

The Resurgence of the Linear Optics Interferometer — Recent Advances & Applications

Si-Hui Tan^{1,*} and Peter P. Rohde^{2,†}

¹*Singapore University of Technology and Design, 8 Somapah Road, Singapore*

²*Centre for Quantum Software & Information (CQSI), Faculty of Engineering & Information Technology, University of Technology Sydney, NSW 2007, Australia*

(Dated: April 16, 2017)

Contents

I. Introduction	1
II. Mathematical background	1
III. Optical encoding of quantum information on single-photons	2
1. Polarisation	2
2. Dual-rail	2
3. Time-bins	2
IV. Efficient circuit decompositions of linear optics networks	2
V. Reconstructing the linear optical network	2
VI. Experimental implementation	3
A. State preparation	3
1. Single-photons	3
2. Bell pairs	3
3. Coherent states	3
4. Squeezed states	3
B. Linear optics networks	3
1. Bulk-optics	3
2. Waveguides	3
3. Time-bins	3
C. Measurement	3
1. Photodetection	3
2. Homodyning	3
VII. Applications for linear optics interferometry	3
A. Linear optics quantum computation	3
B. Boson-sampling	3
C. Quantum metrology	3
D. Encrypted quantum computation	3
VIII. State of the art	3
IX. Conclusion	3
Acknowledgments	3
References	3

I. INTRODUCTION

Si-Hui can colour code things she adds like this
And Peter can do it like this

*sihui.tan@sutd.edu.sg

†dr.rohde@gmail.com; URL: <http://www.peterrohde.org>

Let's add comments and questions like this

Technical advancements have been made on many fronts. It is possible now to put single-photon sources and linear-optical networks on a silica chip. The advantage of using such integrated photonics over bulk optics is that it is more stable against phase fluctuations, and miniaturized. This increases the scalability of optical implementations of quantum information protocols.

II. MATHEMATICAL BACKGROUND

Mathematical representation for LO networks, and very basic background on quantum optics

A idealized single photon in a quantum interferometer is described by its creation operator \hat{a}_j^\dagger , where j is the label of the mode the photon is in within the interferometer. The creation and annihilation operators satisfy the bosonic commutator relationship $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{j,k}$. A similar commutator relationship can be written up when more degrees of freedom, such as polarization, orbital angular momentum, and time-bins (Tillmann et al., 2015; Bozinovic et al., 2013; Nicolas et al., 2014; Humphreys et al., 2013; Donohue et al., 2013), are present. When multiple photons are present, they experience quantum interference when all quantum labels are the same.

The action of a $2d$ -port linear optical interferometer (with an equal number of input and output ports) is expressed as an application of unitary operations on the creation operators,

$$b_i^\dagger = \sum_{j=1}^d U_{ij} a_j^\dagger, \quad (1)$$

where a_j^\dagger and b_i^\dagger are the creation operators of a single input and output photon in the j -th and i -th modes respectively, and $U \in SU(d)$. All such transformation can be expressed as sequence of beamsplitters and waveplates (Reck et al., 1994). In the case when photons have additional labels, for instance, if they have internal labels on top of spatial labels, it is also possible to derive an analogous decomposition, known as a cosine-sine decomposition (Dhand and Goyal, 2015), that realizes the unitary transformation on the photons into a sequence of

beamsplitters and internal transformations. Toolkits using group theory are being developed to deal with partial distinguishabilities among interfering photons (Tan et al., 2013; de Guise et al., 2014, 2016). Others use quantum-to-classical transitions to explain multiparticle interference (Ra et al., 2013). *Experimental implementation* [Walmsley, Jennewein]

III. OPTICAL ENCODING OF QUANTUM INFORMATION ON SINGLE-PHOTONS

1. Polarisation
2. Dual-rail
3. Time-bins

IV. EFFICIENT CIRCUIT DECOMPOSITIONS OF LINEAR OPTICS NETWORKS

The task of implementing an arbitrary quantum computation on linear optics comes down to implementing an arbitrary $n \times n$ unitary matrix. If a non-unitary transformation is desired, it can be embedded within a unitary matrix with larger dimensions. An algorithm for expressing an arbitrary unitary matrix *exactly* in terms of a sequence of $\mathcal{O}(n^2)$ beamsplitters and phase-shifters exists (Reck et al., 1994). Alternatively, Mach-Zedner interferometer can also be used as building blocks instead of beamsplitters and phase shifters (Reck et al., 1994; Englert et al., 2001). Later, it has been shown that any nontrivial beam splitter, that does more than swapping modes around or add phases to them, is universal for linear optics (Bouland and Aaronson, 2014). However, they do not provide a construction for arbitrary unitaries.

If the linear optical transformations can be realized on various degrees of freedom of light, then it is possible to realize a $n \times n$ arbitrary unitary transformation, where $n = n_s n_p$ for n_s spatial modes, and n_p internal modes, by a sequence of $\mathcal{O}(n_s^2 n_p)$ beamsplitters and $\mathcal{O}(n_s^2)$ internal transformations (Dhand and Goyal, 2015). Their approach reduces the required number of beamsplitters but increases the total number of optical elements needed increases by a factor of 2.

