*3.5.1. Scattershot boson-sampling analysis.* The scattershot method is a recipe for overcoming the exponentialy decaying efficiency of parallel SPDC sources. However, there is still a major problem to consider – the fidelity. Here, we discuss it from two aspects under the influence of coupling and detection efficiency.

On one hand, the problem results from the large number (*m* in total) of SPDC sources that SBS needs. As we mentioned before, we will record the instance when both *n* heralding detectors and *n* detectors at the output modes are fired. However, single photon pairs might be generated from more than *n* SPDC sources and coupled into the interferometer without being consciously noticed by us (without our awareness), as long as the extra photons were not successfully heralded due to the detection inefficiency. If an unheralded photon is detected in the output, but a correct heralded photon is lost in the interferometer, a wrong sample will be taken. Promoting the detection efficiency can relax the problem, whereas to put an end to “dark photons” we need high speed vacuum shutters to close the input modes whose corresponding triggering detectors did not click [38].

On the other hand, SPDC sources exhibit higher-order emission, leading to the probability that two or three photon pairs might be generated. The following two approaches may be helpful: attenuating the brightness of the source by decreasing the pump power of the SPDCs, or employing photon-number-resolving detectors [34].

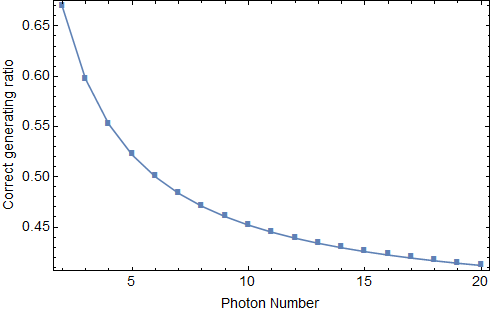
When the detectors are perfect but non-number-resolving, the probability that *n* heralding detectors are triggered, and all of them result from single photon pairs generated by *n* SPDC (ignoring the third and higher order terms) is

. (12)

The total probability that *n* SPDC sources generate photons is given by

. (13)

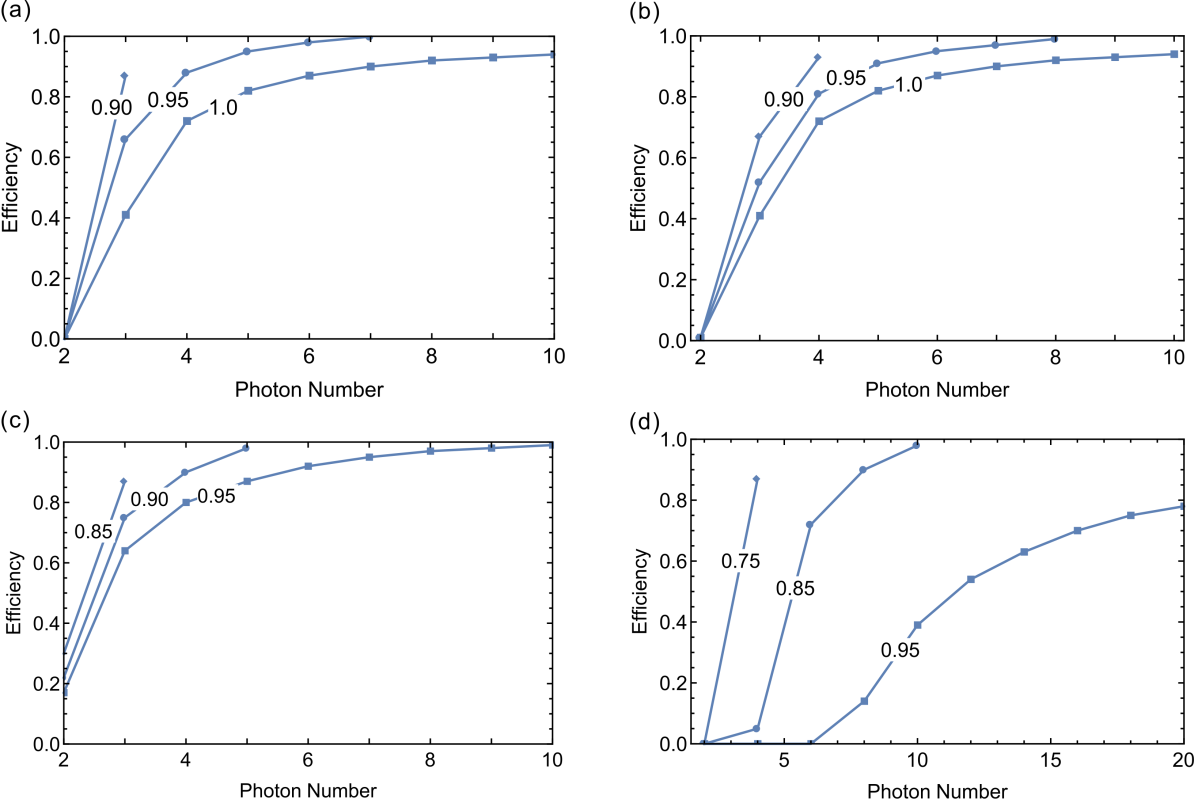
Nevertheless, we cannot distinguish the correct preparation of *n* single photons from the total occasions of *n* heralded sources. As shown in figure 10, the correct generation ratio defined as  decreases rapidly with increasing *n*.



