Part I
Introduction to quantum mechanics

I. OVERVIEW

II. QUANTUM STATES

A. State vectors

$$
\begin{aligned}
|\psi\rangle &= \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \\
&= \alpha\,|0\rangle + \beta\,|1\rangle\,.
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\langle\psi| &= \begin{pmatrix} \alpha \\ \beta \end{pmatrix}^{\dagger} \\
&= \begin{pmatrix} \alpha^*, \beta^* \end{pmatrix} \\
&= \alpha^*\,\langle 0| + \beta^*\,\langle 1|\,.
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
|\psi\rangle &= \sum_n \alpha_n\,|n\rangle\,, \\
|\phi\rangle &= \sum_n \beta_n\,|n\rangle\,,
\end{aligned} \tag{3}
$$

The *overlap* between two states is defined as,

$$\langle\psi|\phi\rangle = \sum_n \alpha\beta^*, \tag{4}$$

where,

$$\sum_n |\alpha_n|^2 = 1, \tag{5}$$

for normalisation. This is equivalent to writing,

$$\langle\psi|\psi\rangle = 1. \tag{6}$$

B. Density operators

$$\hat{\rho} = \begin{pmatrix} a & c \\ c^* & b \end{pmatrix}.$$

$$\mathrm{tr}(\hat{\rho}) = 1, \tag{7}$$

for normalisation.

$$
\begin{aligned}
\hat{\rho} &= |\psi\rangle\,\langle\psi| \\
&= \begin{pmatrix} |\alpha|^2 & \alpha\beta \\ \alpha^*\beta^* & |\beta|^2 \end{pmatrix}.
\end{aligned}
$$

$$\hat{\rho} = \sum_i p_i \hat{\rho}_i, \tag{8}$$

where the probabilities are normalised such that,

$$\sum_i p_i = 1. \tag{9}$$

C. Reduced states

$$\hat{\rho}_A = \mathrm{tr}_B(\hat{\rho}_{A,B}). \tag{10}$$

$$\mathrm{tr}_B(\hat{\rho}_A \otimes \hat{\rho}_B) = \mathrm{tr}(\hat{\rho}_B)\cdot\hat{\rho}_A. \tag{11}$$

From the cyclic property of the trace, it follows that,

$$\mathrm{tr}(|\psi\rangle\,\langle\phi|) = \langle\psi|\phi\rangle\,. \tag{12}$$

III. EVOLUTION

IV. MEASUREMENT