V. RECONSTRUCTING THE LINEAR OPTICAL NETWORK

In many practical situations, the structure of a linear optical device in terms of its constituent beamsplitters and phase shifters is known once it is built. However, owing to manufacturing imperfections, a precise characterization of these devices is still needed post-production. One of the ways, this can be done is via a quantum process tomography (Mitchell et al., 2003; O'Brien et al., 2004; Lobino et al., 2008; Rahimi-Keshari et al., 2011). However, quantum process tomography is an expensive

method in terms of number of measurements required to characterize the network, and it becomes impractical for large optical networks which can now be as large as 900 modes (check citations for number of modes). Alternative characterization protocols have been developed using quantum interference of various quantum light sources (Laing and O'Brien, 2012; Rahimi-Keshari et al., 2013) in the linear optical device.

Generally, the unitary matrix of the $d \times d$ linear optical device are complex numbers $U_{ij} = r_{ij}e^{i\theta_{ij}}$, where $0 \leq r_{ij} \leq 1$, and $0 \leq \theta_{ij} \leq 2\pi$. The scheme in (Laing and O'Brien, 2012) relies on injecting one-photon and two-photon states into the linear optical network with correlated photon detection. First, they note some equivalencies: two unitaries U and U' are equivalent if there exist two diagonal unitary matrices D_1^U and D_2^U such that $U' = D_1^U U D_2^U$, because these diagonal matrices are regarded as unknown and trivial phases on the input and output ports of the network, to which the one-photon and two-photon data are insensitive to. This reduces the first row and first column elements to real numbers, i.e. $\theta_{1j} = \theta_{i1} = 0$. Second, the photon statistics remain unchanged under the complex conjugation of U . Thus, the imaginary part of $M_{2,2}$ must be non-negative. Then assuming that the first two rows and columns are non-vanishing, and that there is no total loss in the interferometer, the matrix to be recovered is

$$U = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1m} \\ r_{21} & r_{22}e^{i\theta_{22}} & \dots & r_{2m}e^{i\theta_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2}e^{i\theta_{m2}} & \dots & r_{mm}e^{i\theta_{mm}} \end{pmatrix}. \quad (2)$$

In the paper, they showed that it is possible to write the parameters r_{ij} , and θ_{ij} for $i, j \geq 2$ in terms of the $2m - 1$ real parameters of the first column and row, and the visibility of two-photon inputs. The remaining $2m - 1$ real parameters can be found via one-photon transmissions. An increased accuracy of the characterization is possible by estimating and correcting systematic errors that arise due to mode mismatch (Dhand et al., 2016). Others have used numerical methods to find the closest parameters that yield the observed visibilities (Spagnolo et al., 2016; Tillmann et al., 2016).

Another characterization method was presented that is similar to (Laing and O'Brien, 2012) with the important exception that coherent states are to be used instead (Rahimi-Keshari et al., 2013; Heilmann et al., 2015). Such states are produced by a standard laser source, thus reducing the resource needed. The r_{jk} terms are found by the square root of the ratios of output intensity at the k th port to the input intensity at the j th port. The remaining phases θ_{ij} are found by the interference pattern given by a two-mode coherent state $|\alpha_1\rangle|\alpha_2\rangle$ created by splitting a single coherent state on a 50-50 beamsplitter, and then imparting a relative phase, ϕ , between them. The states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are input into port 1, and j

respectively. The output intensity at the j th port is

$$I_k = I(r_{1k}^2 + r_{jk}^2 + 2r_{1k}r_{kj}\cos(\phi + \theta_{jk})) , \quad (3)$$

where $\theta_{jk} = 0$ for $k = 1$. By scanning the phase shift ϕ and then locating the maximum value of I_k for $j = 2, \dots, m$, all unknown phases can be found via $\theta_{jk} = 2\pi - \phi$. An elegant extension of the scheme of Rahimi-Keshari *et al.* removes the need for precise control of the phase shift ϕ (Heilmann *et al.*, 2015) by suggesting instead to plot the output intensity I_k with respect to the input intensity I . In time, the natural drift in the laser source will cause this plot to trace out an ellipse, known as a Lissajous figure. whose orientation and direction of evolution will give the phase θ_{jk} and its sign respectively.

VI. EXPERIMENTAL IMPLEMENTATION

A. State preparation

Cat state–Schoelkopf group

1. Single-photons
2. Bell pairs
3. Coherent states
4. Squeezed states

B. Linear optics networks

1. Bulk-optics
2. Waveguides
3. Time-bins

Discuss fibre-loop architecture

C. Measurement

1. Photodetection

Discuss number-resolved and bucket detectors, multiplexed detection, APDs, current micropillar detectors

2. Homodyning

VII. APPLICATIONS FOR LINEAR OPTICS INTERFEROMETRY

A. Linear optics quantum computation

B. Boson-sampling

C. Quantum metrology

Discuss NOON states - Heisenberg limited
Discuss MORDOR scheme

D. Encrypted quantum computation

VIII. STATE OF THE ART

Discuss where experiments are at at the moment

IX. CONCLUSION

Acknowledgments

P.P.R. is funded by an ARC Future Fellowship (project FT160100397).

