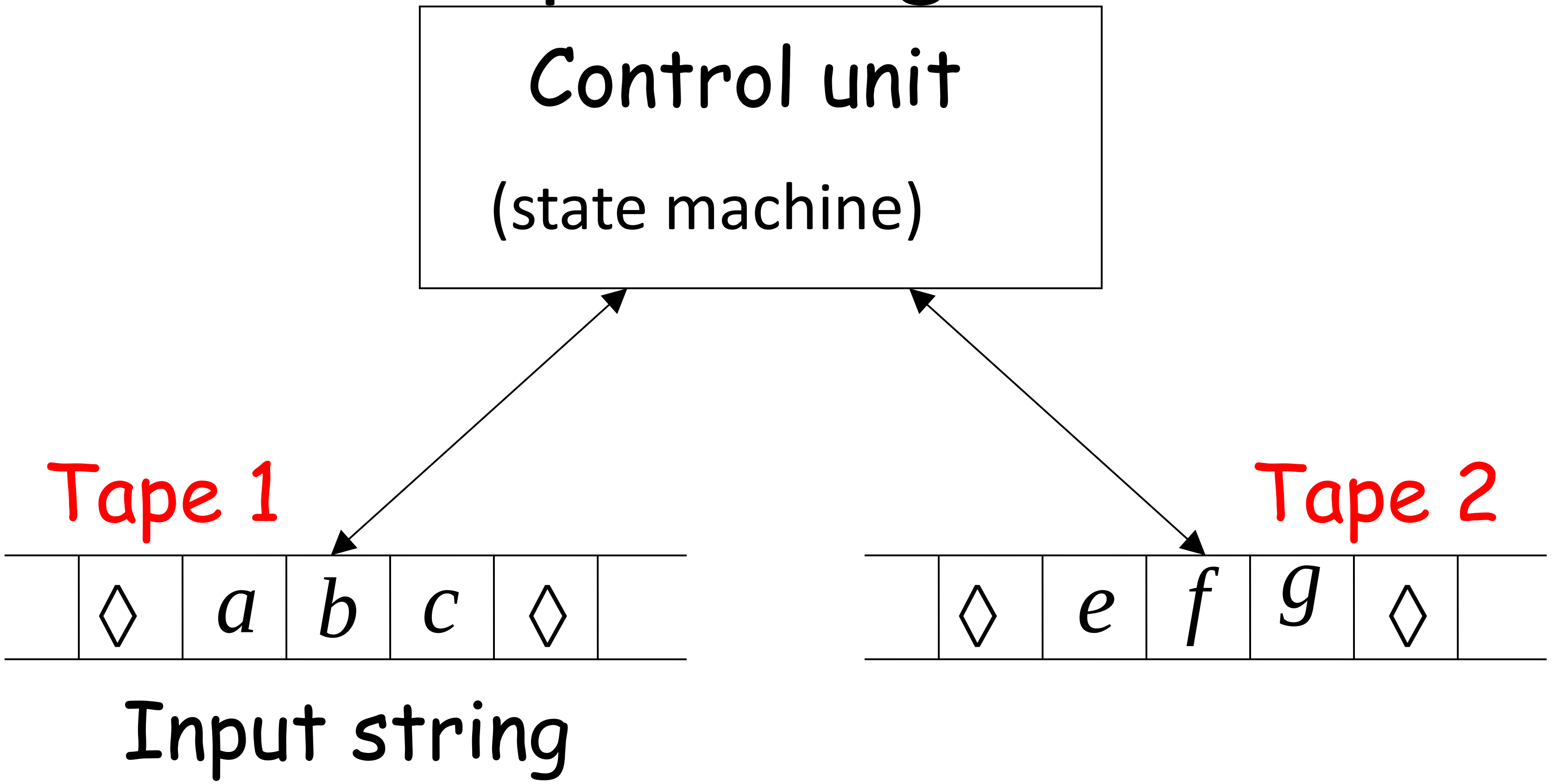
	#	a	b	c	d			Input File
	#	0	0	1	0			head position
	#	e	f	g				Tape
	#	0	1	0				head position

