

DS-GA 1008: Deep Learning, Spring 2019

Homework Assignment 2

Due: 6pm on Friday, Mar 1, 2019

The search for truth is more precious than its possession.

Albert Einstein (1879 - 1955)

1. Fundamentals

1.1. Convolution

Table 1 depicts two matrices. The one on the left represents an 5×5 single-channel image \mathbf{A} . The one on the right represents a 3×3 convolution kernel \mathbf{B} .

- (a) What is the dimensionality of the output if we forward propagate the image over the given convolution kernel with no padding and stride of 1?

Answer: 3×3 (2 pts)

- (b) Give a general formula of the output width O in terms of the input width I , kernel width K , stride S , and padding P (both in the beginning and in the end). Note that the same formula holds for the height. Make sure that your answer in part (a) is consistent with your formula.

Answer: $O = \frac{I-K+2P}{S} + 1$. (10 pts)

- (c) Compute the output \mathbf{C} of forward propagating the image over the given convolution kernel. Assume that the bias term of the convolution is zero.

Answer:

109	92	72
108	85	74
110	74	79

 (3 pts)

- (d) Suppose the gradient backpropagated from the layers above this layer is a 3×3 matrix of all 1s. Write the value of the gradient (w.r.t. the input image) backpropagated out of this layer.

$$\mathbf{A} = \begin{bmatrix} 4 & 5 & 2 & 2 & 1 \\ 3 & 3 & 2 & 2 & 4 \\ 4 & 3 & 4 & 1 & 1 \\ 5 & 1 & 4 & 1 & 2 \\ 5 & 1 & 3 & 1 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & 3 & 3 \\ 5 & 5 & 5 \\ 2 & 4 & 3 \end{bmatrix}$$

Table 1: Image Matrix (5×5) and a convolution kernel (3×3).

Hint: You are given that $\frac{\partial E}{\partial C_{ij}} = 1$ for some scalar error E and $i, j \in \{1, 2, 3\}$. You need to compute $\frac{\partial E}{\partial A_{ij}}$ for $i, j \in \{1, \dots, 5\}$. The chain rule should help!

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Answer:

4	7	10	6	3
9	17	25	16	8
11	23	34	23	11
7	16	24	17	8
2	6	9	7	3

(10 pts)

1.2. Pooling

The pooling is a technique for sub-sampling and comes in different flavors, for example max-pooling, average pooling, LP-pooling.

- (a) List the `torch.nn` modules for the 2D versions of these pooling techniques and read on what they do.

Answer: `torch.nn.MaxPool2d`, `torch.nn.AvgPool2d`, `torch.nn.LPPool2d` (3 pts)

- (b) Denote the k -th input feature maps to a pooling module as $\mathbf{X}^k \in \mathbb{R}^{H_{\text{in}} \times W_{\text{in}}}$ where H_{in} and W_{in} represent the input height and width, respectively. Let $\mathbf{Y}^k \in \mathbb{R}^{H_{\text{out}} \times W_{\text{out}}}$ denote the k -th output feature map of the module where H_{out} and W_{out} represent the output height and width, respectively. Let $S_{i,j}^k$ be a list of the indexes of elements in the sub-region of \mathbf{X}^k used for generating $\mathbf{Y}_{i,j}^k$, the (i,j) -th entry of \mathbf{Y}^k . Using this notation, give formulas for $\mathbf{Y}_{i,j}^k$ from three pooling modules.

Answer:

max-pooling: $\mathbf{Y}_{i,j}^k = \max_{(x,y) \in S_{i,j}^k} \mathbf{X}_{(x,y)}^k$ (5 pts),

average pooling: $\mathbf{Y}_{i,j}^k = \frac{1}{|S_{i,j}^k|} \sum_{(x,y) \in S_{i,j}^k} \mathbf{X}_{(x,y)}^k$ (5pts),

LP-pooling: $\mathbf{Y}_{i,j}^k = \left(\sum_{(x,y) \in S_{i,j}^k} \mathbf{X}_{(x,y)}^k{}^p \right)^{1/p}$ (5 pts)

- (c) Write out the result of applying a max-pooling module with kernel size of 2 and stride of 1 to \mathbf{C} from Part 1.1.

Answer:

109	92
110	85

(2 pts)

- (d) Show how and why max-pooling and average pooling can be expressed in terms of LP-pooling.

Answer: At $p = \infty$, one gets max-pooling. At $p = 1$, one gets Sum Pooling (which is proportional to average pooling). (5 pts)

2. Programming

Complete exercises in `DS-GA-1008-HW_assignment_2.ipynb`.

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3. Evaluation

Homework is worth a total of 100 points.

- Part 1 - 50 points
- Part 2 - 50 points

4. Submission

You are required to write up your solutions to Part 1 using \LaTeX .

Submit the following files to NYU classes by the deadline (6pm on Friday, March 1, 2019):

- `First-name_Last-name_netID_A2.pdf` file for Part 1 (containing a read-only link to your Overleaf project for Part 1)
- `First-name_Last-name_netID_A2.tex` file for Part 1
- `First-name_Last-name_netID_A2.ipynb` file for Part 2

5. Disclaimers

You are allowed to discuss problems with other students in the class but have to write up your solutions on your own.

As feedback might be provided during the first days, the current homework assignment might be undergoing some minor changes. We'll notify you if this happens.