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On the mass of M31

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ABSTRACT

Recent work by several groups has established the properties of the dwarf satellites to M31. We reexamine the reported kinematics of this group employing a fresh technique we have developed previously. By calculating the distribution of a χ statistic (which we define in the paper) for the M31 system, we conclude that the total mass (disc plus halo) of the primary is unlikely to be as great as that of our own Milky Way. In fact the χ distribution for M31 indicates that, like NGC 3992, it does not have a massive halo. In contrast, the analysis of the satellites of NGC 1961 and NGC 5084 provides strong evidence for massive haloes surrounding both spiral galaxies.

Key words: galaxies: haloes – galaxies: individual: M31 – galaxies: kinematics and dynamics – dark matter.

'There are reasons, increasing in number and quality, to believe that the masses of ordinary galaxies may have been underestimated by a factor of 10 or more' (Ostriker, Peebles & Yahil 1974).

'... there is no independent observational dynamical evidence to support the view that the masses of galaxies are very much larger than those derived in investigations of single galaxies which have been carried out over many years' (Burbidge 1975).

1 INTRODUCTION

According to Mateo (1998) more dwarf galaxies have been discovered as Local Group members since 1971 than during the preceding 222 yr. A large fraction of these systems are bound to M31. Recently, Evans & Wilkinson (2000, hereafter EW), Evans et al. (2000) and Côté, Sargent & Olszewski (2000) have independently estimated the total mass associated with M31 (disc plus halo) and found that it is probably less massive than our Milky Way. Here we present an alternative way of estimating the total mass of M31 using these same data but employing a novel technique we have developed (Erickson, Gottesman & Hunter 1987, 1999; hereafter EGH1 and EGH2, respectively).

The use of the kinematics of dwarf satellite galaxies to investigate the masses and extents of galaxy haloes is a very powerful technique (see EGH1; EGH2; also Zaritsky et al. 1993, 1997, hereafter ZSFW1 and ZSFW2, respectively). Almost all studies have been limited to describing the average properties of a generic galaxy because very few individual systems posses enough satellites to allow unique inferences. Owing to the limitations imposed by projection effects, comparisons between observations and models (see EGH2, and Section 3.1 below) indicate that about seven satellites are required before the signature of a massive halo is likely to be observed. We will discuss this further below. In EGH2 we studied the statistics of over 71 satellite–primary pairs. Unfortunately, very few of the primaries had as many as three satellites. However, NGC 1961 (Gottesman, Hunter & Shostak 1983, hereafter GHS) and NGC 5084 (Carignan et al. 1997), as individual galaxies, meet the requirements for a substantial number of satellites (at least eight for NGC 1961 and seven for NGC 5084). In the Local Group, M31 is surrounded by a cloud of dwarf systems (see EW, for example) and it is our purpose to examine the kinematics of its satellites using the methods we have developed in our earlier papers, and to compare the properties of M31 with those of NGC 1961 and NGC 5084. In the sections that follow, we will briefly review our methodology, discuss the data for the three galaxies being compared and draw some conclusions about the properties of M31.

2 METHODS

We have extended the method introduced by van Moorsel (1982, 1987) for the study of binary galaxies to the study of massive primaries with dwarf satellites in orbit (greater detail can be found in EGH2). We treat each group as if it were a set of multiple satellite–primary pairs. We estimate the large-scale mass, M, of the spiral (obtained from the orbital properties of each satellite, which we will call the orbital mass) in terms of the dimensionless function χ as

$$M\chi \equiv r_{\mathrm{p}} V_r^2 \big/ G \qquad (1)$$

where r_{p} and V_r are, respectively, the projected radial separation, and the difference between the projected radial velocity of each satellite and primary. For r_{p}, we use the chord length distance that separates the satellite from M31 rather than the arc length. Thus, $r_{\mathrm{p}} = 2R\sin(\phi/2)$. R is the distance to the satellite and ϕ is the angle between M31 and each satellite calculated from standard spherical trigonometry (Smart 1977). The data for NGC 1961 and NGC 5084 in this paper were obtained from a literature search using the

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NASA/IPAC Extragalactic Data base (NED) and differ slightly from EGH2.

If each object is idealized as a point mass, bound to the primary (as discussed in EGH2), the function χ (which measures the magnitude of the projection effects) depends on the true anomaly, ν, the angle between the line of nodes and the major axis of the orbit, ω, the inclination of the orbital plane, i, and the eccentricity of the orbit, e,

$$\chi = \sin^2 i \frac{[\cos(\nu + \omega) + e \cos\omega]^2}{1 + e\cos\nu}[1 - \sin^2(\nu + \omega)\sin^2 i]^{1/2}. \quad (2)$$

As an estimate of the importance of any halo, we have the observed value, $\chi_{\rm obs}$, where

$$\chi_{\rm obs} = r_{\rm p} V_r^2 \big/ G(m_1 + m_2) \quad (3)$$

where m_1 and m_2 are the masses determined from the rotation of the individual primary and each satellite, respectively. The sum of these masses is an estimate of the total mass on a small scale. For our systems, $m_2/m_1 \ll 1$; hence, m_2 can be ignored and the subscripts dropped. $\chi_{\rm obs}$ is related to χ by

$$\chi_{\rm obs} = (M/m)\chi. \quad (4)$$

Thus, if the mass determined from the rotation curve, m (which we will call the rotation mass), is the total mass of the galaxy, the two estimates of this mass will be the same and the distribution of $\chi_{\rm obs}$ will be the same as that of χ.

