

Kap 02, 03 Posisjon – Hastighet – Akselerasjon

Hastighet

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Akselerasjon

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Hastighet

$$\vec{v} = \vec{v}_0 + \int_0^t \vec{a} \cdot dt$$

$$\vec{v} = \vec{v}_0 + \vec{a} \cdot t$$

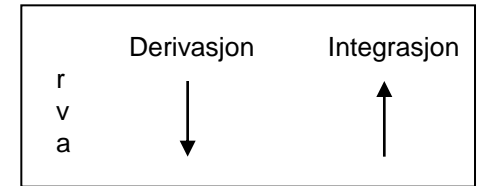
Konstant akselerasjon

Forflytning

$$\vec{r} = \vec{r}_0 + \int_0^t \vec{v} \cdot dt$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \cdot \vec{a} \cdot t^2$$

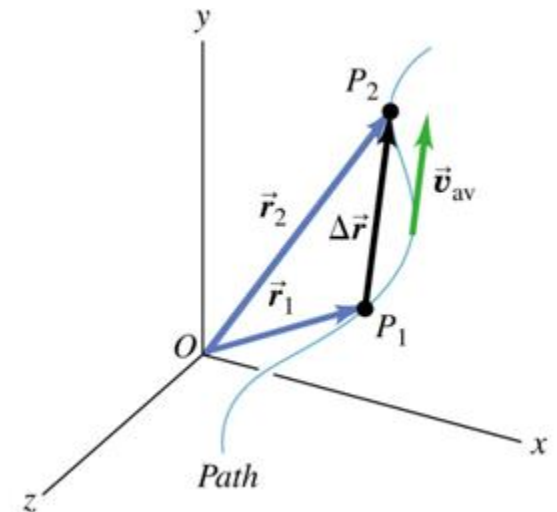
Konstant akselerasjon



Rettlinjet bevegelse og konstant akselerasjon

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

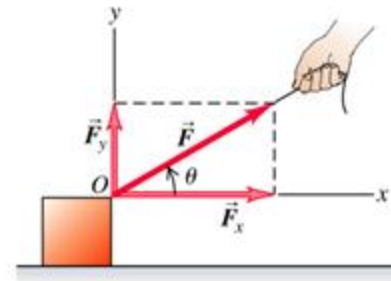
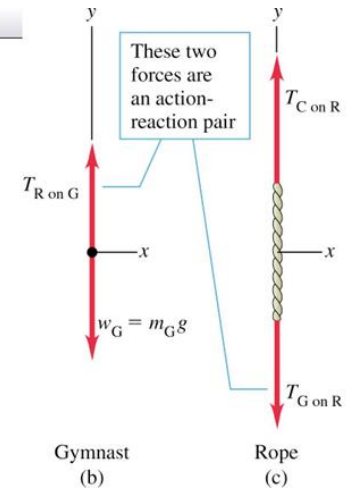
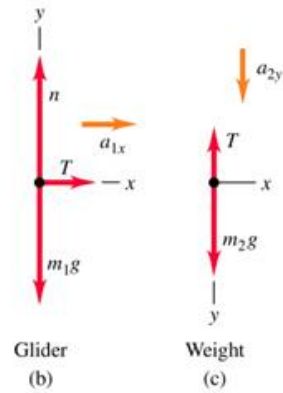
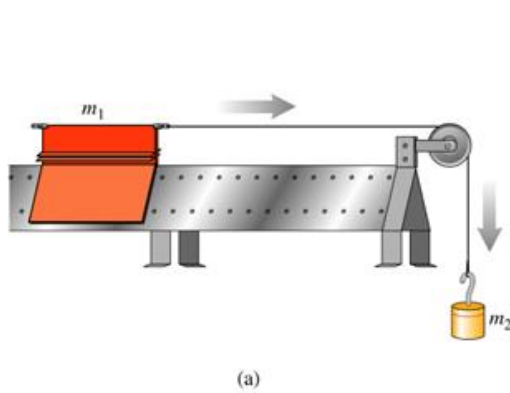
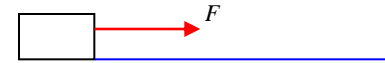
$$x = x_0 + \frac{v_{0x} + v_x}{2} t$$



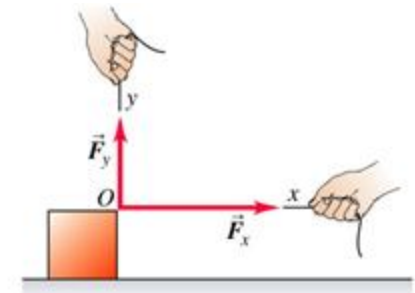
Kap 04, 05 Newtons lover

Newtons lover

1. $\vec{F} = \vec{0} \Rightarrow D\vec{v} = \vec{0}$
2. $\vec{F} = m\vec{a}$
3. $\vec{F} = -\vec{F}'$



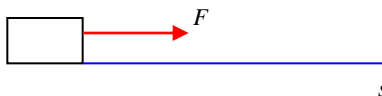
Component vectors: \vec{F}_x and \vec{F}_y
 Components: $F_x = F \cos \theta$ and $F_y = F \sin \theta$



Component vectors \vec{F}_x and \vec{F}_y together have the same effect as original force \vec{F} .

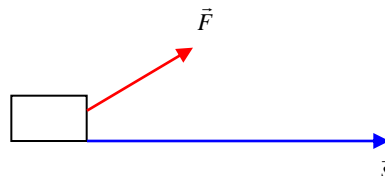
Kap 06 Arbeid – Kinetisk Energi

Def av arbeid W
ved å flytte et objekt med en konstant kraft F
en rettlinjert strekning s
når F og s peker samme vei



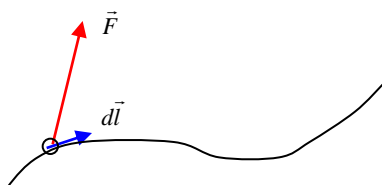
$$W = F \cdot s$$

Def av arbeid W
ved å flytte et objekt med en konstant kraft F
en rettlinjert strekning s



$$W = \vec{F} \cdot \vec{s}$$

Def av arbeid W
ved å flytte et objekt med en varierende kraft F
fra punktet P_1 til punktet P_2 langs en kurve l