**Figure 10.** The correct generation ratio of *n* single photons in *n*2 pairs of SPDCs.

For example, for *n*=6, almost half the input states are erroneous, where some two-photon events are mixed. Without photon-number-resolving detectors, we cannot exclude this erroneous component before coupling photons into the interferometer. Consider the first possible remedy, attenuating the pump power to reducing the generation probability of higher-order terms at the cost of the brightness of the SPS. For simplicity, les us consider the example of *n*=20. To ensure the correct rate of input states is >0.9, we should keep *p*<0.006, much smaller than the scalable condition that *p* equals to, leading to the decrease of source brightness of more than 10 orders of magnitude. What does it imply? If all the coupling and detection processes are perfect, it implies we obtain 6 counts a day for 76 MHz repeat frequency. Let alone beating classical simulation, it is too difficult to keep the system stable in the data collection period. Therefore, only the second remedy – photon-number-resolving detectors – remains probable and useful to implement scalable SBS.

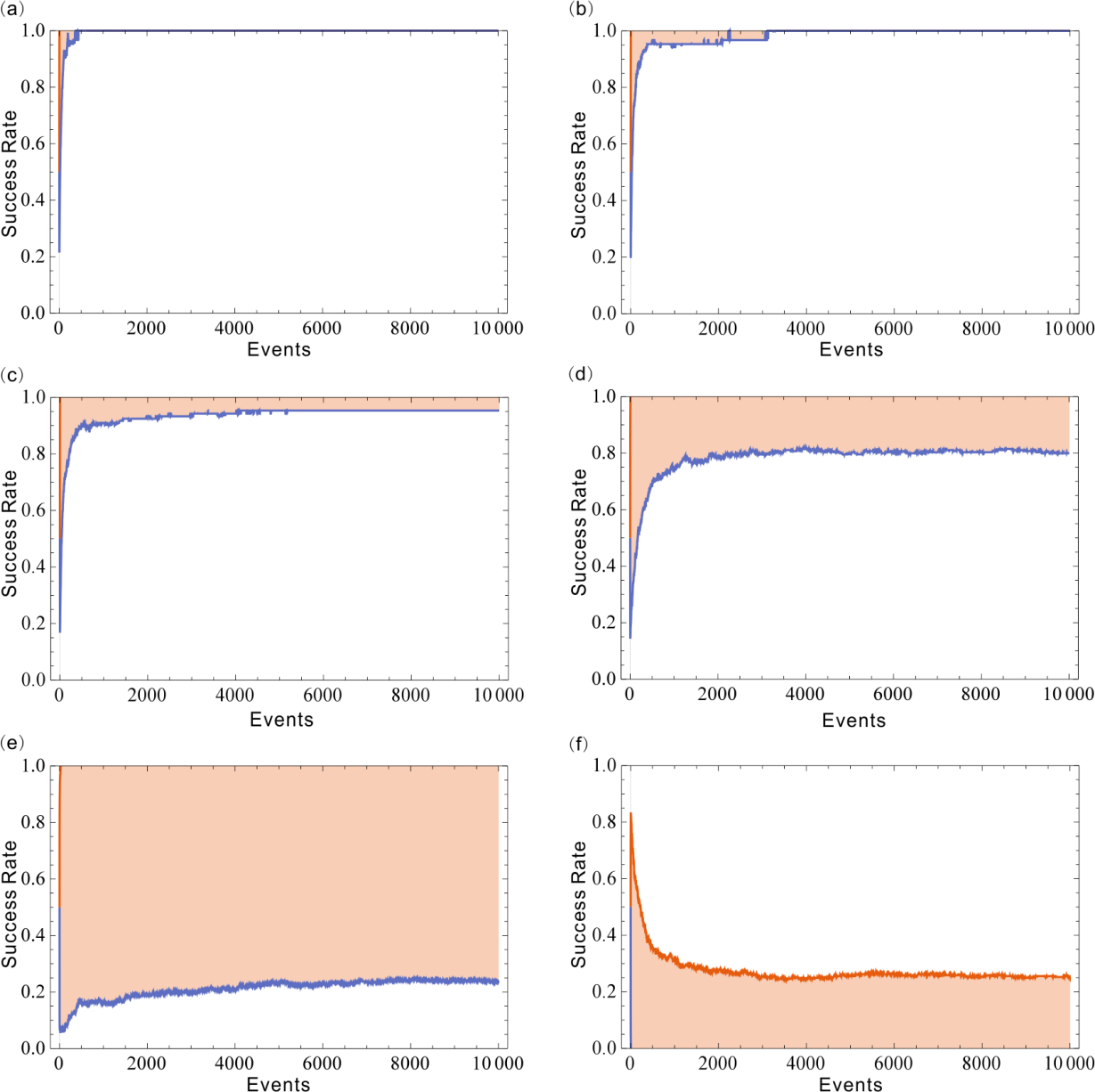
We take shutters and photon number resolving detectors into consideration with different configurations. As shown in figure 11, only if both of them are exploited, the rigorous requirements in detection efficiency and coupling efficiency of the quantum system can be greatly loosened. In this case, 20 photon SBS seems viable if the detection efficiency and coupling efficiency of the quantum system reach 0.95 and 0.8, respectively.



