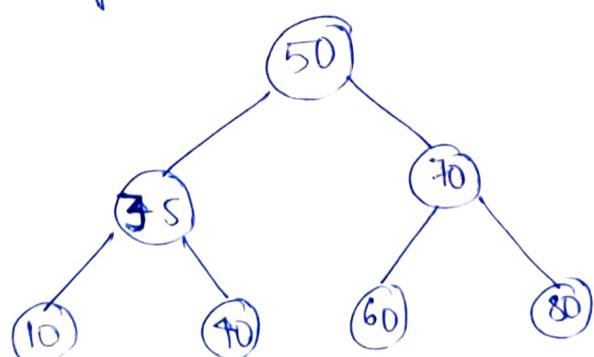


Binary Search Trees

	Array (unsorted)	Array (sorted)	Linked list	BST (Balanced)	Hash Table
Search	$O(n)$	$O(\log n)$	$O(n)$	$O(\log n)$	$O(1)$
Insert	$O(1)$	$O(n)$	$O(1)$ $O(n)$ in sorted	$O(\log n)$	$O(1)$
Delete	$O(n)$	$O(n)$	$O(n)$	$O(\log n)$	$O(1)$
Find closest	$O(n)$	$O(\log n)$	$O(n)$	$O(\log n)$	$O(1)$
sorted Traversal	$O(n \log n)$	$O(n)$	$O(n \log n)$, $O(n)$ in already sorted	$O(n)$	$O(n \log n)$.

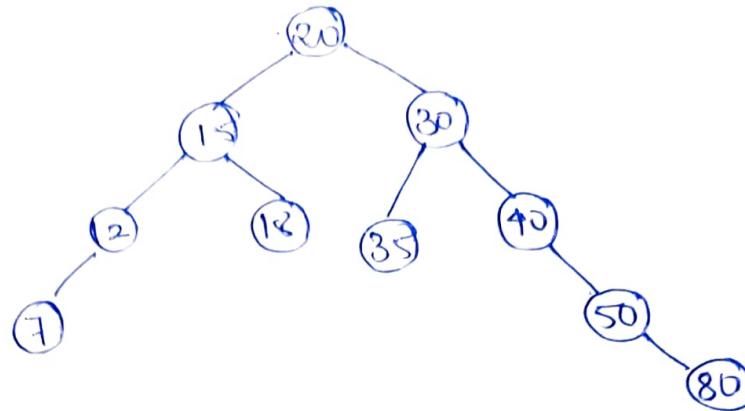
- ① BST is preferred when we've to find closest i.e ceil, floor etc.
- ② Data is organised in such a way so that it reduces more than half.
- ③ Data is organised on the binary search algo of arrays.
- ④ keys on the left are smaller than and keys on the right are greater.



in C++, implemented as map, set, multimap & multiset

Create a BST

input 20, 15, 30, 10, 50, 12, 18, 35, 80, 7



Search in a BST

```
bool searchBST ( Node* root , int val ) {  
    if (!root) return false;  
    if (root->data == val) return true;  
    if (root->data > val) {  
        return searchBST ( root->left , val );  
    }  
    return searchBST ( root->right , val );  
}
```

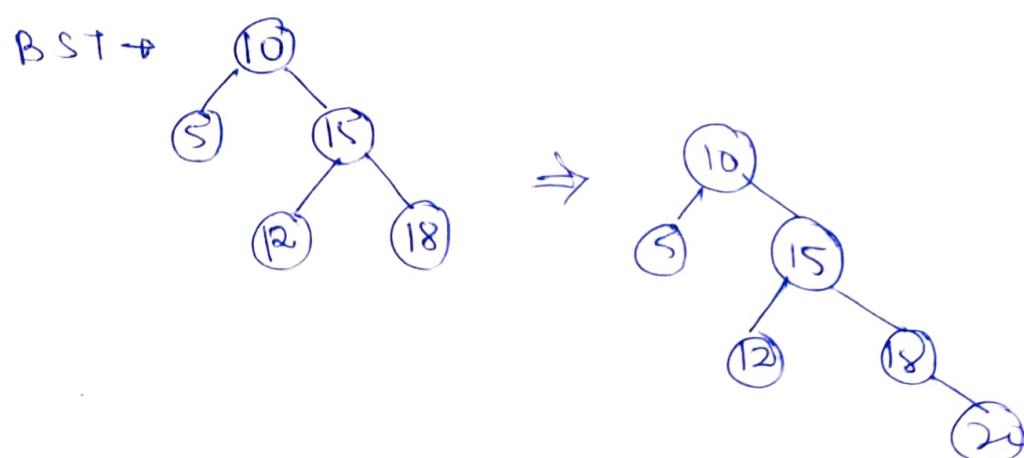
worst case time - $O(n)$

Aux space - $O(n)$.