If values for the orbital elements in equation (2) are chosen randomly, the theoretical distribution of χ, $P(\chi)$, is found to be sharply peaked for small values of χ. That is, there is a strong tendency to underestimate the orbital mass, M, by almost an order of magnitude, because small values of $r_{\rm p}$ and V_r are highly favoured. Furthermore, as long as the rotation curve reflects all the mass of the galaxy, and the satellites are bound, $P(\chi) \rightarrow 0$ as $\chi \rightarrow (1 + e)$, which must be less than two. This analysis employs point masses, but in EGH2 we showed that this assumption is not critical to the conclusion.

The method has several advantages and we comment on two (see EGH2 for details). The ratio, $\chi_{\rm obs}$, is independent of the distance scale, and it compares directly the total mass of the primary estimated simultaneously at several different radii. Thus, we can separate the notion of a massive galaxy from that of a galaxy with a massive halo. It is quite certain that the spiral galaxies NGC 1961 and 5084 are very much more massive than M31, but that in itself implies nothing about the existence of heavy haloes for any or all of the systems. We explore this further in the sections that follow.

3 THE GALAXIES

3.1 NGC 1961 and NGC 5084

In Fig. 1 we show the observed distribution of $\chi_{\rm obs}$, $N(\chi)$, for the combined, independent, published sets of data (EGH2 and ZSFW2), augmented by five new satellite–primary pairs found by Ellington (2001). In all cases the projected separations of the satellites are less than 300 kpc (we adopt a Hubble constant of $H_0 = 67$ km s^{-1} Mpc^{-1}). The properties of Ellington's groups are shown in Table 1. A small fraction, 14 per cent, of the 71 data points have $\chi_{\rm obs} > 2$. If contamination by non-physical pairs is not a problem and all the satellites are bound, then it is these few points ($\chi_{\rm obs} > 2$) that provide information about mass distributions that extend well beyond the radius of the discs (haloes).

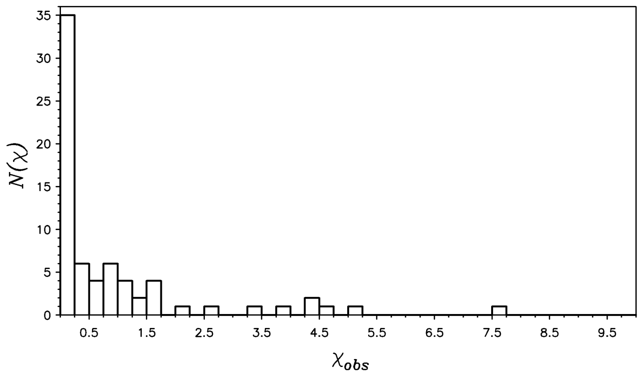

Figure 1. The $N(\chi)$ distribution for 71 primary satellite pairs discussed in EGH2 augmented by Ellington (2001). The ordinate is $N(\chi)$; the abscissa is $\chi_{\rm obs}$ in units of 0.25. The data are restricted to $r_{\rm p} \leqslant 300$ kpc. Ten points show $\chi_{\rm obs} > 2$. One point is off the scale with $\chi_{\rm obs} = 10.6$.

Table 1. New primary–satellite systems (Ellington 2001).

Primary	Satellites	$r_{\rm p}$	m	M	$\chi_{\rm obs}$
NGC 2532	CG 0198	87	16	2.9	0.18
UGC 1448	SBS 0155+018	43	13	58	4.36
NGC 3677	CGCG 242−036	94	92	18	0.2
NGC 1097	NGC 1097a	16	57	1.7	0.03
	ESO 416−G032	223	57	3.2	0.05

Col. 1: name of primary galaxy; **Col. 2:** name of associated satellite(s); **Col. 3:** projected primary–satellite separation, in kpc, based on a Hubble constant $H_0 = 67$ km s^{-1} Mpc^{-1}; **Col. 4:** rotation-based mass of primary from H I profile (see EGH2), in units of 10^{10} M$_\odot$; **Col. 5:** mass of primary from orbital properties of satellites, in units of 10^{10} M$_\odot$; **Col. 6:** the value of $\chi_{\rm obs} = (M/m)\chi$ for each primary–satellite pair.

