The Resurgence of the Linear Optics Interferometer — Recent Advances & Applications

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(Dated: April 20, 2017)

Contents

I. INTRODUCTION

Si-Hui can colour code things she adds like this
And Peter can do it like this

Let's add comments and questions like this

Technical advancements have been made on many fronts. It is possible now to put single-photon sources and linear-optical networks on a silica chip. The advantage of using such integrated photonics over bulk optics is that it is more stable against phase fluctuations, and miniaturized. This increases the scalability of optical implementations of quantum information protocols.

II. MATHEMATICAL BACKGROUND

Mathematical representation for LO networks, and very basic background on quantum optics

A idealized single photon in a quantum interferometer is described by its creation operator $\hat{a}_j^\dagger$, where j is the label of the mode the photon is in within the interferometer. The creation and annihilation operators satisfy the bosonic comutator relationship $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{j,k}$. A similar commutator relationship can be written up when more degrees of freedom, such as polarization, orbital angular momentum, and time-bins (Tillmann et al., 2015; Bozinovic et al., 2013; Nicolas et al., 2014; Humphreys et al., 2013; Donohue et al., 2013), are present. When multiple photons are present, they experience quantum interference when all quantum labels are the same.

The action of a $2d$-port linear optical interferometer (with an equal number of input and output ports) is expressed as an application of unitary operations on the creation operators,

$$b_i^\dagger = \sum_{j=1}^{d} U_{ij} a_j^\dagger \ , \qquad (1)$$

where $a_j^\dagger$ and $b_i^\dagger$ are the creation operators of a single input and output photon in the j-th and i-th modes respectively, and $U \in SU(d)$. All such transformation can be expressed as sequence of beamsplitters and waveplates (Reck et al., 1994). In the case when photons have additional labels, for instance, if they have internal labels on top of spatial labels, it is also possible to derive an analogous decomposition, known as a cosine-sine decomposition (Dhand and Goyal, 2015), that realizes the unitary transformation on the photons into a sequence of

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beamsplitters and internal transformations. Toolkits using group theory are being developed to deal with partial distinguishabilities among interfering photons (Tan et al., 2013; de Guise et al., 2014, 2016). Others use quantum-to-classical transitions to explain multiparticle interference (Ra et al., 2013). Experimental implementation [Walmsley, Jennewein]

III. OPTICAL ENCODING OF QUANTUM INFORMATION ON SINGLE-PHOTONS

1. Polarisation

2. Dual-rail

3. Time-bins

Time-bin qubits is quantum information that is encoded on the time-of-arrival of single photons. It was first conceived for a single-photon passing through a Mach-Zedner interferometer with its two paths having different lengths (Brendel et al., 1999). If the photon passes through the shorter (longer) path, then it will arrive at the output port in an "early" ("late") time bin. Thus, photon will exit the interferometer in a state that is a superposition of these two states. Owing to difficulties in implementing qubit operations in this basis, it was at that time mostly used for demonstrating quantum communication over long distance (Thew et al., 2002; Marcikic et al., 2004). With the advent of faster optical components, and single-photon detectors, it has become feasible to perform any single qubit operations, and a post-selected CPHASE gate on time-bin qubits (Humphreys et al., 2013). At the same time, an ultrafast measurement technique for recovering time-bin qubits was also demonstrated (Donohue et al., 2013).

IV. EFFICIENT CIRCUIT DECOMPOSITIONS OF LINEAR OPTICS NETWORKS

The task of implementing an arbitrary quantum computation on linear optics comes down to implementing an arbitrary $n \times n$ unitary matrix. If a non-unitary transformation is desired, it can be embedded within a unitary matrix with larger dimensions. An algorithm for expressing an arbitrary unitary matrix *exactly* in terms of a sequence of $\mathcal{O}(n^2)$ beamsplitters and phase-shifters exists (Reck et al., 1994). Alternatively, Mach-Zedner interferometer can also be used as building blocks instead of beamsplitters and phase shifters (Reck et al., 1994; Englert et al., 2001). Later, it has been shown that any nontrivial beam splitter, that does more than swapping modes around or add phases to them, is universal for linear optics (Bouland and Aaronson, 2014). However, they do not provide a construction for arbitrary unitaries.

If the linear optical transformations can be realized on various degrees of freedom of light, then it is possible to realize a $n \times n$ arbitrary unitary transformation, where $n = n_s n_p$ for n_s spatial modes, and n_p internal modes, by a sequence of $\mathcal{O}(n_s^2 n_p)$ beamsplitters and $\mathcal{O}(n_s^2)$ internal transformations (Dhand and Goyal, 2015). Their approach reduces the required number of beamsplitters but increases the total number of optical elements needed increases by a factor of 2.