best case comp $\rightarrow O(\log n)$.

Insert in BST

I/P $\rightarrow x = 20$



```

1. Node *insert (Node *root, int x) {
    if (root == NULL)
        return new Node (x);

```

```

    if (root->data > x)
        root->left = insert (root->left, x);
    else if (root->data < x)
        root->right = insert (root->right, x);
    return root;
}

```

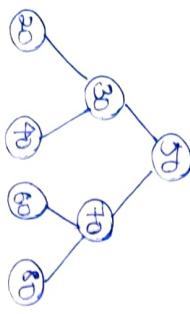
```

Iteration → first search the node, keep track
of parent
and gain the node to the parent.

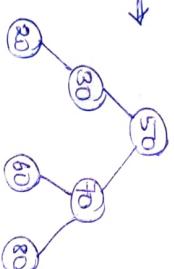
```

Deletion of Node in BST

[I/P + 40]



O/P →



There can be three conditions

- ① both child are null & deleted node is the leaf node. → delete node and return NULL
- ② left child is null → delete node & return right child
- ③ both child are not null

→ get the successor of the root

→ left most child of the right subtree

root->key = succ->key

and we will delete the succ->key.

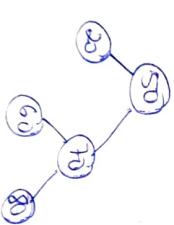
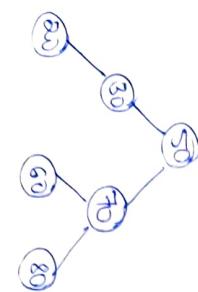
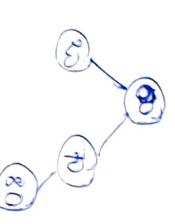
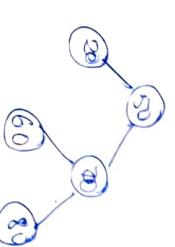
Algorithm

- ① getting the successor node:-

```

Node *getSuccessor (Node *curr) {
    curr = curr->right;
    while (curr != NULL && curr->left)
        curr = curr->left;
}

```



I/P → x = 30

O/P →



if true root is deleted
it can be replaced by
minimum value

If the root node is to be deleted, it can be
will be replaced by closest lower value or closest
higher value.

- * You can make a rule that either you'll pick
least greater value or of closest smaller value.
- * As the inorder traversal of BST is always
sorted,
if we pick the smallest closest
→ in-order predecessor
→ greatest closest → in-order successor.

Node * delNode (Node * root, int x) {

if (!root) return root;

if (root->key > x)

root->left = delNode (root->left, x);

else if (root->key < x)

root->right = delNode (root->right, x);

else {

if (root->left == NULL) {

Node * temp = root->right;

delete root;

return temp;

else if (root->right == NULL) {

Node * temp = root->left;

delete root;

return temp;

else {

Node * succ = getSuccessor (root);

if root->key = succ->key

root->right = delNode (root->right, succ->key);

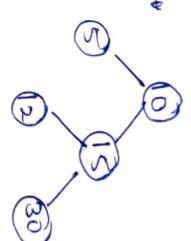
return root;

}

time complexity = O(n)

• closest smaller value.

$x = 4 \rightarrow \text{Node} - 12$



$x = 4$ Same tree

$x = 30$

$x = 30$

Naive solution

Traverses the whole tree and maintains closest smaller

Efficient solution

• use the concept of binary search.

Node * find (Node * root, int x) {

Node * res = NULL;

while (!root) {

if (root->key == x) {

return root;

if (root->key > x)

root = root->left;

else {

res = root;

root = root->right;

}

Floor of BST

code of BST

→ greater than or equal to given key (constant).

