Kap 02, 03   Posisjon – Hastighet – Akselerasjon

	Derivasjon	Integrasjon
r		
v	↓	↑
a		

Hastighet $\vec{v} = \dfrac{d\vec{r}}{dt}$

Akselerasjon $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2}$

Hastighet $\vec{v} = \vec{v}_0 + \int_0^t \vec{a} \cdot dt$

$\vec{v} = \vec{v}_0 + \vec{a} \cdot t$ Konstant akselerasjon

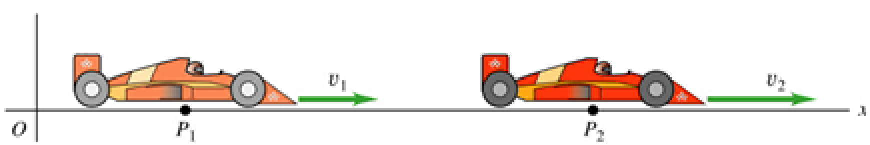
Forflytning $\vec{r} = \vec{r}_0 + \int_0^t \vec{v} \cdot dt$

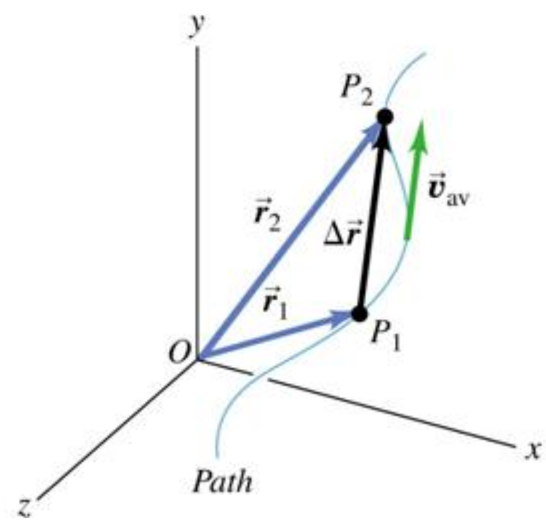
$\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \dfrac{1}{2} \cdot \vec{a} \cdot t^2$ Konstant akselerasjon

Rettlinjet bevegelse og konstant akselerasjon

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0)$$

$$x = x_0 + \frac{v_{0x} + v_x}{2} t$$

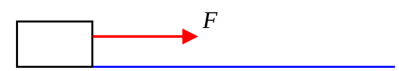


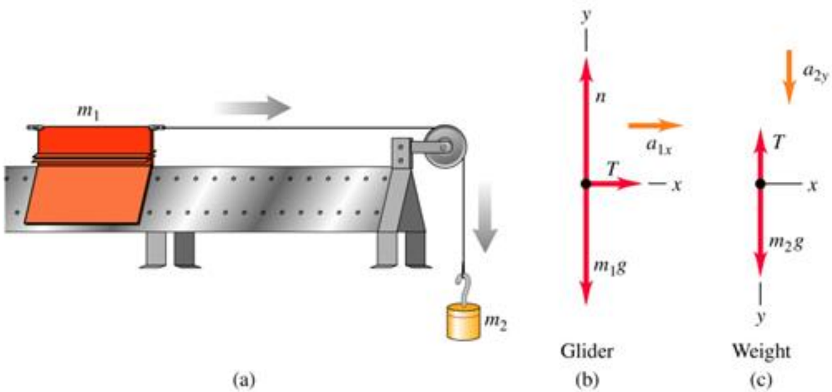


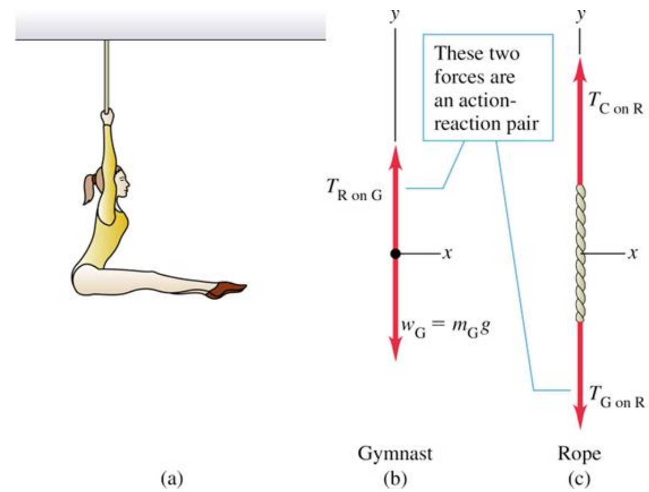
Kap 04, 05   Newtons lover

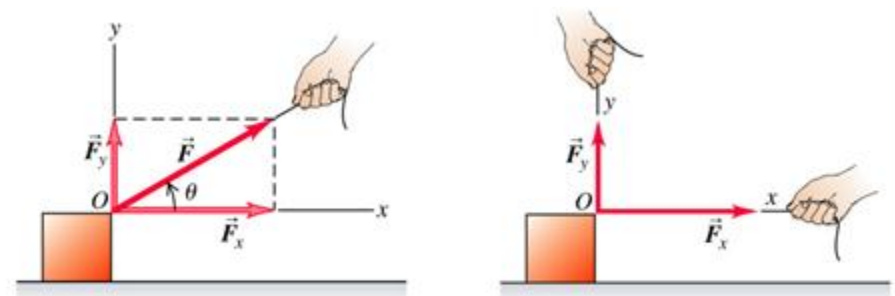
Newtons lover

1. $\boxed{\vec{F} = \vec{0}}$ $\Rightarrow$ $D\vec{v} = \vec{0}$
2. $\boxed{\vec{F} = m\vec{a}}$
3. $\boxed{\vec{F} = -\vec{F}'}$







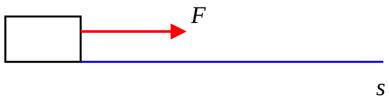


Component vectors: $\vec{F}_x$ and $\vec{F}_y$
Components: $F_x = F\cos\theta$ and $F_y = F\sin\theta$
(a)

Component vectors $\vec{F}_x$ and $\vec{F}_y$ together have the same effect as original force $\vec{F}$.
(b)

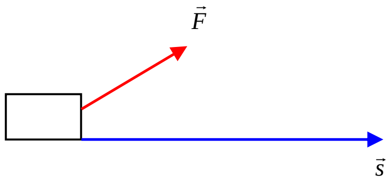
Kap 06   Arbeid – Kinetisk Energi

Def av arbeid W
ved å flytte et objekt med en konstant kraft F
en rettlinjet strekning s
når F og s peker samme vei



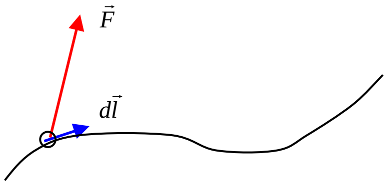
$$W = F \cdot s$$

Def av arbeid W
ved å flytte et objekt med en konstant kraft F
en rettlinjet strekning s



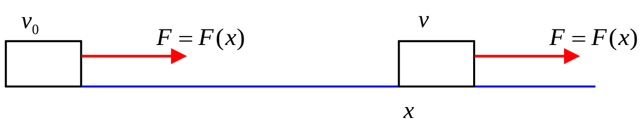
$$W = \vec{F} \cdot \vec{s}$$

Def av arbeid W
ved å flytte et objekt med en varierende kraft F
fra punktet P_1 til punktet P_2 langs en kurve l



$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$

Endring av kinetisk energi ΔK =
Det arbeidet som må utføres på et objekt med masse m
for å endre objektets hastighet fra v_1 til v_2



$$\Delta K = W = \int_{P_1}^{P_2} F dx$$

$$= \int_{P_1}^{P_2} m \cdot a \cdot dx = \int_{P_1}^{P_2} m \cdot \frac{dv}{dt} \cdot dx = \int_{P_1}^{P_2} m \cdot dv \cdot \frac{dx}{dt} = \int_{P_1}^{P_2} m \cdot dv \cdot v$$