In EGH2, we made extensive comparisons of our data with those of ZSFW1 and ZSFW2 (ZSFW1 is really a subset of ZSFW2 and we will ignore it in this discussion), for which the data were chosen using criteria very different from those used in EGH1 and its successors. The question we ask is: what is the probability of finding a galaxy–satellite pair with $\chi_{\rm obs}$ greater than two? In EGH2, from the analysis of independent data sets, we found that between 13 per cent and 21 per cent of the satellites had $\chi_{\rm obs}$ values in excess of two. We argued that these differences were not systematic but were measures of precision. Furthermore, as we discuss below in Section 4.1, the data that followed from EGH1 could be modelled marginally by a generic spiral galaxy with halo. The details of the model are given in Section 4.3 of EGH2. A parameter of interest to our discussion here is the eccentricity of each satellite orbit, which ranged from 0.5 to 0.9. The model, appropriately defined, was just successful in reproducing the observations, but qualitatively the initial peak was low and the tail was short. Thus the 21 per cent of the values we observed with $\chi_{\rm obs} > 2$ is probably at the high extreme and the 14 per cent we found for the combined data we show in Fig. 1 is a conservative but not unrealistic estimate. On this basis, we assume a probability of one in seven. Thus, primary galaxies must have about seven satellites before we can have a reasonable expectation of finding one with a large value of $\chi_{\rm obs}$. A probability as large as one in five would create an even greater problem in consideration of M31, as we will show. (Note that in EGH2 we discussed a probability distribution, whereas here we discuss counts, $N(\chi)$.)

Two galaxies that we have studied that meet this criterion of having at least seven satellites are NGC 1961 (GHS) and NGC 5084

(Carignan et al. 1997). As we have noted, the essence of our technique requires two independent methods for estimating the total mass of a galaxy. (1) We must be able to measure the rotation velocities of the disc. This provides a mass at relatively small radii. (2) We must be able to measure the systemic velocities of the satellites to the galaxies in question. This enables us to estimate the halo masses at larger radii.

For NGC 5084, the properties of the disc were measured by Gottesman & Hawarden (1986) and the satellites were investigated by Carignan et al. (1997). For NGC 1961, the H I was extensively observed by Shostak et al. (1982). However, the nature of the major axis data made it difficult to estimate $V(R_{\max})$. EGH2 employed a value consistent with the lower range of the Dutch data. Here, in addition, we consider the width of the total, or global, H I spectrum. We find that the maximum velocity lies within a range of 367 and 450 km s^{-1}. Adopting $V(R_{\max}) = 409$ km s^{-1} yields a mass of $1.40 \times 10^{12} M_{\odot}$ (see Table 2). This is greater than used in EGH2 and produces smaller values of χ.

In Figs 2 and 3 we show the $N(\chi)$ distributions for these two galaxies; the data for all satellites are plotted. In both cases, and more so if the data are combined, there is clear evidence for the existence of massive haloes with sizes greater than 100 kpc in the case of NGC 5084, and greater than 300 kpc for NGC 1961. The data for M31 are shown in Fig. 4.

3.2 The M31 group

The Local Group is dominated by our own Milky Way and by M31 (NGC 224). They are separated by 770 kpc and each is surrounded by a group of dwarf satellites. Our prime references for the properties of M31 and the M31 group have been EW, Evans et al. (2000), Côté et al. (2000) and Irwin (private communication). However, to ensure uniformity our ultimate authority has been NED, which we use throughout this paper. (Dr Wilkinson has informed us that the position given by Evans et al. (2000) for And vii is in error; our values are correct.)

In Table 2, we list the properties of the three primary galaxies, while in Tables 3–5 we list the properties of each of the satellite groups. However, before considering the $N(\chi)$ distribution for M31 a few comments are required.

First, what satellites should be included in the analysis? We are concerned in particular with the Pegasus and IC 1613 dwarfs. These galaxies lie at projected separations in excess of 400 kpc from M31, which is more than one-half the distance to the Milky Way. We presume the orbital period of the satellites to be at most a Hubble time. Employing a simple Keplerian calculation, the radius of the largest orbit is proportional to $M^{1/3}$. If we adopt the mass of NGC 5084 ($170 \times 10^{10}\, M_{\odot}$) as a fiducial indicator, the maximum radius that satisfies this condition is about 350 kpc. Employing this size

Table 2. Primary galaxies.

Galaxy	D	$V_{\rm sys}$	$V(R_{\max})$	$R_{\max}$	Mass (m)
NGC 224	0.77	−301	235	30	38
NGC 1961	58	3934	409	36	140
NGC 5084	30	1726	328	68	170

Col. 1: name of primary galaxy; **Col. 2:** with the exception of NGC 224 (M31), distance (Mpc) using the heliocentric, systemic velocity (in units of km s^{-1}) shown in **Col. 3** corrected to the CMB frame and $H_0 = 67$ km s^{-1} Mpc^{-1}; **Col. 4:** rotation velocity, measured at the maximum radius (in kpc) observed – shown in **Col. 5**; **Col. 6:** the resultant mass using a Keplerian calculation, in units of $10^{10}\, M_{\odot}$.

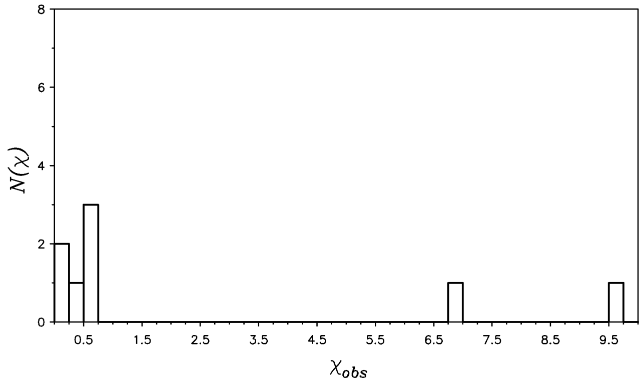

Figure 2. The $N(\chi)$ distribution for NGC 1961. The data are taken from Tables 2 and 3. The maximum projected separation for the satellites is 528 kpc. One point is off the scale with $\chi_{\rm obs} = 12.54$.

