Continuous variable

(Dated: March 1, 2018)

I. TELEPORTATION

Fig. 1

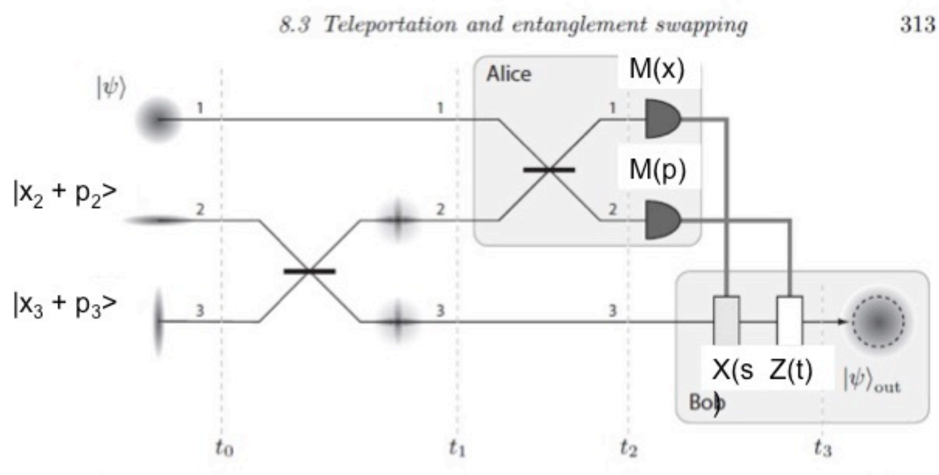

FIG. 1.

Protocol extended to CV by? ? ?

$$|p\rangle = \int dx e^{i/\hbar px} |x\rangle , \qquad |x\rangle = \int dx e^{i/\hbar xp} |p\rangle \tag{1}$$

Displacement operator

$$X(s) = e^{-i/\hbar x \hat{p}} \tag{2}$$

$$X(s) |x\rangle = \int dp e^{-i/\hbar px} e^{-i/\hbar x \hat{p}} |p\rangle \tag{3}$$

$$= dp e^{-i/hbar p(x+s)} |p\rangle \tag{4}$$

$$= |x+s\rangle \tag{5}$$

This is the continuous-variable generalization of the Pauli X operator, in that it displaces the value of the state by amount s.

The analogue of the Pauli Z operator adds a state-dependent phase

$$Z(t) = e^{i/\hbar t \hat{x}} \tag{6}$$

$$e^{i/\hbar t \hat{x}} |x\rangle = e^{i/\hbar px} |x\rangle \tag{7}$$

and acts on the momentum basis state $|p\rangle$ as

$$Z(t) |p\rangle = |p+t\rangle . \tag{8}$$

Before the squeezing operator is applied, we set $t = 0$.

1. At $t = t_0$, the vacuum states in mode 2 and 3 are squeezed. The quadrature operators become

$$x_2(t_0) \to x_2(0) e^r \qquad p_2 \to p_2(0) e^{-r} \tag{9}$$

$$x_3(t_0) \to x_3(0) e^r \qquad p_3 \to p_3(0) e^{-r} \tag{10}$$

2. The 50:50 beam splitter in modes 2 and 3 creates the approximate Bell state necessary for teleportation, and this induces the quadrature transformations

$$x_2(t_1) = \frac{x_2(t_0) + x_3(t_0)}{\sqrt{2}} \qquad x_3(t_1) = \frac{x_2(t_0) - x_3(t_0)}{\sqrt{2}}$$
$$p_2(t_1) = \frac{p_2(t_0) + p_3(t_0)}{\sqrt{2}} \qquad p_3(t_1) = \frac{p_2(t_0) - p_3(t_0)}{\sqrt{2}} \tag{11}$$