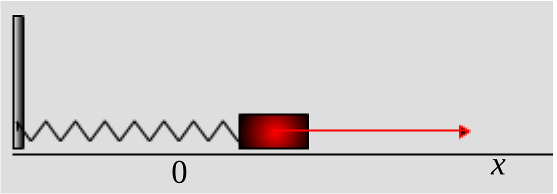
$$= \int_{P_1}^{P_2} m \cdot v \cdot dv = \int_{v_1}^{v_2} m \cdot v \cdot dv = m \int_{v_1}^{v_2} v \cdot dv = m \left[\frac{1}{2} v^2 \right]_{v_1}^{v_2}$$

$$= \frac{1}{2} m v_2^{\ 2} - \frac{1}{2} m v_1^{\ 2}$$

Arbeid – Energi teorem

$$\boxed{\Delta K = W = \frac{1}{2} m v_2^{\ 2} - \frac{1}{2} m v_1^{\ 2}}$$

Kap 06   Arbeid – Kinetisk Energi



Arbeid W ved strekk av en elastisk fjær

$$W = \int_0^x F dx = \int_0^x kx dx = \frac{1}{2} kx^2$$

Gjennomsnitts-effekt P_a
Energi (arbeid) pr tidsenhet
Måle-enhet Watt   W = J/s

$$P_a = \frac{\Delta W}{\Delta t}$$

Momentan-effekt P
Energi (arbeid) pr tidsenhet
Måle-enhet Watt   W = J/s

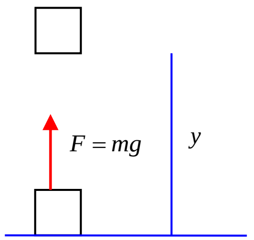
$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Kap 07   Potensiell energi og energi-bevaring

Arbeid utført i tyngdefeltet
ved å løfte en strekning y
med en konstant kraft F = mg (gir ingen fartsendring)

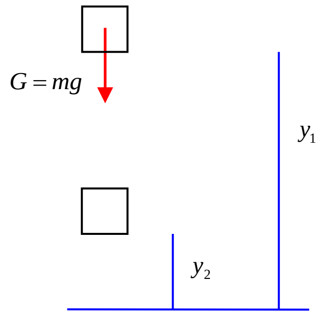
Dette arbeidet er uavhengig av veien

$$\boxed{W = F \cdot h = mgy}$$



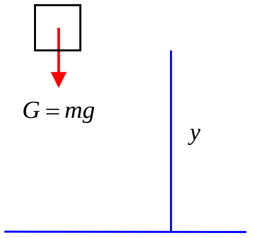
Arbeid som tyngden utfører
ved et fall fra y_1 til y_2

$$W_{grav} = F \cdot s = mgy_1 - mgy_2$$



Definisjon av gravitasjons-potensiell energi U_{grav}

$$U_{grav} = mgy$$



Kap 07   Potensiell energi og energi-bevaring

Sammenheng mellom arbeid W_{grav} utført av tyngden og endring i gravitasjons-potensiell energi U_{grav}

$$W_{grav} = F \cdot s = mgy_1 - mgy_2 = U_{grav,1} - U_{grav,2} = -(U_{grav,2} - U_{grav,1}) = -\Delta U_{grav}$$

Bevaring av mekanisk energi (kinetisk energi + potensiell energi) i tyngdefeltet

$$\left.\begin{aligned} W_{tot} &= W_{grav} \\ W_{tot} &= \Delta K \\ W_{grav} &= -\Delta U_{grav} \end{aligned}\right\} \Rightarrow \Delta K = -\Delta U_{grav} \Rightarrow \Delta K + \Delta U_{grav} = 0$$

$$\Downarrow$$

$$K_1 + U_{grav,1} = K_2 + U_{grav,2}$$

$$\boxed{\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2}$$

Bevaring av mekanisk energi (kinetisk energi + potensiell energi) for en elastisk fjær

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Hvis i tillegg rotasjon,
må tilføyes rotasjons-energi (se kap 10)

Ikke bevaring av mekanisk energi (kinetisk energi + potensiell energi)

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

Total energi-bevaring

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$

Kraft og potensiell energi

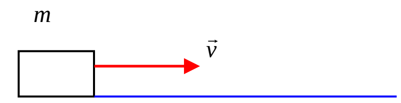
$$W = -dU$$

$$Fdx = -dU$$

$$F = -\frac{dU}{dx}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right]$$

Kap 08 Moment (bevegelsesmengde) – Impuls



Moment (bevegelsesmengde)

$$\vec{p} = m\vec{v} \qquad \vec{P} = \sum_i m\vec{v}_i$$

Newtons 2.lov

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\sum \vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

Hvis massen m endrer seg

Impuls

$$\vec{J} = \sum \vec{F} \cdot \Delta t = \frac{\Delta\vec{p}}{\Delta t}\Delta t = \Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

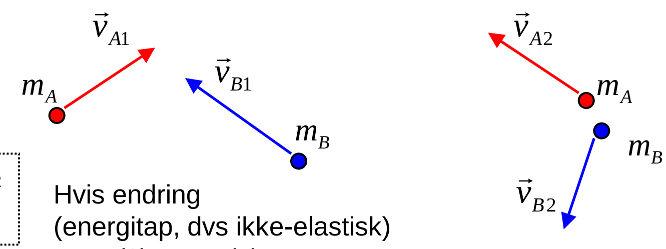
Bevaring av moment

$$\left.\begin{array}{l} \sum \vec{F} = \dfrac{d\vec{p}}{dt} \\ \sum \vec{F} = \vec{0} \end{array}\right\} \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{konstant}$$

Kollisjoner (bevaring av moment)
Ingen ytre krefter virker på system
bestående av m_1 og m_2

Totalt moment før kollisjon = Totalt moment etter kollisjon

$$\boxed{m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = m_A\vec{v}_{A2} + m_B\vec{v}_{B2}}$$



Elastisk kollisjon:
Da gjelder i tillegg bevaring av mekanisk energi (her kinetisk energi)

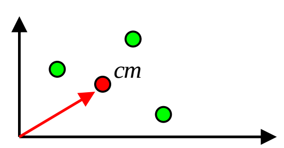
$$\frac{1}{2}m_A v_{A1}^{\ 2} + \frac{1}{2}m_B v_{B1}^{\ 2} = \frac{1}{2}m_A v_{A2}^{\ 2} + \frac{1}{2}m_B v_{B2}^{\ 2}$$

Hvis endring
(energitap, dvs ikke-elastisk)
Etter (2) – Før (1)

Fullstendig uelastisk:
Henger sammen etter kollisjonen

Masse-senter

$$\boxed{\vec{r}_{cm} = \frac{1}{M}\sum_i m_i\vec{r}_i} \qquad M = \sum_i m_i \qquad \boxed{\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm}$$



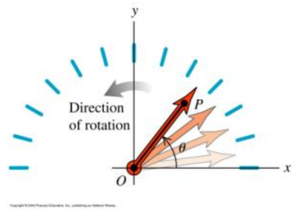
Newtons 2.lov for et utstrakt legeme

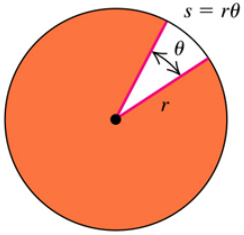
$$\boxed{\sum \vec{F} = m\vec{a}_{cm}} \qquad M\vec{v}_{cm} = \sum_i m_i\vec{v}_i = \vec{P}$$

