WAVE MOTION

Wave is a propagation of a disturbance that transfers energy through matter or space

Types :→ ① Mechanical waves
oscillation requires medium to travel. (ex. air, water)

② Electromagnetic waves
Requires no medium to travel.

③ Matter waves ; ④ Gravitational waves.

Mechanical Waves :→

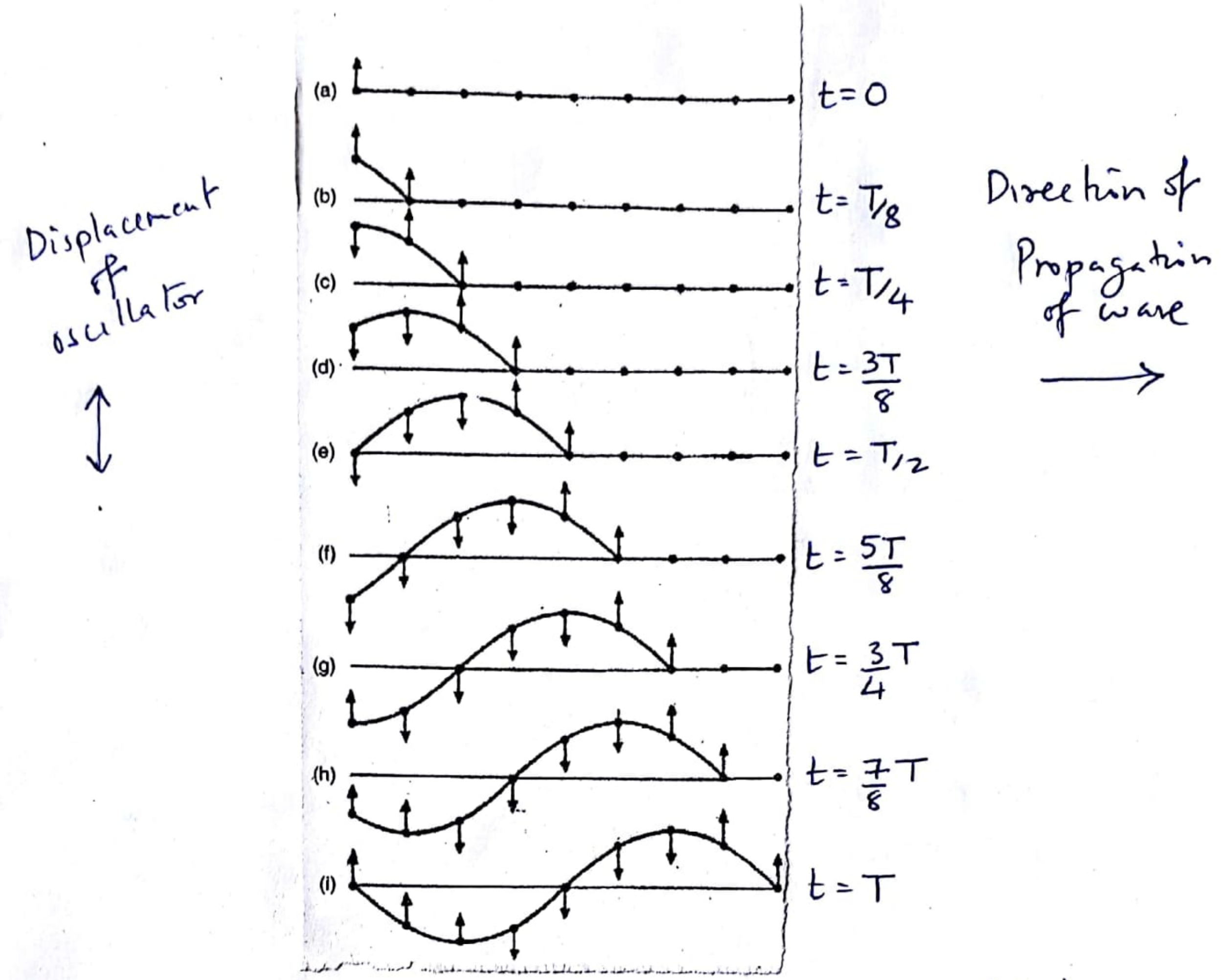
① Requires initial energy input, certain amount of work needs to be done to create the wave.

② Wave will propagate with this energy in the medium. The material of the medium does not move far away from the equilibrium position i.e the medium does not translate.

③ The wave will travel until all its energy is transferred.

④ If the medium is non-dispersive, (all waves with different frequencies travel at the same speed) the waves will not change their shape.

TRANSVERSE WAVE.



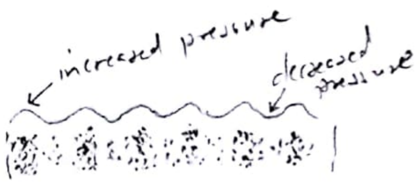
The displacement of the medium is perpendicular to the direction of propagation of waves.