$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$

Endring av kinetisk energi $\Delta K =$
Det arbeidet som må utføres på et objekt med masse m
for å endre objektets hastighet fra v_1 til v_2



$$\begin{aligned} \Delta K = W &= \int_{P_1}^{P_2} F dx \\ &= \int_{P_1}^{P_2} m \cdot a \cdot dx = \int_{P_1}^{P_2} m \cdot \frac{dv}{dt} \cdot dx = \int_{P_1}^{P_2} m \cdot dv \cdot \frac{dx}{dt} = \int_{P_1}^{P_2} m \cdot dv \cdot v \\ &= \int_{P_1}^{P_2} m \cdot v \cdot dv = \int_{v_1}^{v_2} m \cdot v \cdot dv = m \int_{v_1}^{v_2} v \cdot dv = m \left[\frac{1}{2} v^2 \right]_{v_1}^{v_2} \\ &= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \end{aligned}$$

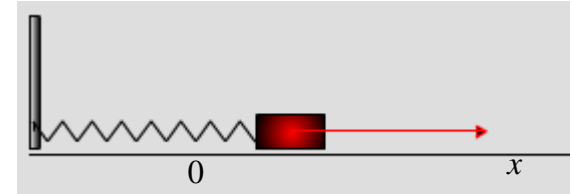
Arbeid – Energi teorem

$$\Delta K = W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Kap 06 Arbeid – Kinetisk Energi

Arbeid W ved strekk av en elastisk fjær

$$W = \int_0^x F dx = \int_0^x kx dx = \frac{1}{2} kx^2$$



Gjennomsnitts-effekt P_a

Energi (arbeid) pr tidsenhet

Måle-enhet Watt $W = J/s$

$$P_a = \frac{\Delta W}{\Delta t}$$

Momentan-effekt P

Energi (arbeid) pr tidsenhet

Måle-enhet Watt $W = J/s$

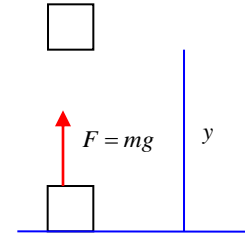
$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Kap 07 Potensiell energi og energi-bevaring

Arbeid utført i tyngdefeltet
ved å løfte en strekning y
med en konstant kraft $F = mg$ (gir ingen fartsendring)

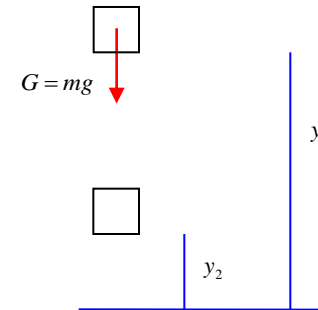
Dette arbeidet er uavhengig av veien

$$W = F \cdot h = mgy$$



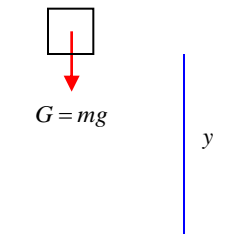
Arbeid som tyngden utfører
ved et fall fra y_1 til y_2

$$W_{grav} = F \cdot s = mgy_1 - mgy_2$$



Definisjon av gravitasjons-potensiell energi U_{grav}

$$U_{grav} = mgy$$



Kap 07 Potensiell energi og energi-bevaring

Sammenheng mellom arbeid W_{grav} utført av tyngden og endring i gravitasjons-potensiell energi U_{grav}

$$W_{\text{grav}} = F \cdot s = mgy_1 - mgy_2 = U_{\text{grav},1} - U_{\text{grav},2} = -(U_{\text{grav},2} - U_{\text{grav},1}) = -\Delta U_{\text{grav}}$$

Bevaring av mekanisk energi (kinetisk energi + potensiell energi) i tyngdefeltet

$$\left. \begin{aligned} W_{\text{tot}} &= W_{\text{grav}} \\ W_{\text{tot}} &= \Delta K \\ W_{\text{grav}} &= -\Delta U_{\text{grav}} \end{aligned} \right\} \Rightarrow \Delta K = -\Delta U_{\text{grav}} \Rightarrow \Delta K + \Delta U_{\text{grav}} = 0$$

\Downarrow

$$K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

Bevaring av mekanisk energi (kinetisk energi + potensiell energi) for en elastisk fjær

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Hvis i tillegg rotasjon, må tilføyes rotasjons-energi (se kap 10)

Ikke bevaring av mekanisk energi (kinetisk energi + potensiell energi)

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

Total energi-bevaring

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$

Kraft og potensiell energi

$$W = -dU$$

$$Fdx = -dU$$

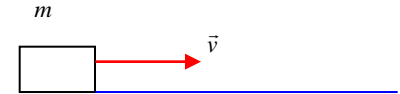
$$F = -\frac{dU}{dx}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right]$$

Kap 08 Moment (bevegelsesmengde) – Impuls

Moment (bevegelsesmengde)

$$\vec{p} = m\vec{v} \quad \vec{P} = \sum_i m\vec{v}_i$$



Newtons 2.lov

$$\sum \vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\sum \vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m \frac{d\vec{v}}{dt}$$

Hvis massen m endrer seg

Impuls

$$\vec{J} = \sum \vec{F} \cdot \Delta t = \frac{\Delta \vec{p}}{\Delta t} \Delta t = \Delta \vec{p} = \vec{p}_2 - \vec{p}_1$$

Bevaring av moment

$$\left. \begin{aligned} \sum \vec{F} &= \frac{d\vec{p}}{dt} \\ \sum \vec{F} &= \vec{0} \end{aligned} \right\} \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{konstant}$$

Kollisjoner (bevaring av moment)

Ingen ytre krefter virker på system
bestående av m_1 og m_2

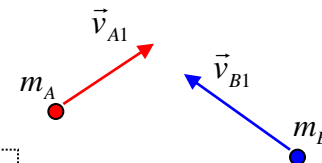
Totalt moment før kollisjon = Totalt
moment etter kollisjon

$$m_A \vec{v}_{A1} + m_B \vec{v}_{B1} = m_A \vec{v}_{A2} + m_B \vec{v}_{B2}$$

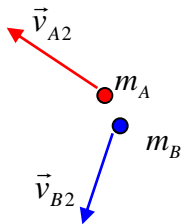
Elastisk kollisjon:

Da gjelder i tillegg bevaring av mekanisk
energi (her kinetisk energi)

$$\frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2 = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2$$



Hvis endring
(energitap, dvs ikke-elastisk)
Etter (2) – Før (1)



Fullstendig uelastisk:

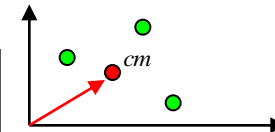
Henger sammen etter kollisjonen

Masse-senter

$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$M = \sum_i m_i$$

$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r} dm$$



Newtons 2.lov for et utstrakt legeme

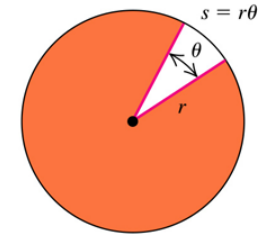
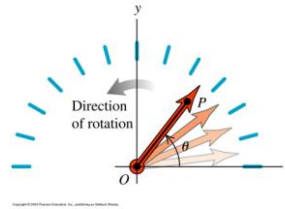
$$\sum \vec{F} = m\vec{a}_{cm}$$

$$M\vec{v}_{cm} = \sum_i m_i \vec{v}_i = \vec{P}$$

Kap 09 Rotasjon

Vinkel

$$\theta = \frac{s}{r}$$



Vinkelhastighet

$$\omega = \frac{d\theta}{dt}$$

Vinkelakselerasjon

$$\alpha = \frac{d\omega}{dt}$$

Benevning

$\theta \leftrightarrow$ ubenevnt eller rad

$v \leftrightarrow m/s$

$\omega \leftrightarrow s^{-1}$ eller rad/s

$a \leftrightarrow s^{-2}$ eller rad/s²

Bevegelses-ligninger

$$\omega(t) = \omega_0 + \int_0^t \alpha(t) dt$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t) dt$$

Konstant vinkelaks.

$$\omega = \omega_0 + \alpha \cdot t$$

$$\theta = \theta_0 + \omega_0 \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2$$

$$\omega^2 = \omega_0^2 + 2 \cdot \alpha \cdot (\theta - \theta_0)$$

Analogi mellom hastighet / akselerasjon og vinkelhastighet / vinkelakselerasjon

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$s \leftrightarrow \theta$$

$$v \leftrightarrow \omega$$

$$a \leftrightarrow \alpha$$

$$v(t) = v_0 + \int_0^t a(t) dt$$

$$s(t) = s_0 + \int_0^t v(t) dt$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t) dt$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t) dt$$



Generelt

$$v(t) = v_0 + at$$

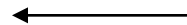
$$s(t) = s_0 + v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$



a konstant

α konstant

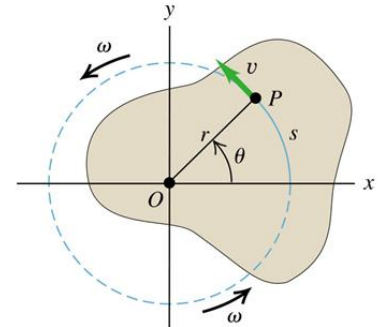
Kap 09 Rotasjon

Vinkelhastighet

$$\omega = \dot{\theta} = \frac{v}{r}$$

Hastighet

$$v = r\omega$$



Vinkelakselerasjon

$$\alpha = \dot{\omega} = \ddot{\theta} = \frac{a}{r}$$

Akselerasjon

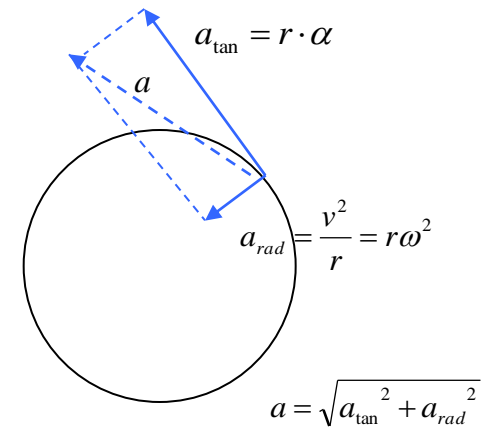
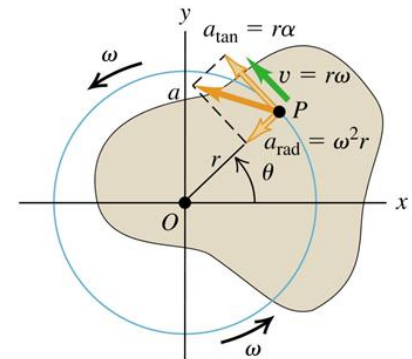
$$a_{\text{tan}} = r \cdot \alpha$$

Akselerasjon

$$a_{\text{tan}} = \dot{v} = \frac{dv}{dt} = r \cdot \frac{d\omega}{dt} = r \cdot \alpha$$

$$a_{\text{rad}} = \frac{v^2}{r} = r\omega^2$$

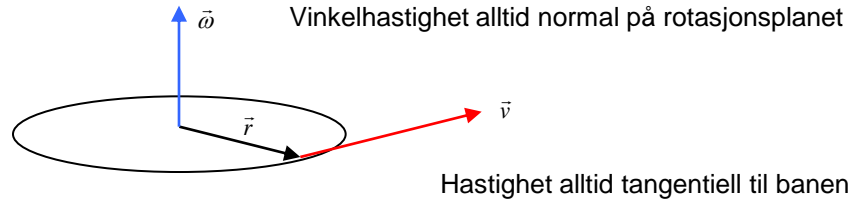
$$a_{\text{rad}} = \frac{v^2}{r} = r\omega^2$$



Vinkelhastighet og vinkelakselerasjon som vektor

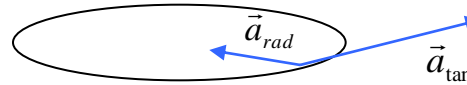
Hastighet

$$\vec{v} = \vec{\omega} \times \vec{r}$$

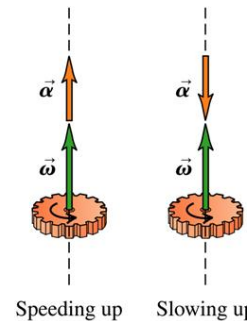
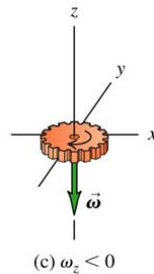
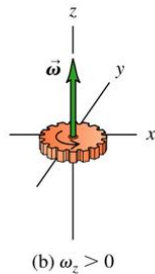
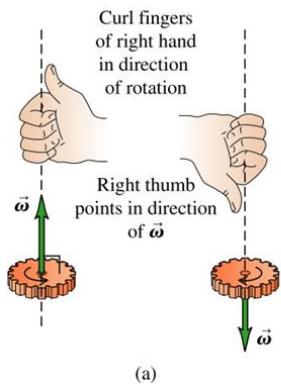
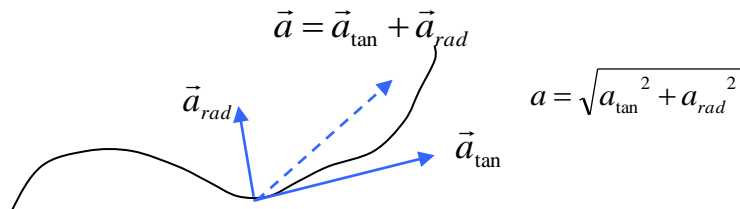


Akselerasjon

$$\begin{aligned}\vec{a} &= \dot{\vec{v}} \\ &= \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}} \\ &= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \\ &= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \\ &= \underbrace{\vec{\alpha} \times \vec{r}}_{\text{tangentiell akselerasjon}} + \underbrace{\vec{\omega} \times \vec{\omega} \times \vec{r}}_{\text{radiell akselerasjon}} \\ &= \vec{a}_{\text{tan}} + \vec{a}_{\text{rad}}\end{aligned}$$



Tangentiell-akselerasjon alltid tangentiell til banen.
Radiell-akselerasjon alltid rettet inn mot sentrum.