Kap 09 Rotasjon

Vinkel

$$\theta = \frac{s}{r}$$





Vinkelhastighet

$$\omega = \frac{d\theta}{dt}$$

Vinkelakselerasjon

$$\alpha = \frac{d\omega}{dt}$$

Benevning

θ	$\leftrightarrow$	ubenevnt eller rad	
v	$\leftrightarrow$	m/s	
ω	$\leftrightarrow$	s^{-1}	eller rad/s
a	$\leftrightarrow$	s^{-2}	eller rad/s^2

Bevegelses-ligninger

$$\omega(t) = \omega_0 + \int_0^t \alpha(t)dt$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t)dt$$

Konstant vinkelaks.

$$\omega = \omega_0 + \alpha \cdot t$$

$$\theta = \theta_0 + \omega_0 \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2$$

$$\omega^2 = \omega_0^{\ 2} + 2 \cdot \alpha \cdot (\theta - \theta_0)$$

Analogi mellom hastighet / akselerasjon og vinkelhastighet / vinkelakselerasjon

$$v = \frac{ds}{dt} \qquad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} \qquad \alpha = \frac{d\omega}{dt}$$

$s \leftrightarrow \theta$

$v \leftrightarrow \omega$

$a \leftrightarrow \alpha$

$$v(t) = v_0 + \int_0^t a(t)dt \qquad \omega(t) = \omega_0 + \int_0^t \alpha(t)dt$$

$$s(t) = s_0 + \int_0^t v(t)dt \qquad \theta(t) = \theta_0 + \int_0^t \omega(t)dt$$

$\longleftarrow$ Generelt

$$v(t) = v_0 + at \qquad \omega(t) = \omega_0 + \alpha t$$

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2 \qquad \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

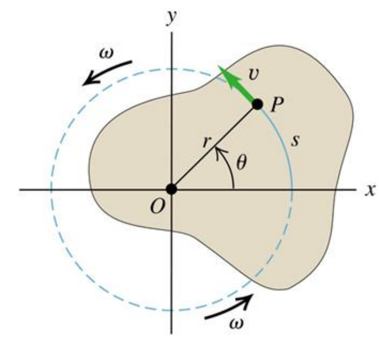
$$v^2 = v_0^{\ 2} + 2a(s - s_0) \qquad \omega^2 = \omega_0^{\ 2} + 2\alpha(\theta - \theta_0)$$

$\longleftarrow$ a konstant $\quad$ α konstant

Kap 09 Rotasjon

Vinkelhastighet $\omega = \dot{\theta} = \dfrac{v}{r}$

Hastighet $\boxed{v = r\omega}$



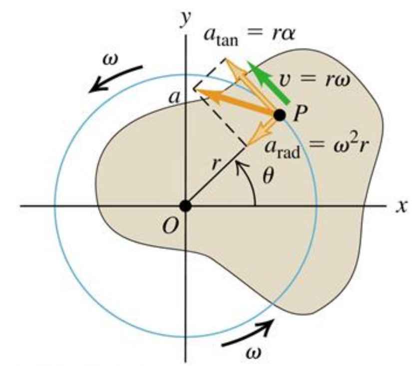
Vinkelakselerasjon $\alpha = \dot{\omega} = \ddot{\theta} = \dfrac{a}{r}$

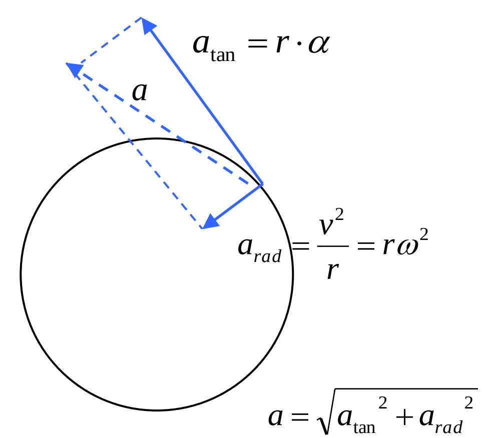
Akselerasjon $a_{\tan} = \dot{v} = \dfrac{dv}{dt} = r \cdot \dfrac{d\omega}{dt} = r \cdot \alpha$

$a_{rad} = \dfrac{v^2}{r} = r\omega^2$

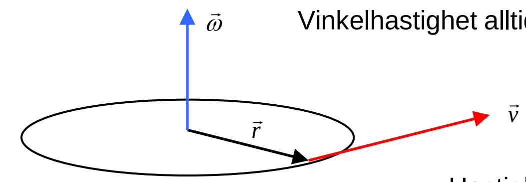
Akselerasjon

$$\boxed{\begin{gathered} a_{\tan} = r \cdot \alpha \\ a_{rad} = \frac{v^2}{r} = r\omega^2 \end{gathered}}$$





Vinkelhastighet og vinkelakselerasjon som vektor



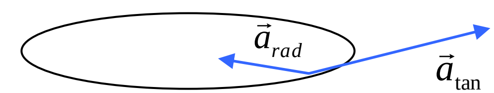
Vinkelhastighet alltid normal på rotasjonsplanet

Hastighet alltid tangentiell til banen

Hastighet $\qquad \vec{v} = \vec{\omega} \times \vec{r}$

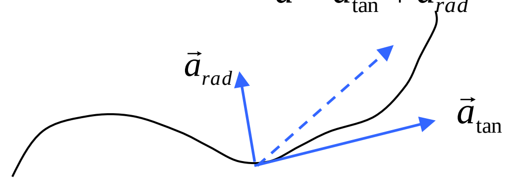
Akselerasjon

$$
\begin{aligned}
\vec{a} &= \dot{\vec{v}} \\
&= \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}} \\
&= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}} \\
&= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \\
&= \underbrace{\vec{\alpha} \times \vec{r}}_{\text{tangensiell akselerasjon}} + \underbrace{\vec{\omega} \times \vec{\omega} \times \vec{r}}_{\text{radielll akselerasjon}} \\
&= \vec{a}_{\tan} + \vec{a}_{rad}
\end{aligned}
$$

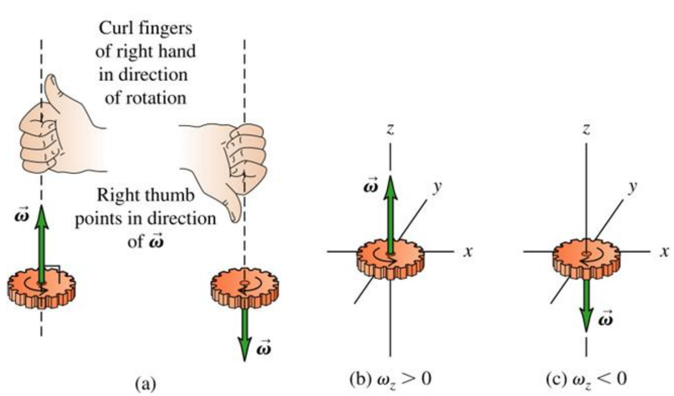


Tangentiell-akselerasjon alltid tangentiell til banen.
Radiell-akselerasjon alltid rettet inn mot sentrum.