Node *getceil (Node *root, int x) {

if (!root) return NULL; res = NULL;

while (root) {

if (root->key == x)

return root;

if (root->key < x) {

root = root->right;

res = getceil (root->right);

}

else {

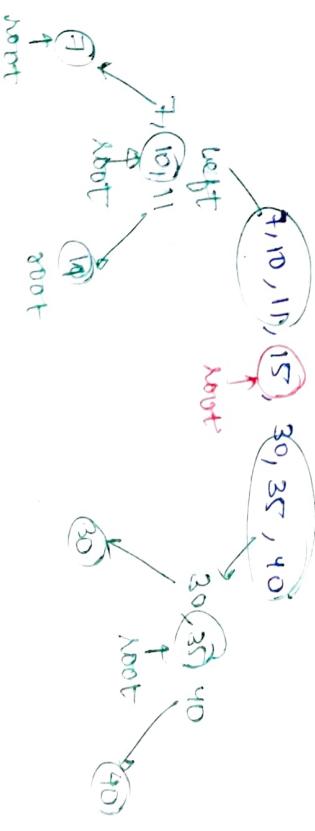
res = root;

root = root->left;

}

return res;

- ① height and structure is decided by height.
- ② if the keys are known prior, we can select one key and select the middle key as root, and recursively do the same for left and right subtrees.

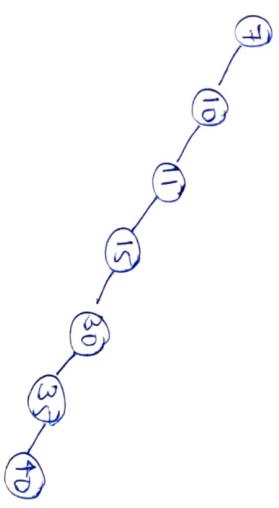


→ height = (log n)

keep one height as O(log n).

Example

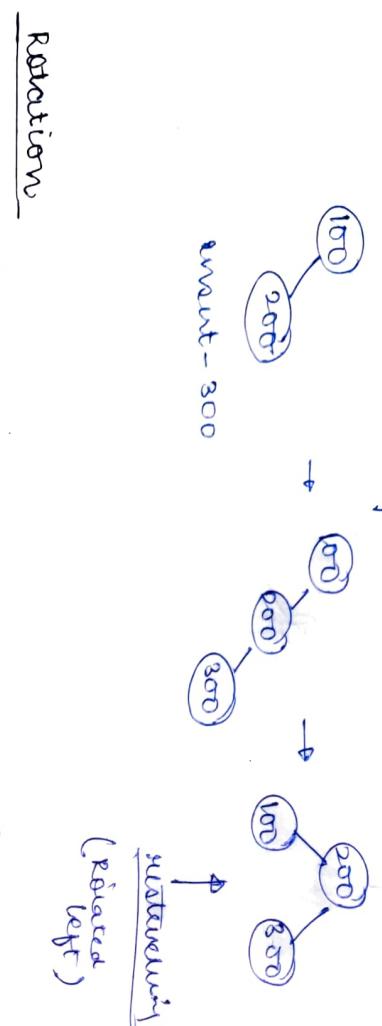
order - 1 - 7, 10, 11, 15, 30, 35, 40.



Self Balancing BST

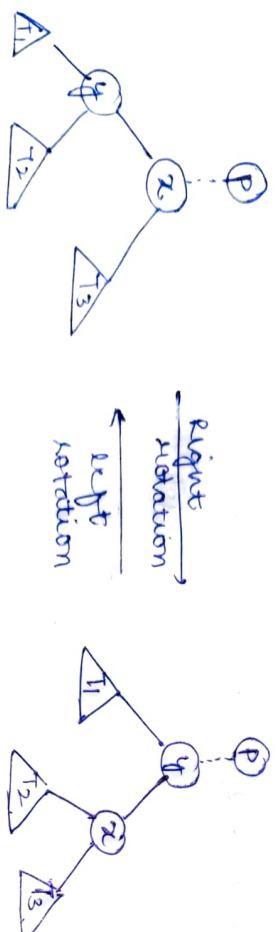
→ Now to maintain height when node is inserted
in random way.