- When one end of the rope is fixed & other end is given periodic up & down pulls, the disturbance propagates along the length of the rope but the particles oscillate up and down.

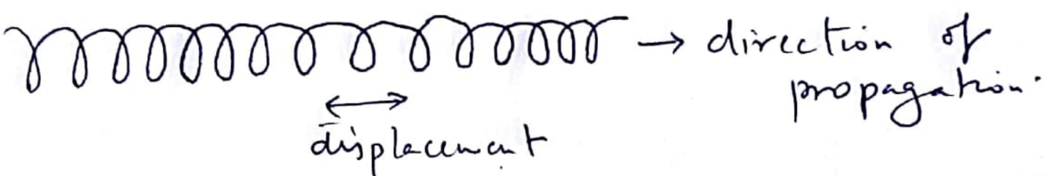
Transverse waves require shearing force in medium. Hence, they can be propagated only in the medium which will support a shearing stress i.e. mainly solids. Mechanical transverse waves cannot travel/propagate in a liquid.

Longitudinal waves

The displacement of the medium is parallel to the propagation of wave



ex. → sound waves in air.



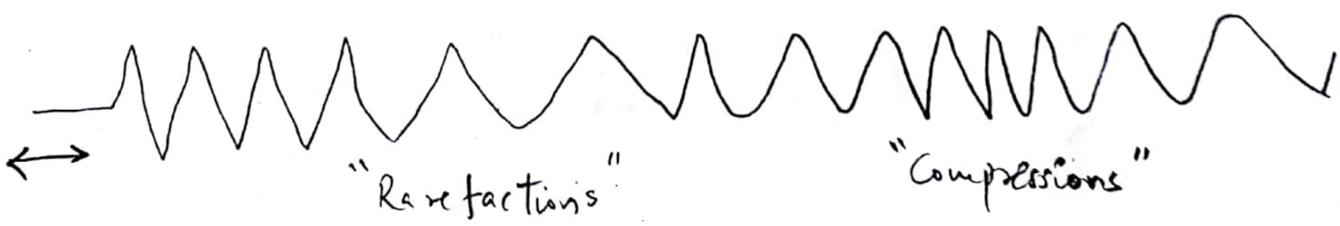
long tube of air with piston at one end.

disturbance travelling in a spring parallel to its length.

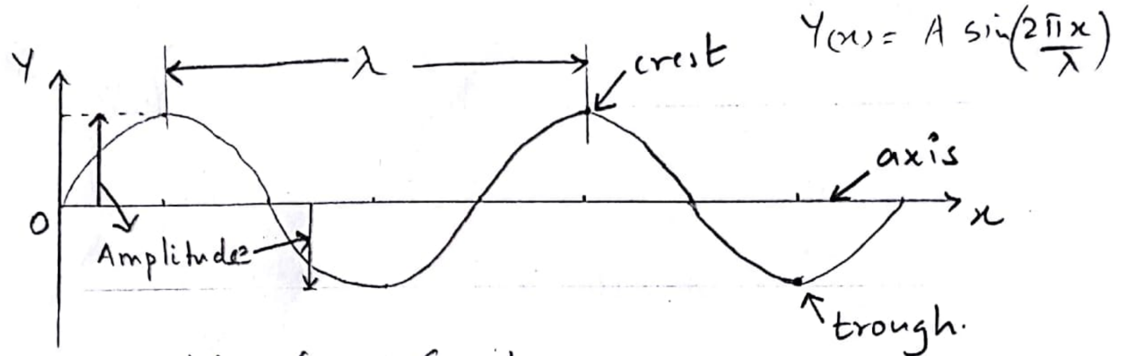
Pressure variation propagating in a liquid

Longitudinal waves can travel in any medium as they do not require shearing stress.

If one end of a stretched spring is given an oscillation parallel to the length of spring, the coils of the spring start exerting forces on each other. The coils oscillate back and forth parallel to the spring. "Compressions", which is crowding together of matter & "rarefactions" which is spreading out of matter travel along the spring & pressure at compression point is higher than at rarefaction point



Characteristics of a Wave



Waveform Graph.