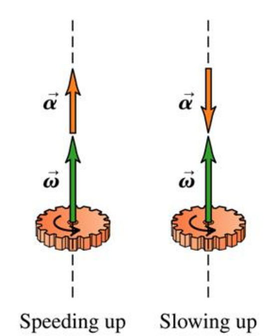
$$\vec{a} = \vec{a}_{\tan} + \vec{a}_{rad}$$



$$a = \sqrt{a_{\tan}^{\ 2} + a_{rad}^{\ 2}}$$

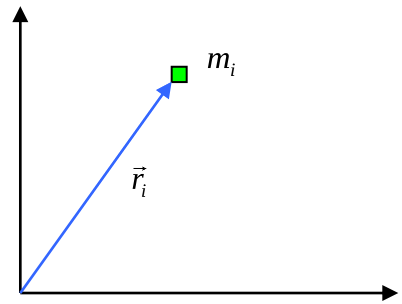


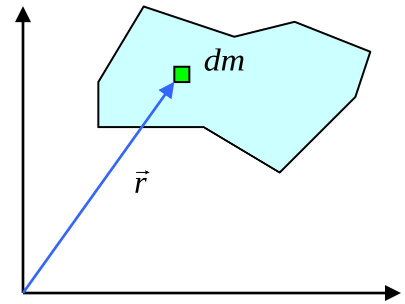
(a) (b) $\omega_z > 0$ (c) $\omega_z < 0$



Speeding up Slowing up

Kap 09 Rotasjon

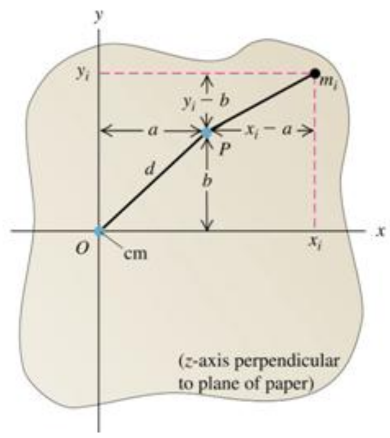




Treghetsmoment $\boxed{I = \sum_i m_i r_i^2}$ $\boxed{I = \int r^2 dm}$

Kinetisk rotasjonsenergi $\boxed{K = \frac{1}{2} I\omega^2}$

Parallellakse - teorem $\boxed{I_P = I_{cm} + Md^2}$



Benevning

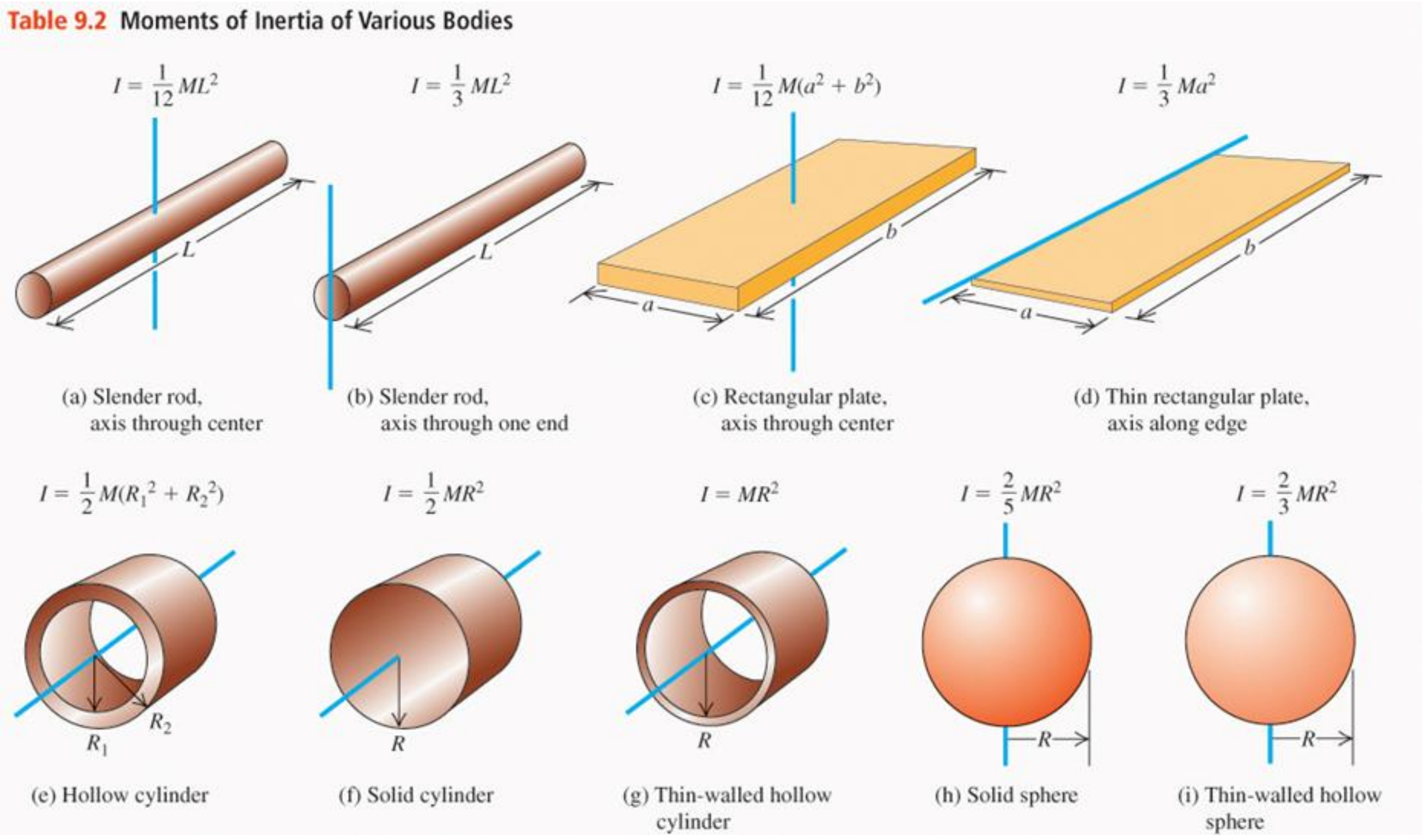
$I \leftrightarrow kgm^2$

$K \leftrightarrow J$

Normalakse - teorem $I_O = I_x + I_y$

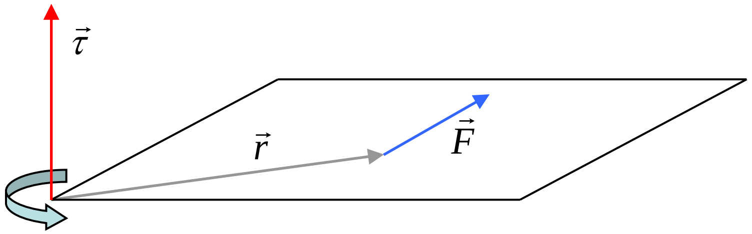
Treghets-moment for noen spesielle legemer med akse gjennom sentrum

Stav med masse M og lengde L	$I = \frac{1}{12} ML^2$
Rektangulær plate med masse M og sider a og b	$I = \frac{1}{12} M(a^2 + b^2)$
Hul sylinder med masse M, indre radius R_1, ytre radius R_2	$I = \frac{1}{2} M(R_1^2 + R_2^2)$
Massiv sylinder med masse M og radius R	$I = \frac{1}{2} MR^2$
Massiv kule med masse M og radius R	$I = \frac{2}{5} MR^2$
Tynnvegget kule med masse M og radius R	$I = \frac{2}{3} MR^2$



Kap 10 Rotasjonsdynamikk

Kraftmoment / Angulært moment



Kraftmoment $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau_{cm} = I_{cm}\alpha$

$\tau_o = I_o\alpha$ når $\begin{cases} 1. & \vec{a}_o = \vec{0} \\ 2. & \vec{a}_o \,||\, \vec{r}_{cm} \\ 3. & \vec{r}_{cm} = \vec{0} \end{cases}$

Kinetisk energi $K = K_{trans} + K_{rot} = \frac{1}{2}mv_{cm}{}^2 + \frac{1}{2}I_{cm}\omega^2$

$K = \frac{1}{2}I_o\omega^2$ når $v_o = \vec{0}$

Arbeid / Effekt $W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I\omega d\omega = \frac{1}{2}I\omega_2{}^2 - \frac{1}{2}I\omega_1{}^2$