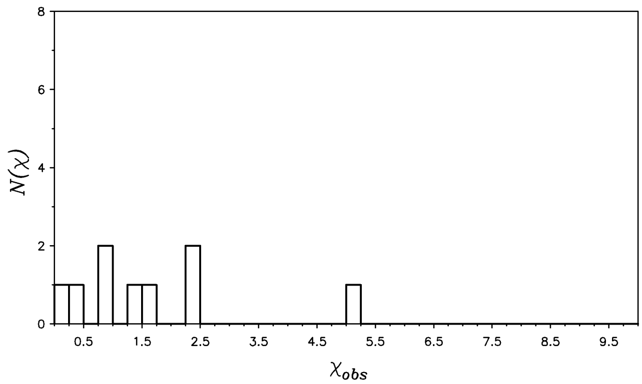

Figure 3. The $N(\chi)$ distribution for NGC 5084. The data are taken from Tables 2 and 4. The maximum projected separation for the satellites is 162 kpc.

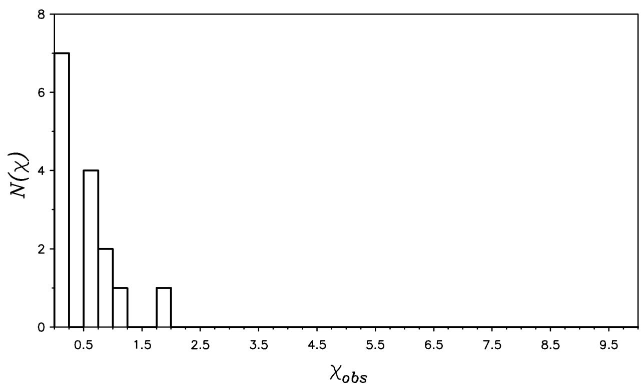

Figure 4. The $N(\chi)$ distribution for M31 as listed in Table 5. Data for M31, the primary, is taken from Table 2. The maximum projected separation for the satellites is 520 kpc.

may be in error, but it will allow us to compare the three galaxies based on the initial assumption that all, including M31, are massive systems. It is unreasonable to expect a two-body calculation to be applicable for satellites at such large distances from the primary (M31) in the nearby presence of our own massive galaxy. However, as we will show, even if these two distant dwarf galaxies are included in the discussion, the M31 group is unusual.

Secondly, Andromeda is a highly-resolved system with an optical diameter in excess of three degrees. This makes it unlike any of the

Table 3. NGC 1961 satellites and measured χ_{obs} values.

Galaxy	ΔV	r_{p}	M	χ_{obs}
CGCG 329–011	+174	129	91	0.65
CGCG 329–009	−62	152	14	0.10
LEDA 138826	−134	188	78	0.56
UGC 3342	+40	231	8.6	0.06
UGC 3344	+348	342	960	6.88
UGC 3349 NOTES01)	+358	456	1357	9.72
CGCG 307–021	+67	464	48	0.35
UGC 3349	+380	522	1749	12.54
CGCG 329–010	+82	528	82	0.59

Col. 1: name of the satellite galaxy taken from NED; **Col. 2:** radial velocity difference (satellite–primary), in units of km s^{-1}; **Col. 3:** projected separation, in kpc; **Col. 4:** the resultant orbital mass using the data in Cols 2 and 3, in units of 10^{10} M$_\odot$; **Col. 5:** $\chi_{\mathrm{obs}} = (M/m)\chi$, the ratio of the mass determined from the kinematics of the satellite in Col. 4 to the mass of the primary from Table 2.

Table 4. NGC 5084 satellites and measured χ_{obs} values.

Galaxy	ΔV	r_{p}	M	χ_{obs}
G1	−40	162	6.0	0.04
G2	−238	97	128	0.75
G3	−295	71	143	0.85
G4	−635	94	877	5.2
G5	+479	51	273	1.6
G6	−387	61	212	1.3
⟨G7–G8⟩	+356	134	394	2.3
G9	+167	103	67	0.39

Col. 1: name of the satellite galaxy taken from Carignan et al. (1997); **Col. 2:** radial velocity difference (satellite–primary), in units of km s^{-1}; **Col. 3:** projected separation, in kpc; **Col. 4:** the resultant orbital mass using data in Cols 2 and 3, in units of 10^{10} M$_\odot$; **Col. 5:** $\chi_{\mathrm{obs}} = (M/m)\chi$, the ratio of the mass determined from the kinematics of the satellite in Col. 4 to the mass of the primary from Table 2.

Table 5. M31 satellites and measured χ_{obs} values.