3. The second beam splitter induces the transforma-

tion

$$x_1(t_2) = \frac{x_1(t_1) - x_2(t_1)}{\sqrt{2}} \tag{12}$$

$$= \frac{x_1(t_1)}{\sqrt{2}} - \frac{x_2(t_0) + x_3(t_0)}{2} \tag{13}$$

$$p_2(t_2) = \frac{p_1(t_1) + p_2(t_1)}{\sqrt{2}} \tag{14}$$

$$= \frac{p_1(t_0)}{\sqrt{2}} + \frac{p_2(t_0) + p_3(t_0)}{2} \tag{15}$$

Now, if we rearrange Eq. (13) to make $x_2(t_0)$ the subject, and identify that $x_1(t_1) = x_1(t_0)$

$$x_2(t_0) = -2x_1(t_2) + \sqrt{2}x_1(t_1) - x_3(t_0) \tag{16}$$

$$p_2(t_0) = -2p_2(t_2) + \sqrt{2}p_1(t_0) - p_3(t_0) \tag{17}$$

Now, for Bob, $x_3(t_2) = x_3(t_1), p_3(t_2) = p_3(t_1)$, whose state is now

$$x_3(t_2) = \frac{-2x_1(t_2) + \sqrt{2}x_1(t_1) - x_3(t_0) - x_3(t_0)}{\sqrt{2}}$$

$$= -\sqrt{2}x_1(t_2) + x_1(t_0) - \sqrt{2}x_3(0)e^{-r} \tag{18}$$

$$p_3(t_2) = \frac{p_2(t_0) - p_3(t_0)}{\sqrt{2}} \tag{19}$$

$$= \frac{-2p_2(t_2) + \sqrt{2}p_1(t_0) - p_3(t_0) - p_3(t_0)}{\sqrt{2}} \tag{20}$$

$$= p_1(t_0) - \sqrt{2}p_2(t_2) - \sqrt{2}p_3(t_0) \tag{21}$$

4. Alice measures the position quadrature $x_1(t_2)$ in mode 1, yielding the outcome $s/\sqrt{2}$, and she measures the momentum quadrature $p_2(t_2)$ in mode 2, yielding the outcome $t/\sqrt{2}$. She sends these measurement outcomes to Bob. applies the displacements

$$X(u) = \exp(-2is\hat{p}_3) \qquad Z(t) = \exp(2it\hat{x}_3) \tag{22}$$

The output quadratures at Bob's mode become

$$x_3 = x_1 - \frac{\sqrt{2}x_3}{e^r} \qquad p_3 = p_1 - \frac{\sqrt{2}p_3}{e^r} \tag{23}$$

Here the input quadratures are teleported to the output quadratures up to a factor $\frac{\sqrt{2}p_3}{e^r}$. In the limit of infinite squeezing $\to \infty$, the teleportated state is perfect.

II. ENTANGLEMENT SWAPPING

III. ENTANGLEMENT DISTILLATION

Entanglement distribution between distant parties is an essential component to most quantum communication protocols[?] . As it has been discussed, entanglement is a resource in a quantum network, and it is of interest for distant parties to share maximally entangled states.

In practice the transmission channel is never perfect, and noise due to interaction with the environment or imperfect gate operations will reduce the entanglement of a state. If one has several copies of some less than maximally entangled state available, it is possible for two parties to concentrate or distill the entanglement.

In contrast to the qubit version, however, it has been proven that for Gaussian states it is impossible to distill more entanglement by using Gaussian operations[? ? ?] . In order to to distill from Gaussian input states, non-Gaussian operations are necessary.

Proposal[?] , one variant experimentally realized by photon-subtraction[?] .

Circuit model	CV cluster state
Pauli X	$X(s) = \exp[-is\hat{p}]$
Pauli Z	$Z(t) = \exp[it\hat{q}]$
Phase gate	$\hat{P}(\eta) = \exp[i\eta\hat{x}^2]$
Hadamard	$F = \exp[i\frac{\pi}{8}(\hat{p}^2 + \hat{q}^2)]$
C_Z	$C_Z = \exp[\frac{i}{2}q_1 \otimes q_2]$
CNOT	$C_X = \exp[-2i\hat{x}_1 \otimes \hat{p}_2]$

TABLE I.

IV. QUANTUM COMPUTING

A. Quantum gates

Gates for the qubit circuit model and the analogous operations for CV are summarised in Table I.[?]