$P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$

Angulært moment $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

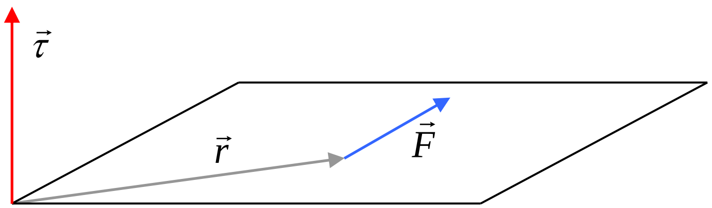
$\vec{L} = I\vec{\omega}$

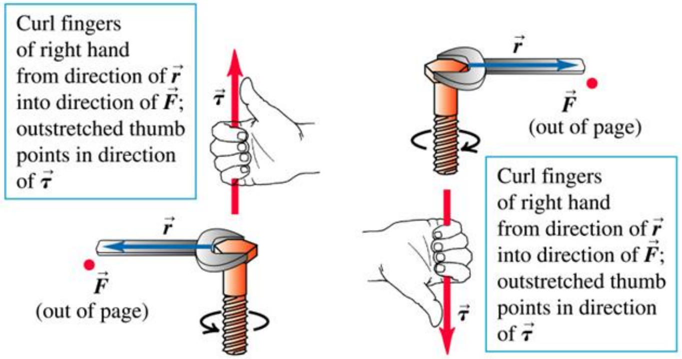
$\dot{\vec{L}}_o = \vec{\tau}_o$ når $\begin{cases} 1. & \vec{v}_o = \vec{0} \\ 2. & \vec{v}_{cm} = \vec{0} \\ 3. & \vec{v}_o \,||\, \vec{v}_{cm} \end{cases}$

Gyroskop $\Omega = \frac{d\varphi}{dt} = \frac{\frac{dL}{L}}{dt} = \frac{\frac{dL}{dt}}{L} = \frac{\tau}{L} = \frac{wR}{I\omega}$

Kraftmoment

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

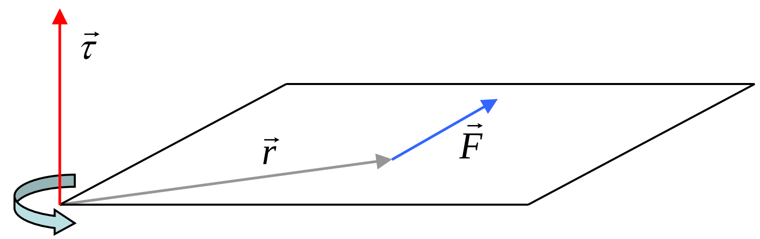


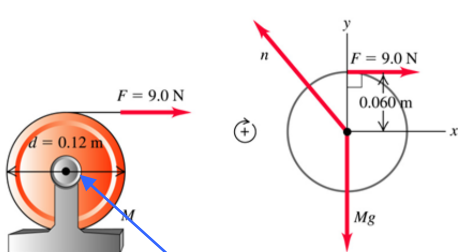
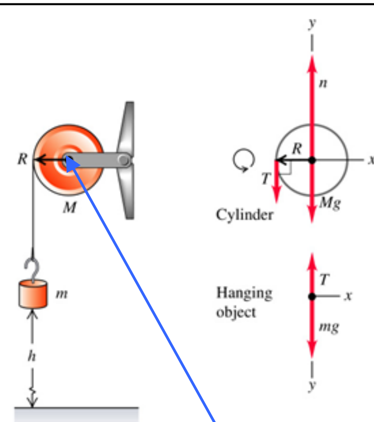
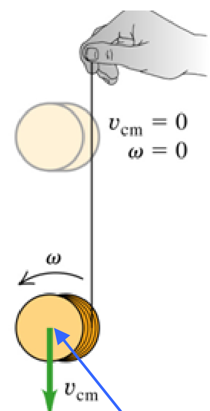
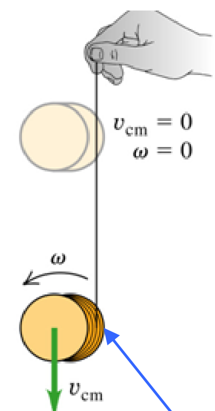


Sammenheng mellom kraftmoment og vinkel-akselerasjon

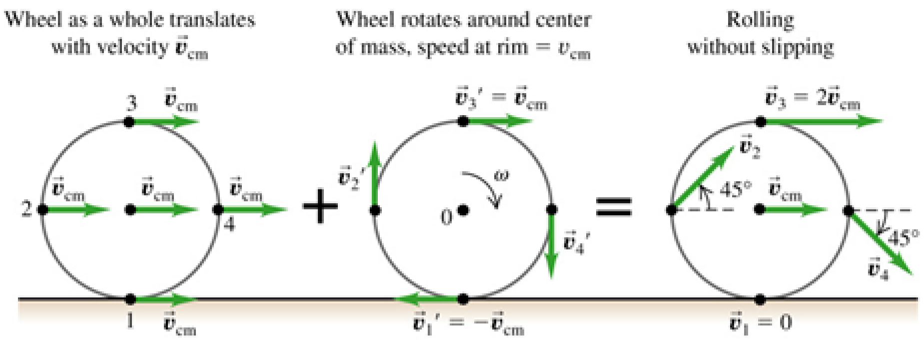
$$\tau_{\text{cm}} = I_{cm}\alpha$$

$$\tau_o = I_o\alpha \qquad \text{når} \quad \begin{cases} 1. \quad \vec{a}_o = \vec{0} \\ 2. \quad \vec{a}_o \parallel \vec{r}_{cm} \\ 3. \quad \vec{r}_{cm} = \vec{0} \end{cases}$$



1		2	3
			

Kap 10 Rotasjonsdynamikk



Kinetisk energi

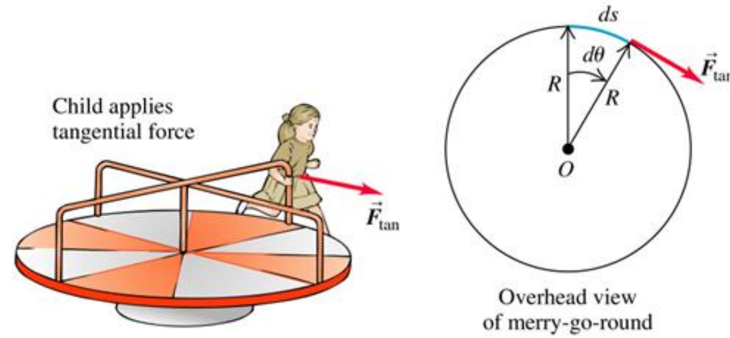
$$K = K_{trans} + K_{rot} = \frac{1}{2} m v_{cm}{}^2 + \frac{1}{2} I_{cm} \omega^2$$

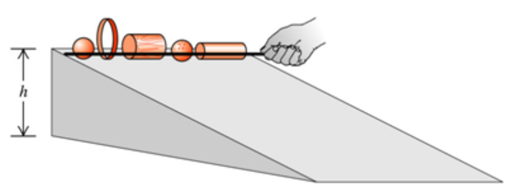
$$K = \frac{1}{2} I_o \omega^2 \qquad \text{når } \mathrm{v_o} = \vec{0}$$

Arbeid / Effekt

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I\omega d\omega = \frac{1}{2} I\omega_2{}^2 - \frac{1}{2} I\omega_1{}^2$$