① Time Period :→ [T]
Time needed for one complete cycle of vibration to pass in a given point. [unit: seconds]

② Wavelength :→ [λ]
Wavelength is the spatial period (distance) over which the wave's shape repeats.
Distance between two successive crests.
[unit: metre]

③ Amplitude :→ [A]
The maximum displacement in a waveform is known as amplitude

④ Frequency :→ [f] or [ν]
The number of waves that pass a given point in one second. [unit: sec^{-1}, Hertz]
(particle frequency)

⑤ velocity :→ [v]
Distance that the waves move in 1 second.
$v = \frac{\lambda}{T}$; $v = f\lambda$ [unit: m/s]

⑥ Phase angle :→ [ϕ]
Phase represents the displacement of particle at any point within a time period of wave
[unit: radians or degrees]

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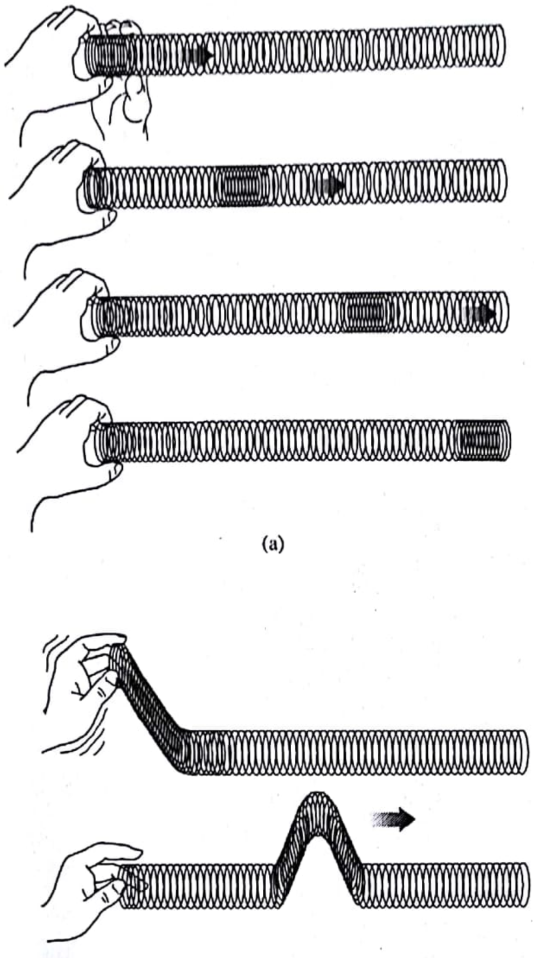


Figure 2.1 (a) A longitudinal wave in a spring. (b) A transverse wave in a spring.

electromagnetic wave (or even the quantum-mechanical probability amplitude of a matter wave).

Since the disturbance is moving, it must be a function of both position and time;

$$\psi(x, t) = f(x, t) \tag{2.1}$$

where $f(x, t)$ corresponds to some specific function or wave shape. This is represented in Fig. 2.3*a*, which shows a pulse traveling in the stationary coordinate system S at a speed v. The shape of the disturbance at any instant, say, $t = 0$, can be found by holding time constant at that value. In this case,

$$\psi(x, t)\big|_{t=0} = f(x, 0) = f(x) \tag{2.2}$$

represents the **profile** of the wave at that time. For example, if $f(x) = e^{-ax^2}$, where a is a constant, the profile has the shape of a bell; that is, it is a **Gaussian function**. (Squaring the x makes it symmetrical around the $x = 0$ axis.) Setting $t = 0$ is analogous to taking a "photograph" of the pulse as it travels by.

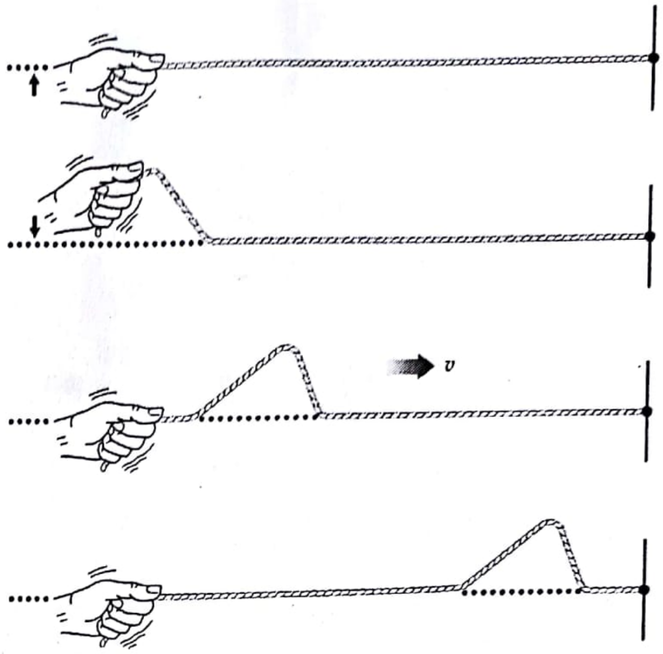
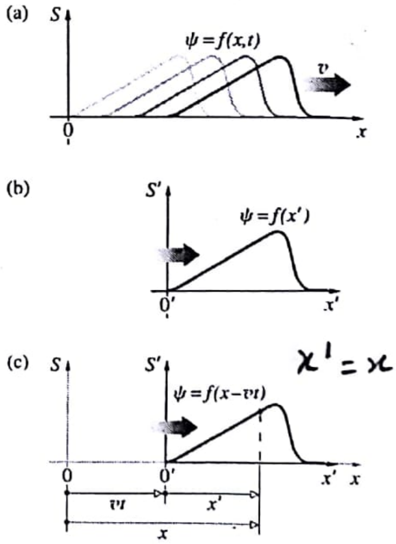


Figure 2.2 A wave on a string.