The operators X, Z can be interpreted as displacement operators

$$D(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \tag{24}$$

which correspond to a translation in phase space. X is a translation in position, and Z is a displacement in momentum.

The X and Z operators can be written as X = D(s/2), Z = D(it/2)

The Fourier gate F is the Gaussian analogue of the qubit Hadamard gate, which corresponds to a $\pi/2$ rotation in phase space, and transforms the quadrature eigenstates from one to another:

$$F = \exp(i\pi/4)\exp(\frac{i\pi}{4}\hat{a}^\dagger\hat{a}) = (\hat{p}^2 + \hat{x}^2) \tag{25}$$

$$F\left|s\right\rangle_x = \left|s\right\rangle_p \tag{26}$$

The controlled-phase gate, C_Z is a two-mode Gaussian gate. It is defined as

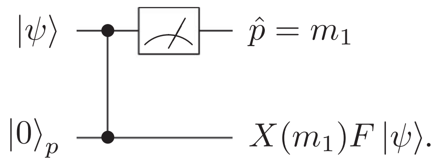

FIG. 2.

$$E_z = \exp\left(\frac{i}{2}\hat{x}_1 \otimes \hat{x}_2\right). \tag{27}$$

It transforms on the quadrature eigenstates as

$$C_Z\,|s\rangle_1\,|t\rangle_2 = \exp(is_1 t_2)/2\,|s\rangle_1\,|t\rangle_2 \tag{28}$$

It transforms the momentum quadratures according to

$$\hat{p}_1 \to \hat{p}_1 + \hat{x}_2, \qquad \hat{p}_2 \to \hat{p}_2 + \hat{x}_1 \tag{29}$$

B. Cluster state/ measurement based

[?] The advantage of this approach is that both the cluster state preparation and comutation can be performed deterministically.

Once the state has been created, a sequence of one-mode (separable) measurement

The one-qubit teleportation give insight into how the cluster-state evolves in this model

Consider perfect squeezing, where the second mode is a momentum eigenstate $|0\rangle_p$. Initially, the state is

$$|\psi\rangle\,|0\rangle_p = \frac{1}{\sqrt{2\pi}}\int dx_1 dx_2 \psi(x_1)\,|x_1\rangle\,|x_2\rangle \tag{30}$$

Applying the C_Z gate results in

$$\frac{1}{2\pi}\int dx_1 dx_2 \psi(x_1)\exp(ix_1x_2/2)\,|x_1\rangle\,|x_2\rangle \tag{31}$$

After measuring $\hat{p}_1$, the state is projected onto $|m_1\rangle\langle m_1|$, we have
$\langle m_1|q_1\rangle = 1/2\pi\exp(-iq_1m_1/2)$

$$\frac{1}{4\pi}\int dx_1 dx_2 \psi(x_1)\exp(ix_1(q_2 - m_1))q_2 \tag{32}$$

Applying the correction $X(m_1)$ gives back the initial state $|\psi\rangle$

We can now consider the teleportation of a quantum gate, which is the key of the measurement-based quantum computing. Consider a variation of the above circuit, where the only difference is the addition of a unitary that is diagonal in the computational basis, and

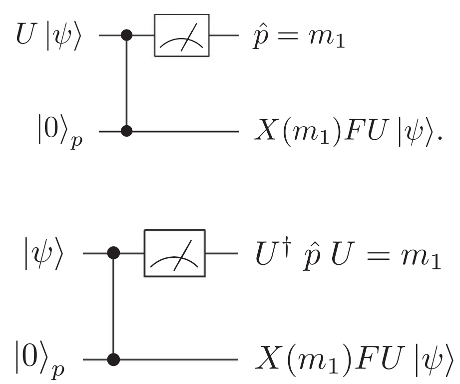

FIG. 3.

therefore commutes with the CPHASE gate, for example $U = \exp(if(\hat{x}))$.

We have just shown that by performing a measurement in the basis $U\hat{x}U^\dagger$, we can absorb the gate into the measurement.

- addition of any non-gaussian projective measurement allows universal QC using CV cluster states
- multimode Gaussian operations can be made in any order

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(33)