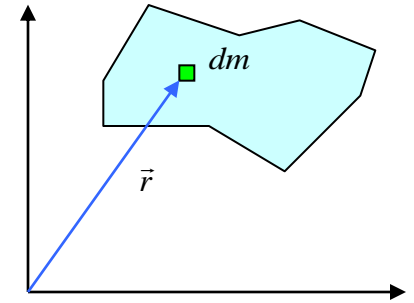
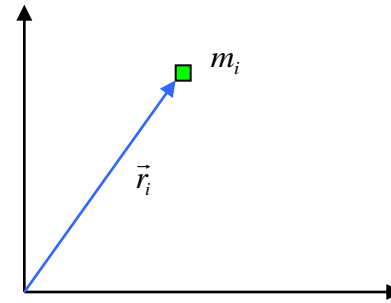


Kap 09 Rotasjon

Treghetsmoment

$$I = \sum_i m_i r_i^2$$

$$I = \int r^2 dm$$

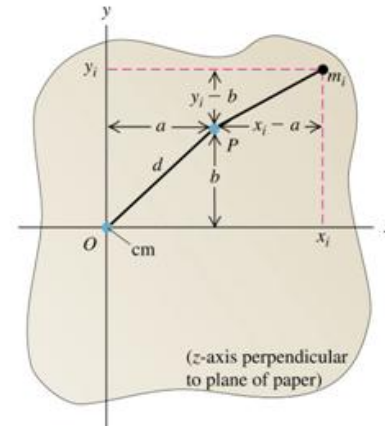


Kinetisk rotasjonsenergi

$$K = \frac{1}{2} I \omega^2$$

Parallellakse - teorem

$$I_p = I_{cm} + M d^2$$



Benevning

$$I \leftrightarrow \text{kgm}^2$$

$$K \leftrightarrow J$$

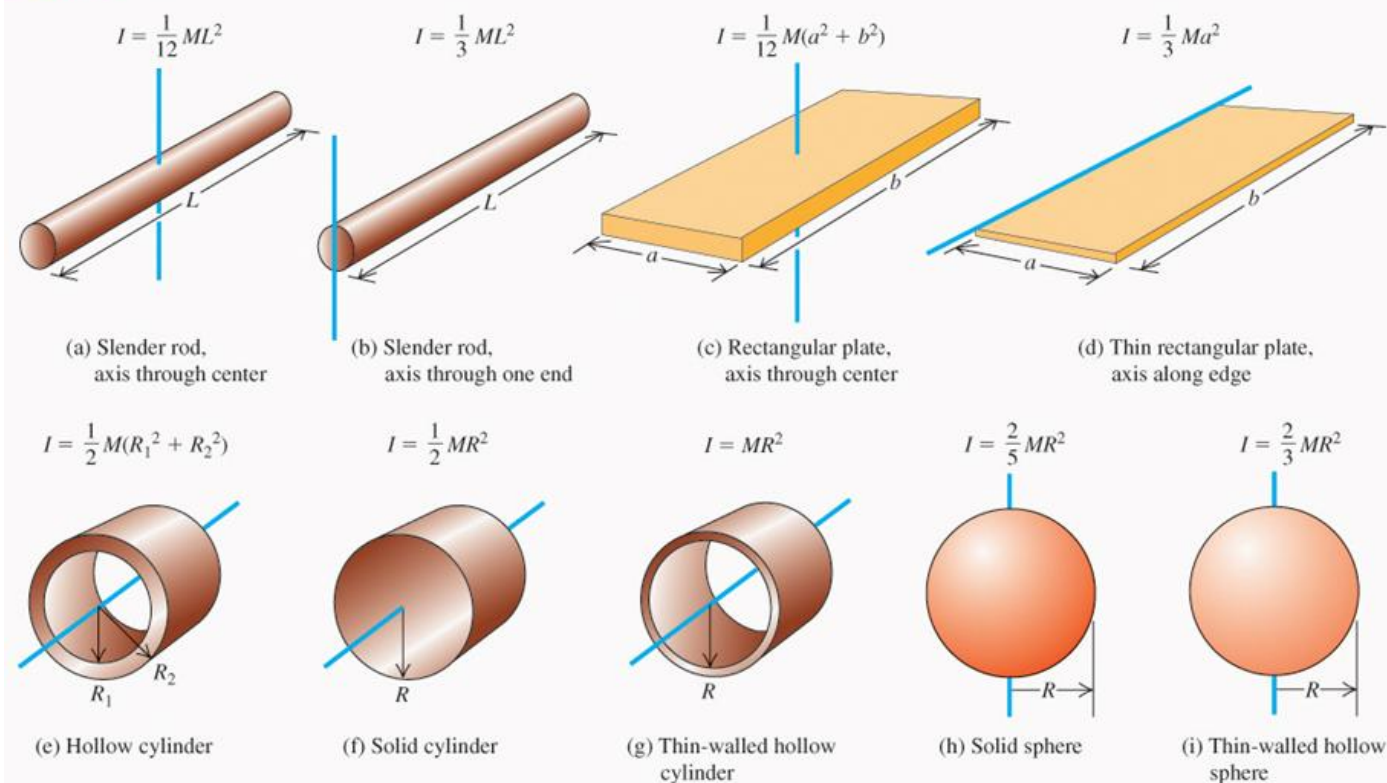
Normalakse - teorem

$$I_o = I_x + I_y$$

Treghets-moment for noen spesielle legemer med akse gjennom sentrum

Stav med masse M og lengde L	$I = \frac{1}{12} ML^2$
Rektangulær plate med masse M og sider a og b	$I = \frac{1}{12} M(a^2 + b^2)$
Hul sylinder med masse M, indre radius R_1 , ytre radius R_2	$I = \frac{1}{2} M(R_1^2 + R_2^2)$
Massiv sylinder med masse M og radius R	$I = \frac{1}{2} MR^2$
Massiv kule med masse M og radius R	$I = \frac{2}{5} MR^2$
Tynnvegget kule med masse M og radius R	$I = \frac{2}{3} MR^2$