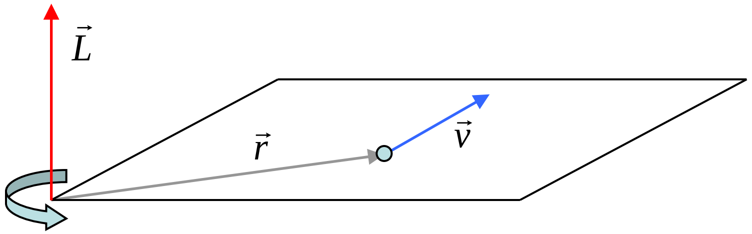
$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$





$$0 + M\cdot g\cdot h = \frac{1}{2}\cdot M\cdot v_{cm}{}^2 + \frac{1}{2}\cdot\left(c\cdot M\cdot R^2\right)\cdot\left(\frac{v_{cm}}{R}\right)^2$$

$$M\cdot g\cdot h = \frac{M\cdot v_{cm}{}^2\cdot(c+1)}{2} \qquad v_{cm} = \sqrt{\frac{2\cdot g\cdot h}{c+1}}$$

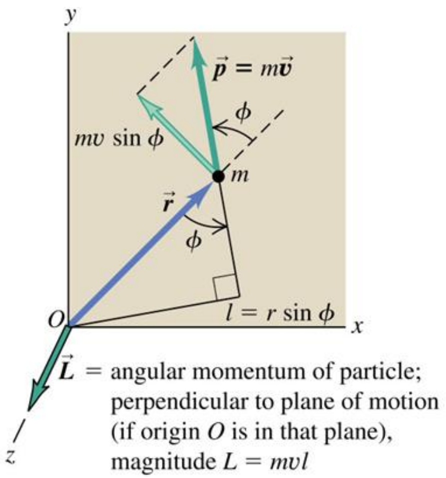


Angulært moment

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$\vec{L} = I\vec{\omega}$$

$$\dot{\vec{L}}_o = \vec{\tau}_o \qquad \text{når} \quad \begin{cases} 1. \quad \vec{v}_o = \vec{0} \\ 2. \quad \vec{v}_{cm} = \vec{0} \\ 3. \quad \vec{v}_o \parallel \vec{v}_{cm} \end{cases}$$

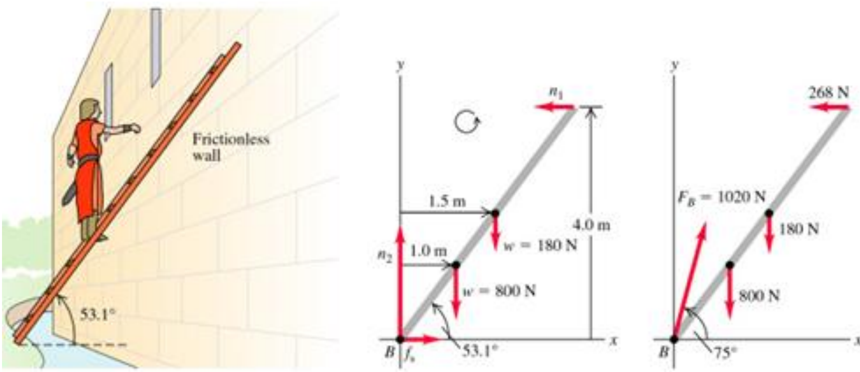


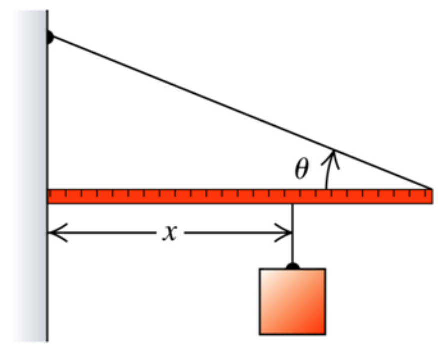
Kap 11   Likevekt og elastisitet

Betingelse for likevekt

$$\sum \vec{F} = \vec{0}$$ Summen av alle ytrekrefter er lik null

$$\sum \vec{\tau} = \vec{0}$$ Summen av alle ytrekraftmomenter om en vilkårlig akse er lik null





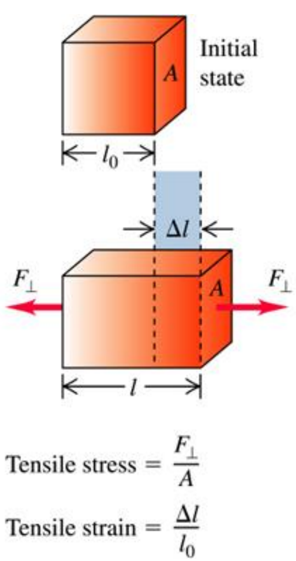
Kap 11 Likevekt og elastisitet

Hookes lov:
Den ytre kraften (Stress) på et system
er proporsjonal med deformasjonen (Strain) av systemet.
Proporsjonalitets-konstanten kalles elastisitetsmodulen.

$$\boxed{\frac{Stress}{Strain} = Elastic \bmod ulus}$$

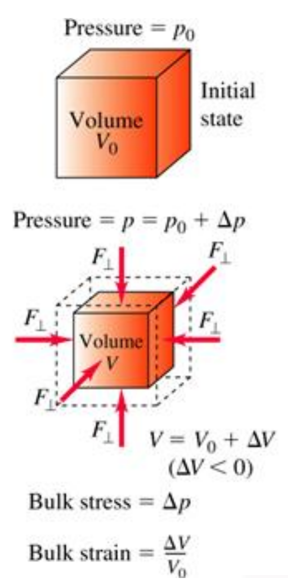
Strekk-stress og strekk-strain:
Elastisitetsmodulen kalles for Youngs modulus.

$$\boxed{Y = \frac{Tensile\ stress}{Tensile\ strain} = \frac{\frac{F_\perp}{A}}{\frac{\Delta l}{l_0}} = \frac{F_\perp}{A}\frac{l_0}{\Delta l}}$$



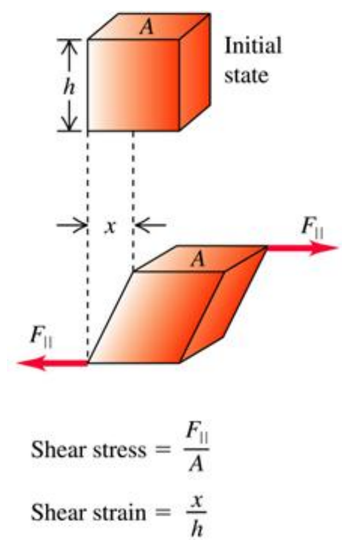
Bulk-stress og bulk-strain:
Elastisitetsmodulen kalles for Bulke modulus.

$$\boxed{B = \frac{Bulk\ stress}{Bulk\ strain} = -\frac{\Delta p}{\frac{\Delta V}{V_0}}}$$



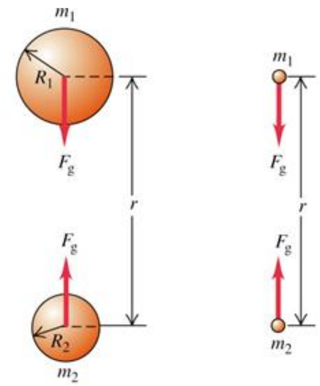
Share-stress og share-strain:
Elastisitetsmodulen kalles for Share modulus.

$$\boxed{S = \frac{Share\ stress}{Share\ strain} = \frac{\frac{F_\perp}{A}}{\frac{x}{h}} = \frac{F_\perp}{A}\frac{h}{x}}$$



Kap 13 Gravitasjon

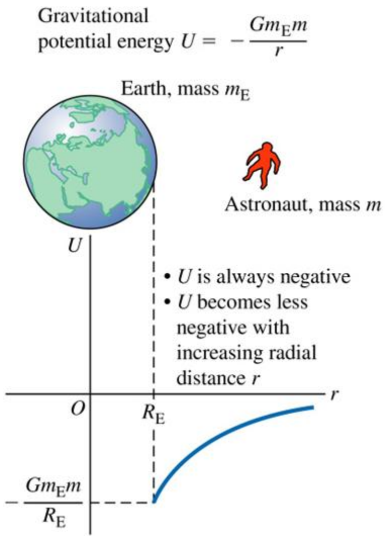
Gravitasjonskraft $\boxed{F_g = G\frac{m_1 m_2}{r^2}}$ $G = 6.67\,10^{-11}\cdot\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}$