→ Do some restructuring



(Rotated left)

sustaining



soft balancing

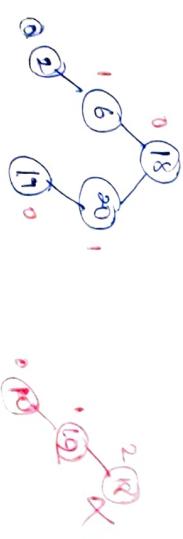
→ AVL tree (very strict in terms of height).
→ Red black tree

AVL Tree

- ① For every node the difference of left subtree and right subtree should not exceed 1

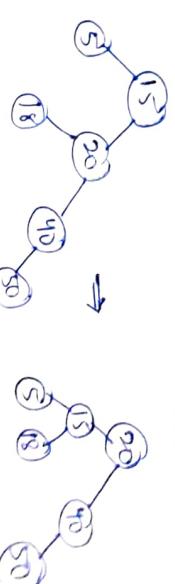
Balance factor = $|l - r|$

$$BF \leq 1$$



right-left case

→ make it right-right



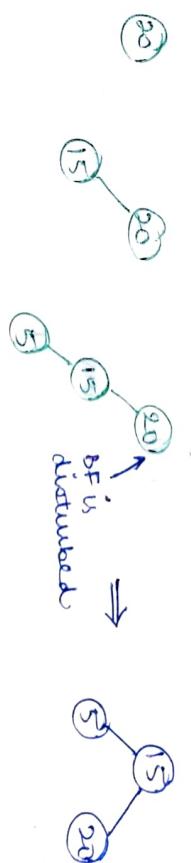
Insert operation

→ Perform normal BST insert.
→ Traverse all ancestors of newly inserted node from node to root

→ If find an unbalanced node, check for any of the below cases

- ① left left) single rotation
- ② right right) single rotation
- ③ left right) double rotation
- ④ right left) double rotation

Order - 20, 15, 5, 40, 50, 18



I(40)

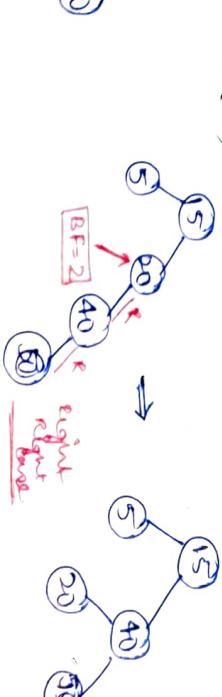
I(50)

I(20)

I(15)

I(5)

I(18)



time complexity

insertion - $\Theta(\log n) * \Theta(\log n)$

deletion

traversing to
top to find
unbalanced
ancestor

- 1) To maintain sorted streams of data.
- 2) To implement doubly ended priority queue.
- 3) To solve problems like:
 - (a) count smaller / greater in a stream
 - (b) floor / ceil / greater / smaller in a stream

$$n < c \log(n+2) + b$$

$$c \approx 1.4405$$

$$b \approx -1.3277$$

Red Black BST

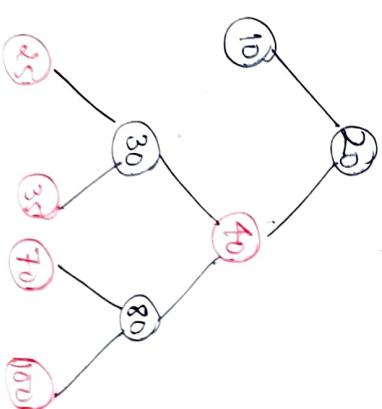
every node is either black or red

root is always black

No two consecutive reds.

Number of black nodes from every node to all of its descendants leaves should be same

- ① slower as compared to AVL tree, and have more complex insert and delete operations
- ② search is more complex.