Table 9.2 Moments of Inertia of Various Bodies



Kap 10 Rotasjonsdynamikk

Kraftmoment / Angulært moment

Kraftmoment

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau_{cm} = I_{cm} \alpha$$

$$\tau_o = I_o \alpha \quad \text{når} \quad \begin{cases} 1. \quad \vec{a}_o = \vec{0} \\ 2. \quad \vec{a}_o \parallel \vec{r}_{cm} \\ 3. \quad \vec{r}_{cm} = \vec{0} \end{cases}$$

Kinetisk energi

$$K = K_{trans} + K_{rot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$K = \frac{1}{2} I_o \omega^2 \quad \text{når} \quad \vec{v}_o = \vec{0}$$

Arbeid / Effekt

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I \omega d\omega = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$$

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau \omega$$

Angulært moment

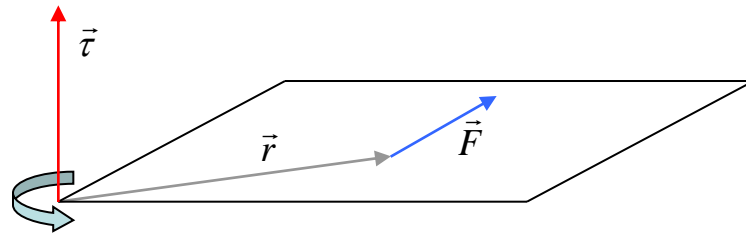
$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m \vec{v}$$

$$\vec{L} = I \vec{\omega}$$

$$\dot{\vec{L}}_o = \vec{\tau}_o \quad \text{når} \quad \begin{cases} 1. \quad \vec{v}_o = \vec{0} \\ 2. \quad \vec{v}_{cm} = \vec{0} \\ 3. \quad \vec{v}_o \parallel \vec{v}_{cm} \end{cases}$$

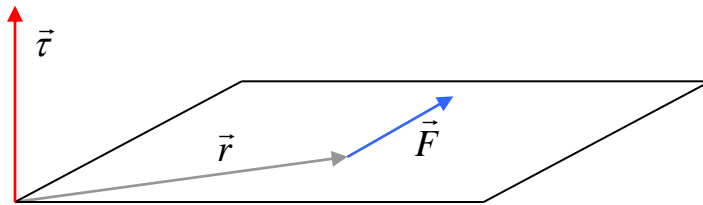
Gyroskop

$$\Omega = \frac{d\varphi}{dt} = \frac{\frac{dL}{L}}{\frac{dt}{dt}} = \frac{\frac{dL}{dt}}{L} = \frac{\tau}{L} = \frac{wR}{I\omega}$$

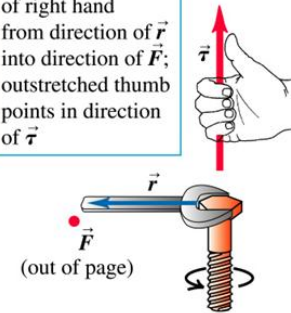


Kraftmoment

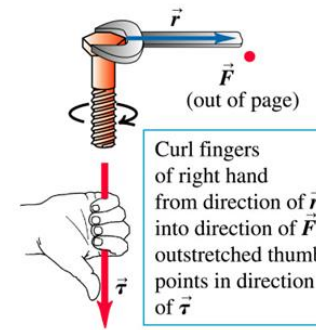
$$\vec{\tau} = \vec{r} \times \vec{F}$$



Curl fingers of right hand from direction of \vec{r} into direction of \vec{F} ; outstretched thumb points in direction of $\vec{\tau}$



Curl fingers of right hand from direction of \vec{r} into direction of \vec{F} ; outstretched thumb points in direction of $\vec{\tau}$

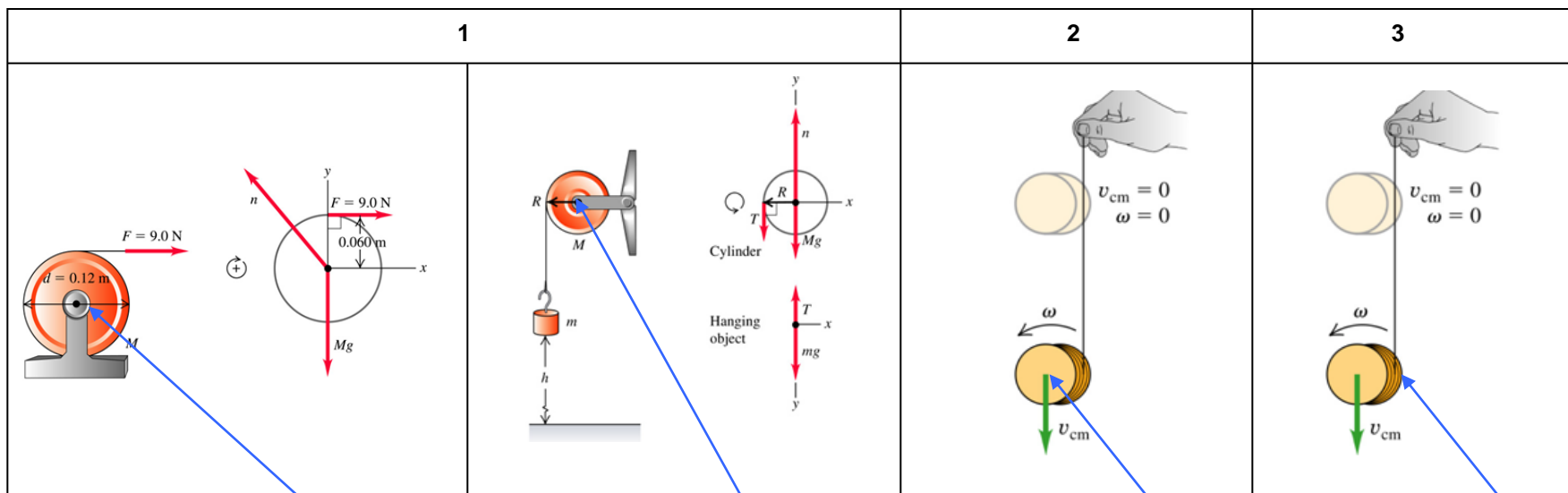
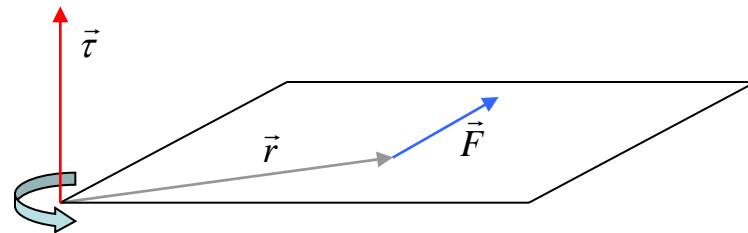


