FIG. 1 Quantum clock synchronization schemes. (top) The proposal by Jozsa, Dowling, and co-workers where a singlet state is shared and measured in an ensemble of qubits (Jozsa *et al.*, 2000). (bottom) The proposal by Lukin, Ye, and co-workers where a GHZ state is generated between satellites to obtain measure the frequency drift at the Heisenberg limit (Komar *et al.*, 2014).

1. Quantum clock synchronization

Clock synchronization is a fundamental task that is has widespread applications, ranging from navigation, telecommunications, financial transactions, the internet, and many scientific applications. Of these, the Global Positioning System (GPS) has become a day-to-day necessity for a large fraction of the human population as it has been increasingly built into smartphones and other devices. The GPS system famously relies upon very precise clock synchronization to perform its task through a process of quadrangulation from several satellites. Due to the high speed of light, one requires in practice highly synchronized clocks which are accurate to a centralized time standard to the $\sim$ ns level. This allows positioning to be performed to the $\sim$ m level, which is acceptable for many applications. GPS satellites have atomic clocks that are stable to one part in 10^{13}, so that active corrections can maintain the accurate to the $\sim$ ns level. The great success of the GPS system in turn has created a further demand for increasingly precise navigation. For example, autonomous vehicles would immediately benefit from a more precise navigation system. In principle, technology for more stable clocks is already present, with atomic clocks exceeding stabilities of those on satellites being routinely produced, and optical atomic clocks now reaching stabilities of one part in 10^{18} (Ludlow *et al.*, 2015). An outstanding question is then how to synchronize these clocks given their remarkable stabilities.

Several past works have examined the problem of clock synchronization in space (see also Sec. [TB: refer to other section on clock sync]). In the proposal of Jozsa and co-workers, many copies of shared entanglement in a singlet state is first distributed and stored on the clock states of an atomic clock (**?**). The measurement is then made by one of the parties, which collapses the states simultaneously at each party, and the time evolution of the states begins. Classical information is exchanged between them, which reveals the time elapsed since the measurement, which can be used to synchronize the clocks. While the original protocol only allowed allowed clock synchronization between two parties, similar ideas were used to extend it to a multiparty context (**???**). In a more recent proposal, a shared GHZ state is prepared across all the nodes of the quantum network, which allows for a quantum metrologically enhanced detection of the clock signal drift at the Heisenberg limit (**?**). The use of shared resources acts to improve the overall precision, allowing for an optimal scheme for the qubit resources that are used. Several other proposals have also been made, which are quantum versions of Eddington's slow clock transport where the qubit keeps time of the transmission time (**??**).

Experimentally, there have been several demonstrations of the protocol, albeit at relatively short distances. Continuous time-bin entangled photons were used as the entanglement resource to obtain a time-correlation between a distance of 3 km (Valencia *et al.*, 2004), and another technique based on Hong-Ou-Mandel interferometry was performed with a 4 km fiber link (Quan *et al.*, 2016). Several other demonstrations based on NMR (Kong *et al.*, 2017; Zhang *et al.*, 2004) have also been performed.

There are however several outstanding problems with the quantum clock synchronization scheme as presented above. In the scheme of Jozsa and co-workers, if one starts in a perfect singlet state, the scheme works as intended, but if one instead starts in the state

$$\frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - e^{i\delta}|1\rangle_A|0\rangle_B\right) \tag{1}$$

then one obtains an offset to the synchronizations between the two parties. In practice, such a phase could arise from decoherence induced noise, or differences in the basis conventions that the two parties choose. Thus in practice entanglement purification would to be performed to produce a singlet state with $\delta = 0$ before the synchronization protocol is performed. However, it was argued that to perform the entanglement purification quantum circuit correctly, the timing of the quantum gates would need to be controlled, which requires synchronized clocks (Preskill, 2000) – rendering the syncrhonization impossible. It was previously been shown that such a phase cannot be eliminated using asynchronous entanglement purification (Yurtsever and Dowling, 2002), and hence the protocol remains incomplete in the general case where imperfect singlet pairs are shared.

2. Fundamental physics experiments

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