Galaxy	ΔV	r_{p}	M	χ_{obs}
M32	±96	5.4	1.17	0.03
M33	±75	198	25.5	0.67
NGC 147	±118	100	32.2	0.85
NGC 185	±107	95	25.2	0.66
NGC 205	±61	8.2	0.85	0.02
IC 10	−33	246	6.35	0.17
*IC 1613	−60	520	43.9	1.16
LGS 3	−30	266	5.36	0.14
*Peg dwarf	±85	412	68.4	1.8
And i	−85	44	7.42	0.20
And ii	±82	138	21.8	0.57
And iii	−58	67	5.3	0.12
And v	−108	108	28.9	0.76
And vi	−65	264	25.7	0.68
And vii	±22	217	2.53	0.07

Col. 1: name of the satellite galaxy taken from NED, IC 1613 and Peg dwarf are marked * as their membership in the group is uncertain; **Col. 2:** radial velocity difference (satellite–primary), in units of km s^{-1}; **Col. 3:** projected separation, in kpc; **Col. 4:** the resultant mass using a Keplerian calculation, in units of 10^{10} M$_\odot$; **Col. 5:** $\chi_{\mathrm{obs}} = (M/m)\chi$, the ratio of the mass determined from the kinematics of the satellite in Col. 4 to the mass of the primary from Table 2.

other galaxies we have studied. Our lines of sight to the centre of the galaxy and to the satellites are not parallel. Indeed, as independent distances can be measured to the satellites, most studies (see for example EW) are done in an Andromedacentric system. As noted in equation (3), our formalism has been developed for projected velocity and position coordinates. We cannot correct for line-of-sight effects on the observed radial velocities without knowing something about space motions. We expect the effects not to be large and certainly to be less than the factor of three or more that is required to alter our discussion. We have compared a χ'_{obs} calculated using the correct satellite distance from M31, as well as χ_{obs} using projected separations and a distance to M31 of 770 kpc. For our data, the χ_{obs} values from $V_r^2 r_{\mathrm{p}}$ are about 90 per cent of those obtained from $V_r^2 r$.

To gain some insight on this problem it is instructive to consider the special case of a circular orbit of radius r and circular speed V. In order to maximize the χ values at all orbital phases, we let the orbital plane lie perpendicular to the plane of the sky ($i = 90°$). The values of $V_r^2 r_{\mathrm{p}}$ range from 0 to $V^2 r$, with the angle-averaged values being $4/(3\pi)V^2 r = 0.4244 V^2 r$. It should be noted that the coefficient is independent of the ratio r/d, where d is the separation between the centres of the Milky Way and M31. A similar calculation of the angle-averaged value $V_r^2 r = 0.5 V^2 r$. The coefficients $4/3\pi$ and $1/2$ are maximum values because $\pi/2$ is the maximum inclination for any orbit. For this simple example, the angle average of $V_r^2 r_{\mathrm{p}}$ is roughly 85 per cent of the angle average of $V_r^2 r$.

Although it makes no essential difference to our conclusions, we will adhere to projected separations in order to be as consistent as possible with our earlier studies. In Fig. 5 we show the $N(\chi)$ distribution for the thirteen satellites of M31, for which $r_{\mathrm{p}} < 350$ kpc. This should be compared with Fig. 6, which shows the combined data for NGC 1961 and NGC 5084 for which $r_{\mathrm{p}} < 343$ kpc. Unlike these latter systems, M31 shows no χ_{obs} values greater than 2. Only if we include the observations for the Pegasus dwarf, for which $r_{\mathrm{p}} = 412$ kpc, do we get a value as large as 1.8 (see Fig. 4). Given the projected separation, this is surprisingly small, if M31 has a heavy halo. It is considerably less than the χ_{obs} values of five or more seen for NGC 1961 and 5084. In both of these cases we are considering fewer satellites than for M31, and, for NGC 5084 in particular, the satellites, on average, have smaller projected separations than those observed for M31.

The number of galaxies studied by EGH2 was large enough that a separation could be made between systems having $r_{\mathrm{p}} \leqslant 300$ kpc

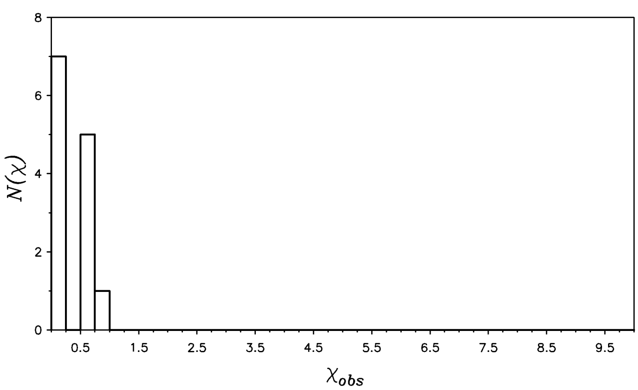

Figure 5. The $N(\chi)$ distribution for M31 as in Fig. 4 except that the maximum projected separation for the satellites is 266 kpc.

