

## Fundamental Algorithms, Problem Set 2

Due February 1 in Recitation

He who learns but does not think is lost. He who thinks but does not learn is in great danger. – Confucius

1. Illustrate the operation of PARTITION( $A, 1, 12$ ) on the array

$$A = (13, 18, 9, 5, 12, 8, 7, 4, 11, 2, 6, 10)$$

(You may use either the text's program or the version given in class, but please specify which you are using.)

2. Let  $L(n)$ , (“L” for lucky) denote the number of comparisons that quicksort does if each time it is applied the pivot lies in the precise center of the array. For example, applying quicksort to an array of length 31, say  $A(1) \cdots A(31)$  objects, there would be 30 comparisons (between  $A(31)$  and all the other  $A(j)$ ) and then  $A(31)$  would end up in the 16<sup>th</sup> place and there would be two recursive calls to quicksort on arrays each of size 15. Find the *precise* value of  $L(1023)$ . (Hint: that's one less than 1024!)
3. You wish to sort five elements, denoted  $a, b, c, d, e$ . Assume that you *already know* that  $a < b$ ,  $c < d$  and  $a < c$ . Sort (by giving the decision tree) the elements with 4 *further* comparisons. (The assumption is actually just a convenience as giving the full decision tree with 120 nodes would be exhausting.) (Think of this as a tricky puzzle – and don't worry if you don't get it!)
4. Babu<sup>1</sup> is trying to sort  $a, b, c, d, e$  with seven comparisons. First he asks “Is  $a < b$ ” and the answer is yes. Now he asks “Is  $a < c$ ?” Argue that (in worst-case) he will not succeed.
5. Illustrate the operation of COUNTING-SORT with  $k = 6$  on the array  $A = (6, 0, 2, 2, 0, 1, 3, 4, 6, 1, 3)$ .
6. You are given a Max-Heap with  $n$  entries. Assume all entries are distinct. Your goal is to find the *third largest* entry. One way would be to EXTRACT-MAX twice and then MAXIMUM. How long does this take? Find a better (by which we always mean faster for  $n$  large) way.

Tell me, what do you plan to do with your one wild and precious life? – Mary Oliver, The Summer Day

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<sup>1</sup>former student, now big cheese at GE Hyderabad