Tyngde $w = F_g = G\frac{m_E m}{R_E{}^2}$

Tyngdeakselerasjon $g = \frac{F_g}{m} = G\frac{m_E}{R_E{}^2}$

Potensiellenergi $U = -G\frac{m_E m}{r}$

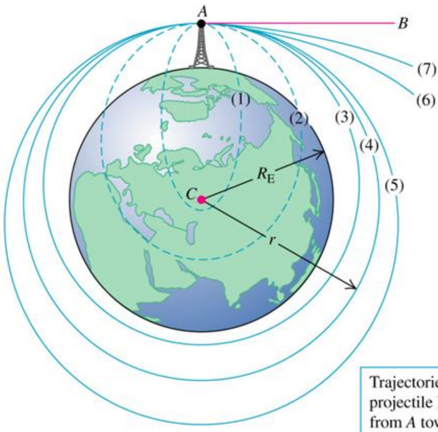


Kap 13   Gravitasjon   -   Satelitt

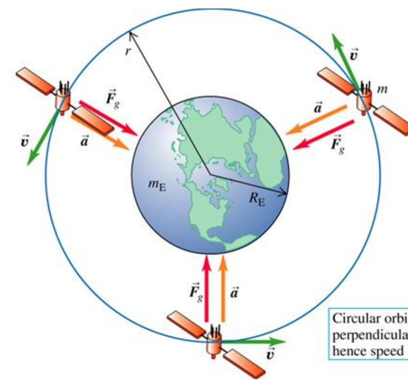
Satelitt – hastighet $$v = \sqrt{G\frac{m_E}{r}}$$

Satelitt – periode $$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{rGm_E}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{Gm_E}}$$

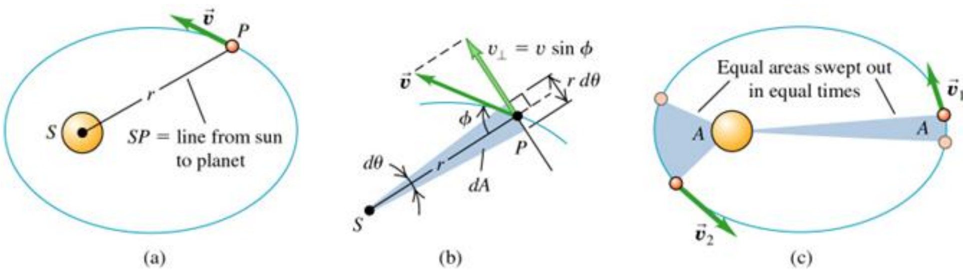
$$T = \frac{2\cdot\pi\cdot a^{\frac{3}{2}}}{\sqrt{G\left(m_p + m_s\right)}}$$



Trajectories (1) through (7): projectile launched from *A* toward *B* with increasing speed

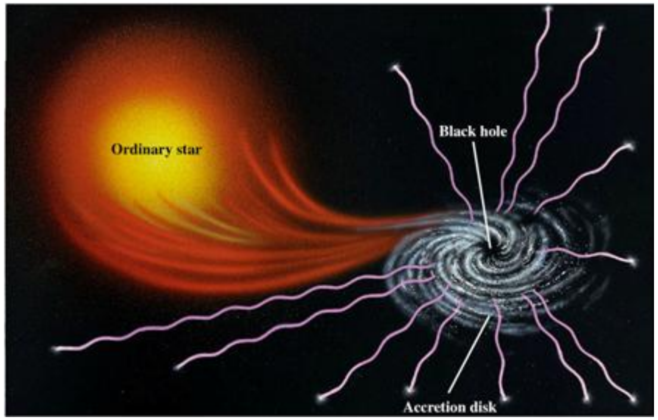


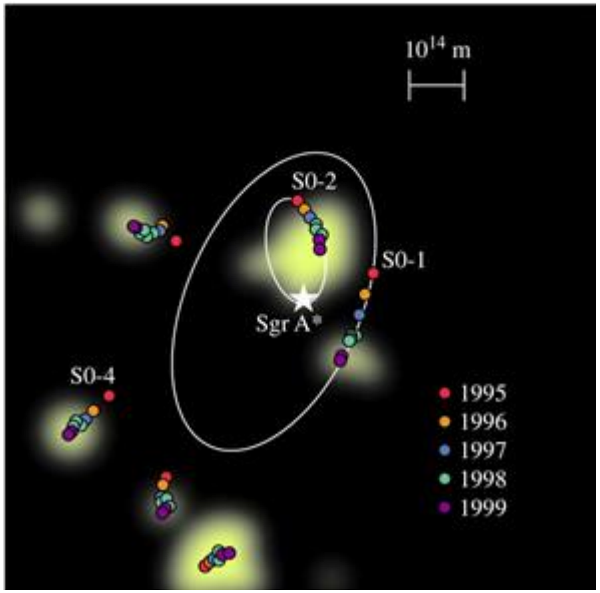
Circular orbit: acceleration $\vec{a}$ perpendicular to velocity $\vec{v}$, hence speed v is constant



Kap 13   Gravitasjon   -   Sort hull

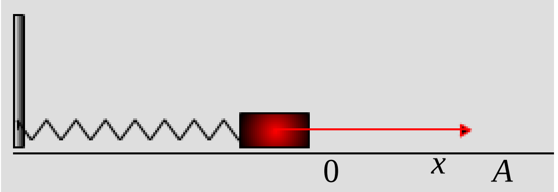
Schwarzschild radius (sort hull) $R_S = \frac{2GM}{c^2}$





Kap 14 Periodisk bevegelse

A	Amplitude	Maxutslag i x – retning		Enhet m
T	Periode (Svingetid)	Tiden for en hel svingning		Enhet s
f	Frekvens	Antall svingninger pr tidsenhet	$f = \frac{1}{T}$	Enhet $Hz = s^{-1}$
ω	Vinkelfrekvens	Rotasjonshastighet	$\omega = 2\pi f$	Enhet s^{-1} (eller rad/s)



SHM $\boxed{F = -kx}$

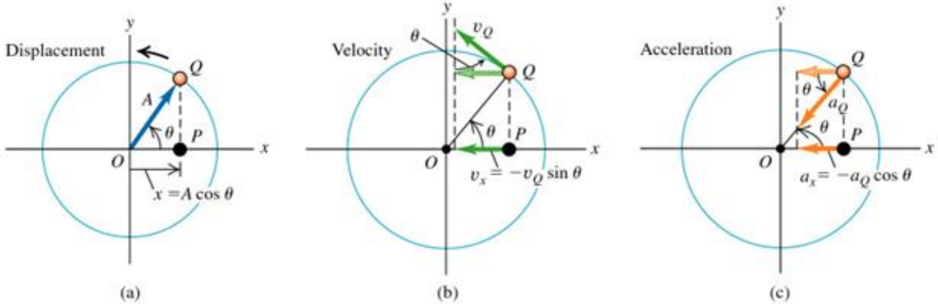
Amplitude A

Periode T

Frekvens $f = \frac{1}{T}$

Vinkelfrekvens $\omega = 2\pi f$

Energi $E = E_k + E_p = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \underbrace{\frac{1}{2}m{v_{max}}^2}_{\text{I origo, Likevektstilling, Max } E_k} = \underbrace{\frac{1}{2}kA^2}_{\text{I ytterpunktene, Max } E_p}$