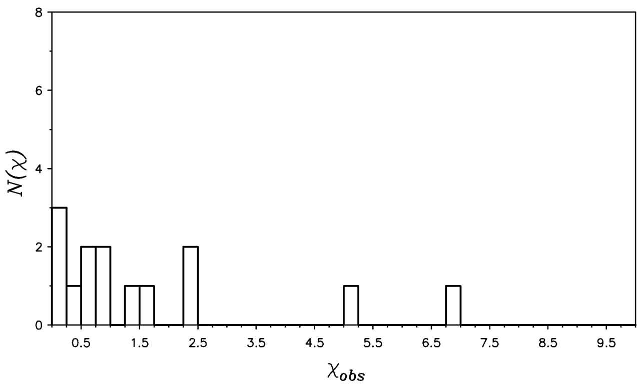

Figure 6. The combined $N(\chi)$ distribution for NGC 1961 and 5084. The data are taken from Tables 2, 3 and 4. The maximum projected separation for the satellites is 342 kpc.

and those having $r_p > 300$ kpc. The first group of 71 points has been shown in Fig. 1. For the second case, 11 of 37 χ_{obs} values were greater than two and of these two had values $10 < \chi_{obs} < 21$. Based upon our studies of bound systems with massive haloes, we expect at least 14 per cent (one in seven, as noted in Section 3.1) of the satellites to show χ_{obs} values between two and ten, if the projected radius of the group is about 300 kpc. None of the satellites of M31 that meet this criterion have χ_{obs} ratios greater than two. Furthermore, if the radius of interest is extended out to 500 kpc there should be larger values of χ_{obs}, some approaching a value of 20. However, $\chi_{obs} = 0.32$ for IC 1613 ($r_p = 520$ kpc). Thus, the data for M31 differ significantly from the general galaxies studied in EGH2; also, which is of more interest, the system differs substantially from NGC 1961 and NGC 5084 – galaxies that we believe show conclusive evidence for extended and massive haloes. If we consider 'satellites' orbiting at distances as large as 530 kpc, then three out of nine dwarfs associated with NGC 1961 have χ_{obs} values greater than two (6.9, 9.7 and 12.5). When we include the two points already noted for NGC 5084 (which has not been surveyed for satellites with such large orbits), a total of five satellites out of a population of 17 indicate the presence of a massive halo. None of the satellites for M31, out of a population of fifteen, meet this criterion. The difference is further emphasized by comparing the rms velocity dispersions of the satellites to the rotation velocities of the primaries. For M31 this ratio is 0.33, while for the combined data of NGC 1961 + NGC 5084 the ratio is 0.88 (for all three galaxies, $r_p \leqslant 412$ kpc). There appear to be two possible conclusions: (1) there is no massive halo associated with M31; or (2) the characteristics of the orbits of the satellites to M31 as a group (including the effects of dynamical friction) are systematically different from those of more isolated 'field' galaxies. Either way, the results for the M31 system are not consistent with those of other massive galaxies.

4 DISCUSSION

4.1 Kinematics

In their discussion on determining the mass of spherical systems, Bahcall & Tremaine (1981, hereafter BT) employed the kinematical properties of a collection of bound particles. They considered an estimator based on the projected mass

$$q \equiv \frac{V_r^2 R}{G}, \tag{5}$$

where both the velocity and radial separation are projected quantities. q serves as an estimator for the mass M of the system. For an arbitrary distribution function they showed that the expectation value of q becomes

$$\langle q \rangle = \frac{\pi M}{32}(3 - 2\langle e^2 \rangle). \tag{6}$$

If all the particles have circular orbits $\langle e^2 \rangle = 0$, while for radial orbits $\langle e^2 \rangle = 1$. These arguments lead them to define two specialized estimators based on the mean of the projected mass $\langle q \rangle = \langle V_r^2 R \rangle / G$:

$$M_I = \frac{16}{\pi G}\langle q \rangle \tag{7}$$

and

$$M_L = \frac{32}{\pi G}\langle q \rangle. \tag{8}$$

For isotropic orbits $\langle e^2 \rangle = 0.5$, and $M_I = M$. For linear orbits $\langle e^2 \rangle = 1$, and $M_L = M$. Equations (6)–(8) show in a very simple way, but clearly, that the estimated mass of M31 depends on the average eccentricity of the orbits of the satellites. The greater the typical eccentricity, the greater the *underestimate* of the mass of the galaxy by $\langle q \rangle$, the estimator of the projected mass. From equation (6), the variation between extremes can be as much as a factor of three. Of course, the analysis above does not consider the influence of a halo with a logarithmic potential.

However, EGH2 did consider the general problem of orbits in massive, isothermal haloes. As the orbits are not closed, the eccentricity was defined, as can be done in the Keplerian problem, as $e \equiv (r_{apg} - r_{prg})/(r_{apg} + r_{prg})$, where r_{apg} and r_{prg} are the apogalactic and perigalactic radii, respectively. In a pure halo potential, the large χ values were found to originate in the vicinity of perigalacticon from orbits with large r_{apg} (that is, where the apogalactic radius approaches that of the halo, $r_{apg} \rightarrow r_H$). In this fashion, EGH were able to explain the large χ values of about 20 that were observed in EGH2 as arising from systems with massive haloes and radii in excess of 300 kpc. The authors found that, as the eccentricity of the orbits varied from 0.90 to 0.00, the maximum value of χ increased from 9.7 to 24.7. The mass of the primary was unchanged for all of these experiments. As the orbital eccentricities increase for a given model, the estimate of the mass (equivalent to $\langle q \rangle$) decreases, as was noted by BT. Thus, one way of 'hiding' mass, as indicated by the $N(\chi)$ distribution, would be to have essentially linear orbits. While the galaxy halo would be massive, the orbits would show an unexpectedly low value for χ_{max}.

