

Fundamental Algorithms, Assignment 6

Due March 8 in Recitation

I am slow to learn and slow to forget that which I have learned.
 My mind is like a piece of steel; very hard to scratch anything
 on it and almost impossible after you get it there to rub it out.
 – Abraham Lincoln

- (Continuation of Problem from Last Assignment)) Consider a Binary Search Tree T with vertices a, b, c, d, e, f, g, h and $ROOT[T] = a$ and with the following values (N means NIL)

vertex	a	b	c	d	e	f	g	h
parent	N	e	e	a	d	g	c	a
left	h	N	N	e	c	N	f	N
right	d	N	g	N	b	N	N	N

- Which is the successor of c . Illustrate how the program **SUCCESSOR** will find it.
 - Which is the minimal element? Illustrate how the program **MIN** will find it.
 - Illustrate the program **DELETE[e]**
- Draw binary search trees of height 2,3,4,5,6 on the set of keys $\{1,4,5,10,16,17,21\}$.
 - What is the difference between the binary-search property and the heap property? (*) Can the heap property be used to print out the keys of an n -node tree in sorted order in $O(n)$ time? Explain how or why not.
 - What would the BST tree look like if you start with the root a_1 with $key[a_1] = 1$ (and nothing else) and then you apply

$$INSERT[a_2], \dots, INSERT[a_n]$$

in that order where $key[a_i] = i$ for each $2 \leq i \leq n$? Suppose the same assumptions of starting with a_1 and the key values but the INSERT commands were done in *reverse* order

$$INSERT[a_n], \dots, INSERT[a_2]$$

5. Let $A[1 \cdots N]$ be an array of numbers. Consider the following program:
(* means times, Assume $N \geq 2$ and that the $A[I]$ are distinct to avoid trivialities.)

KAUSHIK[A]

1. BUILD-MAX-HEAP[A]

2. $I \leftarrow 1$

3. WHILE $I \leq N$

 4. EXTRACT-MAX[A]

 5. $I++$

6. END WHILE

7. Return $A[1]$

- (a) How long does step 4 take as a function of N and I .
- (b) What is the total time for the WHILE loop.
- (c) What is the total time for KAUSHIK[A].
- (d) Describe the value z returned in terms of the original values of the array A .

In the novel I never wrote, I wanted the hero to be a computer programmer because it was the most poetic and romantic occupation I could think of [...] I conceived of him, whose professional life was spent in the sanctum of the night (when, I was told, the computers, too valuable to be unemployed by industry during the day, are free, as it were, to frolic) devising idioms whereby problems might be fed to the machines and emerge, under binary percussion, as the music of truth - I conceived of him as being too fine, translucent, and scrupulous to live in our coarse age. He was to be, if the metaphor is biological, an evolutionary abortion, a mammalian mutation crushed underfoot by dinosaurs, and, if the metaphor is mathematical, a hypothetical ultimate, one digit beyond the last real number. The title of the book was to be $N + 1$.

from *The Music School* by John Updike