Set in C++

- \rightarrow maintain data in sorted order (increasing).
- \rightarrow duplicates are not allowed.

insert() \rightarrow insert element in a set and maintain a sorted order

begin() \rightarrow points to first element of set
end() \rightarrow point to one location beyond last element

subbegin() & send() \rightarrow reverse iterators

find() \rightarrow search an element

```
auto it = s.find();
```

$\text{if } (\text{it} == \text{s.end()}) \rightarrow$ not found

else \rightarrow found

clear() \rightarrow remove all elements

count() \rightarrow returns 0 / 1 (as count of any element ≤ 1).

erase() \rightarrow can remove

 - a single element

 - a range of elements.

erase(it1, it2)

\curvearrowleft not including

- try rebalancing
- rotation

lower_bound() →

auto it = s.lower_bound(x)

↳ gives an iterator to

element if present

↳ if not present, give

the element just greater

than x or

if there is no element

greater than x

it returns s.end()

upper_bound() → auto it = s.upper_bound(x)

↳ if x is present, it returns

iterator to next element

↳ if x is not present - it

returns iterator to -next

greater element

↳ greater than greatest insert

→ s.end()

internally

set uses Red Black tree

insert(), find()

count() → lower_bound()

upper_bound(), erase(vae)

erase(it) — maximized, O(1)

Map in C++ (Ordered)

↳ built using red black tree

↳ store a key value pair

↳ element are ordered on the basis of key

insert() → inserts element into the map

m[10] = 20;

↳ we access something which is not present in map.

cout << m[20];

then it will want a value to the map
key = 20
value = default int + 0

(20,0) → instead

at() → m.at(10) = 300 (updating)

m.at(20)

↳ if 20 is not present, it will throw an exception

m.first() → key

m.second() → values.

upper_bound() → same as set

lower_bound() → same as set.

erase — can pass a key
can pass iterator
can pass range it₁ to it₂

Cutting on left side

$$\text{IP} = \text{arr}[] = \{20, 8, 30, 15, 25, 10\}$$

$\text{O/P} \rightarrow$

$$\{1, 1, 1, 30, 30, 25\}$$

new elements
 $\geq 15, 25, 30$

$$\text{IP} = \text{arr}[] = \{30, 20, 10\}$$

$\geq 1, 30, 20$

smaller
more
move

$$\text{IP} = \{10, 20, 30\}$$

$1, 1, 1$

Approach

Empty self balancing binary tree
 insert arr[0] into s
 point (1) \leftarrow for 1st element (fixed).

ceil + lower
bound

for (int i = 1; i < n; i++) {

 if (s contains a ceiling of arr[i])

 point on ceiling

 else

 point \rightarrow 1

 insert arr[i] into s

3.

K th Smallest Element

Design a DS such that insert, delete, search & kth smallest efficiently.

Naive approach

use a BST and traverse inodes and maintain a count. if count == k, return root \rightarrow kth.

Efficient approach

→ change tree structure of BST, augmented BST

class Node {

 int x; int count;

 Node(x);

 x = x;

 count = count;

};

compare (count+1) with k.

- 1) if same, return root.
- 2) if greater, search for left subtree.
- 3) if smaller, search for right subtree with k as (k - 1 - count+1)



How to maintain count

- ① want something in left, increase the count
- ② while searching on right, * to be count remains unchanged.

Node * insert (Node * root, int x) {

 if (root == NULL)

 return new Node(x);

 if (root->key > x) {

 root->left = insert (root->left, x);

 root->key++;

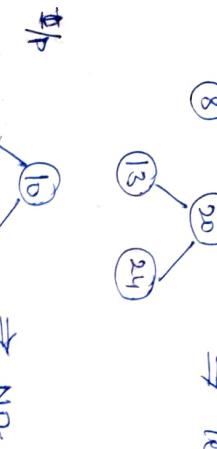
 else if (root->key) root->right =

 insert (root->right, x);

 return root;

check if the binary tree is BST

I/P \rightarrow 8 10 13 20 24



→ NO.

Naive solution (super efficient) :

Do inorder traversal of the tree, if the resulting array is sorted, given tree is BST. (store prev **)

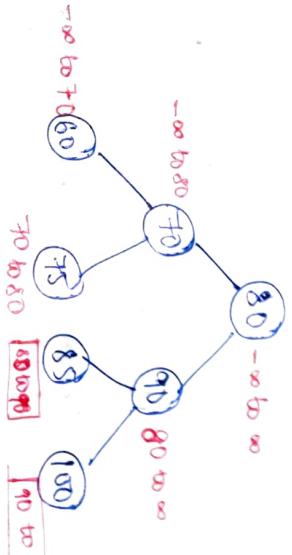
2nd solution

calculate max on left subtree and \Rightarrow left min on right subtree \Rightarrow digit
left < root < right