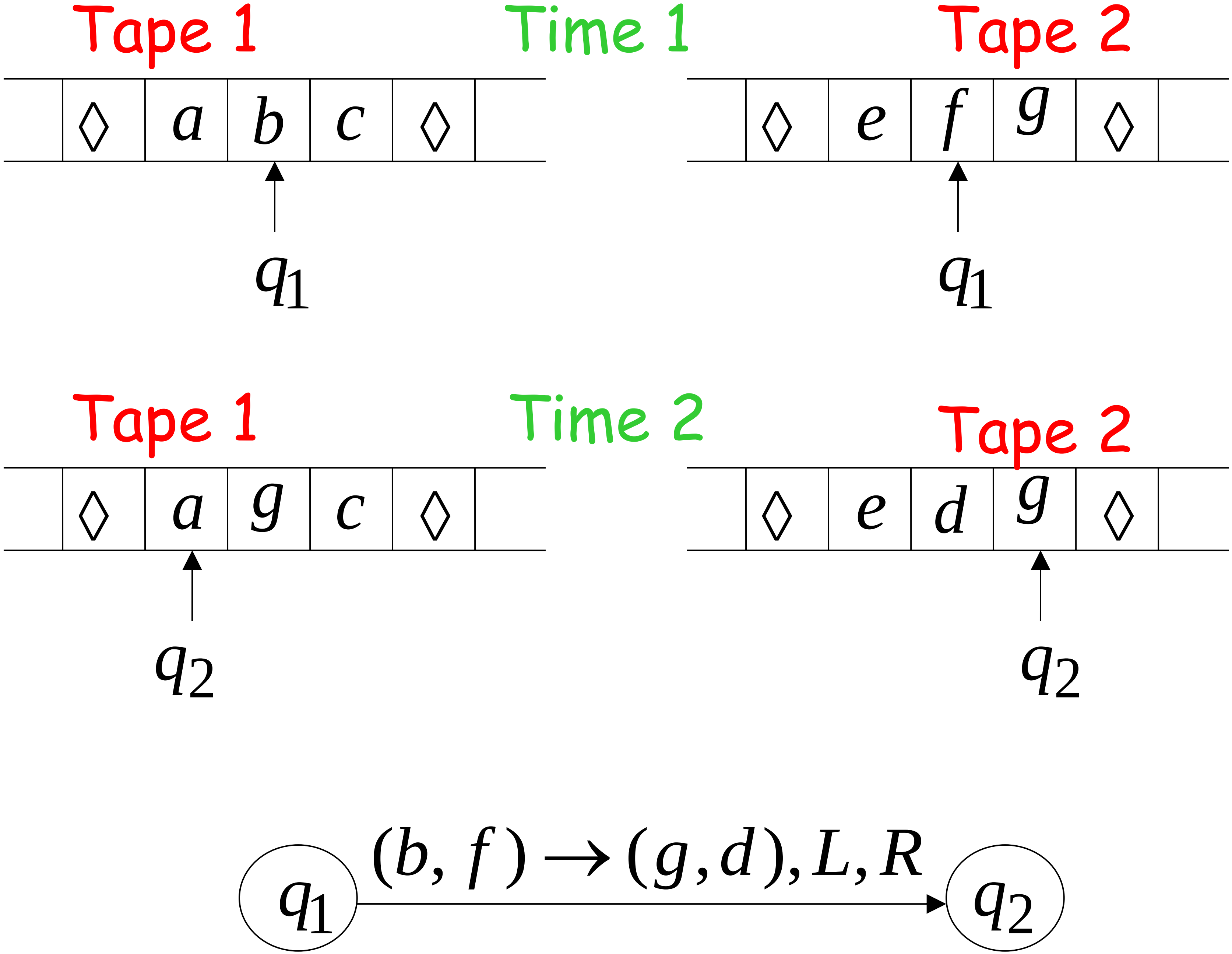
Repeat for each state transition:

1. Return to reference point
2. Find current input file symbol
3. Find current tape symbol
4. Make transition

END OF PROOF

Multi-tape Turing Machines




Tape 1
Time 1
Tape 2
◊ a b c ◊
◊ e f g ◊
q_1
q_1
Tape 1
Time 2
Tape 2
◊ a g c ◊
◊ e d g ◊
q_2
q_2
q_1
$(b, f) \to (g, d), L, R$
q_2

Theorem: Multi-tape machines have the same power with Standard Turing machines

Proof: 1. Multi-tape machines simulate Standard Turing machines

2. Standard Turing machines simulate Multi-tape machines

1. Multi-tape machines simulate Standard Turing Machines:

Trivial: Use just one tape

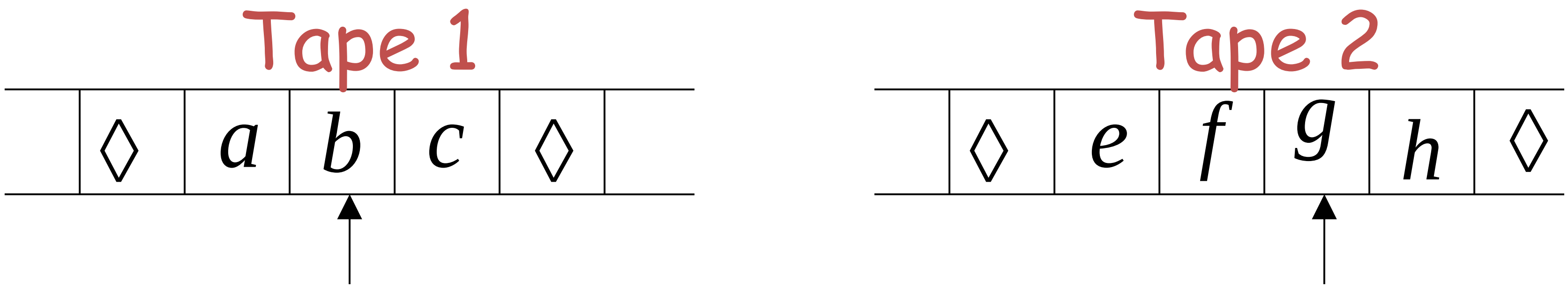
2. Standard Turing machines simulate Multi-tape machines:

Standard machine:

- Uses a multi-track tape to simulate the multiple tapes

- A tape of the Multi-tape machine corresponds to a pair of tracks

Multi-tape Machine



Standard machine with four track tape

		a	b	c				Tape 1
		0	1	0				head position
		e	f	g	h			Tape 2
		0	0	1	0			head position

Reference point

	#	a	b	c				Tape 1
	#	0	1	0				head position
	#	e	f	g	h			Tape 2
	#	0	0	1	0			head position

Repeat for each state transition:

1. Return to reference point
2. Find current symbol in Tape 1
3. Find current symbol in Tape 2
4. Make transition

END OF PROOF

Same power doesn't imply same speed:

$$L = \{a^n b^n\}$$

Standard Turing machine: $O(n^2)$ time

Go back and forth $O(n^2)$ times
to match the a's with the b's

2-tape machine: $O(n)$ time

1. Copy b^n to tape 2 ($O(n)$ steps)

2. Compare a^n on tape 1
and b^n tape 2 ($O(n)$ steps)