In EGH2 we explored a large number of N-body models designed to simulate a range of $N(\chi)$. In order to compare these results with M31, certain adjustments to the models are required. The asymptotic maximum velocity has to be scaled to 235 km s^{-1}; the Hubble constant has to be revised to $H_0 = 67$ km s^{-1} Mpc^{-1}, and the characteristic eccentricity of the orbits has to be increased at least to 0.8. (See the discussion associated with Section 4 of EGH2.) If the apogalactic distance of the satellites is, for example, 350 kpc, then a larger eccentricity implies that perigalacticon is within the disc and the effect of interactions should be severe and clear to the observer. This does not appear to be the case in general. (However, M32 is a notable counter-example, which we will discuss below.)

The first model we discuss (model 5 in EGH2) was chosen to reproduce the data of EGH1 and its successors. The second model (model 6) was developed to satisfy the data of EGH2, but did not reproduce the very largest values of χ_{obs}. The third model (model 3) was formulated to describe the full range of data in EGH2.

Model 5 was a one-galaxy model, a generic disc-plus-halo combination. The halo radius was 90 kpc and the total mass of the system was 9×10^{11} $M_{\odot}$. This model would require many of the satellites of M31 to orbit outside the galactic halo.

Model 6 was constructed to model the data in EGH2. As we will discuss in more detail below, the full range of data in EGH2 required a two-component model, a very massive disc-plus-halo constituent, and an essentially haloless disc. Model 6 was capable of reproducing the data of EGH2 provided the highest values of χ_{obs} were ignored (possible interlopers). The massive component of this model had a halo radius of 225 kpc, and a total mass of 2×10^{12} $M_{\odot}$.

In order for the tail of model $N(\chi)$ distributions to approach the largest values of χ_{obs} found in EGH2, model constituents with halo radii in excess of 300 kpc were required.

Model 3 was the most successful example. Two types of galaxy were required to model the full range of $N(\chi)$. One, a disc with a massive halo, was required to reproduce the high χ_{obs} tail, and the other, a disc-only component, was necessary to match the peak in $N(\chi)$ at small values of χ_{obs}. As in model 6 above, we will assume that the massive element duplicates the properties of M31. For this model, the halo radius was 350 kpc, and the total mass of the system was 3×10^{12} $M_{\odot}$.

Each of these models was developed independently from numerical experiments designed to satisfy different criteria. However, as the data were common and several of the characteristics of the models were common (such as the radius of any disc), it is not surprising that the mass of successful model galaxies scales with halo radius. With regard to M31, however, these models have a singular failure. The maximum value for χ is in the range 9–12. Such models would describe the properties of NGC 1961 and NGC 5084 but not M31. In order to approach a maximum value of $\chi_{obs} \simeq 2$, the halo masses of the model galaxies must be reduced.

We conclude that it is very unlikely that the mass of M31 within a radius of 350 kpc is greater than 6×10^{11} $M_{\odot}$. This solution requires eccentric (near linear) orbits and suggests that the halo dynamics of M31 are quite different from those of the galaxies studied by EGH2 and certainly are quite different from either NGC 1961 or NGC 5084.

The data for M31 bear a strong similarity to those for NGC 3992. We have argued from several perspectives that this galaxy is unlikely to have a massive halo. (See EGH2 page 158 for further references and for a discussion of the data.) Like M31, NGC 3992 has a very-compressed $N(\chi)$ distribution.

Courteau & van den Begh (1999) suggest that IC 1613 (the other questionable dwarf 'satellite' of M31, lying at a slightly larger distance from M31 than Peg) is a free-floating member of the Local Group rather than a satellite of M31. A similar argument can be made for the Peg dwarf, given its large separation from M31 and the nearby presence of the Milky Way. If the radius of any putative halo for M31 is closer to 250 kpc, the mass is not greater than 4×10^{11} $M_{\odot}$. Braun (1991), from H I observations, finds a falling rotation curve with a value of 200 km s^{-1} at a radius of 26 kpc ($M = 2.4 \times 10^{11}$ $M_{\odot}$). No heavy halo is required to reproduce his observations. However, if the rotation curve remained flat out to a distance of 260 kpc (for which there is no observational evidence), the implied mass of an isothermal halo for M31 would be 2.4×10^{12} $M_{\odot}$. Such a system is too massive to explain the nature of the $N(\chi)$ distribution for the satellites. To conclude this part of the discussion, we emphasize that it would be very difficult (unless nature conspires) to have a galaxy system (M31 + satellites) with a total mass larger than 4×10^{11} $M_{\odot}$ and not have at the same time several satellites with $\chi_{obs} > 2$ and χ_{max} substantially greater than 1.8. If M31 has such a large halo mass, the kinematic properties of the satellites should resemble those of NGC 1961 and NGC 5084, which they do not.