$O(n^2)$

Efficient solution

\Rightarrow pass range with every node.
 \Rightarrow for root, range is $-\infty$ to $+\infty$.
 \Rightarrow for left child \Rightarrow range upper bound = root \rightarrow key
for right child \Rightarrow range lower bound = root \rightarrow key.



isBST (root, INT-MIN, INT-MAX);

Implementation of 1st solution

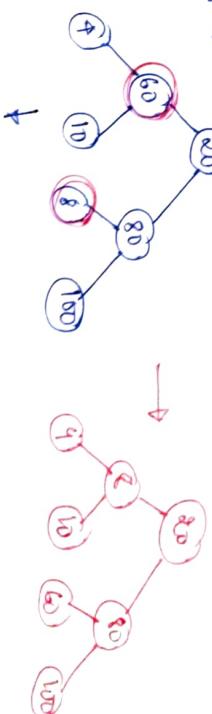
```

int isBST (Node *root, int min, int max);
bool isBST (Node *root) {
    int prev = INT_MIN;
    if (root == NULL) return true;
    if (!isBST (root->left)) return false;
    if (root->key <= prev) return false;
    prev = root->key;
    return isBST (root->right);
}
  
```

return isBST (root->right);

Fix a BST with two nodes swapped

I/P \rightarrow 4 10 15 20 80



+ 60 10 20 80 100

\Rightarrow BST (root->left, min, root->key)
 \Rightarrow BST (root->right, root->key, max)

```

bool isBST (Node *root, int min, int max);
if (root == NULL) return true;
root->key > min &&
root->key < max &&
isBST (root->left, min, root->key)
&& isBST (root->right, root->key, max)
  
```

case-1

4	60	10	20	8	80	100
↑				↑		

not adjacent

case-2

4	8	10	60	20	80	100
↑	↑					

adjacent

- we will update first only one-time

prev = INT_MIN, first = NIL, second = NIL

for (int i=0; i<n; i++) {

 if (arr[i] < prev) {

 if (first == NIL)

 first = prev;

 second = arr[i];

 }

 prev = arr[i];

multiple
time
updates.

Node * prev = NULL, * first = NULL, * sec = NULL;

void fixBST (Node *root) {

 if (root == NULL) return;

 fixBST(root → left);

 if (prev != NULL && root → key < prev → key):

 if (first == NULL)

 first = prev;

 second = root;

 }

 prev = root;

 fixBST(root → right);

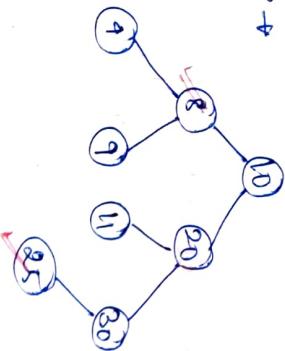
y.

Pairs with given sum in BST

S/P+

$$\text{sum} = 33$$

Yes



New solution

- do an inorder traversal of the tree
- get the sorted array
- use two pointer approach to get sum

$$\begin{aligned} \text{time} &\rightarrow O(n) + O(n) \\ &= O(n) \\ \text{Aux} &\rightarrow O(1) \end{aligned}$$

Efficient solution

- while traversing through the tree, keep adding
 - the nodes in the map table. If $(\text{sum} - \text{root} - \text{key})$ is present in map table return true.

```

bool isPairSum (Node *root, int sum,
               unordered_set<int> &s) {
    if (root == NULL) return false;
    if (pairSum (root->left, sum, s) == true)
        return true;
    if (s.find (sum - root->key)) = s.end())
        return true;
    else s.insert (root->key);
    return false;
}

