Fundamental Algorithms, Assignment 11

Due April 26 in Recitation.

Every block of stone has a statue inside it, and it is the task of the sculptor to discover it. – Michelangelo

1. Consider Prim's Algorithm for MST on the complete graph with vertex set $\{1, \ldots, n\}$. Assume that edge $\{i, j\}$ has weight $(j - i)^3$. Let the root vertex $r = 1$. Show the pattern as Prim's Algorithm is applied. In particular, Let $n = 500$ and consider the situation when the tree created has 211 elements and π and key have been updated.

   (a) What are these 211 elements.

   (b) What are $\pi[309]$ and $key[309]$.

2. Find $d = \gcd(144, 89)$ and x, y with $144x + 89y = 1$. [Remark: This is part of a pattern with two consecutive numbers from the Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$]

3. Find $\frac{311}{507}$ in Z_{1000}.

4. Solve the system
   $x \equiv 34 \bmod 101$
   $x \equiv 59 \bmod 103$.

5. Using the Island-Hopping Method to find 2^{1072} modulo 1073 using a Calculator but NOT using multiple precision arithmetic. (You should never have an intermediate value more than 1072^2.) What does your result tell you about the primality of 1073.

6. (extra from last week!) Suppose that during Kruskal's Algorithm (for MST) and some point we have $SIZE[v] = 35$. What is the interpretation of that in the case when $\pi[v] = v$? What is the interpretation of that in the case when $\pi[v] = u \neq v$? Let w be a vertex. How many different values can $\pi[w]$ have during the course of Kruskal's algorithm? How many different values (as a function of V, the number of vertices) can $SIZE[w]$ have during the course of Kruskal's algorithm? (That is, the maximal number possible.)

The universe is not only queerer than we suppose but queerer than we *can* suppose. – J.B.S. Haldane