Fundamental Algorithms, Assignment 5

Due March 1, in Recitation

What we need is more people who specialize in the impossible.
– Theodore Roethke

1. Some exercises in which n is NOT the data size but we want the answer in terms of n. (Answers in Θ-land, as a power of n times a power of $\ln n$ – or possibly just one of them.)

   (a) How long does `MERGE-SORT` on n^2 items take?

   (b) Suppose that when $n = 2^m$, `ANNA` takes time $\Theta(m^2 2^m)$. How long does it take as a function of n.

   (c) Suppose that when $n = 2^m$, `BOB` takes time $\Theta(5^m)$. How long does it take as a function of n.

   (d) How long does `COUNTING-SORT` take to sort n^2 items with each item in the range 0 to $n^3 - 1$.

   (e) How long does `RADIX-SORT` take to sort n^2 items with each item in the range 0 to $n^3 - 1$ and base n is used.

2. Consider hashing with chaining using as hash function the sum of the numerical values of the letters (A=1,B=2,...,Z=26) mod 7. For example, h(JOE)= 10+15+5mod7 = 2. Starting with an empty table apply the following operations. Show the state of the hash table after each one. (In the case of Search tell what places were examined and in what order.)
   Insert COBB
   Insert RUTH
   Insert ROSE
   Search BUZ
   Insert DOC
   Delete COBB

3. Consider a Binary Search Tree T with vertices a, b, c, d, e, f, g, h and $ROOT[T] = a$ and with the following values (N means NIL)

vertex	a	b	c	d	e	f	g	h
parent	N	e	e	a	d	g	c	a
left	h	N	N	e	c	N	f	N
right	d	N	g	N	b	N	N	N
key	80	170	140	200	150	143	148	70

Draw a nice picture of the tree. Illustrate `INSERT[i]` where `key[i]=100`.

4. Set $N = 2^K$. We'll represent integers $0 \leq x < N$ by $A[0 \cdots (K-1)]$ with $x = \sum_{i=0}^{k-1} A[i]2^i$. (This is the standard binary representation of x, read right to left.) Consider the following algorithms:

   Procedure $FANG[A]$
   $I \leftarrow 0$
   $A[0] + +$
   WHILE ($A[I] = 2$ AND $I < K - 1$)
   $A[I] \leftarrow 0$
   $I + +$
   $A[I] + +$
   END WHILE

   and:

   VIKAS[A]
   FOR $J = 1$ TO $N - 1$
   DO $FANG[A]$
   END FOR

   (a) If the input to $FANG[A]$ is the binary representation of x with $0 \leq x \leq N - 2$ describe what the output (the final value of A) will be.

   (b) For "time" we will mean here the number of times the line: "WHILE ($A[I] = 2$ AND $I < K - 1$)" is reached. We want here the "time" as a function of N. What is the worst-case time for $FANG$? What is the best-case time for $FANG$?

   (c) *Assume* the array A is initially all zeroes. Describe [1] what $VIKAS$ is doing.

   (d) (*) Again *assume* the array A is initially all zeroes and "time" as above. What is the time for $VIKAS$ in Θ-land?

   LaDoll was one of those people who seem, even to those who know them well, digitally enhanced: the bright blond bob cut; the predatory lipstick, the roving algorithmic eyes.
   from *A Visit from the Goon Squad* by Jennifer Egan

[1] If English is not your native language it is especially important that you give clear English explanations – not some formula!!