**Figure 11.** The requirements of coupling and detection efficiency for scattershot boson-sampling to guarantee SNR>2 when increasing photon number (a) without shutters and number-resolving detectors, (b) with shutters but without number-resolving detectors, (c) without shutters but with number-resolving detectors, (d) with shutters and number-resolving detectors. The efficiency denoted on the vertical axis is the coupling efficiency from SPDC sources to the interferometer. The numbers labeled in the broken lines indicate different detection efficiencies of the quantum system.

**Appendix A. Numerically modelling the effect of signal-to-noise ratio**

Here we provide more details on the reasonable range of the signal-to-noise ratio (SNR) of boson-sampling output data. More specifically, we use a Bayesian approach [90] to validate ideal boson-sampling (SNR approaches infinity) against hypothetical mixed sampling (SNR equals some finite value). For the sake of simplicity, the hypothetical sampling is mixing between ideal boson-sampling and thermal state sampling [91]. As shown in figure A.1, given  and , only if , the Bayesian analysis can reach a relatively high success rate. Accordingly, we suppose  as a minimum requirement in the Appendix B.



**Figure A.1**. Bayesian approach to validate ideal boson-sampling (SNR approaches infinity) against hypothetical mixed sampling (SNR equals some finite value). Here we set n=5, m=10. Red (blue) lines correspond to the boson-sampling validation as +1 (-1) standard deviations by averaging over a numerical simulation with 1,000 Haar-uniform unitary matrices. (a) , success rate: ; (b) , success rate: ; (c) , success rate: ; (d) , success rate: ; (e) , success rate: ; (f) , success rate: .

**Appendix B. Details of scattershot boson-sampling analysis**

*Case 1.*

First, let us start from the case without shutters and resolving detectors. When *n* coincidence counting were both recorded at heralding and at the output, the probability that it is a correct boson-sampling signal is given by

 (14)

, where the probability of simultaneously generating s single photons and t twin-photons in *m* sources is

. (15)

Here the subscript c indicates “correct” and (0, 0) indicates without shutter or photon-number-resolving detectors.  is the probability that a non-photon-number-resolving detector will click if a twin-photon is injected, and .

The probability of getting a signal satisfying the condition of *n* coincidence counting is of the form

 (16)

, where the subscript t indicates the total probability of getting a signal.

To ensure SNR larger than two, which is quite a low requirement for correct sampling, we plot the requirement for coupling and detection efficiency when increasing *n* in figure 10(a). As shown in figure 10(a), the requirements for detectors and coupling are rather high if without shutter or photon-number-resolving detector **(CHECK THIS SENTENCE)**. When the coupling efficiency is 0.9 and detection efficiency is 0.95, we can only scale boson-sampling experiments to 4 photons.

*Case 2.*

If employing shutters, but without photon-number-resolving detectors, we can apply some small modifications to the equations (14) and (16) to get



(17)

 (18)

To satisfy SNR>2, we plot the requirements to achieve quantum supremacy with increasing photon number in figure 10(b) **(CHECK THIS SENTENCE)**. From the figure, we can see that when the size of a boson-sampler is relatively small, i.e. *n*=3~4, the requirement for coupling efficiency is relaxed a little by employing the shutter. We can implement 4 photon SBS with 0.8 coupling efficiency and 0.95 detection efficiency without photon-number-resolving detectors.

*Case 3.*

When coming to the case with photon-number-resolving detectors but without shutters, using  to subsititude  in equation (14) and (16), we have

 (19)

 (20)

In the same way, to guarantee SNR>2, we determine the lower bound for coupling efficiency and detection efficiency against the scaling of the SBS device. Comparing figure 10(a) and figure 10(b), if photon-number-resolving detectors of 0.95 detection efficiency were utilized, the quantum system could perform almost as well as a quantum system with perfect photodetectors that were non-photon-number-resolving.

*Case 4.*

Finally, both shutters and photon-number-resolving detectors are available. Just employing equations (17) and (18), and changing to as we have done before, we obtain



(21)



(22)

We are glad to see that, in this case, the rigorous requirements in detection efficiency and coupling efficiency of the quantum system can be substantially loosened. And a SBS of 20 photons appears viable if the detection and coupling efficiency of the device can reach 0.95 and 0.8, respectively.