For the moment we limit ourselves to a wave that *does not change its shape* as it progresses through space. After a time t the pulse has moved along the x-axis a distance vt, but in all other respects it remains unaltered. We now introduce a coordinate system S', that travels along with the pulse (Fig. 2.3*b*) at the speed v. In this system ψ is no longer a function of time, and as we move along with S', we see a stationary constant profile described by Eq. (2.2). Here, the coordinate is x' rather than x, so that

$$\psi = f(x') \tag{2.3}$$



$x' = x - vt$

Figure 2.3 Moving reference frame.

$\therefore\ \psi(x,t) = f(x - vt,\ t)$

One Dimensional Wave Equation

$\psi = f(x, t)$ Describes the wave being a function of space & time

at $t = 0$ $\psi = f(x)$

after certain time 't' if the wave is moving with a velocity v, then, the new position coordinate of the wave can be given by

$x' = x - vt$

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x'} \cdot \frac{\partial x'}{\partial x} = \frac{\partial f}{\partial x'} \quad \left\{ \text{as } \frac{\partial x'}{\partial x} = 1 \right\}$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial f}{\partial x'} \cdot \frac{\partial x'}{\partial t} = -v \frac{\partial f}{\partial x'} \quad \left\{ \text{as } \frac{\partial x'}{\partial t} = -v \right\}$$

$$\therefore \frac{\partial \psi}{\partial t} = \mp v \frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$$

$$\& \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial t}\right) = \frac{\partial}{\partial t}\left(\mp v \frac{\partial f}{\partial x'}\right) \frac{\partial x'}{\partial t} = v^2 \frac{\partial^2 f}{\partial x'^2}$$

$$\therefore \boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}}$$

is the one dimensional wave equation

Q Solve the one dimensional wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$ using separation of variables

& find the solutions.

Ans According to separation of variables method, we assume the solution to be product of functions: where each function depends on only one variable

Let the solution be of the form.

$$\Psi(x,t) = X(x)\, T(t) \quad \ldots (1)$$

$X(x)$ is a function dependent on x alone &

$T(t)$ is a function dependent on t alone.

Substituting in wave equation we get

$$T(t)\frac{\partial^2 X}{\partial x^2} = \frac{1}{v^2} X \frac{\partial^2 T}{\partial t^2}$$

Rearranging we get.

$$\frac{1}{X}\frac{d^2 X}{dx^2} = \frac{1}{v^2 T}\frac{\partial^2 T}{\partial t^2} = -K^2 \ldots (2)$$

Let (2) this be equal to $-K^2$.

$$\therefore \text{ we get } \frac{d^2 X}{dx^2} + K^2 X = 0 \quad \ldots (3)$$

$$\& \quad \frac{d^2 T}{dt^2} + K^2 v^2 T = 0 \quad \text{or} \quad \frac{d^2 T}{dt^2} + \omega^2 T = 0 \quad \ldots (4)$$

where $\omega = Kv = \frac{2\pi v}{\lambda}$ = angular frequency of the wave.

Solution to equation (3) & (4).

$$X = A\cos Kx + B\sin Kx$$

$$T = C\cos\omega t + D\sin\omega t$$

$\therefore \quad \Psi = (A\cos kx + B\sin kx)\,(C\cos\omega t + D\sin\omega t)$

$\Psi_{(x,t)} = a\,(\cos kx - \omega t + \varphi)$

$\Psi_{(x,t)} = a\,(\cos kx + \omega t + \varphi)$

or $\Psi_{(x,t)} = a\,\exp\,[\pm i(kx \pm \omega t + \varphi]$

Q Show that $y(x,t) = a\cos(k(x - vt))$ is a solution to the one dimensional wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \varphi}{\partial t^2} \quad \text{......} \; ①$$

L.H.S :

$$\frac{\partial y}{\partial x} = -ak\,\sin(k(x - vt))$$

$$\frac{\partial^2 y}{\partial x^2} = -ak^2\cos(k(x - vt))$$

RHS.

$$\frac{1}{v^2}\frac{\partial y}{\partial t} = \frac{1}{v^2} + avk\,\sin(k(x - vt))$$

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t} = \frac{1}{v^2} - av^2k^2\cos(k(x - vt)) = -ak^2\cos(k(x - vt))$$

$\therefore$ LHS = RHS & $\therefore$ $y(x,t)$ is a solution to ODWE ①.