Kap 10 Rotasjonsdynamikk

Sammenheng mellom kraftmoment og vinkel-akselerasjon

$$\tau_{\text{cm}} = I_{\text{cm}} \alpha$$

$$\tau_o = I_o \alpha \quad \text{når} \quad \begin{cases} 1. \vec{a}_o = \vec{0} \\ 2. \vec{a}_o \parallel \vec{r}_{\text{cm}} \\ 3. \vec{r}_{\text{cm}} = \vec{0} \end{cases}$$



Kap 10 Rotasjonsdynamikk

Kinetisk energi

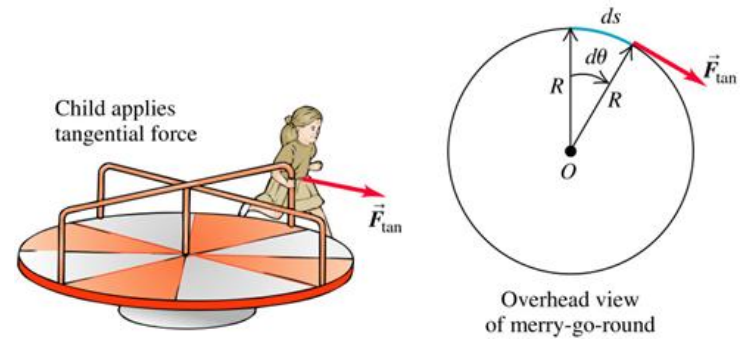
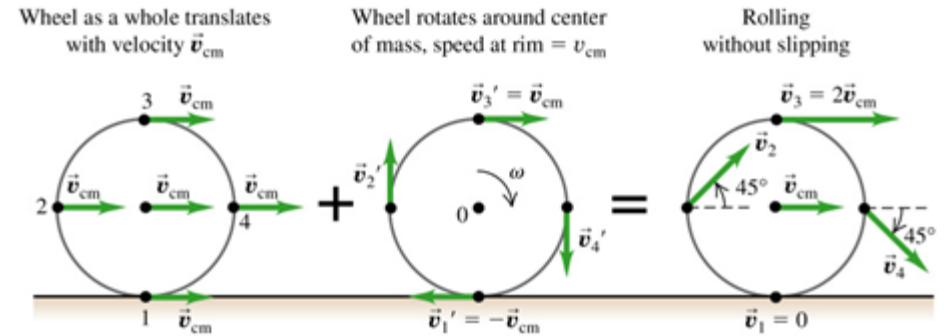
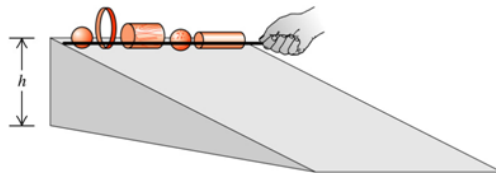
$$K = K_{trans} + K_{rot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$K = \frac{1}{2} I_o \omega^2 \quad \text{når } v_o = \vec{0}$$

Arbeid / Effekt

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I \omega d\omega = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$$

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau \omega$$



$$0 + M \cdot g \cdot h = \frac{1}{2} \cdot M \cdot v_{cm}^2 + \frac{1}{2} \cdot (c \cdot M \cdot R^2) \cdot \left(\frac{v_{cm}}{R} \right)^2$$

$$M \cdot g \cdot h = \frac{M \cdot v_{cm}^2 \cdot (c + 1)}{2}$$

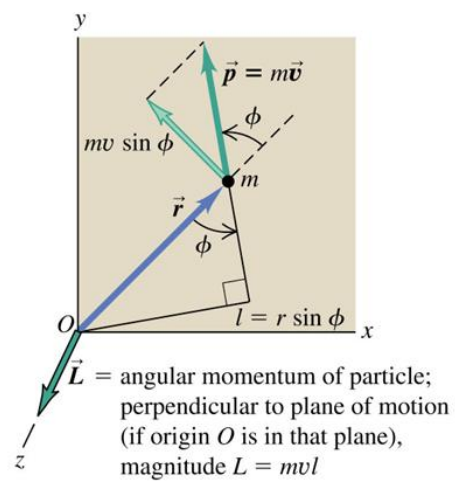
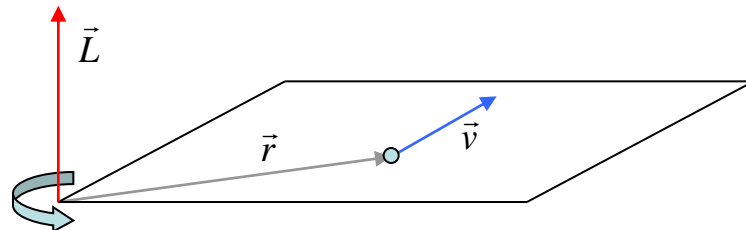
$$v_{cm} = \sqrt{\frac{2 \cdot g \cdot h}{c + 1}}$$

Angulært moment

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$\vec{L} = I\vec{\omega}$$

$$\dot{\vec{L}}_o = \vec{\tau}_o \quad \text{når} \quad \begin{cases} 1. \vec{v}_o = \vec{0} \\ 2. \vec{v}_{cm} = \vec{0} \\ 3. \vec{v}_o \parallel \vec{v}_{cm} \end{cases}$$



Kap 11 Likevekt og elastisitet

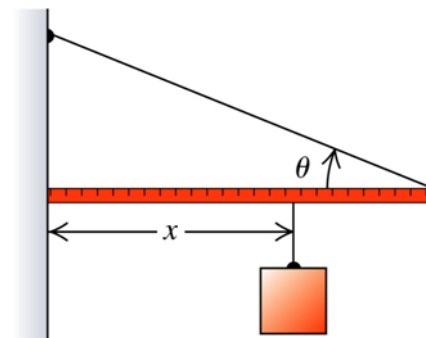
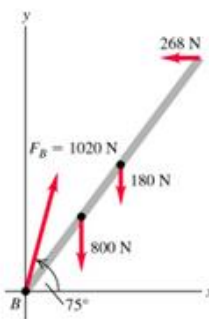
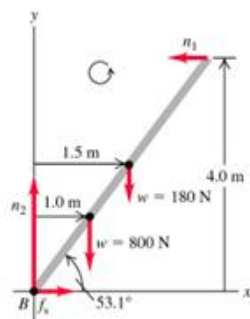
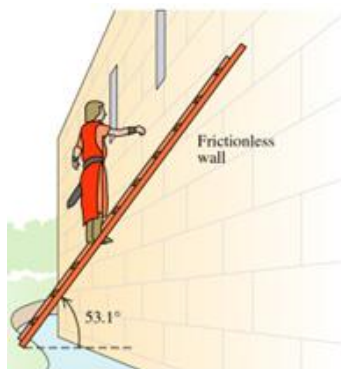
Betingelse for likevekt

$$\sum \vec{F} = \vec{0}$$

Summen av alle ytre krefter er lik null

$$\sum \vec{\tau} = \vec{0}$$

Summen av alle ytre kraftmomenter om en vilkårlig akse er lik null



Kap 11 Likevekt og elastisitet

Hookes lov:

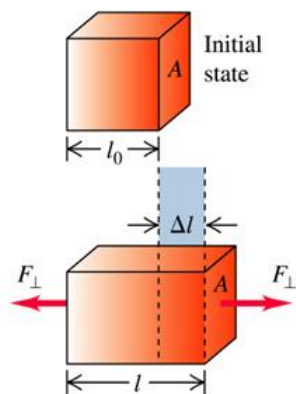
Den ytre kraften (Stress) på et system er proporsjonal med deformasjonen (Strain) av systemet. Proporsjonalitets-konstanten kalles elastisitetsmodulen.

$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus}$$

Strekk-stress og strekk-strain:

Elastisitetsmodulen kalles for Youngs modulus.

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\frac{F_{\perp}}{A}}{\frac{\Delta l}{l_0}} = \frac{F_{\perp}}{A} \frac{l_0}{\Delta l}$$



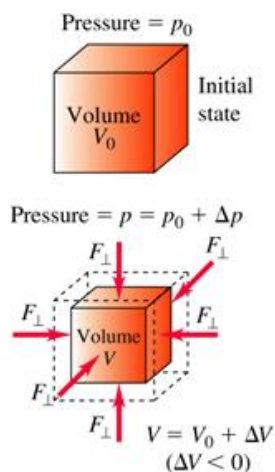
$$\text{Tensile stress} = \frac{F_{\perp}}{A}$$

$$\text{Tensile strain} = \frac{\Delta l}{l_0}$$

Bulk-stress og bulk-strain:

Elastisitetsmodulen kalles for Bulke modulus.

$$B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = - \frac{\Delta p}{\frac{\Delta V}{V_0}}$$



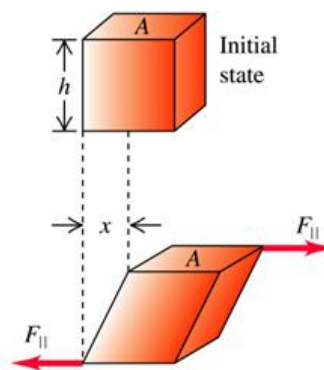
$$\text{Bulk stress} = \Delta p$$

$$\text{Bulk strain} = \frac{\Delta V}{V_0}$$

Share-stress og share-strain:

Elastisitetsmodulen kalles for Share modulus.

$$S = \frac{\text{Share stress}}{\text{Share strain}} = \frac{\frac{F_{\perp}}{A}}{\frac{x}{h}} = \frac{F_{\perp}}{A} \frac{h}{x}$$



$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

$$\text{Shear strain} = \frac{x}{h}$$

Kap 13 Gravitasjon

Gravitasjonskraft

$$F_g = G \frac{m_1 m_2}{r^2}$$

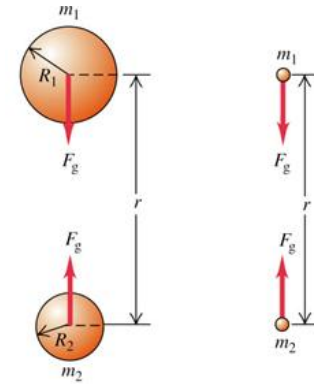
$$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

Tyngde

$$w = F_g = G \frac{m_E m}{R_E^2}$$

Tyngdeakselerasjon

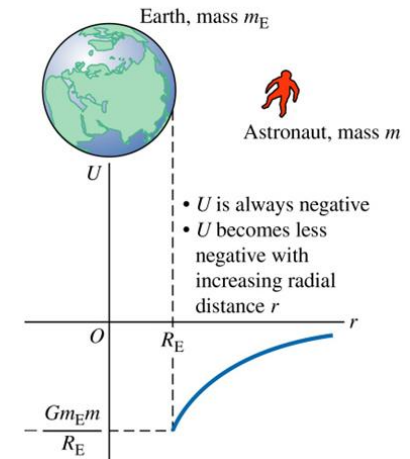
$$g = \frac{F_g}{m} = G \frac{m_E}{R_E^2}$$



Potensiellenergi

$$U = -G \frac{m_E m}{r}$$

$$\text{Gravitational potential energy } U = -\frac{G m_E m}{r}$$



Kap 13 Gravitasjon - Satellitt

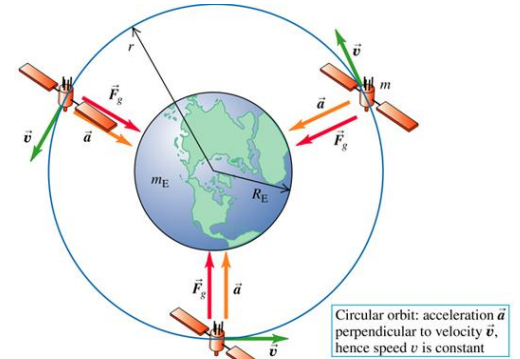
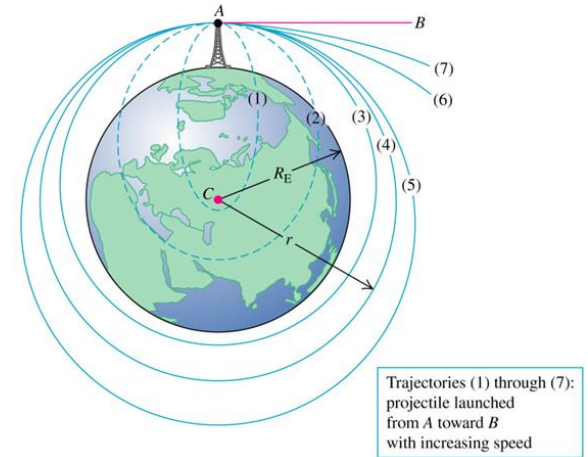
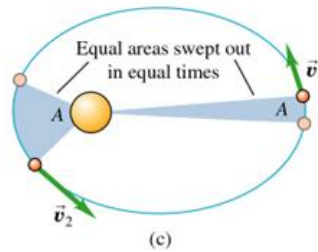
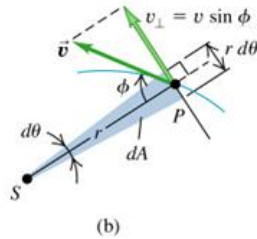
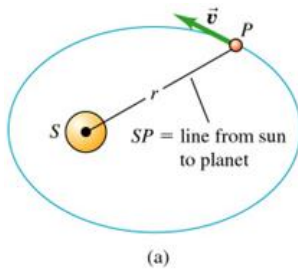
Satelitt – hastighet