V. RECONSTRUCTING THE LINEAR OPTICAL NETWORK

In many practical situations, the structure of a linear optical device in terms of its constituent beamsplitters and phase shifters is known once it is built. However, owing to manufacturing imperfections, a precise characterization of these devices is still needed post-production. One of the ways, this can be done is via a quantum process tomography (Mitchell et al., 2003; O'Brien et al., 2004; Lobino et al., 2008; Rahimi-Keshari et al., 2011) . However, quantum process tomography is an expensive method in terms of number of measurements required to characterize the network, and it becomes impractical for large optical networks which can now be as large as 900 modes (check citations for number of modes). Alternative characterization protocols have been developed using quantum interference of various quantum light sources (Laing and O'Brien, 2012; Rahimi-Keshari et al., 2013) in the linear optical device.

Generally, the unitary matrix of the $d \times d$ linear optical device are complex numbers $U_{ij} = r_{ij} e^{i\theta_{ij}}$, where $0 \leq r_{ij} \leq 1$, and $0 \leq \theta_{ij} \leq 2\pi$. The scheme in (Laing and O'Brien, 2012) relies on injecting one-photon and two-photon states into the linear optical network with correlated photon detection. First, they note some equivalencies: two unitaries U and U' are equivalent if there exist two diagonal unitary matrices D_1^U and D_2^U such that $U' = D_1^U U D_2^U$, because these diagonal matrices are regarded as unknown and trivial phases on the input and output ports of the network, to which the one-photon and two-photon data are insensitive to. This reduces the first row and first column elements to real numbers, i.e. $\theta_{1j} = \theta_{i1} = 0$. Second, the photon statistics remain unchanged under the complex conjugation of U. Thus, the imaginary part of $M_{2,2}$ must be non-negative. Then assuming that the first two rows and columns are non-vanishing, and that there is no total loss in the interferometer, the matrix to be recovered is

$$U = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1m} \\ r_{21} & r_{22}e^{i\theta_{22}} & \dots & r_{2m}e^{i\theta_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2}e^{i\theta_{m2}} & \dots & r_{mm}e^{i\theta_{mm}} \end{pmatrix} . \qquad (2)$$

In the paper, they showed that it is possible to write the paramaters r_{ij}, and θ_{ij} for $i, j \geq 2$ in terms of the $2m-1$ real parameters of the first column and row, and the visibility of two-photon inputs. The remaining $2m-1$ real parameters can be found via one-photon transmissions. An increased accuracy of the characterization is possible by estimating and correcting systematic errors that arise due to mode mismatch (Dhand et al., 2016). Others have used numerical methods to find the closest parameters that yield the observed visibilities (Spagnolo et al., 2016; Tillmann et al., 2016).

Another characterization method was presented that is similar to (Laing and O'Brien, 2012) with the important exception that coherent states are to be used instead (Rahimi-Keshari et al., 2013; Heilmann et al., 2015). Such states are produced by a standard laser source, thus reducing the resource needed. The r_{jk} terms are found by the square root of the ratios of output intensity at the kth port to the input intensity at the jth port. The remaining phases θ_{ij} are found by the interference pattern given by a two-mode coherent state $|\alpha_1\rangle|\alpha_2\rangle$ created by splitting a single coherent state on a 50-50 beamsplitter, and then imparting a relative phase, ϕ, between them. The states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are input into port 1, and j respectively. The output intensity at the jth port is

$$I_k = I(r_{1k}^2 + r_{jk}^2 + 2r_{1k}r_{kj}\cos(\phi + \theta_{jk})) \ , \qquad (3)$$

where $\theta_{jk} = 0$ for $k = 1$. By scanning the phase shift ϕ and then locating the maximum value of I_k for $j = 2, \ldots, m$, all unknown phases can be found via $\theta_{jk} = 2\pi - \phi$. An elegant extension of the scheme of Rahimi-Keshari *et al.* removes the need for precise control of the phase shift ϕ (Heilmann et al., 2015) by suggesting instead to plot the output intensity I_k with respect to the input intensity I. In time, the natural drift in the laser source will cause this plot to trace out an ellipse, known as a Lissajous figure. whose orientation and direction of evolution will give the phase θ_{jk} and its sign respectively.

VI. EXPERIMENTAL IMPLEMENTATION

A. State preparation

1. Single-photons

SPDC, quantum-dot micropillar sources

2. Bell pairs

3. Coherent states

4. Squeezed states

B. Linear optics networks

1. Bulk-optics

2. Waveguides

3. Time-bins

Discuss fibre-loop architecture
Peter to fill this in (Motes et al., 2014)

C. Measurement

1. Photodetection

Discuss number-resolved and bucket detectors, multiplexed detection, APDs, current micropillar detectors

2. Homodyning

VII. APPLICATIONS FOR LINEAR OPTICS INTERFEROMETRY

A. Linear optics quantum computation

B. Boson-sampling

C. Quantum metrology

Discuss NOON states - Heisenberg limited
Discuss MORDOR scheme

D. Encrypted quantum computation

VIII. STATE OF THE ART

Discuss where experiments are at at the moment

IX. CONCLUSION

Acknowledgments

P.P.R. is funded by an ARC Future Fellowship (project FT160100397).

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