```

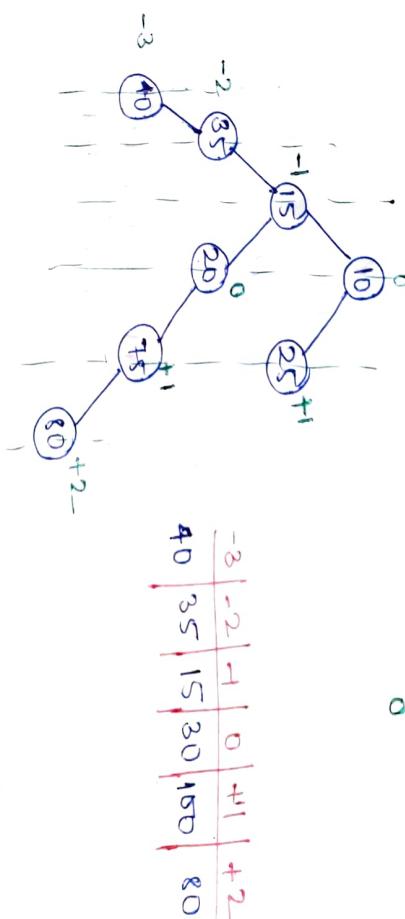
Vertical sum in a Binary Tree

↳ sum of elements having same horizontal distance

right child = +1
left child = -1



S/P



solution use a map (ordered)

```
void vsum (Node *root, int nd, map<int, int > &m)
```

```

if (root == NULL) return;
vsum (root->left, nd-1, m);
m[nd] += root->key;
vsum (root->right, nd+1, m);

```

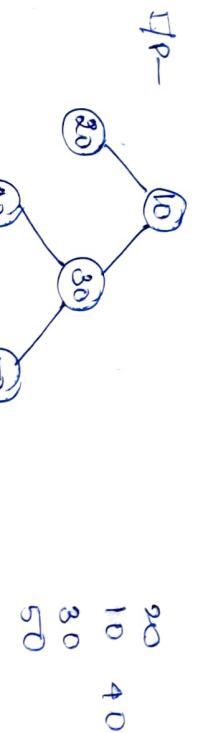
log nd + nd

```

void vsum (Node *root) {
    map<int, int > m;
    vsum (root, 0, m);
    for (auto sum : m)
        cout << sum << endl;
}

```

Vertical Traversal



- we'll use level order traversal, because we have to maintain order also.
- we'll use two data structures

① queue

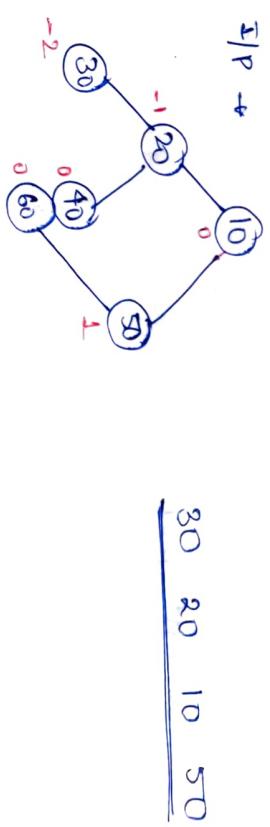
- └ it will contain
 - ─ address of node
 - ─ horizontal distance

② map<int, vector<int>>

```

void VTraversal ( Node *root) {
    map<int, vector<int>> mp;
    queue < pair<Node*, int>> q;
    q.push ({root, 0});
    while (q.empty() == false) {
        auto p = q.front();
        Node *curr = p.first;
        int nd = p.second;
        while (q.empty() == false) {
            if (mp.find(nd) == mp.end ()) {
                mp[nd] = curr->data;
            }
            q.pop();
            if (curr->left) q.push ({curr->left, nd+1});
            if (curr->right) q.push ({curr->right, nd+3});
        }
    }
}
  
```

using the same concept, this time we'll update the mp[nd] every time, to get the bottom most node

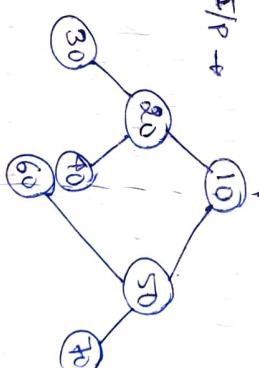


Bottom view of Binary Tree

Bottom view of Binary Tree

I/P →

30 20 60 50 40



I/P →

30 20 60 50 40

→ we'll use the same approach, but this time we'll not push elements instead we'll have only one time insertion for a key and will not update that.

`if (mp.find (nd) == mp.end ())
 mp[nd] = curr->data;`