$$v = \sqrt{G \frac{m_E}{r}}$$

Satelitt – periode

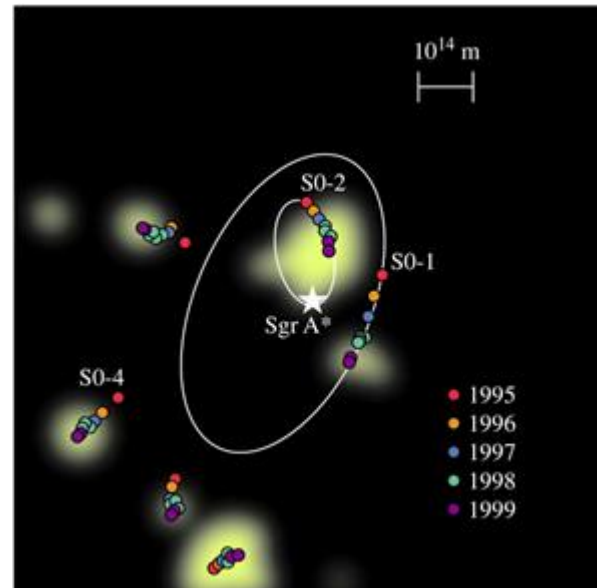
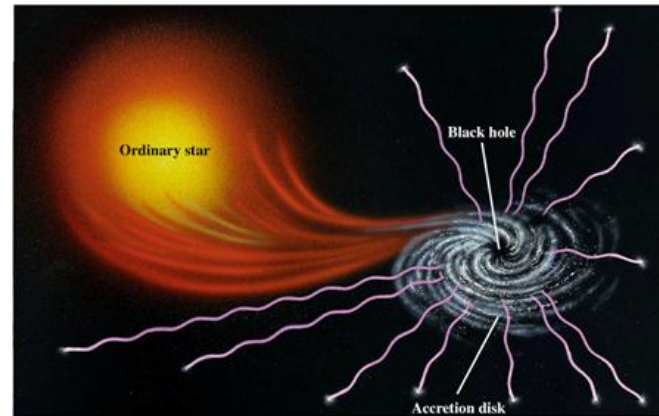
$$T = \frac{2\pi r}{v} = 2\pi \sqrt{\frac{r}{r G m_E}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{G m_E}}$$

$$T = \frac{2 \cdot \pi \cdot a^{\frac{3}{2}}}{\sqrt{G(m_p + m_s)}}$$



Kap 13 Gravitasjon - Sort hull

Schwarzschild radius (sort hull) $R_s = \frac{2GM}{c^2}$



Kap 40 Kvantefysikk

Tilstandsvektor	$ \Psi\rangle$
Dual tilstandsvektor	$\langle\Psi $
Normalisering	$\langle\Psi \Psi\rangle=1$
Utvikling etter orthonormale basisfunksjoner	$ \Psi\rangle=\sum_n c_n \varphi_n\rangle=\sum_n \langle\varphi_n \Psi\rangle \varphi_n\rangle\langle\varphi_m \varphi_n\rangle=\delta_{mn}$
Sannsynlighetsamplitude c_n , sannsynlighet c_n^2	$\sum_n c_n^2=1$
Kompleks konjugering	$\langle\varphi_2 \varphi_1\rangle=\langle\varphi_1 \varphi_2\rangle^*$
Operator	$A \Psi_1\rangle= \Psi_2\rangle$ $\langle\varphi_2 A \varphi_1\rangle=\langle\varphi_1 A\varphi_2\rangle=\langle\varphi_1A^+ A\varphi_2\rangle$
Projeksjonsoperator	$P_n= \varphi_n\rangle\langle\varphi_n P_mP_n=\begin{cases} P_n m=n \\ 0 m\neq n \end{cases}$
Kompletthet	$\sum_n \varphi_n\rangle\langle\varphi_n =I$
Hermitisk operator	$A=A^+$
Unitær operator	$U^+U=I$
Egentilstand	$A \varphi_n\rangle=a_n \varphi_n\rangle$
Operator utviklet etter egenvektorer/egenverdier	$A=\sum_n a_n \varphi_n\rangle\langle\varphi_n $
Operator-forventningsverdi	$\langle A\rangle=\langle\Psi A \Psi\rangle$
Kommutator	$[A,B]=AB-BA$
Uskarpetsrelasjon	$\Delta A\cdot\Delta B\geq\frac{1}{2} \langle[A,B]\rangle \quad \Delta x\cdot\Delta p\geq\frac{\hbar}{2} \quad \Delta E\cdot\Delta t\geq\frac{\hbar}{2}$
Generator for infinitesimale transformasjoner	$U(s)=e^{iKs}K=K^+$
Moment-operator	$p=\frac{\hbar}{i}\nabla$
Hamilton operator	$H=\frac{p^2}{2m}+V=-\frac{\hbar^2}{2m}\nabla^2+V$
Tidsavhengig Schrødingeligning	$i\hbar\frac{\partial}{\partial t} \Psi(t)\rangle=H \Psi(t)\rangle$
Tidsuavhengig Schrødingeligning	$H\psi(\vec{r})=E\psi(\vec{r})$
Partikkel i en boks	$E_n=\frac{p_n^2}{2m}=\frac{n^2\hbar^2}{8mL^2}=\frac{n^2\pi^2\hbar^2}{2mL^2} \quad n=1,2,3,\dots$
Harmonisk oscillator	$E_n=\left(n+\frac{1}{2}\right)\hbar\omega=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} \quad n=0,1,2,3,\dots$

Kap 42 Halvlederfysikk

Tilstandstetthet

$$g(E) = \frac{(2m)^{\frac{3}{2}} V}{2\pi\hbar^3} E^{\frac{1}{2}}$$

Sannsynlighet for okkupert energitilstand

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

Strøm/spennings - relasjon for en ideell p - n overgang

$$I = I_s (e^{\frac{eV}{kT}} - 1)$$