Notes on economics

(Dated: June 6, 2017)

I. FORWARD CONTRACT PRICING MODEL

We wish to price the future value of a fixed number of qubits, that are traded as a time-shared asset, unified with the global network.

The standard forward pricing model is,

$$F(T) = S_0 e^{(r-q)T} - \sum_{t=0}^{T} D_t e^{(r-q)(T-t)}, \tag{1}$$

where T is the time of maturity (at which the contract is executed), r is the risk-free rate of return, q is the cost of carry, and D_t is the expected dividend at time t.

Assume cost-of-carry, q, is zero since maintenance of physical computer assets is negligible and the goods are effectively non-perishable.

Assume that the dollar value of FLOPS, L_t, is decreasing exponentially over time t. This is what we observe for the classical Moore's Law, and it is reasonable to assume a similar exponential trajectory in the future,

$$L_t = L_0 {\gamma_L}^{-t}, \tag{2}$$

where $\gamma_L \geq 1$ characterises the rate of exponential decay.

Let the number of FLOPS associated with the tradable asset grow according to the quantum economic leverage over time, λ_t. That is, as the future network grows, so does our leverage, and thus our classical-equivalent processing power.

Assume the number of qubits in the global network in the future is growing exponentially over time (i.e the rate of progress of quantum technology will observe a Moore's Law-like behaviour – exponential reduction in qubit manufacturing costs over time will yield exponential growth in the number of qubits in existence),

$$N_t = N_0 {\gamma_N}^t, \tag{3}$$

where $\gamma_N \geq 1$ characterises the rate of exponential growth in the number of qubits available to the quantum network.

The leverage then scales as,

$$\begin{aligned} \lambda_t &= \eta_n \frac{f_s(N_t)}{N_t} \\ &= \eta_n \frac{f_s(N_0 {\gamma_N}^t)}{N_0 {\gamma_N}^t}, \end{aligned} \tag{4}$$

where,

$$\eta_n = \frac{n}{f_s(n)}, \tag{5}$$

and n is the number of qubits involved in the transaction, which we treat as a constant, since we are valuing the future price of an asset comprising a fixed number of qubits.

Let the dividend, D_t, be a measure of the dollar value of the n qubits' computational power at a given time t. This scales with the leverage and dollars-per-FLOPS,

$$\begin{aligned} D_t &= L_t \lambda_t \\ &= \frac{\eta_n L_0 {\gamma_L}^{-t} f_s(N_t)}{N_t} \\ &= \frac{\eta_n L_0}{N_0} \cdot \frac{f_s(N_0 {\gamma_N}^t)}{(\gamma_L \gamma_N)^t}. \end{aligned} \tag{6}$$

Let the spot price, S_0, be the present day price ($t = 0$) of FLOPS times the leverage at time T (i.e its future computing power),

$$\begin{aligned} S_0 &= L_0 \lambda_T \\ &= L_0 \eta_n \frac{f_s(N_T)}{N_T} \\ &= L_0 \eta_n \frac{f_s(N_0 {\gamma_N}^T)}{N_0 {\gamma_N}^T} \end{aligned} \tag{7}$$

Then the forward price is,

$$F(T) = \frac{e^{rT} L_0 \eta_n}{N_0} \left[\frac{f_s(N_0 {\gamma_N}^T)}{{\gamma_N}^T} - \sum_{t=0}^{T} \frac{f_s(N_0 {\gamma_N}^t)}{(\gamma_N \gamma_L e)^t} \right]. \tag{8}$$

A. Linear scaling functions

As a first example, to provide a benchmark, let the scaling function be linear in n,

$$f_s(n) = \alpha n, \tag{9}$$

as we would (approximately) observe for clustered classical computers, whereby there is no leverage. Then the forward price reduces to,

$$\begin{aligned} F(T) &= e^{rT} L_0 \left[1 - \sum_{t=0}^{T} (\gamma_L e)^{-t} \right] \\ &= e^{rT} L_0 \left[\frac{1 - (e\gamma_L)^{-T}}{1 - e\gamma_L} \right]. \end{aligned} \tag{10}$$

B. Quadratic scaling functions

Let the scaling function be a simple quadratic polynomial (e.g the quantum resources are being employed for Grover-like algorithms, such as speeding up **NP**-complete optimisation problems),

$$f_s(n) = \alpha n^2 \tag{11}$$

Then the forward price reduces to,

$$F(T) = \frac{e^{rT} L_0 N_0}{n^2} \left[{\gamma_N}^T - \sum_{t=0}^{T} \left(\frac{\gamma_N}{\gamma_L e} \right)^t \right]$$
$$= \frac{e^{rT} L_0 N_0}{n^2} \left[{\gamma_N}^T + \frac{\gamma_N \left(\frac{\gamma_N}{e\gamma_L} \right)^T - e\gamma_L}{e\gamma_L - \gamma_N} \right]. \quad (12)$$

Something I don't understand: arbitrage should enforce the condition that 2 qubits costs twice as much as 1 qubit. This formula doesn't reflect this.

C. Exponential scaling functions

To do