4.2 Dynamical friction

EGH2 included dynamical friction in the model calculations, and its effects cannot be ignored if the satellite mass m_2 is greater than 10^9 $M_{\odot}$. In Model 6 of EGH2, $N(\chi)$ distributions for satellites having mass $m_2 = 2 \times 10^9$ $M_{\odot}$ were compared with distributions for satellites having mass $m_2 = 0.67 \times 10^9$ $M_{\odot}$. After integrating the orbits of the higher-mass satellites for one Hubble time, dynamical friction was shown to truncate the tail at large χ values, and to increase the peak of the distribution at low values of χ. In other words, for the galaxies with massive haloes studied by EGH2, the satellites cannot have masses much larger than 10^9 $M_{\odot}$ or they will spiral into the nucleus of the primary. One uncertain parameter controlling deceleration caused by dynamical friction is the Coulomb logarithmic coefficient Λ, which was taken to be 6.8. One can define an effective satellite mass $M_{sat}\Lambda$, which in our studies was 10^9 $M_{\odot}$, and this value was utilized as the mass of the satellites in our numerical experiments. If M31 has a massive halo, the satellites must have small masses, as defined above. If they have large masses, the existence of a massive halo would be very doubtful. Unfortunately, this is not a sensitive test since most of the satellite masses (that have been measured) are significantly less than 10^9 $M_{\odot}$ (Mateo 1998), with the notable exceptions of IC 10, M32 and M33. The implications for the two groups of satellites need to be considered separately.

For the majority of satellites, the masses are too small to be effected by dynamical friction and cannot be important in truncating the high χ_{obs} tail of $N(\chi)$. If M31 has a substantial halo, the nature of the orbits, and not dynamical friction, is responsible for diminishing the tail in the distribution.

For the massive satellites, the implications are different and interesting. M32 is a special case. It is very close to M31 (the radial separation is estimated to be 5 kpc) and interactions are important (for example Byrd 1976; Bekki et al. 2001). At this small separation, this satellite may be about to disappear owing to tidal and frictional effects.

In contrast, both IC 10 and M33 lie at distances from M31 greater than 190 kpc. At these distances the frictional effect of the halo is unlikely to be large. If the orbits of these systems are deeply plunging (as we have strongly argued may be the case if the halo mass of M31 is large) then dynamical friction is important and the orbits should have decayed over a Hubble time, but they have not. M31 cannot have a very massive halo unless the orbits of these massive satellites have low eccentricity. However, if the orbits of all of the satellites are near circular, then $N(\chi)$ is not consistent with the existence of a massive halo. Of course, the eccentricity of the satellite orbits could change systematically with apogalactic distance, with more circular orbits at larger radii. Such complications are not required to explain the properties of satellite orbits in the general case, as shown by EGH2. Thus, if $\langle e^2 \rangle \approx 0$, the M31 system would be most unusual.

It is clear that the majority of the satellites are not massive enough to have suffered significant frictional deceleration, regardless of the properties of the halo. The orbits of the very few (known) massive satellites should have suffered from dynamical friction. If they have eccentric orbits they should have decayed and now resemble that of the Large Magellanic Cloud (see EGH2) and be on their last few circuits before oblivion. On the contrary, they lie at large radial distances from M31. Either dynamical friction is insignificant or their orbits are not deeply plunging.

We conclude that: (1) the masses of most of the satellites are too small to have suffered a significant deceleration in a massive halo; (2) the massive and distant satellites may not be on highly eccentric orbits if a massive halo exists; or (3) most simply, a massive halo does not exist and frictional decelerations are not expected for any of the satellites without regard to location or mass.

Mass and length are directly proportional to the value of H_0 employed in the calculations. The results, taken from our models and quoted here, have been scaled to $H_0 = 67$ km s^{-1} Mpc^{-1}. Our models were developed in EGH2 using a Hubble constant of $H_0 = 100$ km s^{-1} Mpc^{-1}. However, we investigated the characteristics of these model galaxies for different values of H_0, and no significant differences in our conclusions were found for $50 < H_0 < 100$ km s^{-1} Mpc^{-1}.

5 CONCLUSION

In conclusion, we emphasize that the models for M31 that we have considered were contrived to produce the greatest halo mass consistent with the results of EGH2. We find no convincing evidence for the existence of a massive halo, nor do we require unusual characteristics for the orbits of the satellites. *'Essentia non sunt multiplicanda praeter necessitatum'* (see for example Ockham W., as printed in 1495).

On the basis of the analysis we have presented, we believe that the Keplerian mass, consistent with Braun's 1991 observations of M31, is close to the total mass of the galaxy. Certainly, if NGC 1961 and NGC 5084 are typical of galaxies with massive haloes, then M31 is not of this class. The $N(\chi)$ distribution is too compressed. Our limit is consistent with, but at the lower bounds of, the models presented by EW and Côté et al. (2000). However, we are dealing with a small number of satellites and caution is required before drawing definitive conclusions. The detection of two or three new satellites of M31, with separations less than but velocities relative to M31 greater than that of the Peg dwarf, might alter our conclusions. In addition, the velocities of the satellites must be made more secure. However, in the absence of such data we are in complete sympathy with the suggestion that the Milky way is the massive member of the Local Group (Wilkinson & Evans 1999).

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