References

- M. Tillmann, S.-H. Tan, S. E. Stoeckl, B. C. Sanders, H. de Guise, R. Heilmann, S. Nolte, A. Szameit, and P. Walther, Phys. Rev. X **5**, 041015 (2015), URL <http://link.aps.org/doi/10.1103/PhysRevX.5.041015>.
- N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, A. Willner, and S. Ramachandran, Science **340**, 1545 (2013).
- A. Nicolas, L. Veissier, L. Giner, E. Giacobino, D. Maxein, and J. Laurat, Nat. Photonics **8**, 234 (2014).
- P. C. Humphreys, B. J. Metcalf, J. B. Spring, M. Moore, X.-M. Jin, M. Barbieri, W. S. Kolthammer, and I. A. Walmsley, Phys. Rev. Lett. **111**, 150501 (2013), URL <http://link.aps.org/doi/10.1103/PhysRevLett.111.150501>.
- J. M. Donohue, M. Agnew, J. Lavoie, and K. J. Resch, Phys. Rev. Lett. **111**, 153602 (2013), URL <http://link.aps.org/doi/10.1103/PhysRevLett.111.153602>.
- M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994), URL <http://link.aps.org/doi/10.1103/PhysRevLett.73.58>.
- I. Dhand and S. K. Goyal, Phys. Rev. A **92**, 043813 (2015), URL <http://link.aps.org/doi/10.1103/PhysRevA.92.043813>.
- S.-H. Tan, Y. Y. Gao, H. de Guise, and B. C. Sanders, Phys. Rev. Lett. **110**, 113603 (2013), URL <http://link.aps.org/doi/10.1103/PhysRevLett.110.113603>.
- H. de Guise, S.-H. Tan, I. P. Poulin, and B. C. Sanders, Phys. Rev. A **89**, 063819 (2014), URL <http://link.aps.org/doi/10.1103/PhysRevA.89.063819>.
- H. de Guise, D. Spivak, J. Kulp, and I. Dhand, J. Phys. A:Math. Theor. **49**, 09LT01 (2016).

- Y.-S. Ra, M. C. Tichy, H.-T. Lim, O. Kwon, F. Mintert, A. Buchleitner, and Y.-H. Kim, Proceedings of the National Academy of Sciences **110**, 1227 (2013), eprint <http://www.pnas.org/content/110/4/1227.full.pdf>, URL <http://www.pnas.org/content/110/4/1227.abstract>.
- B.-G. Englert, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. A **63**, 032303 (2001), URL <https://link.aps.org/doi/10.1103/PhysRevA.63.032303>.
- A. Bouland and S. Aaronson, Phys. Rev. A **89**, 062316 (2014), URL <https://link.aps.org/doi/10.1103/PhysRevA.89.062316>.
- M. W. Mitchell, C. W. Ellenor, S. Schneider, and A. M. Steinberg, Phys. Rev. Lett. **91**, 120402 (2003).
- J. L. O'Brien, G. J. Pryde, A. Gilchrist, D. F. V. James, N. K. Langford, T. C. Ralph, and A. G. White, Phys. Rev. Lett. **93**, 080502 (2004), URL <https://link.aps.org/doi/10.1103/PhysRevLett.93.080502>.
- M. Lobino, D. Korystov, C. Kupchak, E. Figueroa, B. C. Sanders, and A. I. Lvovsky, Science **322**, 563 (2008), ISSN 0036-8075, eprint <http://science.sciencemag.org/content/322/5901/563.full.pdf>, URL <http://science.sciencemag.org/content/322/5901/563>.
- S. Rahimi-Keshari, A. Scherer, A. Mann, A. T. Reza-khani, A. I. Lvovsky, and B. C. Sanders, New Journal of Physics **13**, 013006 (2011), URL <http://stacks.iop.org/1367-2630/13/i=1/a=013006>.
- A. Laing and J. O'Brien, arXiv: 1208.2868v1 (2012).
- S. Rahimi-Keshari, M. A. Broome, R. Fickler, A. Fedrizzi, T. C. Ralph, and A. G. White, Opt. Express **21**, 13450 (2013), URL <http://www.opticsexpress.org/abstract.cfm?URI=oe-21-11-13450>.
- I. Dhand, A. Khalid, H. Lu, and B. C. Sanders, Journal of Optics **18**, 035204 (2016), URL <http://stacks.iop.org/2040-8986/18/i=3/a=035204>.
- N. Spagnolo, E. Maiorino, C. Vitelli, M. Bentivegna, A. Crespi, R. Ramponi, P. Mataloni, R. Osellame, and F. Sciarrino (2016), eprint arXiv:1610.03291v1.
- M. Tillmann, C. Schmidt, and P. Walther, Journal of Optics **18**, 114002 (2016), URL <http://stacks.iop.org/2040-8986/18/i=11/a=114002>.
- R. Heilmann, M. Grfe, S. Nolte, and A. Szameit, Science Bulletin **60**, 96 (2015), ISSN 2095-9273, URL <http://www.sciencedirect.com/science/article/pii/S2095927316305400>.