$x = A\cos(\omega t + \varphi)$

Hastighet $v = \pm\sqrt{\frac{k}{m}} \cdot \sqrt{A^2 - x^2}$

Diff.lign. $m\ddot{x} + kx = 0$

Posisjon $\boxed{x = A\cos(\omega t + \varphi)}$

Start i ro i høyre ytterstilling $\varphi = 0 \Rightarrow x = A\cos(\omega t)$

Starter klokka ved passering origo på vei mot høyre $\varphi = -\frac{\pi}{2} \Rightarrow x = A\sin(\omega t)$

Periode $T = \frac{2\pi}{\omega}$

Frekvens $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Vinkelfrekvens $\omega = 2\pi f = \sqrt{\frac{k}{m}}$

Fasevinkel $\varphi = \arctan(-\frac{v_0}{\omega x_0})$

Amplitude $A = \sqrt{{x_0}^2 + \frac{{v_0}^2}{\omega^2}}$

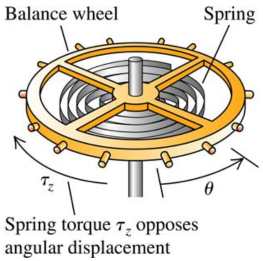
Sirkulær SHM

$\theta = \Theta\cos(\omega t + \varphi)$

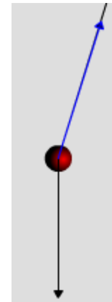
$\omega = \sqrt{\frac{\kappa}{I}}$

$T = 2\pi\sqrt{\frac{I}{\kappa}}$

$f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$



Kap 14   Periodisk bevegelse



For alle diff.lign. av typen $a\ddot{x} + bx = 0$ gjelder

$$\omega = \sqrt{\frac{b}{a}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad x = A\cos(\omega t + \varphi)$$

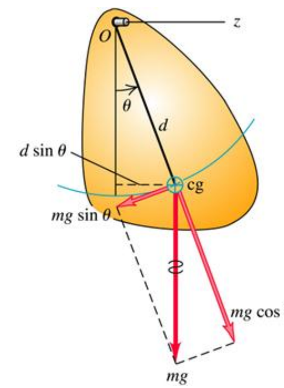
Enkel pendel

$$L\ddot{\theta} + g\theta = 0$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$



Fysisk pendel

$$I\ddot{\theta} + mgd\theta = 0$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

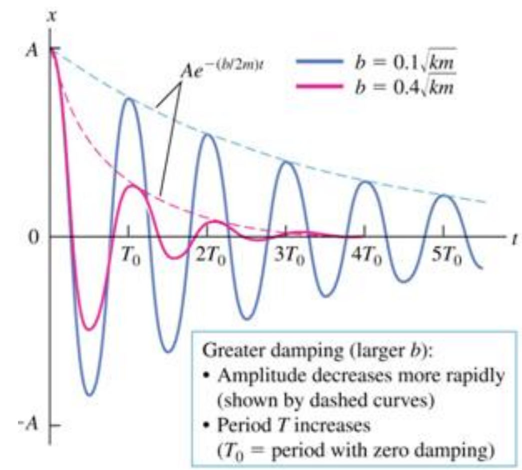
$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

Dempede svingninger

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$x = Ce^{\lambda t} \Rightarrow m\lambda^2 + b\lambda + k = 0 \Rightarrow \lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$x = \begin{cases} c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} & b^2 > 4mk \quad \text{Overdempet} \\ (c_1 + c_2 t)e^{\lambda t} & b^2 = 4mk \quad \text{Kritisk dempet} \\ Ce^{-\frac{b}{2m}t}\cos(\omega' t + \varphi) & b^2 < 4mk \quad \text{Underdempet} \qquad \omega = \sqrt{\omega_0{}^2 - \frac{b^2}{4m^2}} \end{cases}$$

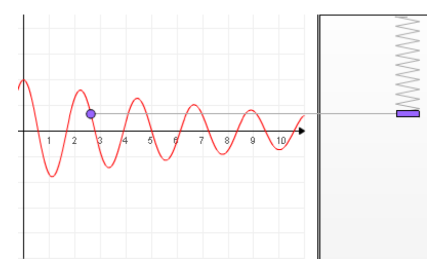


Tvungne svingninger

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos\omega t$$

$$x = x_0 + x_p = x_0 + A\cos(\omega t - \eta)$$

$$A = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + b^2\omega^2}}$$



Kap 40 Kvantefysikk

Tilstandsvektor	$\lvert\Psi\rangle$
Dual tilstandsvektor	$\langle\Psi\rvert$
Normalisering	$\langle\Psi\lvert\Psi\rangle = 1$
Utvikling etter orthonormale basisfunksjoner	$\lvert\Psi\rangle = \sum_n c_n \lvert\varphi_n\rangle = \sum_n \langle\varphi_n\lvert\Psi\rangle\lvert\varphi_n\rangle \langle\varphi_m\lvert\varphi_n\rangle = \delta_{mn}$
Sannsynlighetsamplitude c_n, sannsynlighet c_n^2	$\sum_n c_n^2 = 1$
Kompleks konjugering	$\langle\varphi_2\lvert\varphi_1\rangle = \langle\varphi_1\lvert\varphi_2\rangle^*$
Operator	$A\lvert\Psi_1\rangle = \lvert\Psi_2\rangle$
	$\langle\varphi_2\lvert A\rvert\varphi_1\rangle = \langle\varphi_1\lvert A\varphi_2\rangle = \langle\varphi_1 A^+\lvert A\varphi_2\rangle$
Projeksjonsoperator	$P_n = \lvert\varphi_n\rangle\langle\varphi_n\rvert P_m P_n = \begin{cases} P_n m = n \\ 0 m \neq n \end{cases}$
Kompletthet	$\sum_n \lvert\varphi_n\rangle\langle\varphi_n\rvert = I$
Hermitisk operator	$A = A^+$
Unitær operator	$U^+U = I$
Egentilstand	$A\lvert\varphi_n\rangle = a_n\lvert\varphi_n\rangle$
Operator utviklet etter egenvektorer/egenverdier	$A = \sum_n a_n \lvert\varphi_n\rangle\langle\varphi_n\rvert$
Operator-forventningsverdi	$\langle A\rangle = \langle\Psi\lvert A\rvert\Psi\rangle$
Kommutator	$[A, B] = AB - BA$
Uskarphetsrelasjon	$\Delta A\cdot\Delta B \geq \frac{1}{2}\lvert\langle[A,B]\rangle\rvert \qquad \Delta x\cdot\Delta p \geq \frac{\hbar}{2} \qquad \Delta E\cdot\Delta t \geq \frac{\hbar}{2}$
Generator for infinitesimale transformasjoner	$U(s) = e^{iKs} K = K^+$
Moment-operator	$p = \frac{\hbar}{i}\nabla$
Hamilton operator	$H = \frac{p^2}{2m} + V = -\frac{\hbar^2}{2m}\nabla^2 + V$
Tidsavhengig Schrødingerligning	$i\hbar\frac{\partial}{\partial t}\lvert\Psi(t)\rangle = H\lvert\Psi(t)\rangle$
Tidsuavhengig Schrødingerligning	$H\psi(\vec{r}) = E\psi(\vec{r})$
Partikkel i en boks	$E_n = \frac{p_n^2}{2m} = \frac{n^2h^2}{8mL^2} = \frac{n^2\pi^2\hbar^2}{2mL^2} n = 1,2,3,\ldots$
Harmonisk oscillator	$E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} n = 0,1,2,3,\ldots$

Kap 42   Halvlederfysikk

Tilstandstetthet $$g(E)=\frac{(2m)^{\frac{3}{2}}V}{2\pi\hbar^3}E^{\frac{1}{2}}$$

Sannsynlighet for okkupert energitilstand $$f(E)=\frac{1}{e^{\frac{E-E_F}{kT}}+1}$$

Strøm/spennings-relasjon for en ideell p-n overgang $$I=I_S(\mathrm{e}^{\frac{\mathrm{eV}}{\mathrm{kT}}}-1)$$